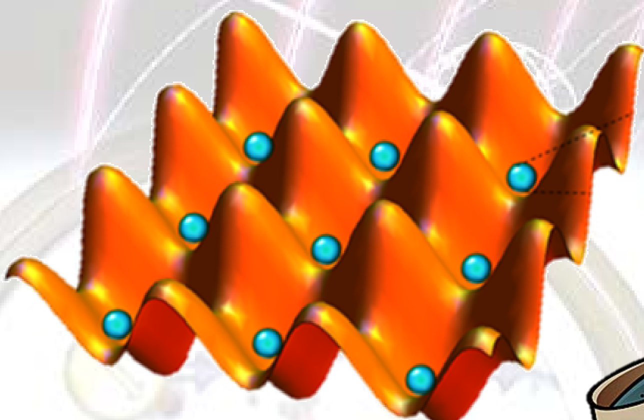
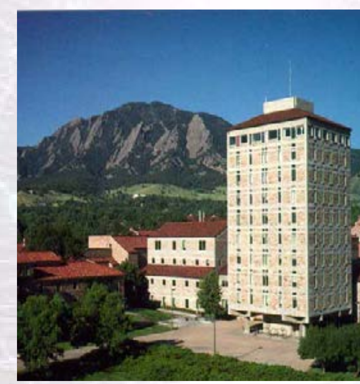
Exploring many-body physics with ultra-cold quantum matter

Ana Maria Rey

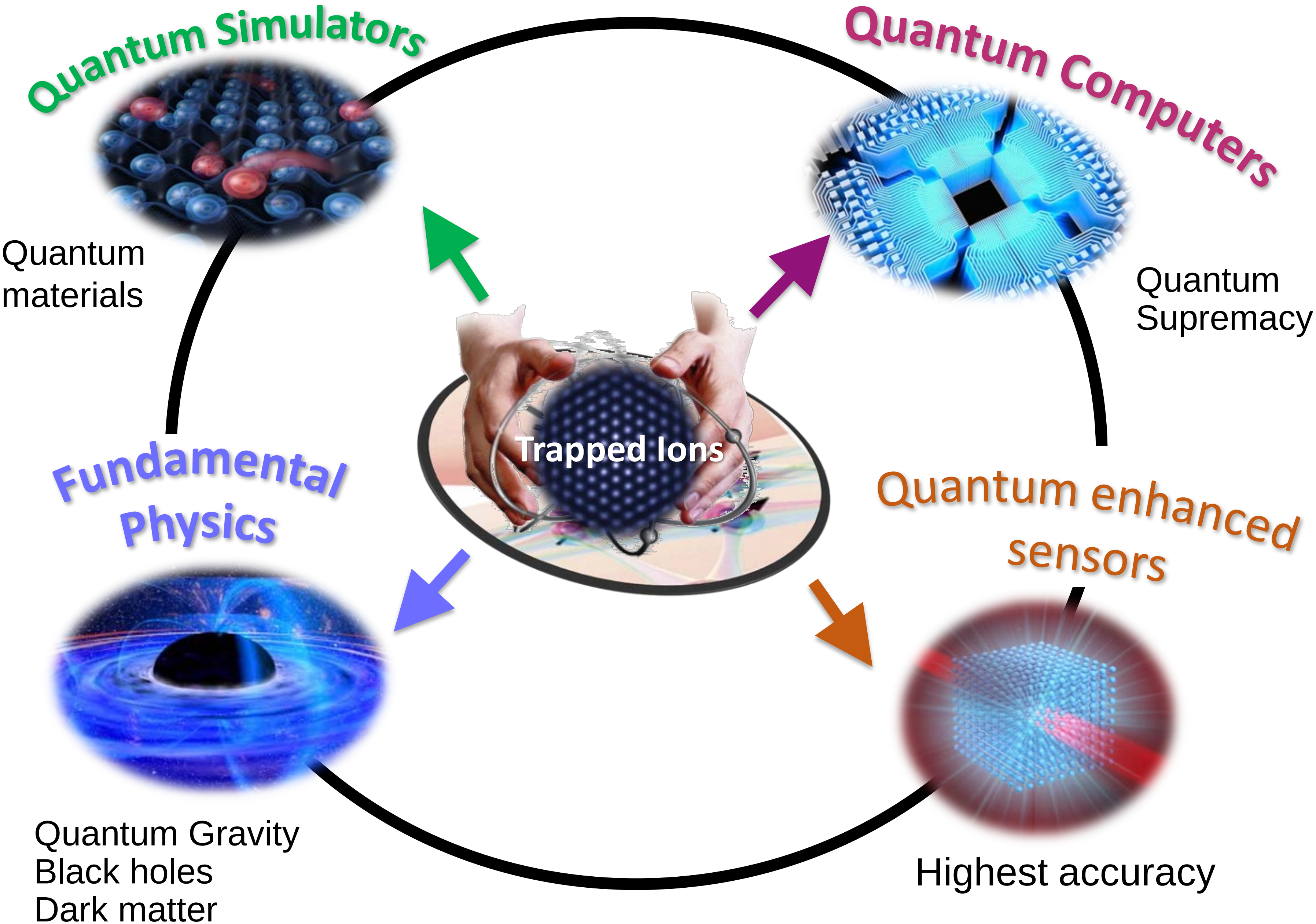




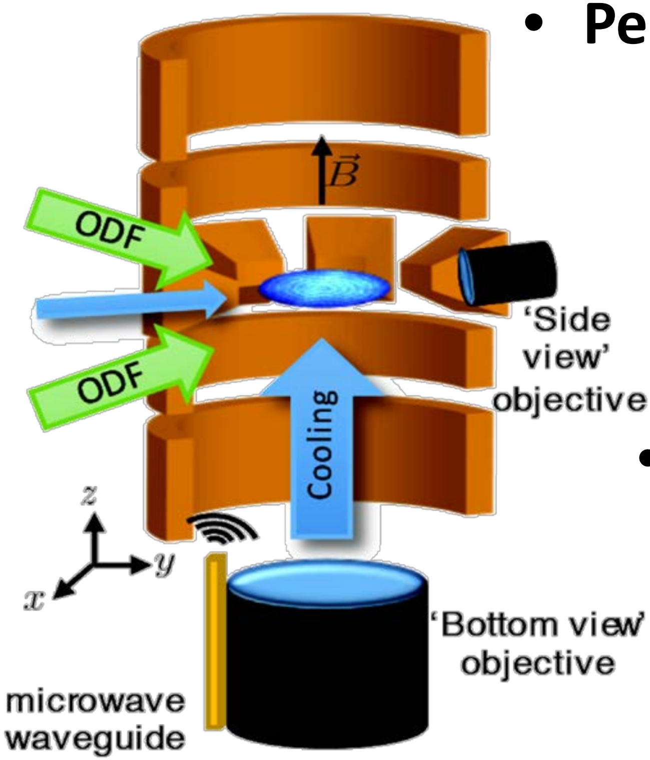
QIP Tutorials

Boulder, Colorado, January 12th, 2019

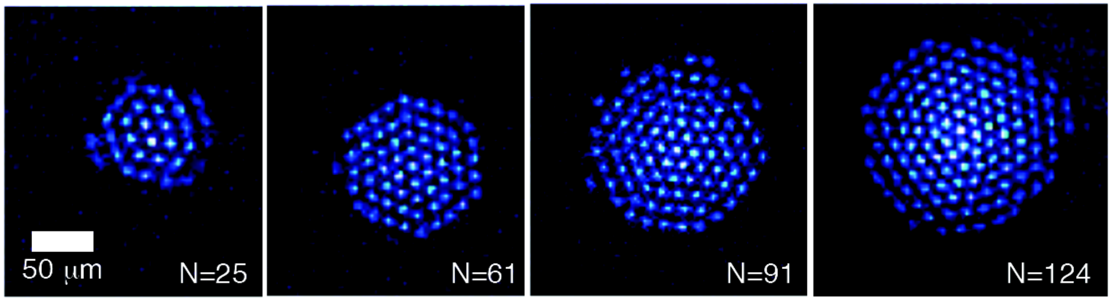
Control of correlated Many-body Quantum Systems



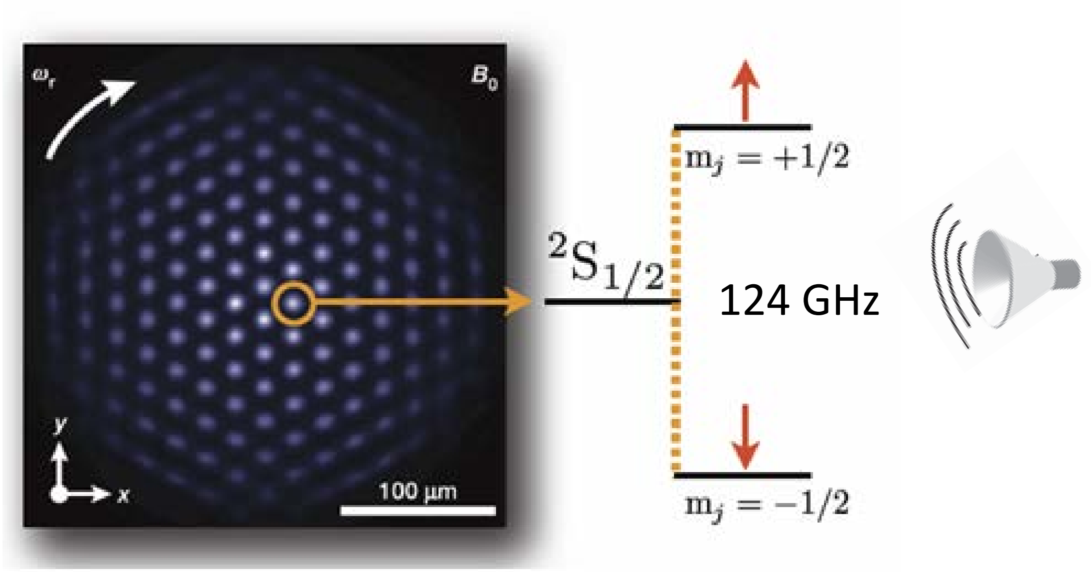
Penning Trap Experiments: $^9Be^+$



- **Penning trap: 2D triangular crystals of 20-300 ions**

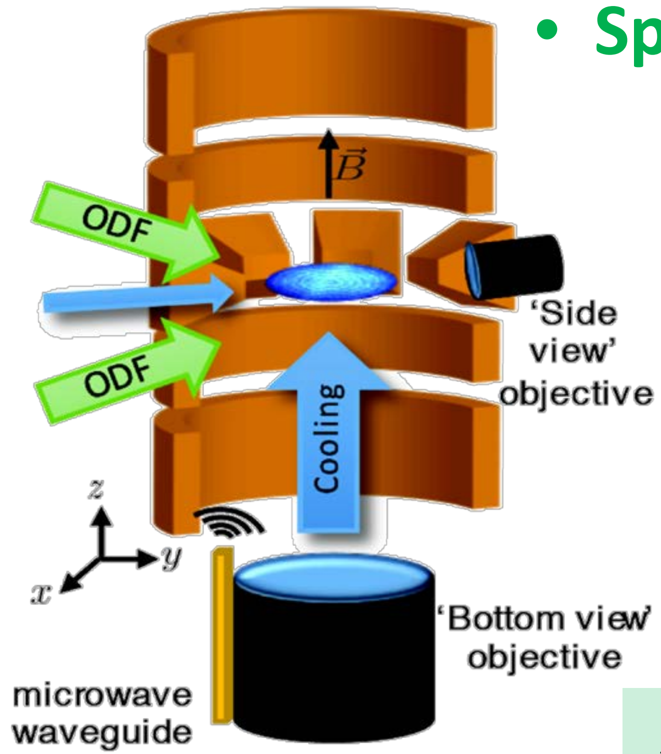


- **Two hyperfine states used as spin ½ system**

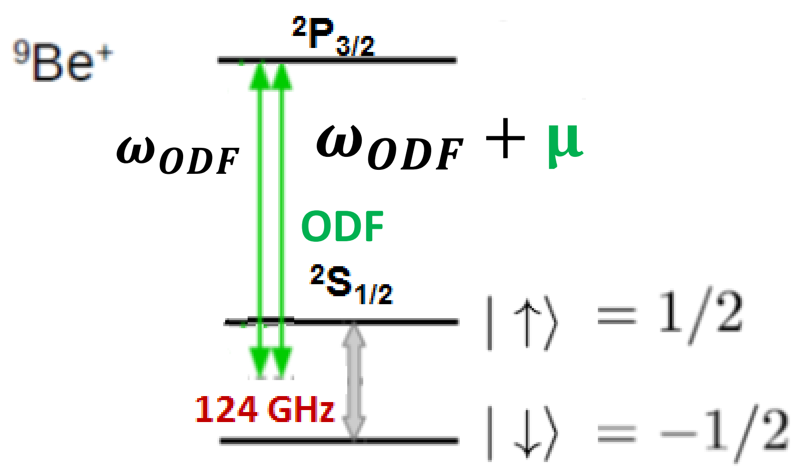


Single qubit gates with 99.9 fidelity

Penning Trap Experiments: $^9Be^+$

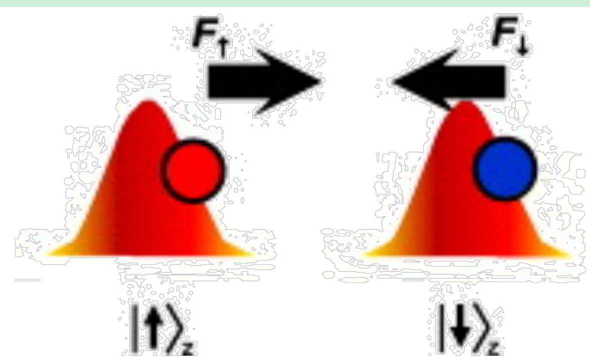


- **Spin–spin interactions generated by lasers**



- **Spin dependent force**

$$\hat{H}_{ODF}(t) = -F_0 \cos(\mu t) \sum_{j=1}^{N} \hat{z}_j \cdot \hat{\sigma}_j^z$$



$\hat{\sigma}_j^z$ Pauli matrix on spin j

Phonons mediate Spin-spin interactions

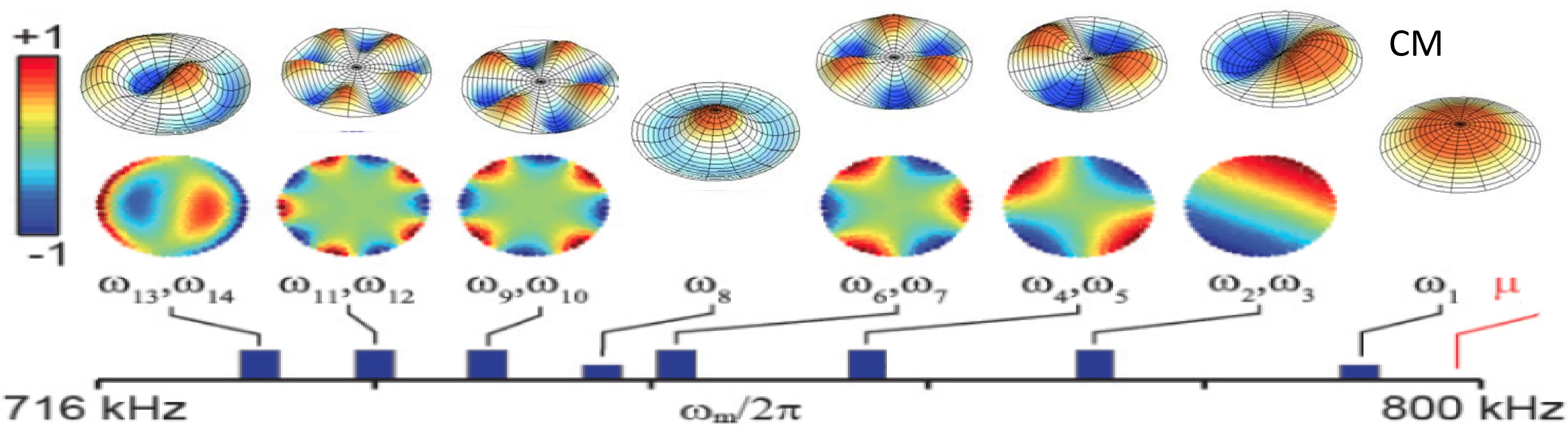
$$\hat{H}_{ODF}(t) = -F_0 \cos(\mu t) \sum_{j=1}^{N} \hat{z}_j \cdot \hat{\sigma}_j^z$$

Written in terms of drumhead eigenmodes

$$\sum_{m=1}^{N} b_{jm} \sqrt{\frac{\hbar}{2M\omega_m}} \left(\hat{a}_m^\dagger e^{i\omega_m t} + \hat{a}_m e^{-i\omega_m t} \right)$$

N drumhead eigenvalues ω_m and eigenvector $\vec{b}_m$

- Ions are not independent: form a crystal due to Coulomb interactions



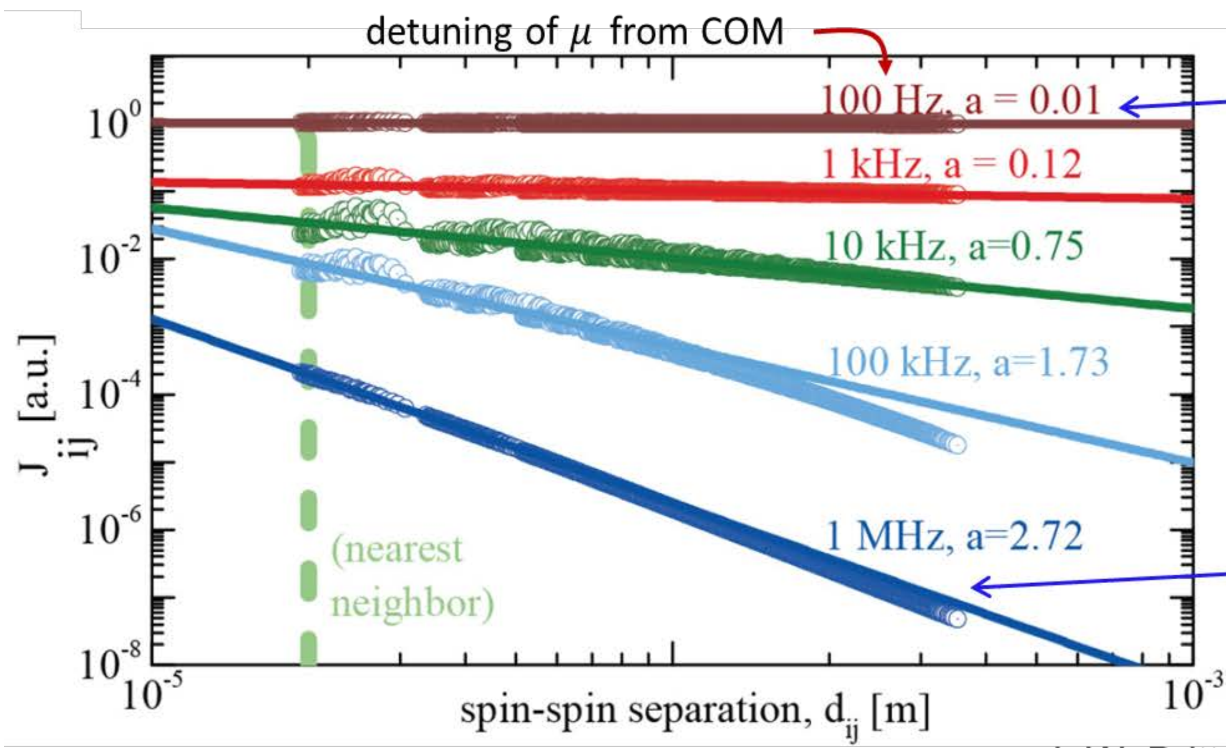
<a href="http://www.acs.psu.edu/drussell/Demos/waves/wavemotion.html">

Phonons mediate Spin-spin interactions

- Far detuning $\boldsymbol{\delta} > |\boldsymbol{\mu} - \boldsymbol{\omega_m}|$: phonons can be adiabatically eliminated
- Effective Hamiltonian: Ising spin model

$$H_{SS} = \frac{1}{N}\sum_{i<j} J_{ij}(t)\sigma_i^z\sigma_j^z \qquad J_{ij} \sim \frac{{F_0}^2}{\hbar}\sum_{\mu} b_{\mu i}\, b_{\mu j}\, \frac{1}{\mu^2 - \omega_\mu^2}$$

$J_{i,j}$ depends on eigenmodes and ODF detuning (μ)



infinite range $J_{i,j} \approx J$

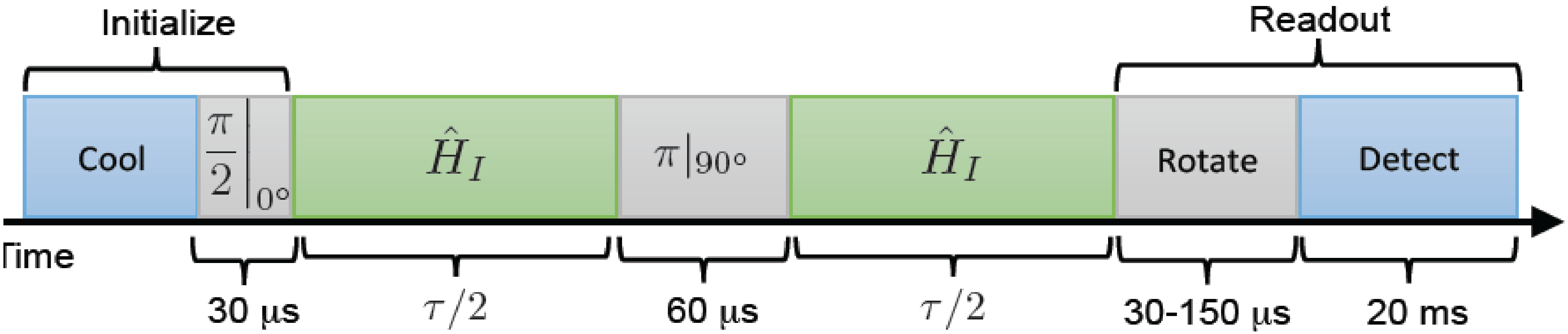
power-law interaction range

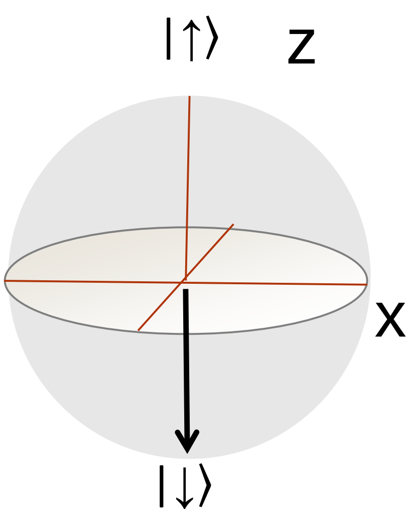
$$J_{i,j} \propto d_{i,j}^{-a}$$

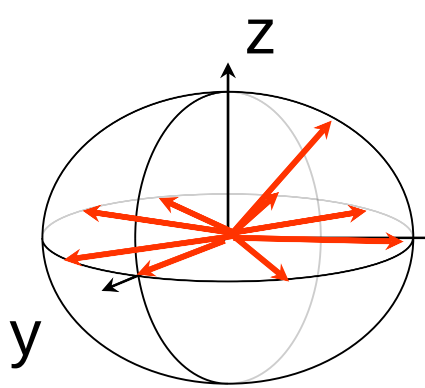
dipole-dipole coupling

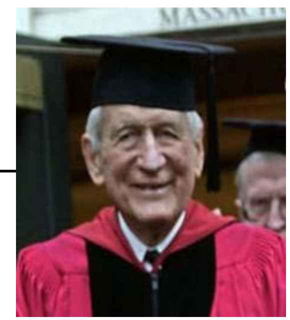
J. W. Britton *et al* Nature 484, 489 (2012)

Probing Spin model with dynamics









N. Ramsey.
Nobel prize
1989

- Initial |↓↓↓↓⟩
- Rotate:θ
- Wait τ/2
- echo
- Wait τ/2
- Read

Goals:

- Verify spin model
- Create strong correlations
- Explore regime intractable to theory

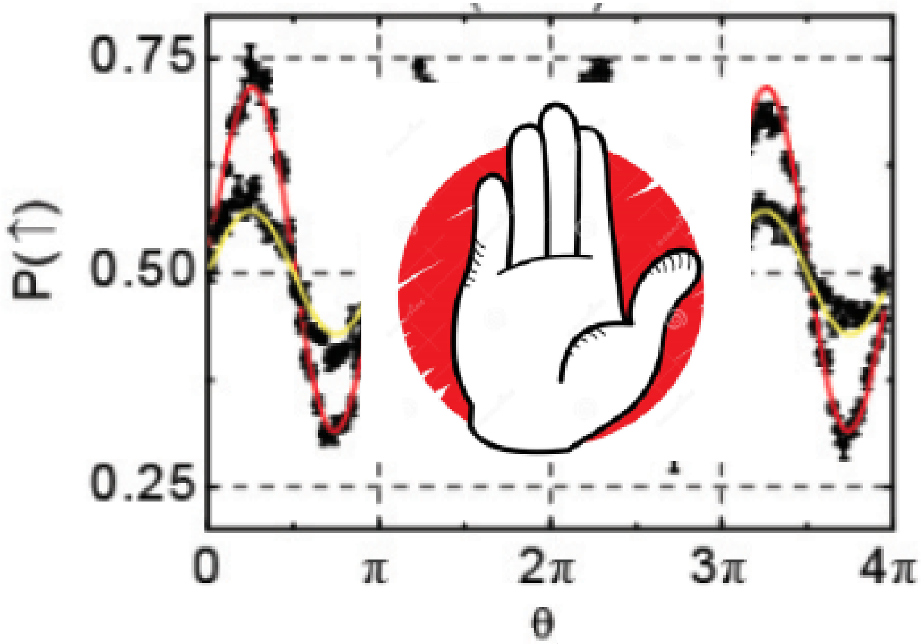
Mean field dynamics: Simple precession

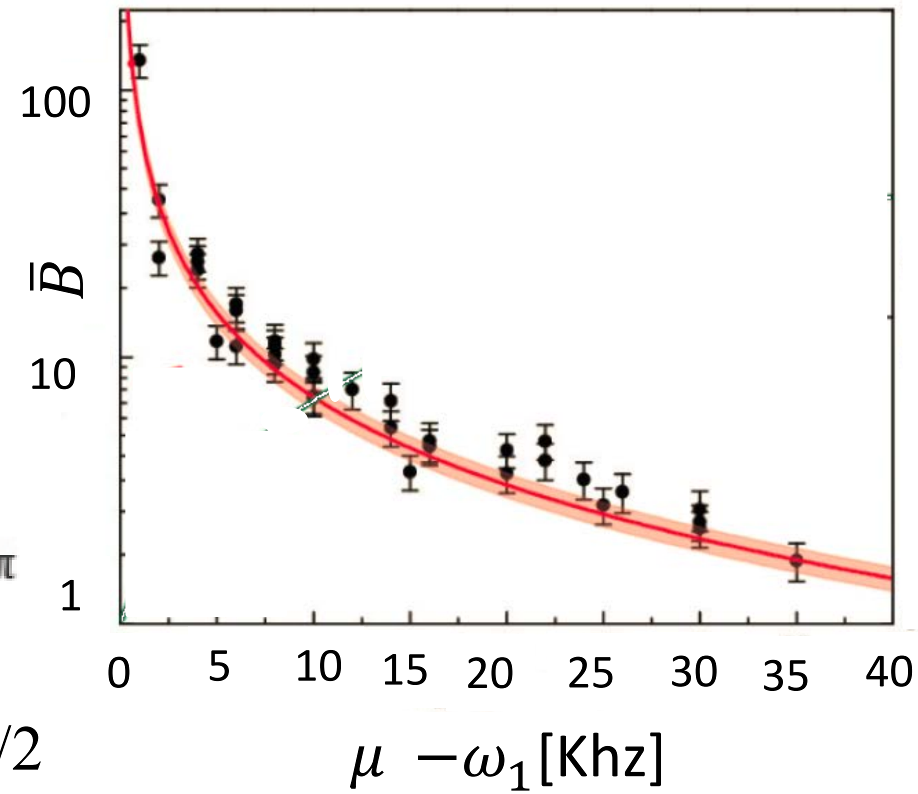
$$H_{SS} = \frac{1}{N}\sum_{i<j} J_{ij}\,\sigma_i^z\sigma_j^z \xrightarrow[\text{limit}]{\text{mean field}} H_{MF} = \sum_j B_j\sigma_j^z \quad B_j = \frac{\cos\theta}{N}\sum_{i\neq j}^{N} J_{ij}$$

precession

Short time $Jt \ll \sqrt{N}$

used to measure $\bar{B} = \sum_j B_j/N$





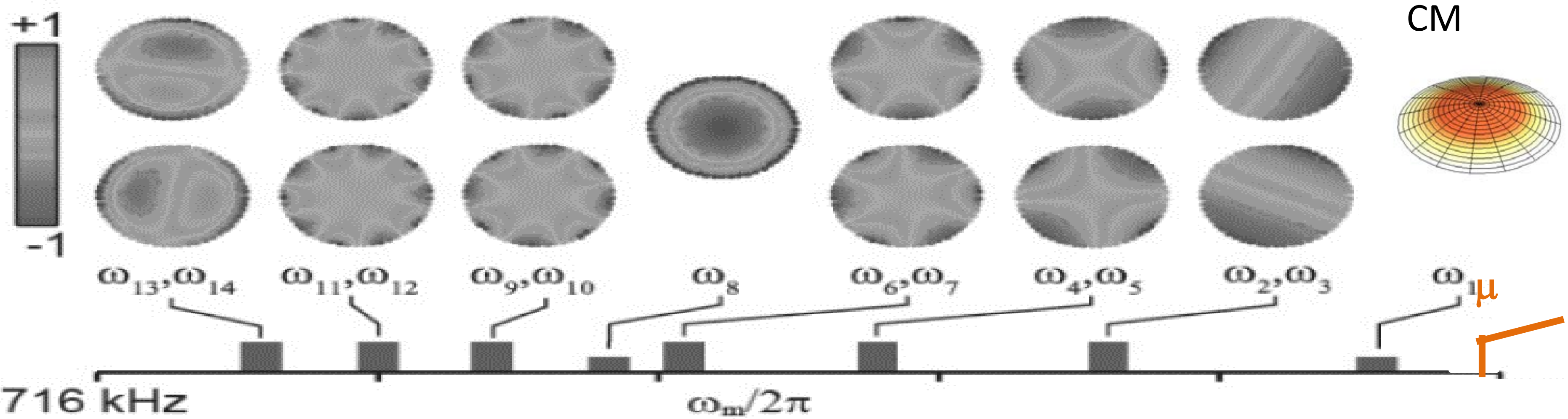
No mean field dynamics at θ=π/2

Not the full story

J. W. Britton *et al* Nature 484, 489 (2012)

Only Excite Center of Mass Mode

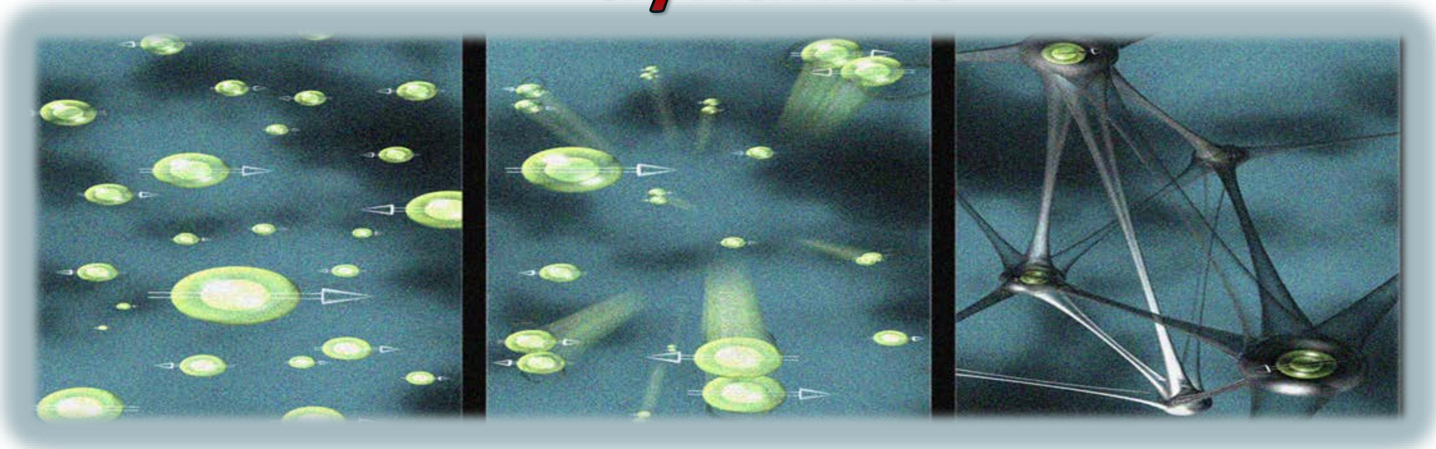
$\delta = \mu - \omega_1$ Detuning



$$\hat{H}_{zz} = \frac{J}{N}(\hat{S}_z)^2$$

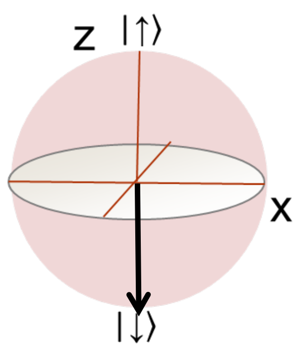
Collective Ising Model

Entanglement generation via many-body dynamics



Prepare uncorrelated state

$|\rightarrow \cdots \rightarrow\rangle$



Evolve: Interaction Hamiltonian

$$\hat{H}_{OAT} = \frac{J}{N}(\hat{S}_z)^2$$

One-Axis Twisting

Entangled state

Trapped Ions: NIST,..

Cavity QEDs: MIT, Stanford,..

Bose Einstein Condensates: Heidelberg, Georgia Tech, Max-Planck-Institute, Hannover,...

How entanglement builds up?

Bloch Sphere: From Spin ½ to Spin S

Spin ½ $\quad |\uparrow\rangle, |\downarrow\rangle$

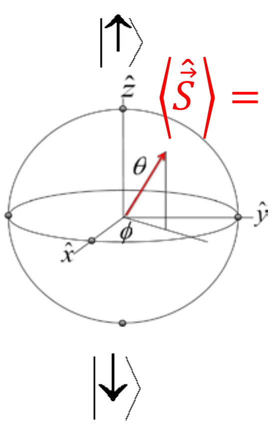
Dimension 2

$$|\psi\rangle = \cos\left(\tfrac{\theta}{2}\right)|\uparrow\rangle + e^{i\phi}\sin\left(\tfrac{\theta}{2}\right)|\downarrow\rangle$$

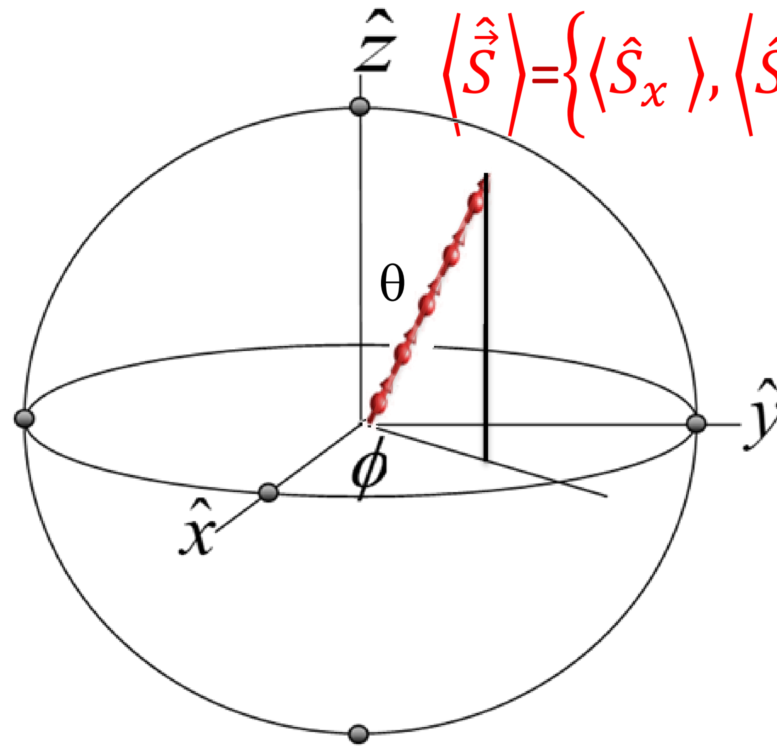
Spin S=N/2 $\quad \left|S = \frac{N}{2}, S_z = M\right\rangle$

Dimension: N+1 $\quad M = \{-\frac{N}{2}, \ldots, \frac{N}{2}\}$

$$|\psi\rangle = \sum_M C_M |M\rangle$$

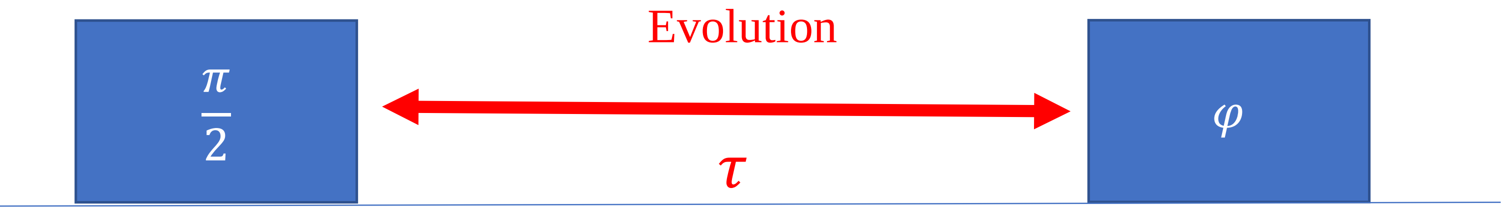


$$\langle \hat{\vec{S}} \rangle = \{\langle \hat{S}_x \rangle, \langle \hat{S}_y \rangle, \langle \hat{S}_z \rangle\}$$



$$\langle \hat{\vec{S}} \rangle = \{\langle \hat{S}_x \rangle, \langle \hat{S}_y \rangle, \langle \hat{S}_z \rangle\}$$

Spin Magnetization



Projects coherence onto the spin population

Contrast or transverse magnetization

$$\mathcal{C} = \left|\left\langle \hat{S}^{+}(\tau)\right\rangle\right| = \sqrt{\left\langle \hat{S}_{x}(\tau)\right\rangle^{2} + \left\langle \hat{S}_{y}(\tau)\right\rangle^{2}} = \left|\langle \hat{S}_{x}(\tau)\rangle\right|$$

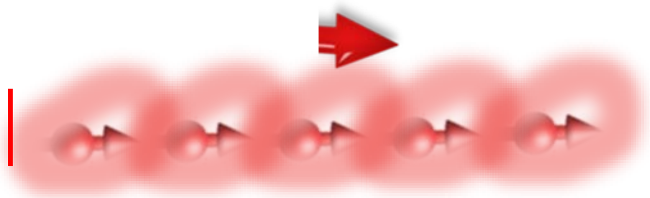
$$\left\langle \hat{S}_{y}(\tau)\right\rangle = \langle \hat{S}_{z}(\tau)\rangle = 0$$

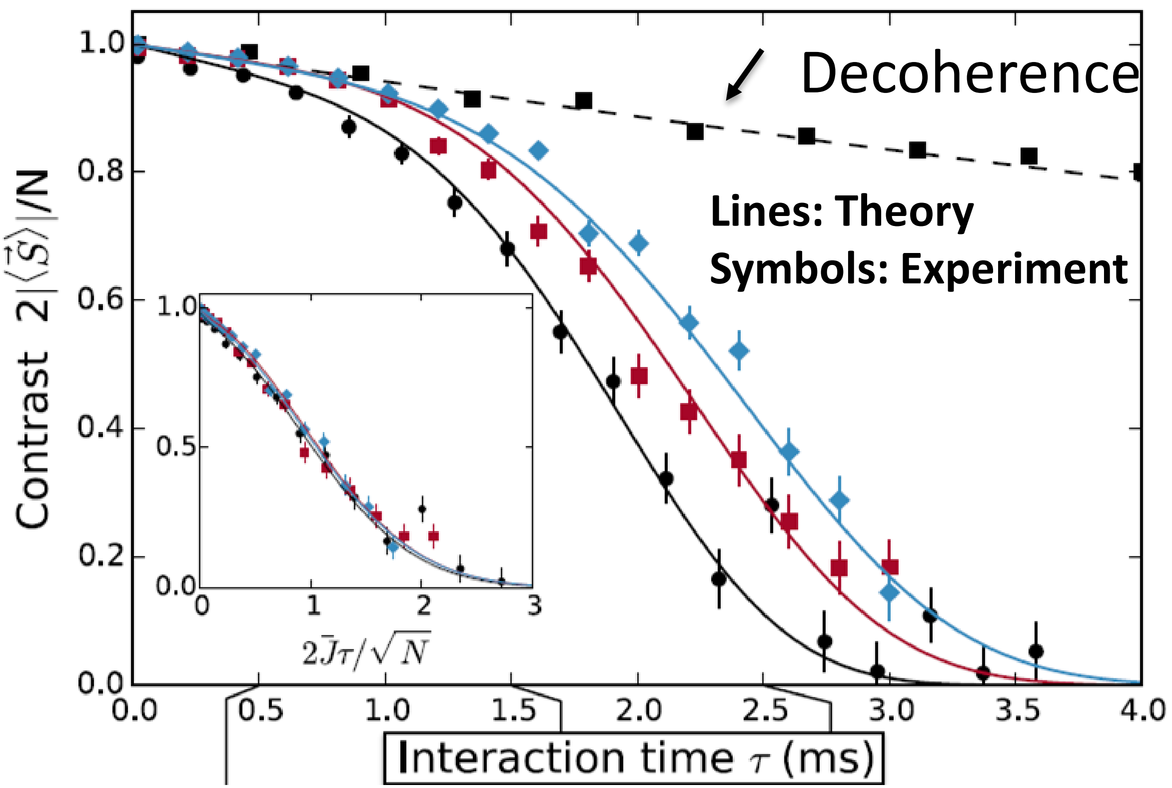
At the mean field limit $\mathcal{C}$ is constant. But that is not the case when quantum correlations are included

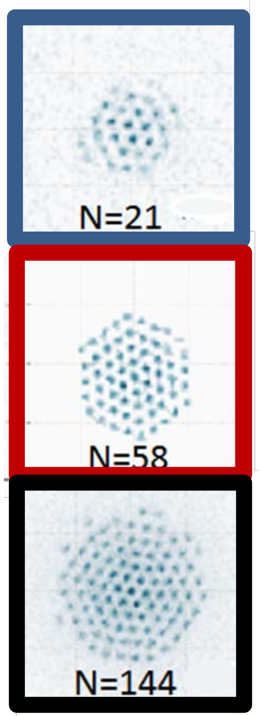
Magnetization Measurement

- Coherent spin demagnetization: Bloch vector length $|\langle S_x \rangle|$ vs time

$$\left|\langle \hat{S}_x \rangle\right| = \left|\frac{N}{2} \cos^{N-1}(J\tau/N)\right|$$







Bohnet *et al.*, Science, 352, 1297(2016).

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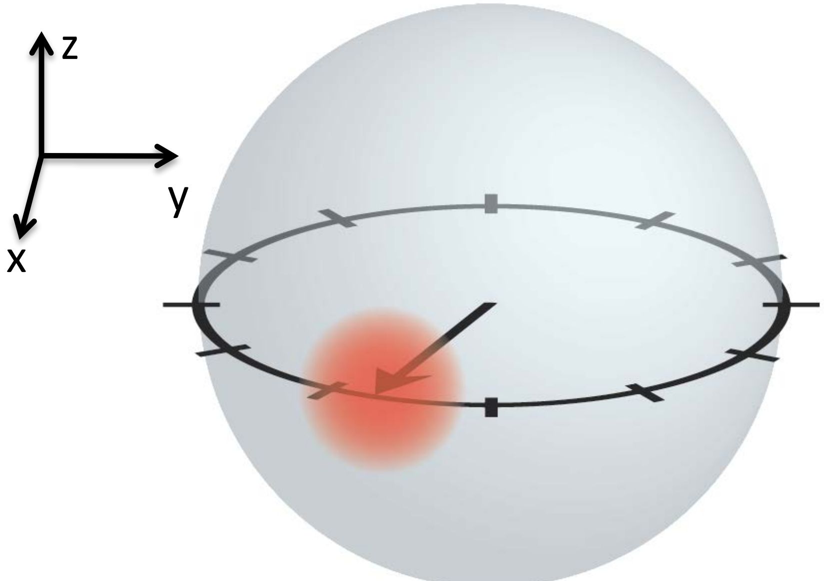
Heisenberg Uncertainty Relations

Quantum Mechanics Sets the Size of the "Uncertainty Blob"

$$\left\langle \Delta \hat{S}_y \right\rangle \langle \Delta \hat{S}_z \rangle \geq \hbar |\langle \hat{S}_x \rangle / 2|$$

Quantum Fuzziness



Coherent Spin states $\bigotimes \frac{(|\uparrow\rangle + |\downarrow\rangle)}{\sqrt{2}}$

Uncorrelated

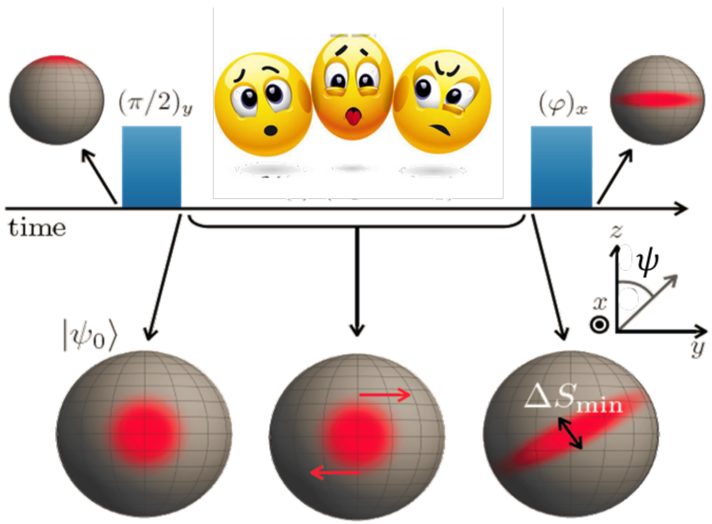
$\langle \hat{S}_x \rangle = N/2$

$\left\langle \hat{S}_{y,z} \right\rangle = 0$

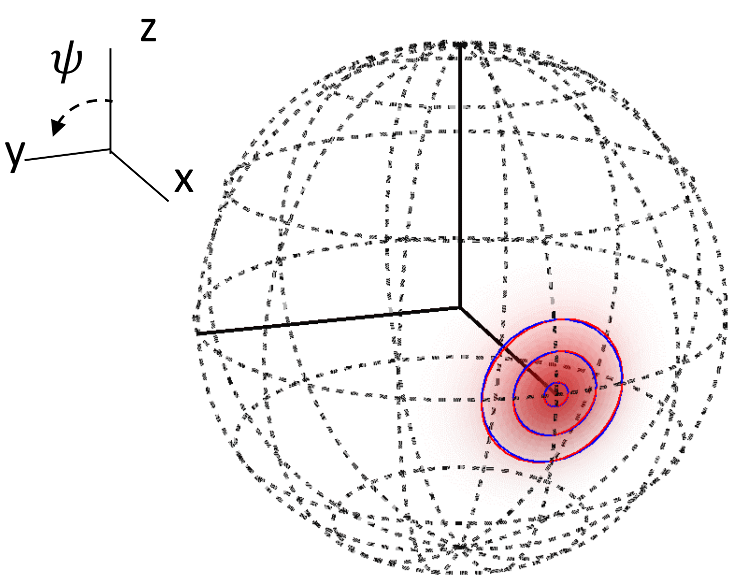
$\langle \Delta \hat{S}_x \rangle = 0$

$\left\langle \Delta \hat{S}_{y,z} \right\rangle = \sqrt{\frac{N}{4}}$

Beyond mean-field: Simple picture



$$\hat{H}_{zz} \sim \left(\frac{J}{N} \langle \hat{S}_z \rangle \right) \hat{S}_z$$



Spin Squeezing Parameter

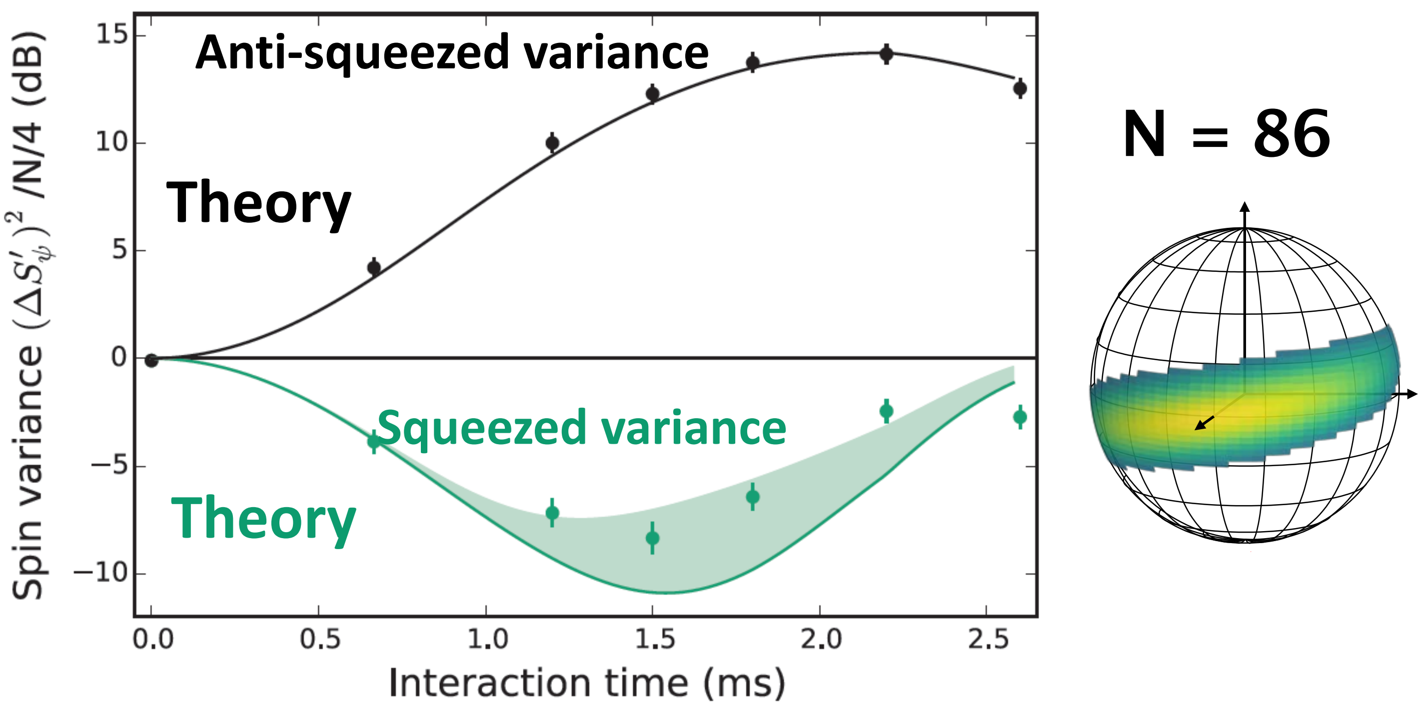
$$\xi(\psi) = \frac{\sqrt{N}\Delta S^{\psi}}{\overline{\langle \hat{S}^x \rangle}} \quad \xi^2 < 1$$

A. Sørensen *et al* Nature 409, 63 (2001)

- Entanglement witness
- Enhanced sensitivity
- Useful only for Gaussian states

Comparison with Experiment

- largest inferred squeezing: -6.0 dB



Bohnet *et al*., Science, 352, 1297(2016).

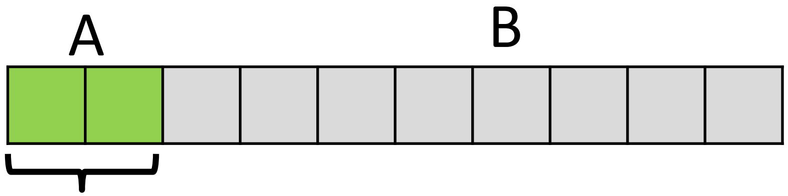
- Disappearance of squeezing at longer time does not mean no entanglement
- Squeezing is only useful for Gaussian states
- How can we quantify entanglement?

Entanglement Entropy

$\hat{\rho}$: Density Matrix of the close system

$\mathrm{Tr}(\hat{\rho}\,) = 1 \quad \mathrm{Tr}(\hat{\rho}^2) = 1$ Purity

$\hat{\rho}_A = \mathrm{Tr}_B\, \hat{\rho}$

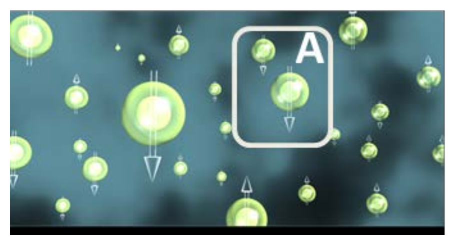


Reduce density Matrix of subsystem A

Renyi entropy: Purity of the A subsystem

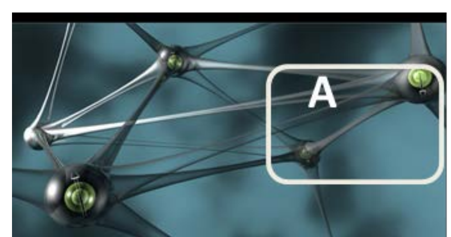
$S_A = -\ln[\mathrm{Tr}(\rho_A^2)]$

Product state $\hat{\rho} = \otimes_i \hat{\rho}_i$ $S_A = 0$



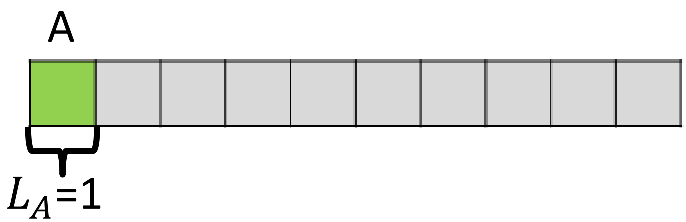
Do not lose information when I cut

Entangled state $S_A > 0$

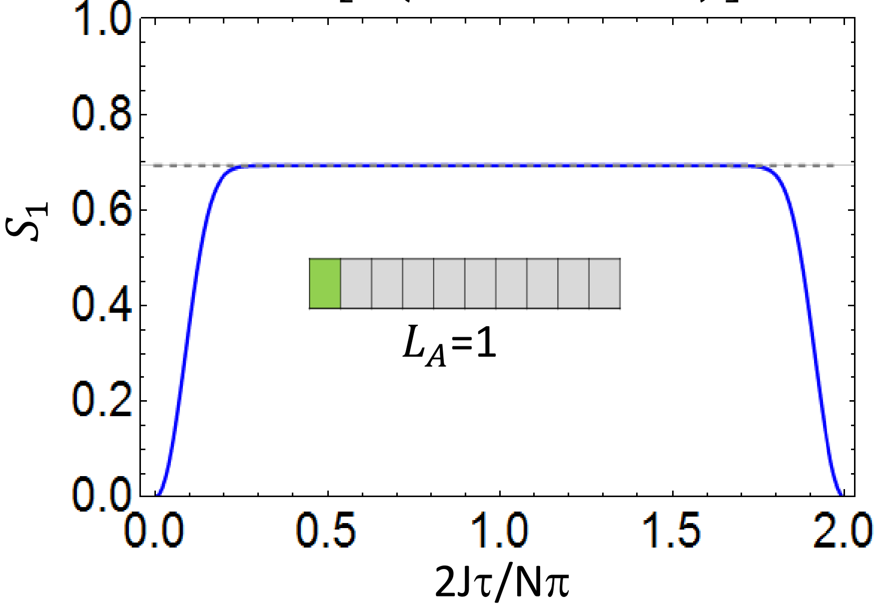


Lose information when I cut

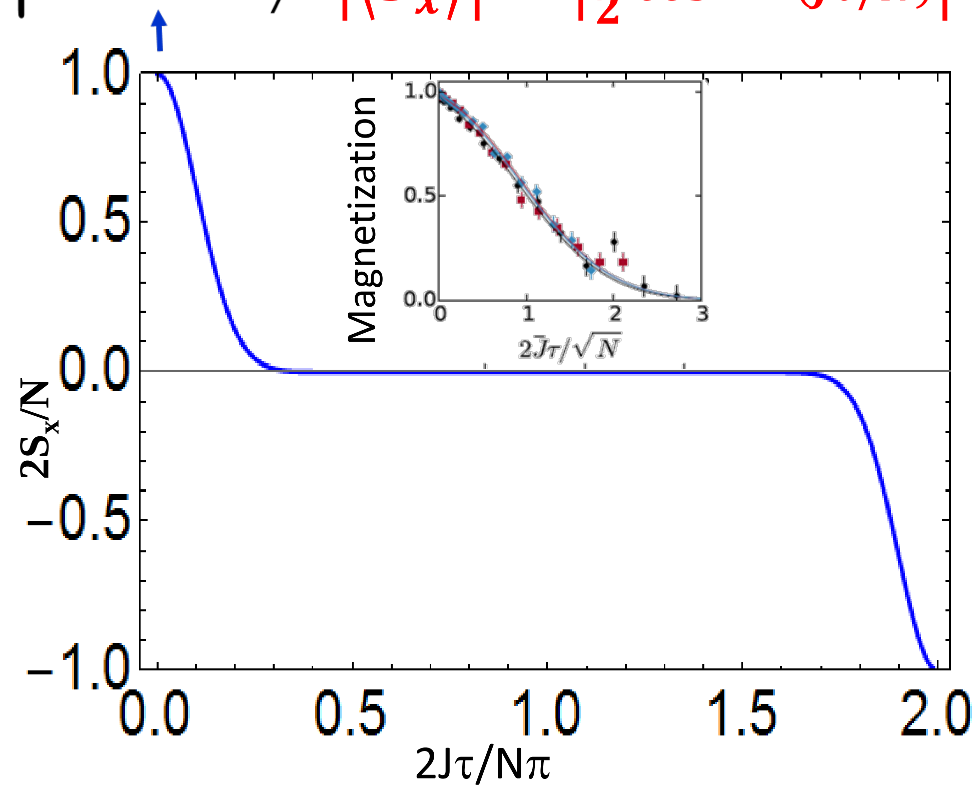
Magnetization: One spin Renyi entropy



$$S_1 = -\log\left[\frac{1}{2}\left(1 + \left(\frac{2}{N}\langle \hat{S}_x\rangle\right)^2\right)\right]$$

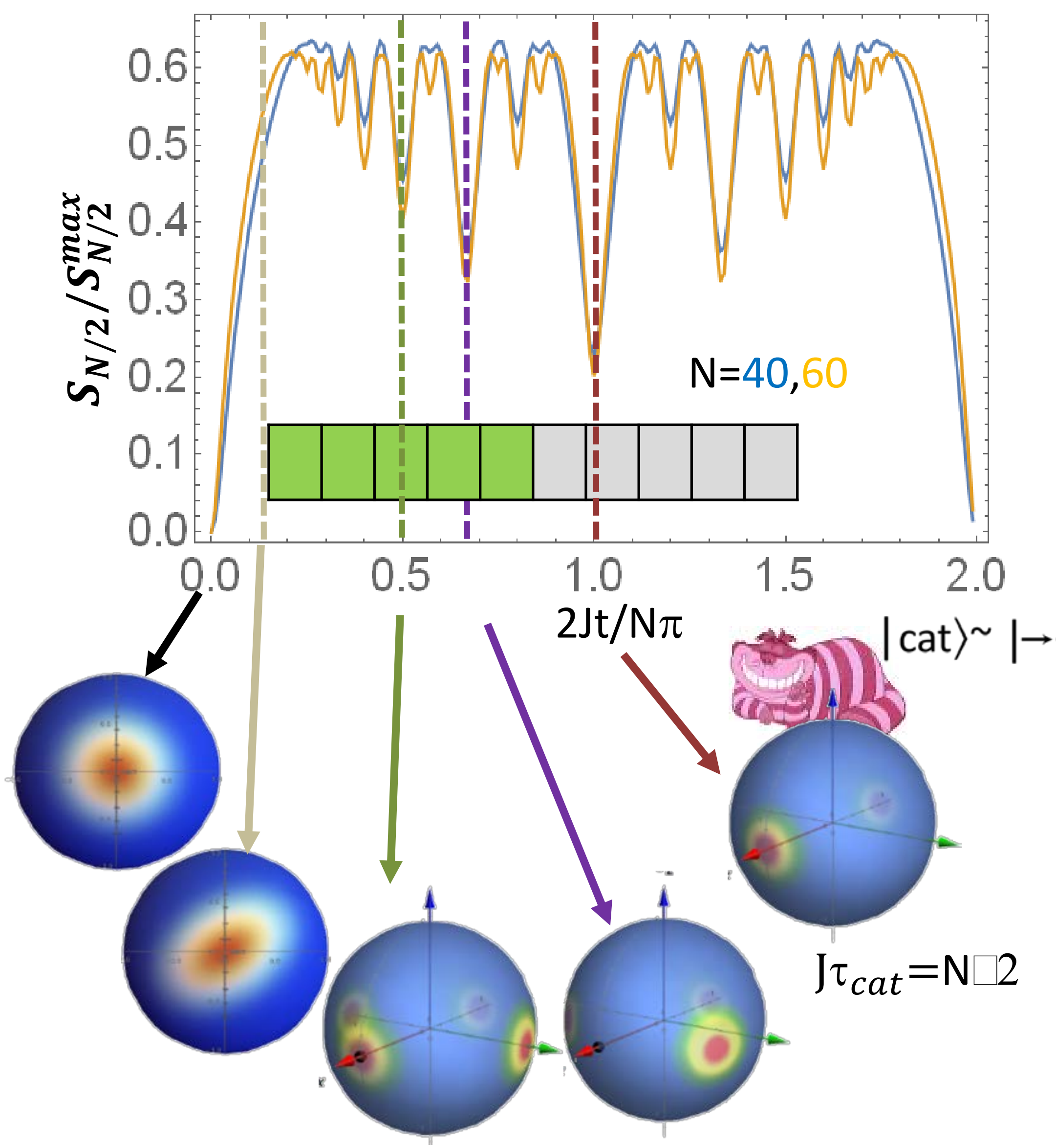


$|\rightarrow\rightarrow\rightarrow\rightarrow\rightarrow\rangle$ $\left|\langle \hat{S}_x\rangle\right| = \left|\frac{N}{2}\cos^{N-1}(J\tau/N)\right|$



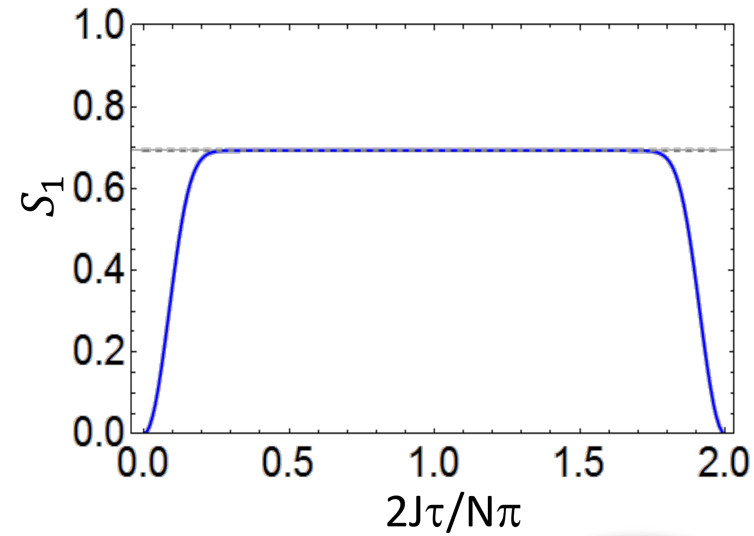
Entanglement in ALL-to-All Ising

More Information



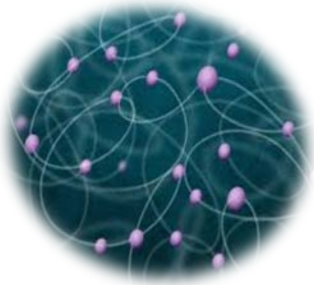
|cat⟩~ |→→→→→⟩ + |←←←←←⟩

Jτ$_{cat}$=N□2



Boring

Quantum Thermalization in closed systems

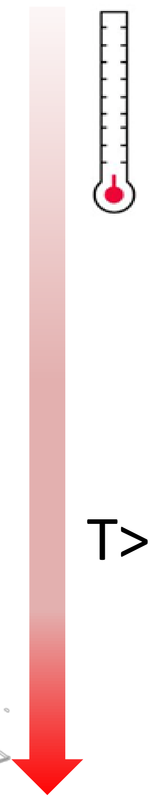


Entanglement

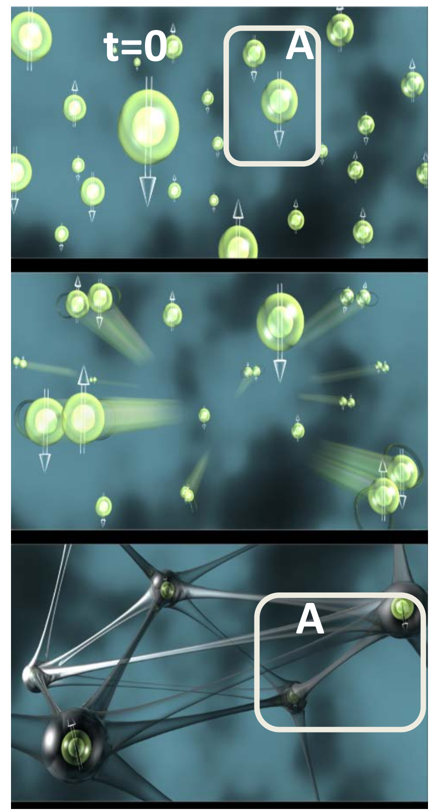
$\hat{\rho}\,(0) = \otimes_i \hat{\rho}_i$ Product state

Apparent loss of information in local observables

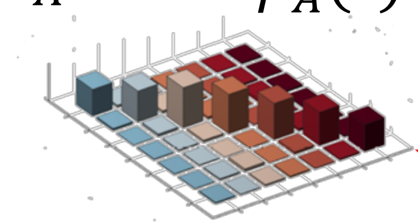
$S_A = 0$ $\hat{\rho}_A(0)$



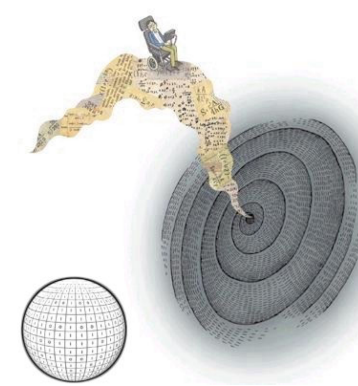
T>0



$S_A > 0$ $\hat{\rho}_A(t)$



Entangled state



Black Hole Information paradox

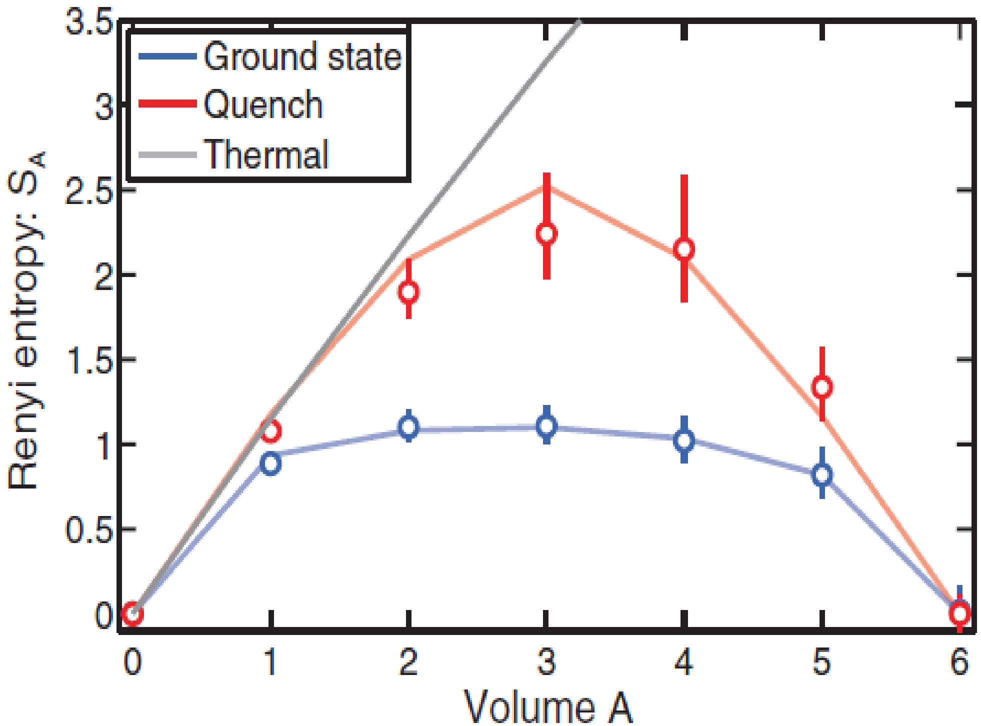
D'Alessio *et al*, Adv. in Phys.(2016)

Scrambling of Quantum Information

Information not loss but Scrambled

Spread over many-body degrees of freedom, becoming inaccessible to local measurements

Kaufman *et al,* Science(2016)



Greiner group:
quantum gas microscope

✓ Single site addressing
✓ Only in small systems L=6

But entanglement entropy is hard to measure in large systems

Brydges,…., P. Zoller, R. Blatt, C. F. Roos, arXiv:1806.05747

How to measure scrambling?

Out-of-time-Order-Correlators OTOCs

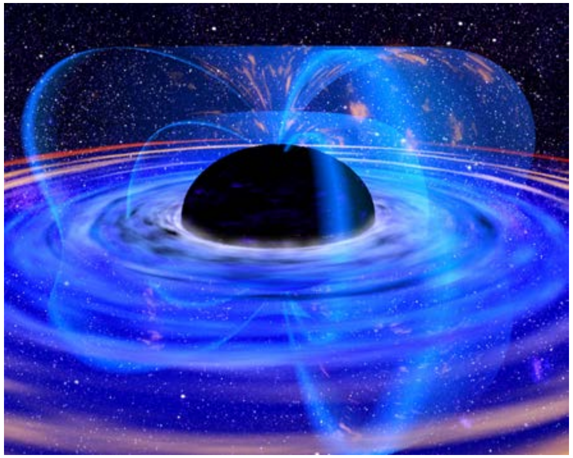
$$F(t) = \langle \hat{W}^\dagger(t)\hat{V}^\dagger(0)\hat{W}(t)\hat{V}(0) \rangle$$

$$F(t) = 1 - C(t) \qquad C(t) = \langle |\ [\hat{W}(t), \hat{V}(0)]\ |^2 \rangle$$

Measurement of the degree of non-commutativity of $\hat{V}(0)$ and the time evolved version of $\hat{W}(t)$

[Hayden-Preskill, Sekino-Susskind, Shenker-Stanford ’13, Kitaev ‘14]

OTOCs and Quantum Gravity



- Black holes scramble quantum information as fast as possible
  - Fast scramblers: $C(t) \sim e^{\lambda t}$

$$\lambda \leq 2\pi T$$

- Bound of growth of quantum chaos: λ Lyapunov exponent

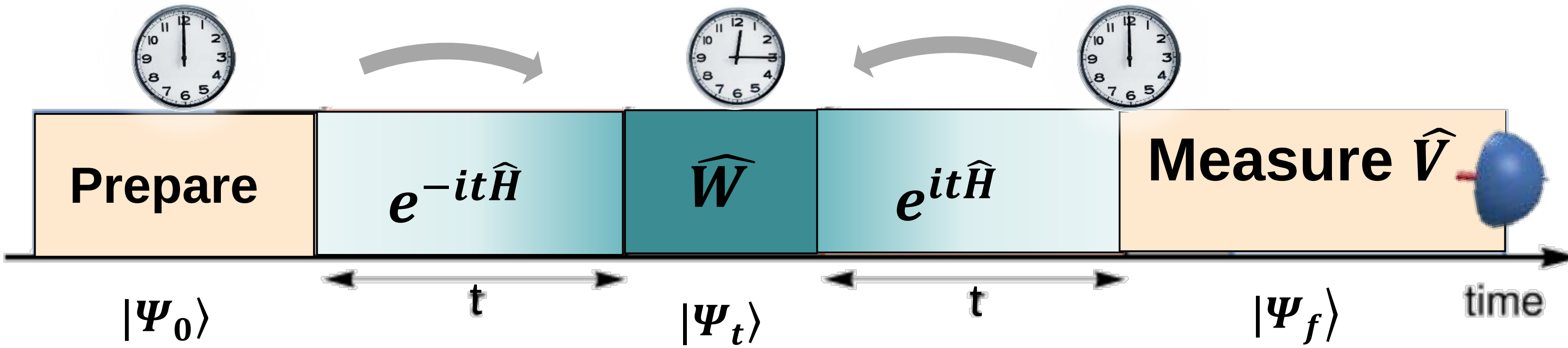
[Maldacena-Shenker-Stanford][Martinis'16]

Can we access OTOCS? [Swingle et al 16]

[Yao et al16]

[Zhu et al16]

Measuring OTOCS



$\hat{W}$ and $\hat{V}$ Two commuting operators

$$F(t) = \langle \Psi_0 |\, e^{it\hat{H}}\, \hat{W}^\dagger\, e^{-it\hat{H}}\, \hat{V}^\dagger\, e^{it\hat{H}}\, \hat{W}\, e^{-it\hat{H}}\, |\Psi_0\rangle$$

$$\hat{V}|\Psi_0\rangle = |\Psi_0\rangle$$

$$= \langle \Psi_0 |\, \underbrace{e^{it\hat{H}}\, \hat{W}^\dagger\, e^{-it\hat{H}}}_{\hat{W}_t^\dagger}\, \hat{V}^\dagger\, \underbrace{e^{it\hat{H}}\, \hat{W}\, e^{-it\hat{H}}}_{\hat{W}_t}\, \hat{V} |\Psi_0\rangle$$

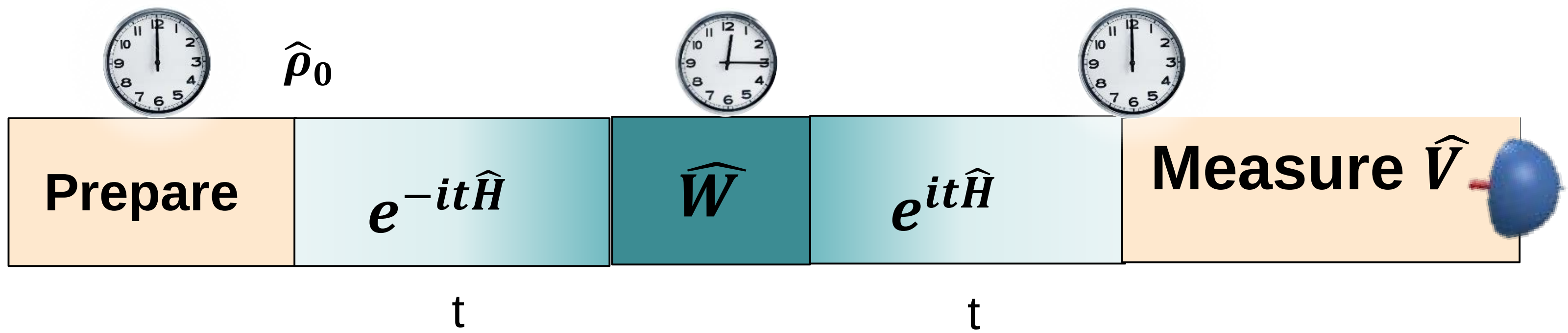
$$= \left\langle \hat{W}_t^\dagger\, \hat{V}^\dagger \hat{W}_t \hat{V} \right\rangle$$

OTOCS

Garttner *et al Nat. Physics (2017)*
Garttner *et al PRL (2018)*

Multi-quantum Coherences in NMR

M. Munowitz and M. Mehring , Sol. St. Com., 64, 605 (1987)



$\widehat{W}=e^{i\phi\hat{S}_z}$ In NMR states are highly mixed $\hat{\rho}_0=(1+\varepsilon\hat{S}_z)$

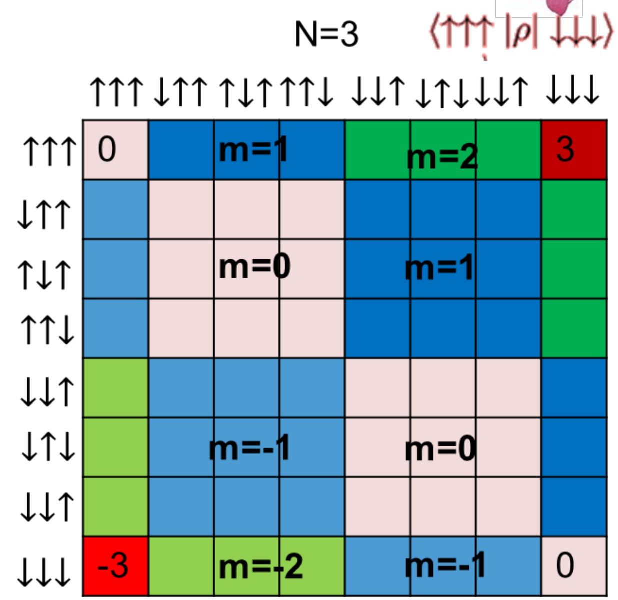
$\hat{V}=\hat{\rho}_0$ **Fourier transform gives the Multi-Quantum spectrum.**

$$\Im_\phi(\tau)=\sum_{m=-N}^{N} I_m\, e^{-im\phi}$$ **Multi-quantum intensities**

$$I_m=\mathrm{Tr}[\hat{\rho}_{-m}(t)\hat{\rho}_m(t)]$$

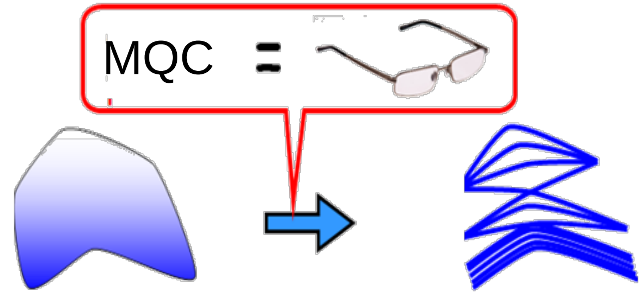
$$\hat{\rho}=\sum_m \hat{\rho}_m$$

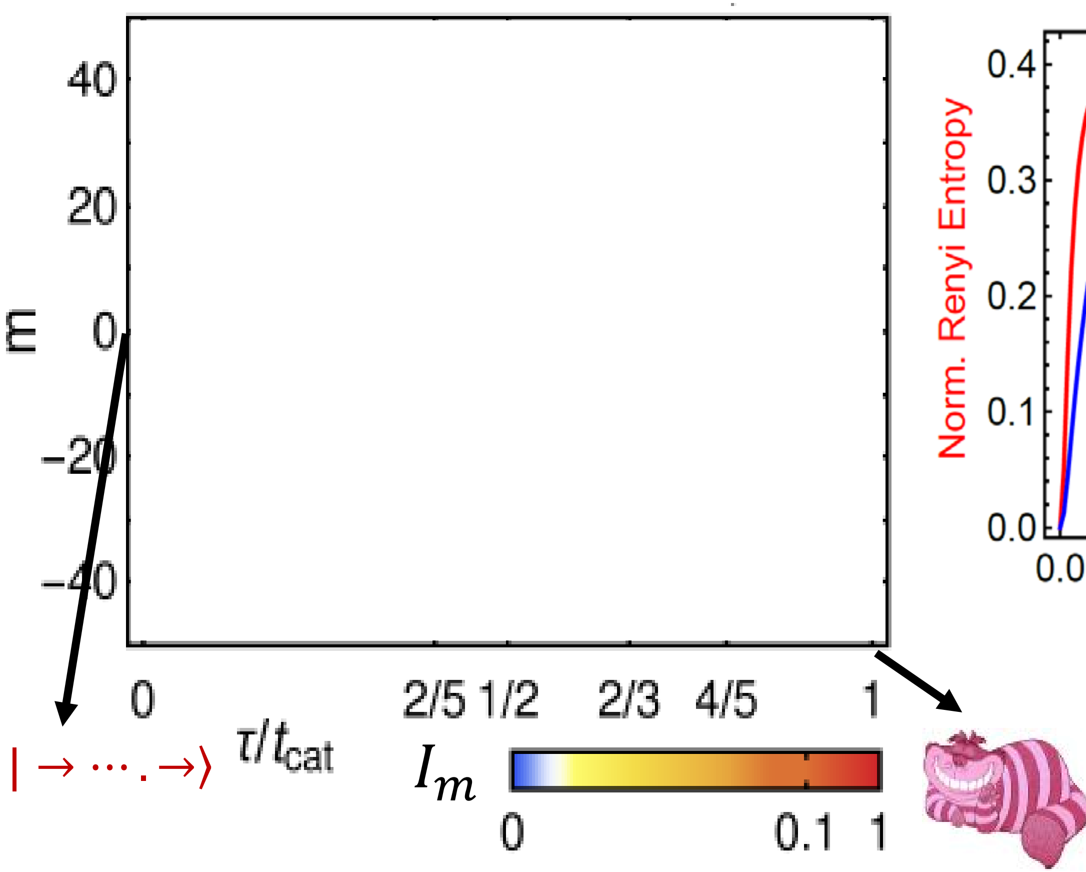
$\hat{\rho}_m$ **: all matrix elements with coherences between states differing in S_z by *m***

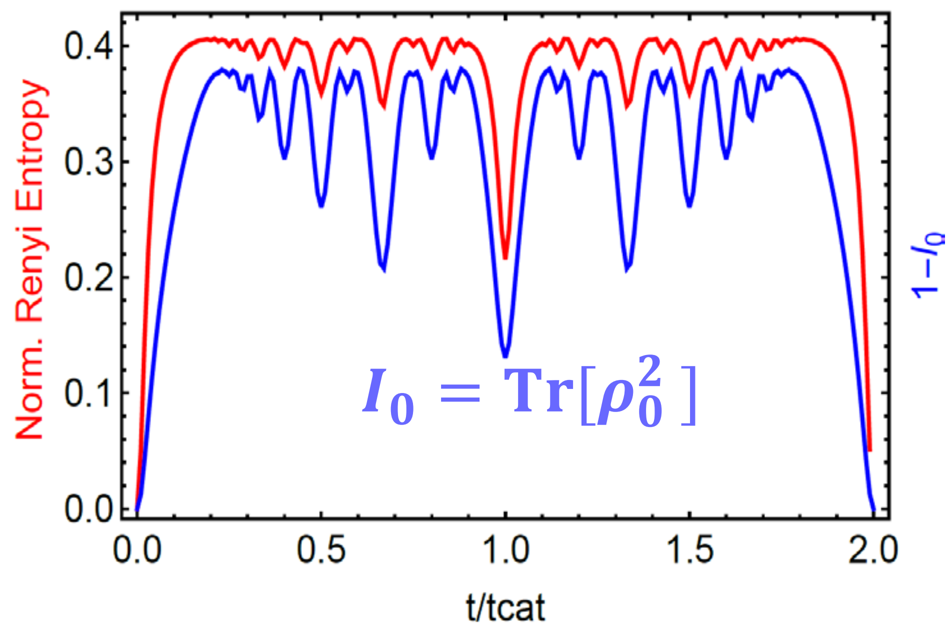


Multi-quantum Coherences (MQC)

Detailed structure of the state …





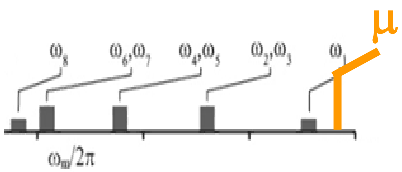


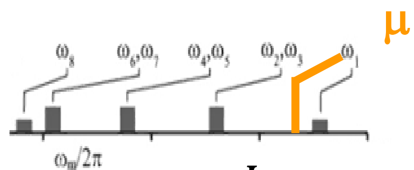
Measuring OTOCs (MQC) in trapped ions

Requirements:

1) Invert many-body time evolution.

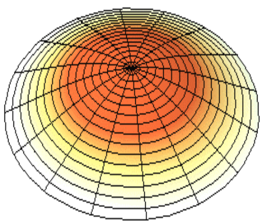
$$\hat{H}_{zz} = \frac{J}{N}(\hat{S}_z)^2$$

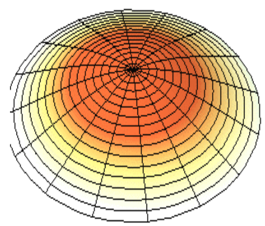




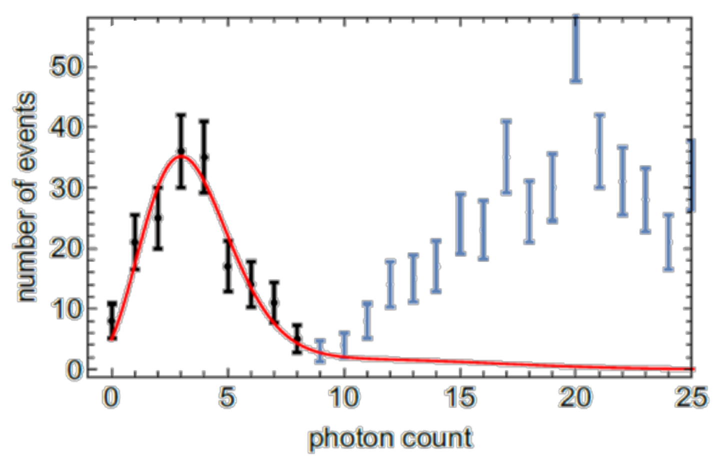
$$J \sim \frac{J_0}{\delta}$$

$$J \sim -\frac{J_0}{\delta}$$





2) Measure initial state.

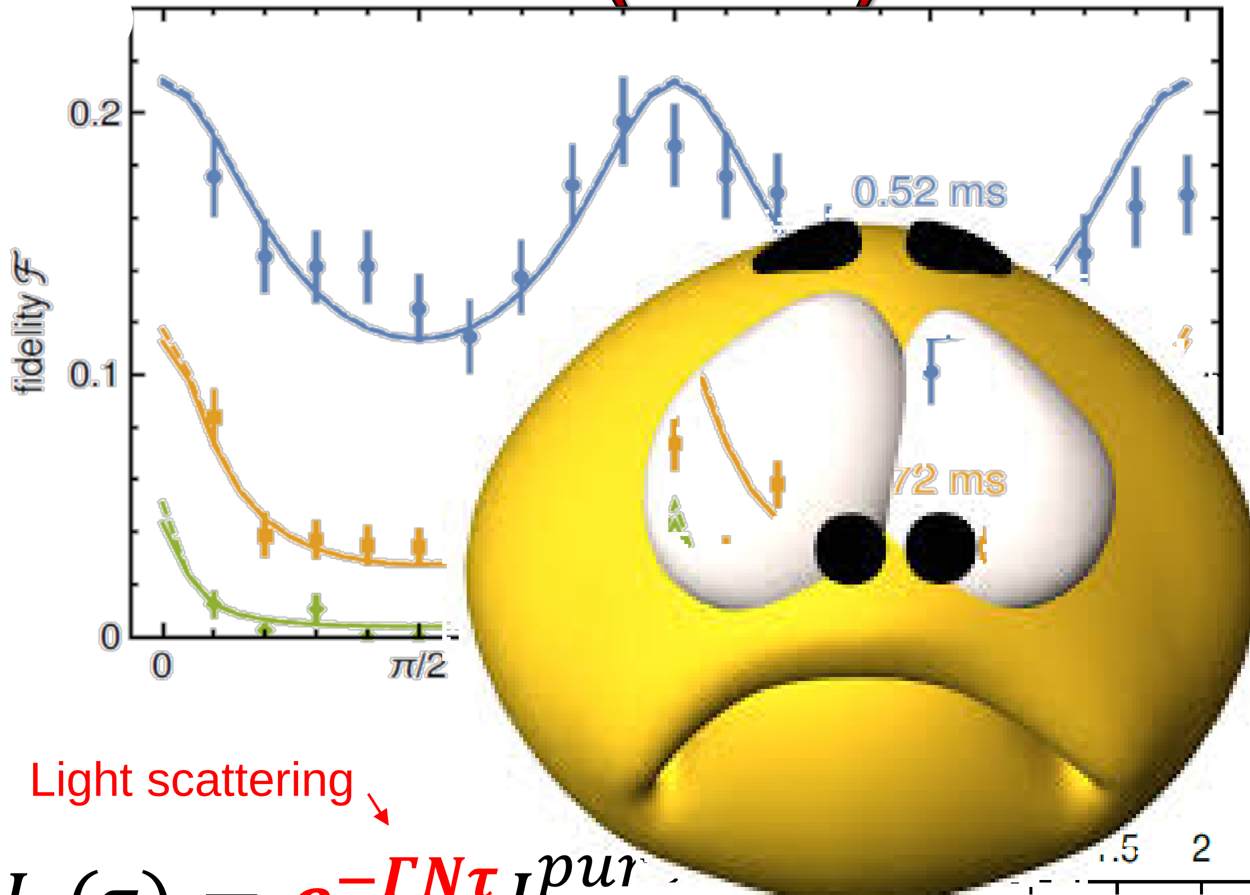


Fidelity:

Probability of all down

$$\hat{\rho}_0^z = |\downarrow \cdots \downarrow\rangle\langle\downarrow \cdots \downarrow|$$

OTOCS (MQC) Measurements



N=48

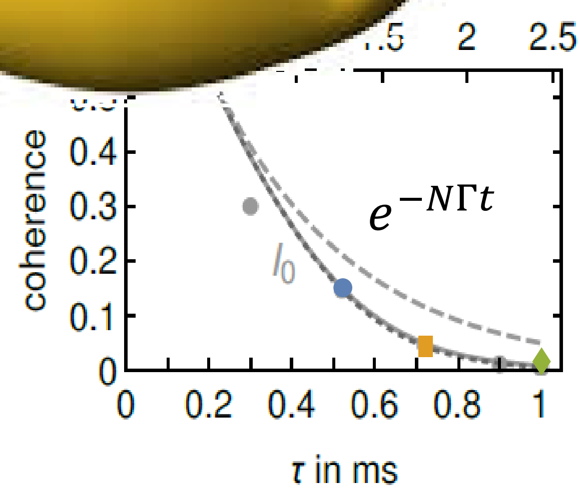
Solid lines: decoherence + phonons

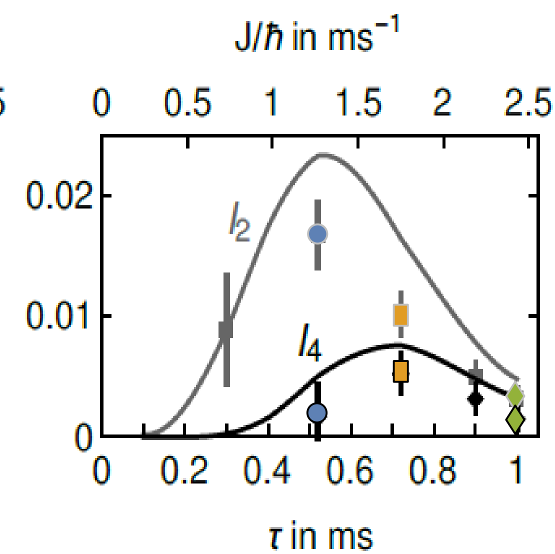
6.5 % τ_{cat}~1ms

Light scattering

$$I_0(\tau) = {\color{red}e^{-\Gamma N \tau}} I_0^{pure}$$

$$\boldsymbol{I_0^{pure}(\tau) = (1 + J^2\tau^2)^{-1}}$$





Garttner et *al Nature Physics*, doi:10.1038/nphys4119

Magnetization Measurements

Fidelity measurements decays too fast due to decoherence

Measure magnetization instead of fidelity

$\boldsymbol{V = \hat{\rho}_0}$ ➡ $\boldsymbol{V = \hat{S}_x}$

I_m ➡ A_m

$$A_m(\tau) = e^{-\Gamma\tau} A_m^{pure}$$

Less sensitive to decoherence.

Also an OTOC but what information A_m gives us?

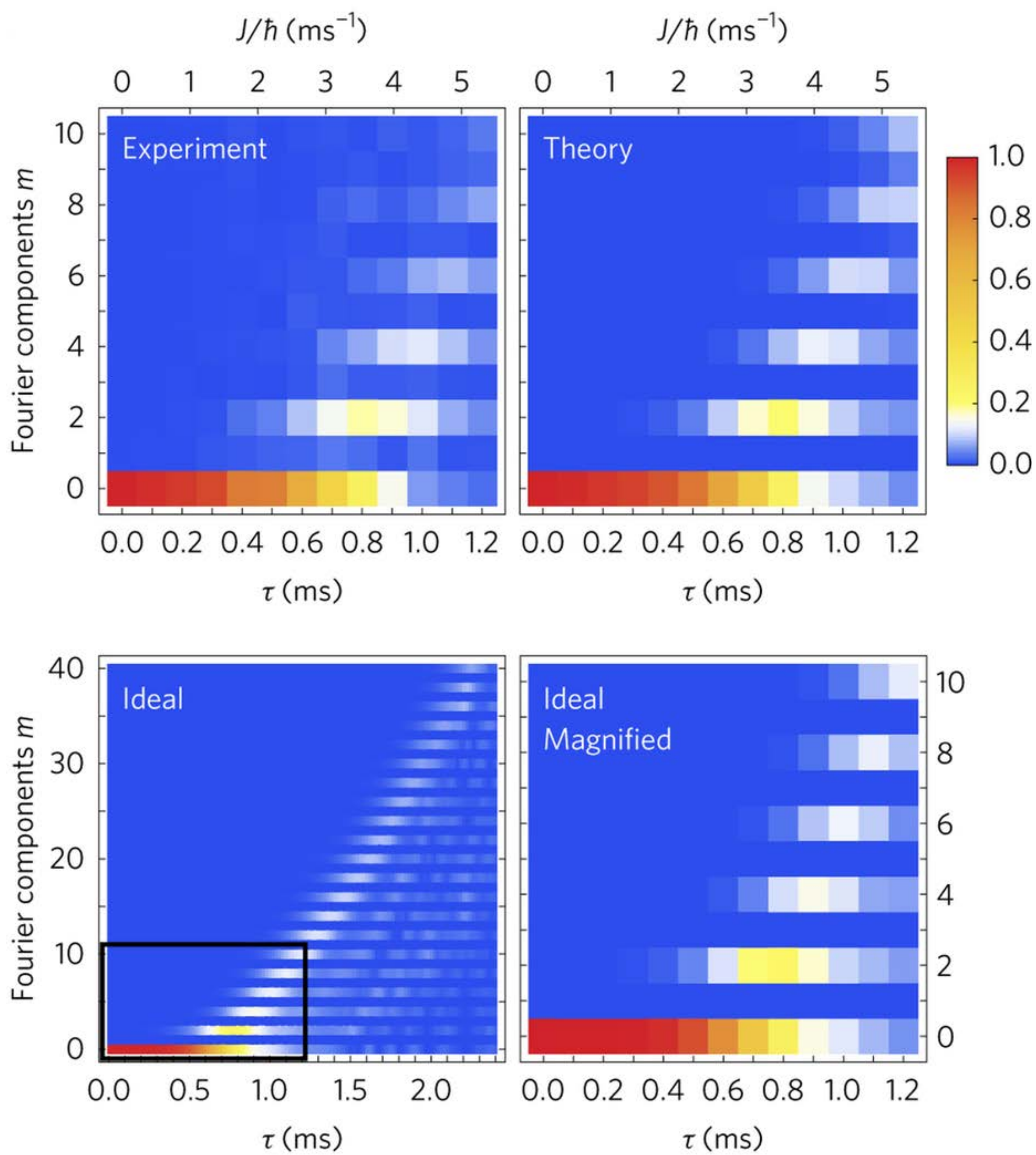
A non-zero A_m >0 implies $\langle \sigma_1^{\alpha_1} \sigma_2^{\alpha_2} \cdots \sigma_m^{\alpha_m} \rangle > 0$

Signal buildup of at least m-body correlations.

Magnetization Measurements

N=111 $\quad$ 7.3 % τ_{cat}=1 ms

- Successful benchmark
- Decoherence under control
- Access features of Hamiltonian and prepared states

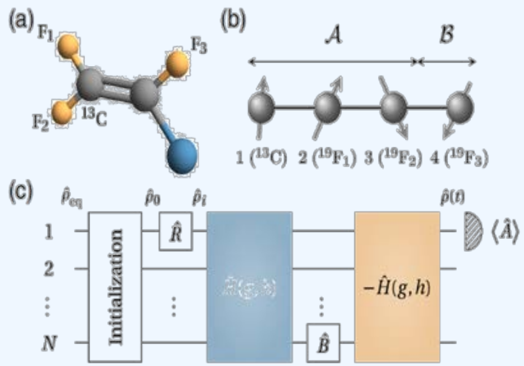


Up to m=8 significant correlations!!

Garttner et *al Nature Physics*, doi:10.1038/nphys4119

Other recent OTOC Measurements

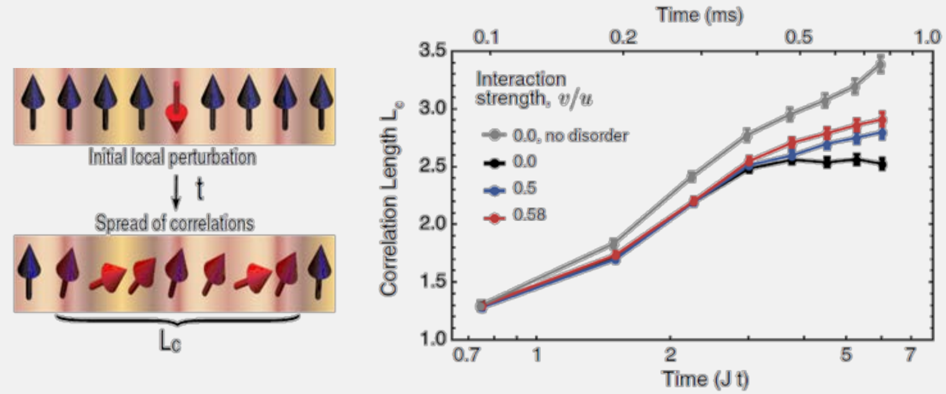
Scrambling in 4 nuclear spins in NMR, China



Selected for a Viewpoint in *Physics*

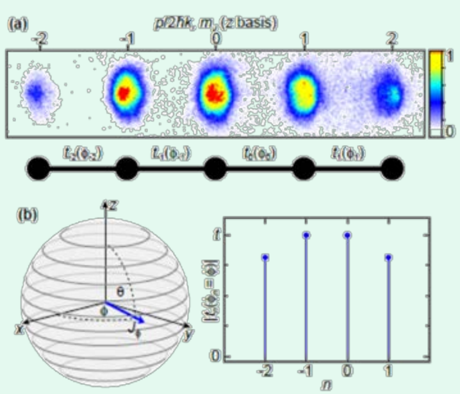
PHYSICAL REVIEW X **7**, 031011 (2017)

Probing localization with OTOCs MIT



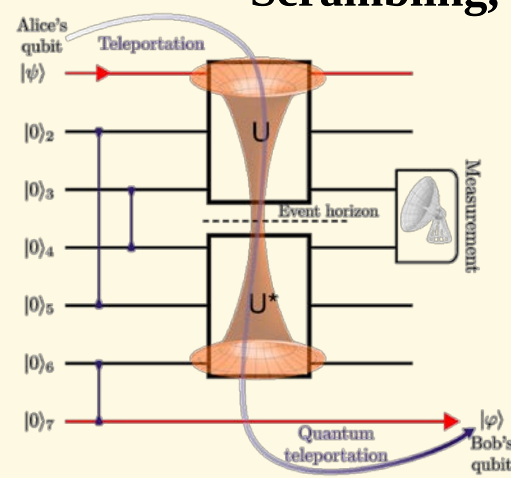
PHYSICAL REVIEW LETTERS **120**, 070501 (2018)

Chaos in a kicked BEC, U. of Illinois



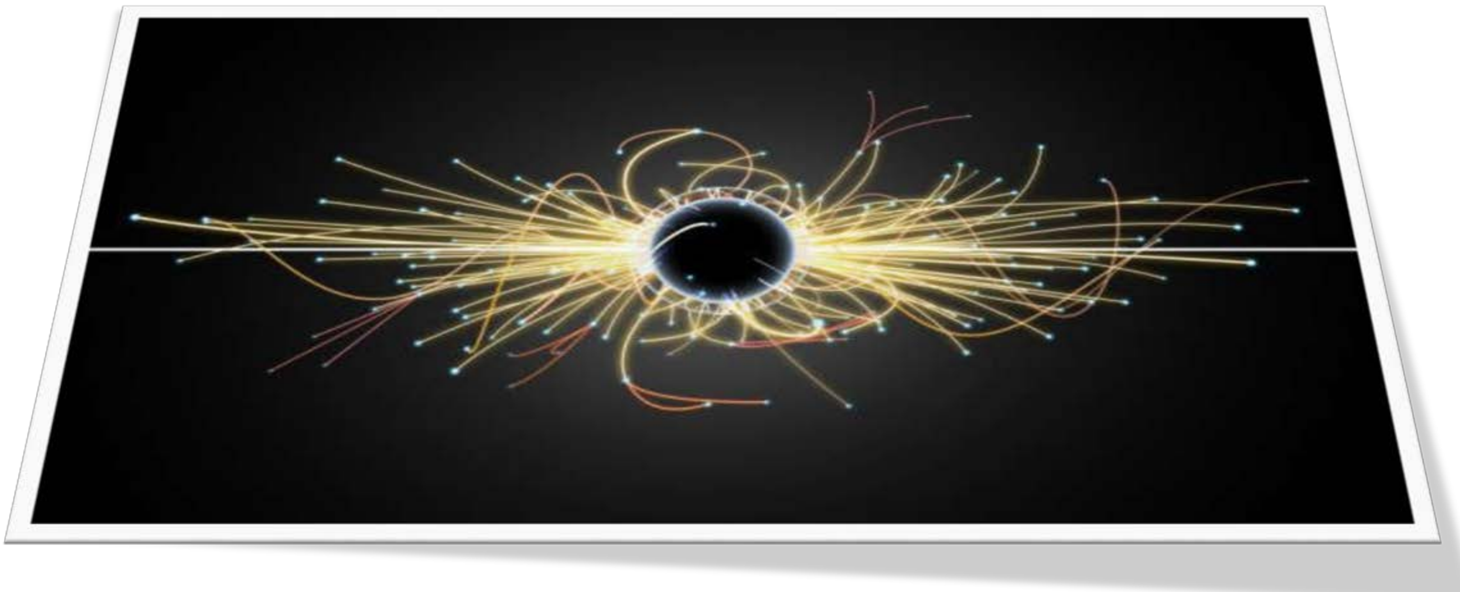
arXiv:1705.06714v1

7 Ions: Verified Information Scrambling, JQI



arXiv:1806.02807

But… The there is no chaos in the Ising Model



Can we simulate fast scrambling and analogs of black holes with trapped ions?

- Add Transverse Field
- Involve Phonons

Hamiltonian in the rotating frame

$$\hat{H} = -\delta \hat{a}^{\dagger}\hat{a} - \frac{g}{\sqrt{N}}(\hat{a}^{\dagger}+\hat{a})\hat{S}_z = -\delta \hat{b}^{\dagger}\hat{b} + \frac{J}{N}(\hat{S}_z)^2$$

$\hat{z}_i \propto (\hat{a}^{\dagger}+\hat{a})/\sqrt{N}$

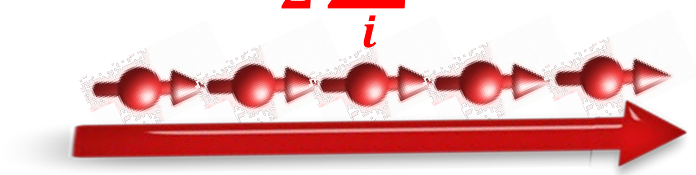
$\hat{a}^{\dagger}$: CM phonons creation operator

$$\hat{b} = \left(\hat{a} - \frac{g}{\delta\sqrt{N}}\hat{S}_z\right)$$

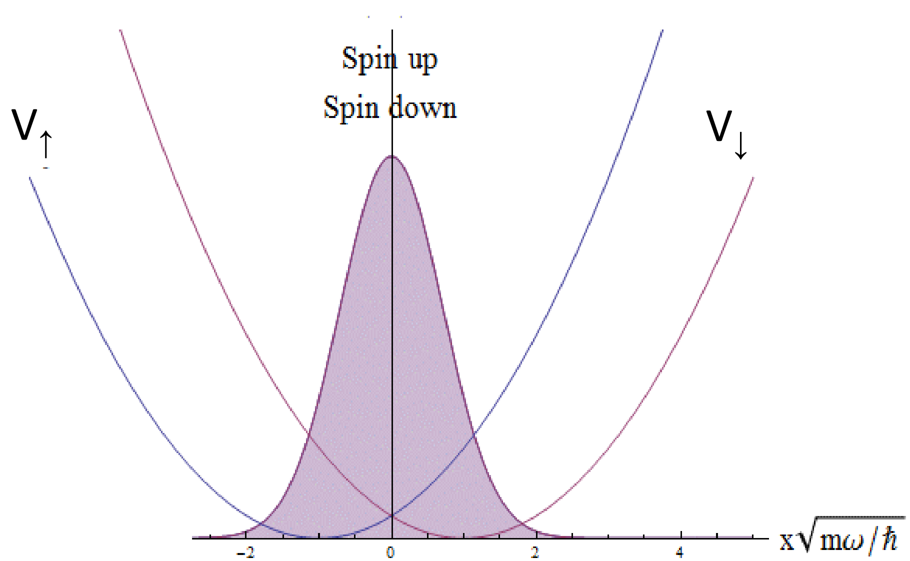
$$\hat{S}^{x,y,z} = \frac{1}{2}\sum_i \hat{\sigma}_i^{x,y,z}$$

Collective spin: S=N/2

$$J = g^2/\delta$$



Displaced Harmonic Oscillator



At $t\delta = 2\pi n$ Decoupling points

$$\hat{H}_{ZZ} = \frac{J}{N}(\hat{S}_z)^2$$

Collective Ising Model

Add Transverse Field

$$\hat{H} = -\delta \hat{a}^\dagger \hat{a} - \frac{g}{\sqrt{N}}(\hat{a}^\dagger + \hat{a})\hat{S}_z - B\hat{S}_x$$

Phonons | Spin-Phonon | Transverse Field

No decoupling points $[\hat{H}_{ODF}, \hat{S}^x] \neq 0$

We can NOT eliminate the phonons

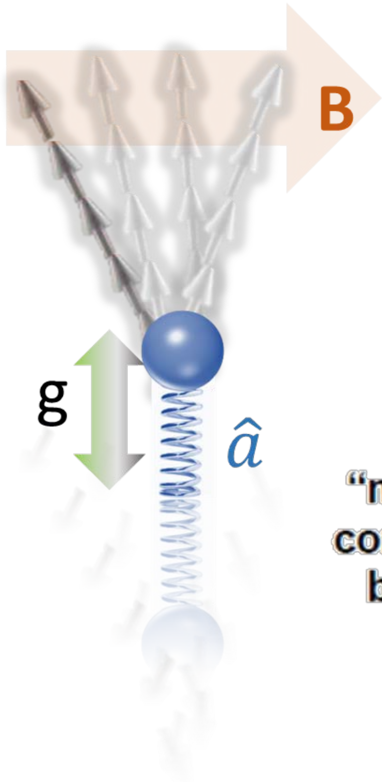
$$\hat{H} = \frac{J}{N}(\hat{S}_z)^2 \quad B\hat{S}_x$$

Dicke Model

$$\hat{H} = \delta \hat{a}^\dagger \hat{a} + \frac{2g}{\sqrt{N}}(\hat{a}^\dagger + \hat{a})\hat{S}_z - B\hat{S}_x$$

Phonons | Spin-Phonon | Transverse Field

$\hat{a}^\dagger$: CM phonons creation operator



"molecules interacting with a common radiation field cannot be treated as independent"
R.H. Dicke (1953)

On the Superradiant Phase Transition for Molecules in a Quantized Radiation Field: the Dicke Maser Model

KLAUS HEPP

Physics Department, ETH, Zürich, 8049 Switzerland

AND

ELLIOTT H. LIEB*

Mathematics Department, MIT, Cambridge, Mass. 02139, USA

A system of N two-level molecules coupled to finitely many modes of a quantized radiation field via a truncated dipolar interaction is investigated. The thermodynamic and correlation functions can be exactly computed in the limit $N \to \infty$. The system exhibits a second order phase transition from normal to superradiance. Different effective Hamiltonians with linear Heisenberg equations of motion become asymptotically exact in the limit $N \to \infty$.

ANNALS OF PHYSICS: 76, 360–404 (1973)

Safavi-Naini et al (arXiv:1807.03178)PRL 2018

Dicke Model

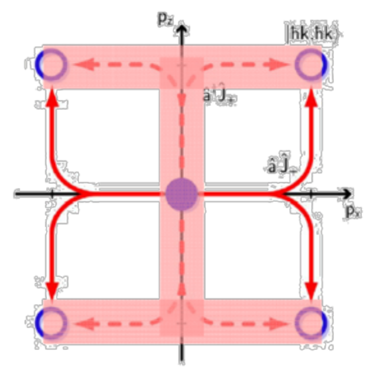
PHYSICAL REVIEW A **75**, 013804 (2007)

Proposed realization of the Dicke-model quantum phase transition in an optical cavity QED system

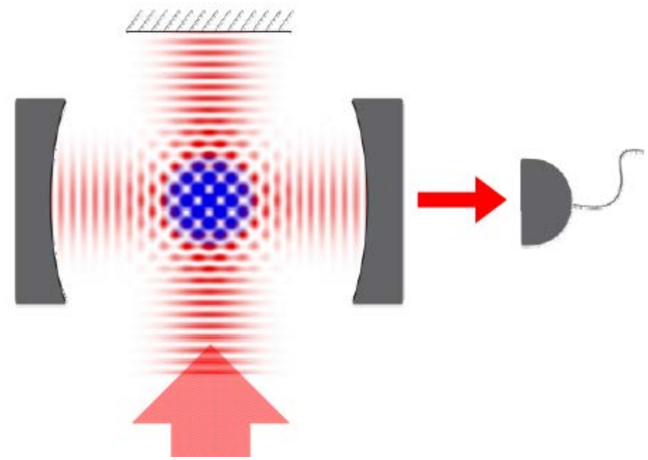
F. Dimer,[1] B. Estienne,[2] A. S. Parkins,[3,*] and H. J. Carmichael[1]
[1]*Department of Physics, University of Auckland, Private Bag 92019, Auckland, New Zealand*
[2]*Laboratoire de Physique Théorique et Hautes Energies, Université Pierre et Marie Curie, 4 place Jussieu, F-75252 Paris Cedex 05, France*
[3]*Norman Bridge Laboratory of Physics 12-33, California Institute of Technology, Pasadena, California 91125, USA*
(Received 18 July 2006; published 8 January 2007)

Renew interest in cold-atoms

T. Esslinger group 2010:
Effective Dicke Model in a BEC
(self-organization)



$\hat{J}_+ = \sum_i |\pm\hbar k, \pm\hbar k\rangle_{i\,i}\langle 0,0| = \hat{J}_-^\dagger$



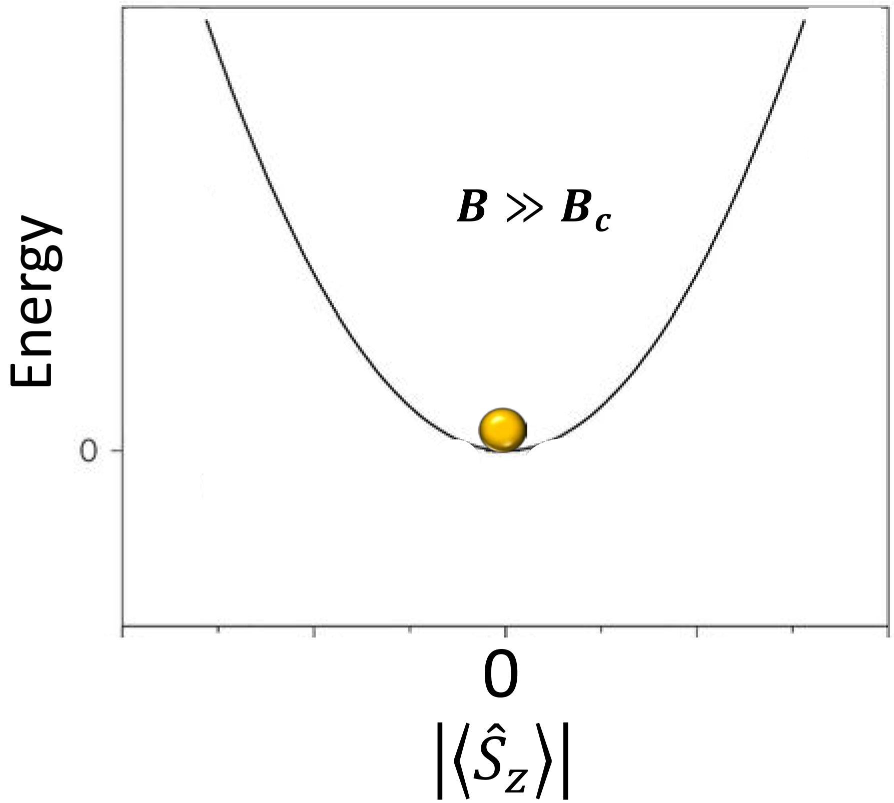
Dicke Model: Full Controllability

$$\hat{H} = -\delta \hat{a}^\dagger \hat{a} - \frac{g}{\sqrt{N}}(\hat{a}^\dagger + \hat{a})\hat{S}_z - B(t)\hat{S}_x$$

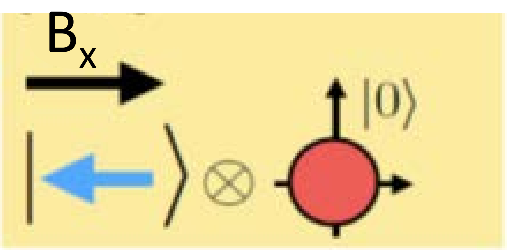
δ<0 Phonons   Spin-Phonon   Transverse Field

Normal to superradiant second order phase transition at $B_c = J$

$$\hat{H} = -\delta \hat{b}^\dagger \hat{b} + \frac{J}{N}(\hat{S}_z)^2 - B(t)\hat{S}_x \qquad J = g^2/\delta \qquad \hat{b} = \left(\hat{a} - \frac{g}{\delta\sqrt{N}}\hat{S}_z\right)$$



Normal: $B \gg B_c$



Paramagnetic,
No phonons
Decoupled spin/phonon

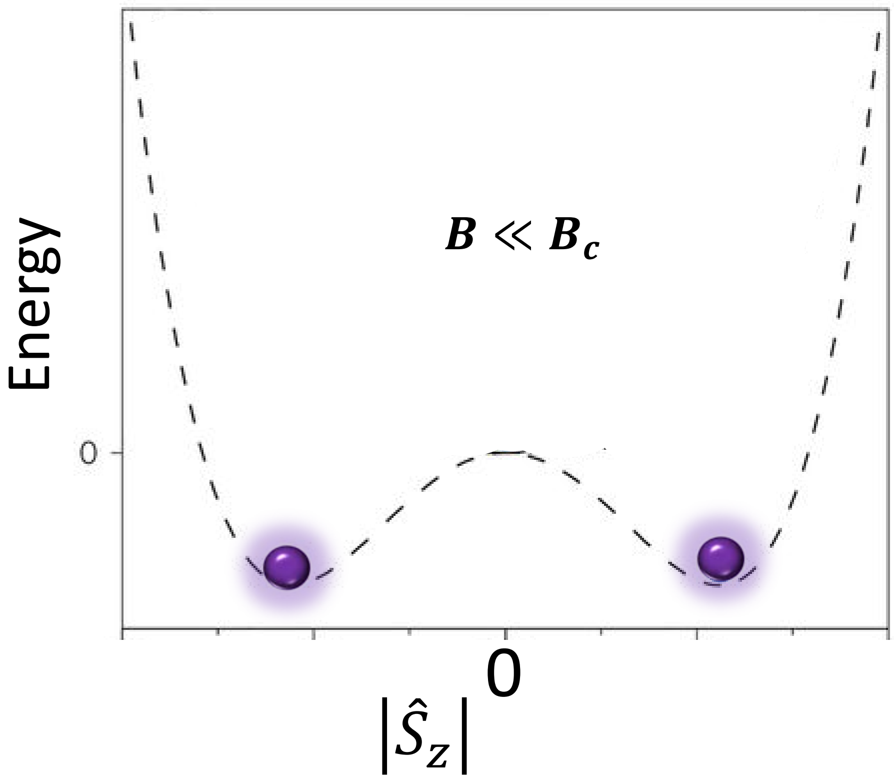
Dicke Model: Full Controllability

$$\hat{H} = -\delta\hat{a}^\dagger\hat{a} - \frac{g}{\sqrt{N}}(\hat{a}^\dagger + \hat{a})\hat{S}_z - B(t)\hat{S}_x$$

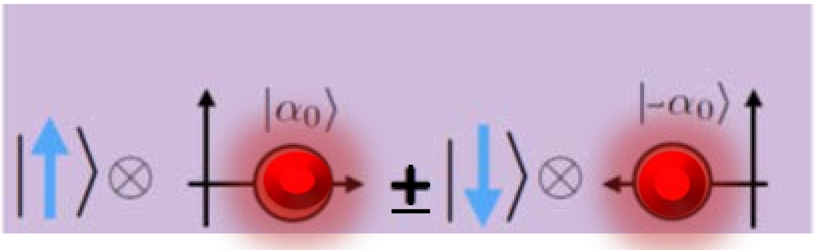
δ<0 Phonons Spin-Phonon Transverse Field

Normal to superradiant second order phase transition at $B_c = g^2/\delta$

$$\hat{H} = -\delta\hat{b}^\dagger\hat{b} + \frac{J}{N}(\hat{S}_z)^2 - B(t)\hat{S}_x \qquad J = g^2/\delta \qquad \hat{b} = \left(\hat{a} - \frac{g}{\delta\sqrt{N}}\hat{S}_z\right)$$

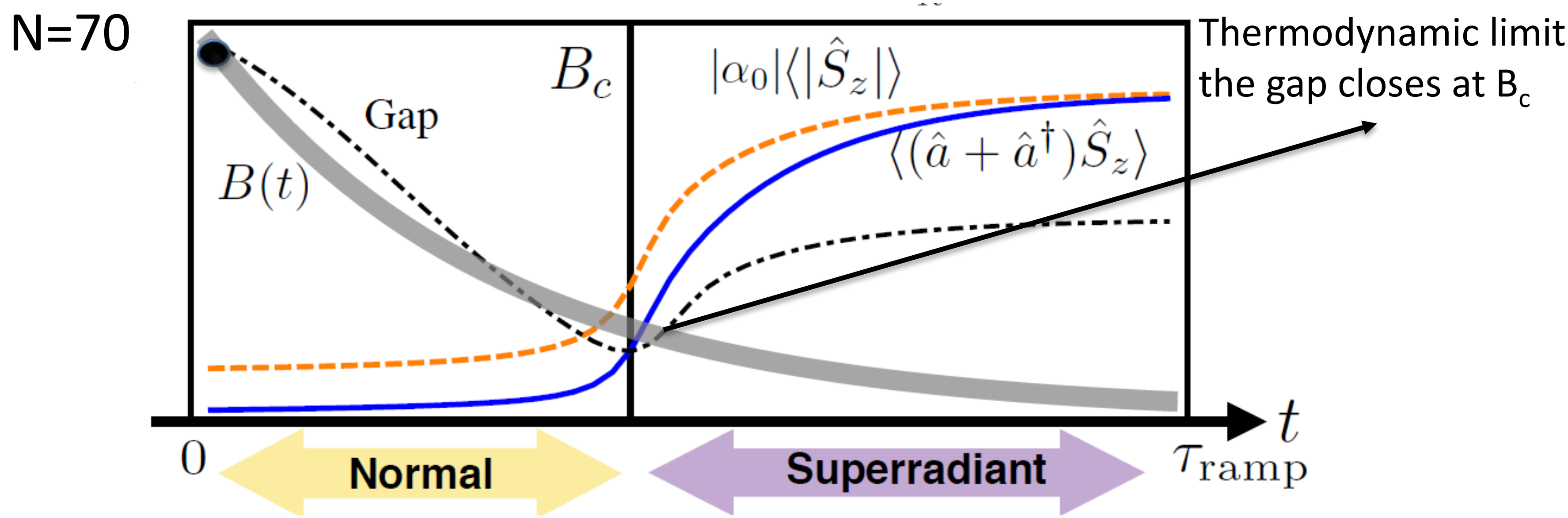


Superradiant: $B \ll B_c$



Ferromagnetic,
Spin/phonon Entanglement
Macroscopic phonon population ∝ N

Exploring Dicke Model: Slow quenches



Paramagnetic,
decoupled spin/phonon

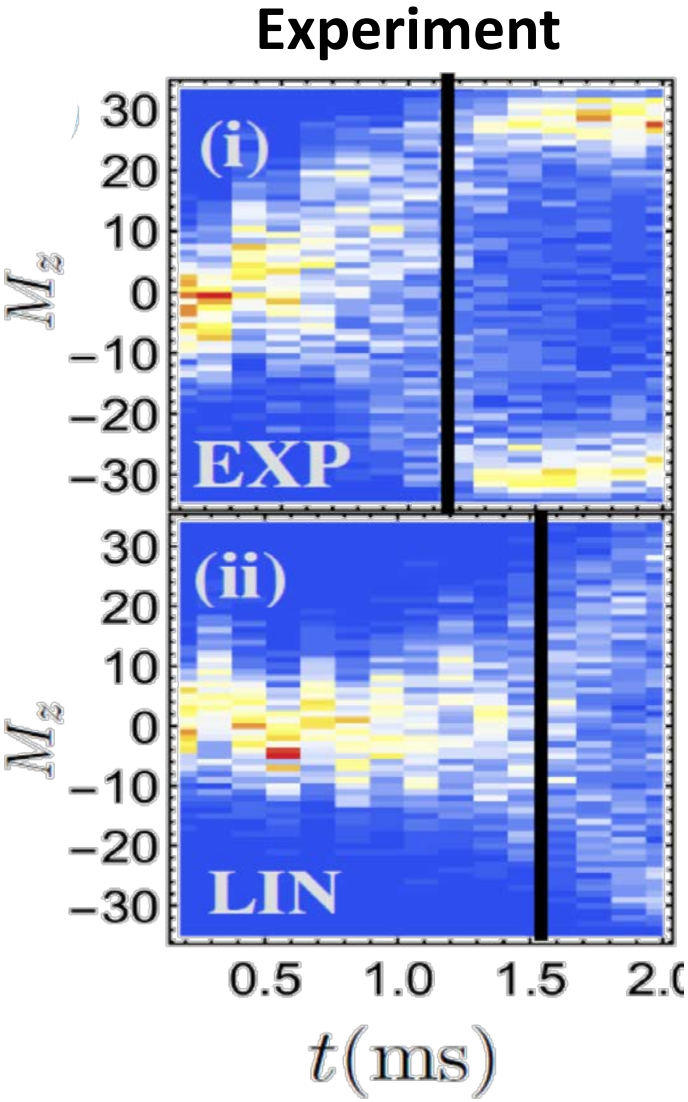
Ferromagnetic, Spin/phonon Entanglement
Macroscopic phonon population $\propto$ N

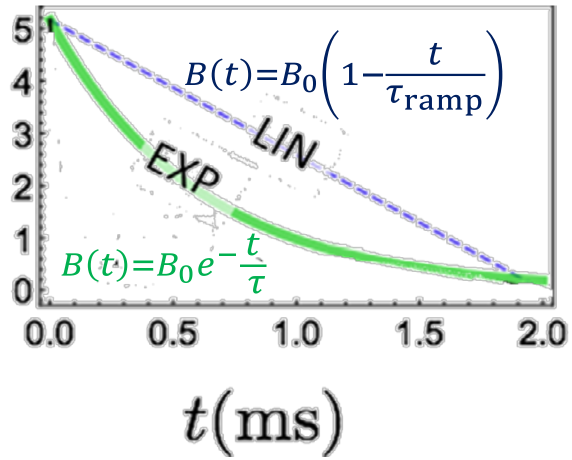
- Initial state (large B): Spin aligned along x, thermal phonon state $\bar{n}\sim 6$

$$|\boldsymbol{\Psi}(\mathbf{0})\rangle = |\leftarrow \cdots \leftarrow\rangle \otimes |\vec{n}\rangle$$

- Optimal slow ramps to follow adiabatic state
- Trade-off: adiabaticity vs decoherence (light scattering) Ramp limited to ~2 ms ramps

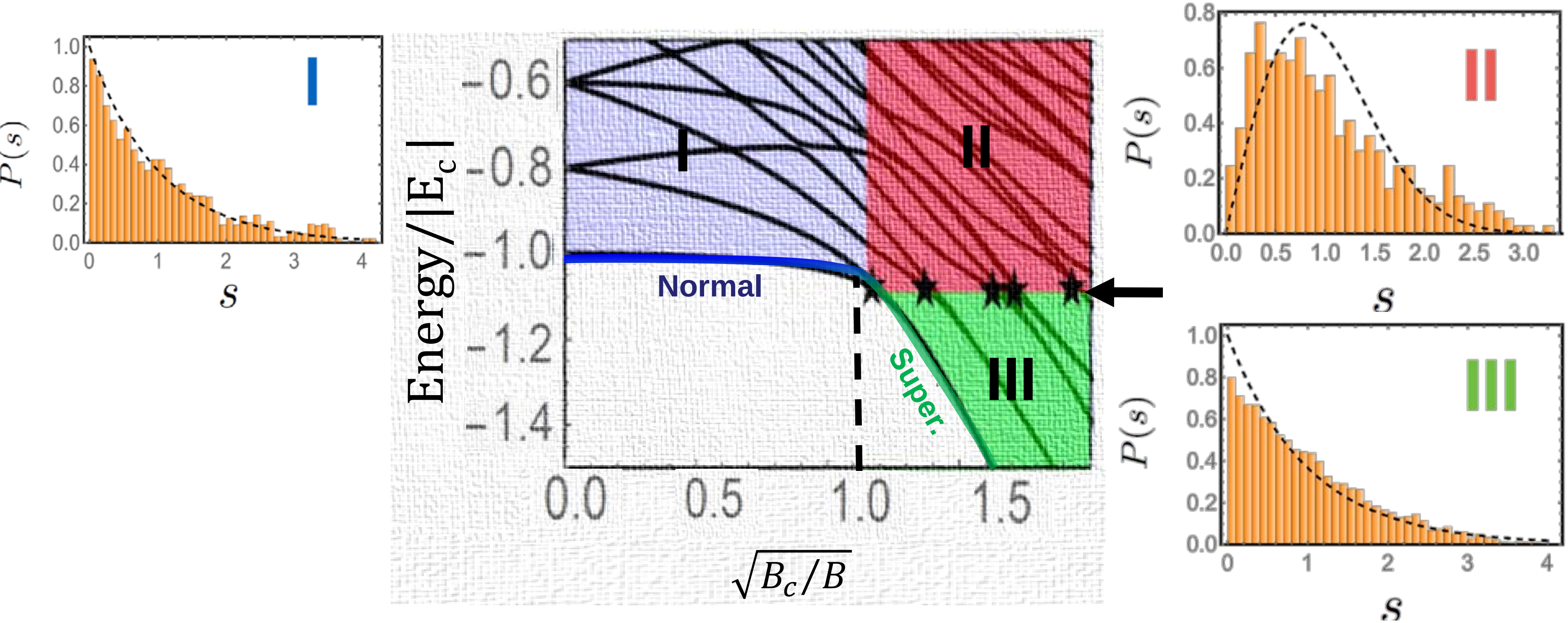
Slow Quenches through the critical point





- Measure full spin distributions (global fluorescence)
- Benchmark the quantum simulator

Dicke Model: Rich Physics



- Excited State Phase Transition at E_c =-NB/2 and $B>B_c$
  Singularity in the energy level structure
- $B>B_c$ **Poissonian Integrable**
- **E>Ec: Wignger-Dyson: Chaos**
  E<Ec: mixture (Wigner Dyson/ Poissonian)

Emary&Brandes, PRE (2003)
Brandes,PRE (2013)
Altland & Haake, PRL (2012)

Dicke Model: Rich Physics

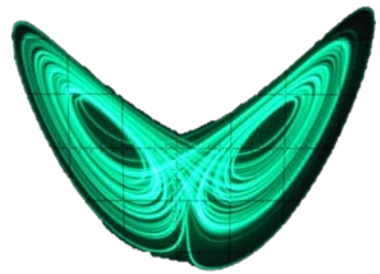
Connection to classical Chaos

- Solve mean field equations for $\vec{x} = \{\langle \hat{S}_x \rangle, \langle \hat{S}_y \rangle, \langle \hat{S}_z \rangle, \langle \hat{a} \rangle_R, \langle \hat{a} \rangle_{Im}\}$

$$|\vec{x}(t) - \vec{x}(0)| = \Delta x(t) = \Delta x(0) e^{\lambda_L t}$$

λ_L: Lyapunov Exponent

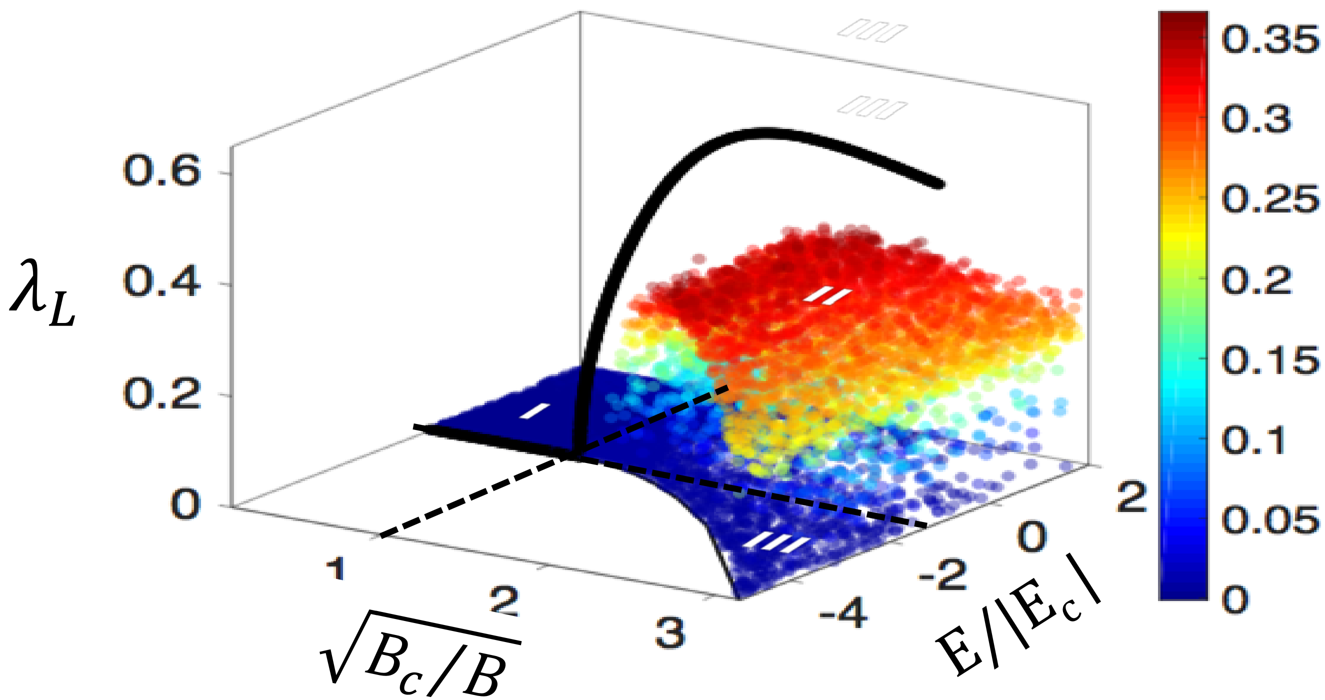
$\lambda_L > 0$: Signature of classical chaos

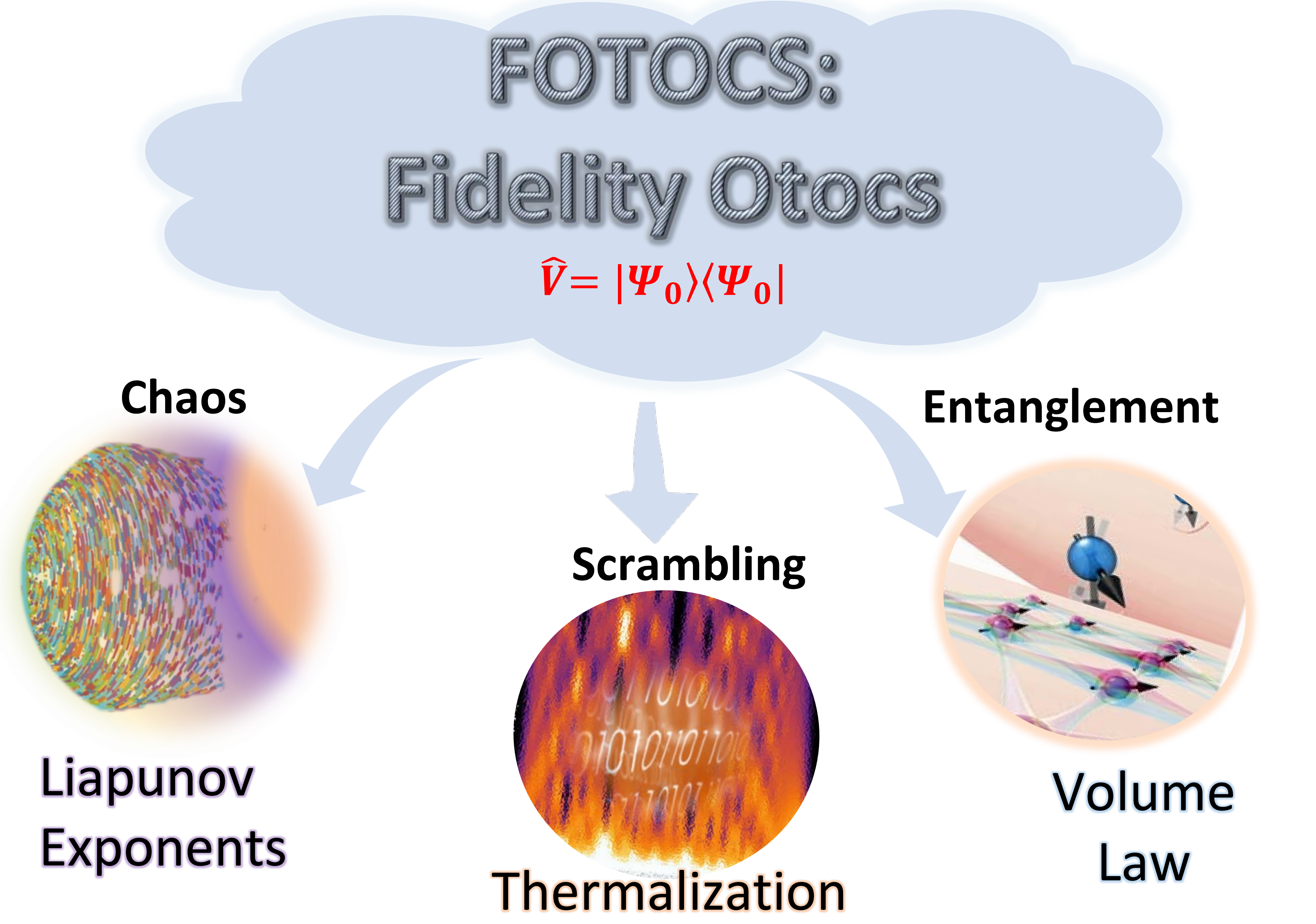


Butterfly effect
Strogatz Book

- State $|\boldsymbol{\Psi_0^c}\rangle = |\rightarrow \cdots \rightarrow\rangle \otimes |\mathbf{0}\rangle$ $\langle \Psi_{\mathbf{0}}^{\boldsymbol{c}} | \hat{H} | \Psi_{\mathbf{0}}^{\boldsymbol{c}} \rangle = E_c$

Maximal Exponent

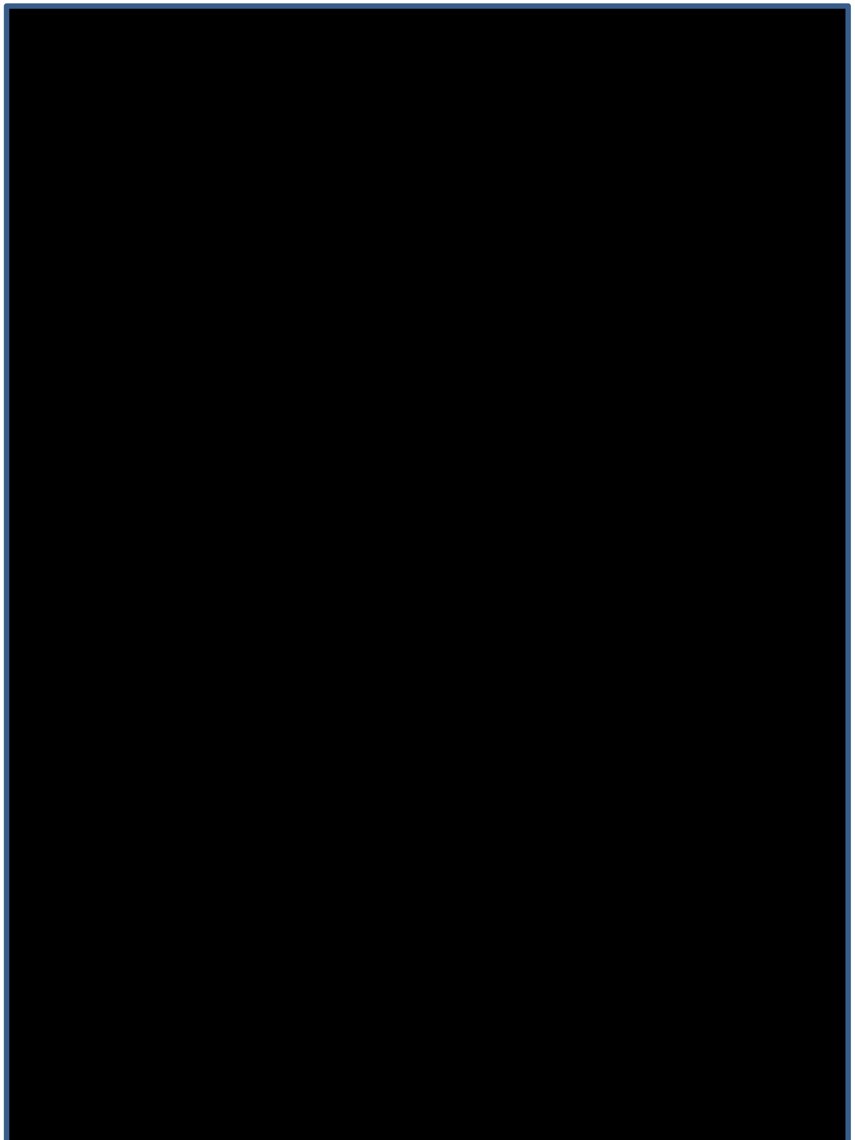




FOTOCS:
Fidelity Otocs
$\hat{V} = |\Psi_0\rangle\langle\Psi_0|$
Chaos
Entanglement
Scrambling
Liapunov
Exponents
Thermalization
Volume
Law

FOTOCS

Air: Arcimboldo 1566



Water: Arcimboldo 1566

Fotocs and Quantum Chaos

$\widehat{W}=e^{i\delta\phi\hat{G}}$

$\delta\phi$<<1 Small Perturbation

$\hat{V}=|\Psi_0\rangle\langle\Psi_0|$

$$F_G(\delta\phi,t)\approx 1-\delta\phi^2\left(\langle\Psi_t|\hat{G}^2|\Psi_t\rangle-\langle\Psi_t|\hat{G}\,|\Psi_t\rangle^2\right)\equiv 1-\delta\phi^2\,\Delta^2 G$$

Great Insight:

- ✓ Provide a semi-classical picture of the scrambling dynamics
  - ➢ Variance can be computed by phase space methods
  - ➢ Compute large systems intractable with numerical methods
- ✓ Connect classical and quantum Liapunov exponents

$\langle\delta G\rangle_c\sim e^{\lambda_L t}$ $\quad 1-F_G(\delta\phi,t)\sim(\delta\phi^2)e^{\lambda_Q t}\equiv\delta\phi^2\,e^{2\lambda_L t}$ $\quad \lambda_Q=2\lambda_L$

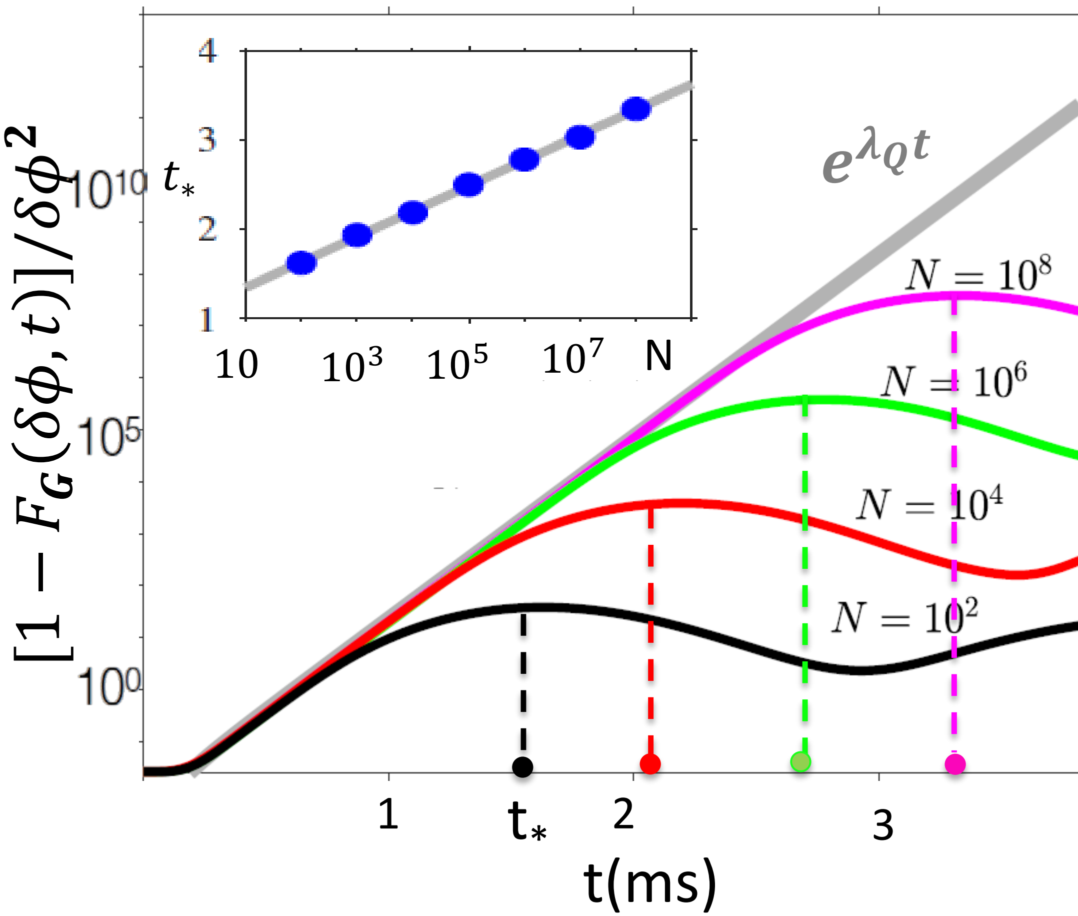
Classical — Quantum

Schmitt et al arXiv:1802.06796
Rozenbaum et al, PRL (2017)

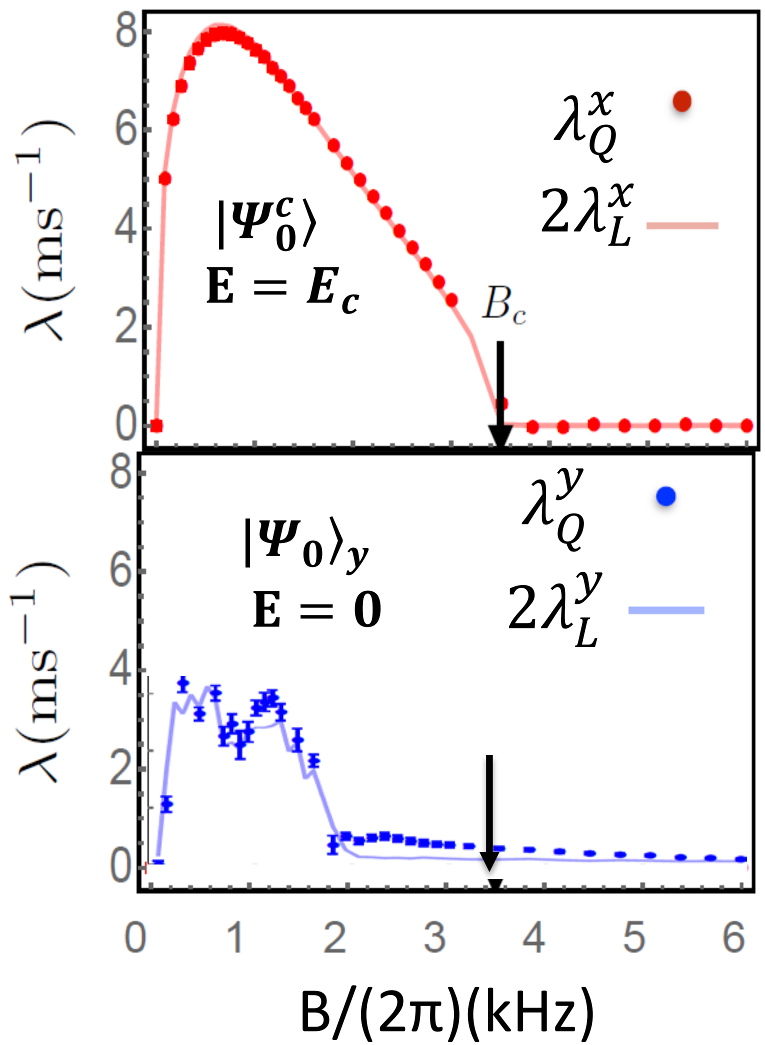
Fotocs and Quantum Chaos

$$\hat{G} = \hat{X} = \frac{1}{2}(\hat{a} + \hat{a}^\dagger)$$

$$|\Psi_0^c\rangle = |\rightarrow \cdots \rightarrow\rangle \otimes |0\rangle$$



- Fast scrambling in Dicke M.
- Scrambling time $\lambda_Q t_* \sim \log N$



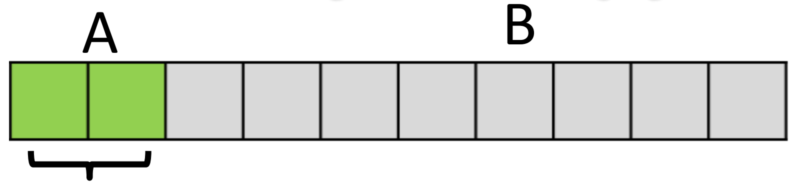
At the critical energy scrambling is faster

$$\lambda_Q = 2\lambda_L$$

FOTOCs & Renyi Entropy

$\hat{\rho}$: Density Matrix

$\mathrm{Tr}(\hat{\rho}^{\,2}) = 1$



$$\hat{\rho}_A = \mathrm{Tr}_B\, \hat{\rho}$$

Reduce density Matrix of A

Renyi entropy: Purity of $\hat{\rho}_A$ $\quad S_A = -\log[Tr(\rho_A^2)]$

$$e^{-S_A} = \sum_{W \epsilon B} \mathrm{Tr}\left[W_t^\dagger\, \hat{O}\; e^{-\beta H}\, \hat{O}^\dagger W_t\, \hat{O}\, e^{-\beta H}\, \hat{O}^\dagger \right]$$

$$V = \hat{O}\, e^{-\beta H}\, \hat{O}^\dagger$$

H. Zhai: Science Bulletin(2017)

B. Yoshida: JHEP02 (2016)

Sum over complete set of operators acting on B :Exponential 4^B terms

At $\beta \to \infty$ $\quad V = \hat{O}\, |\Psi_g\rangle\langle\Psi_g|\, \hat{O}^\dagger = |\Psi_0\rangle\langle\Psi_0|$ $\quad$ FOTOC

Probing entanglement entropy via randomized measurements: Up to N=20 P. Zoller, R. Blatt, C. F. Roos, arXiv:1806.05747

FOTOCs & Renyi Entropy

Spins

$$\hat{\rho} = \sum \varrho^{\,n',n}_{\,m',m} |m'\rangle\langle m| \otimes |n'\rangle\langle n|$$

Phonons

$$\hat{G}_{S_r}|m_r\rangle = (e_r \cdot \hat{\vec{S}})|m_r\rangle = m_r|m_r\rangle \qquad \hat{G}_n|n\rangle = \hat{a}^\dagger \hat{a}|n\rangle = n|n\rangle$$

Spin Renyi Entropy: $S_2(\hat{\rho}_S) = -\log(\mathrm{Tr}[\hat{\rho}_S^2])$: Tracing over phonons

$$\mathrm{Tr}[\hat{\rho}_S^2] = I_0^{\hat{S}_r} + I_0^{\hat{n}} - D_{\mathrm{diag}}^{\hat{S}_r,\hat{n}} + C_{\mathrm{off}}^{\hat{S}_r,\hat{n}}$$

Multi-Quantum Intensities. $\hat{V} = |\Psi_0\rangle\langle\Psi_0|$

$$I_0^{\hat{S}_r}(t) = \frac{1}{2\pi}\int_0^{2\pi} F_{G_{S_r}}(\phi,t)\,d\phi \qquad I_0^{\hat{n}}(t) = \frac{1}{2\pi}\int_0^{2\pi} F_{G_n}(\phi,t)\,d\phi \qquad \hat{W}_G = e^{i\phi\hat{G}}$$

Purely-diagonal elements: $D_{\mathrm{diag}}^{\hat{S}_r,\hat{n}} = \sum (\varrho^{n,n}_{m,m})^2 \sim 1/(N n_{ph})$

Off-diagonal elements:

$$C_{\mathrm{off}}^{\hat{S}_r,\hat{n}} = \sum_{m \neq m', n \neq n'} \varrho^{\,n,n'}_{\,m,m}\,\varrho^{\,n',n}_{\,m',m'} = \sum_{m \neq m', n \neq n'} \varrho^{\,n,n}_{\,m,m'}\,\varrho^{\,n',n'}_{\,m',m} \longrightarrow 0$$

Dephase for $t < t_c \sim \lambda_Q^{-1}$

For $t > t_c$: randomize

FOTOCs & Renyi Entropy

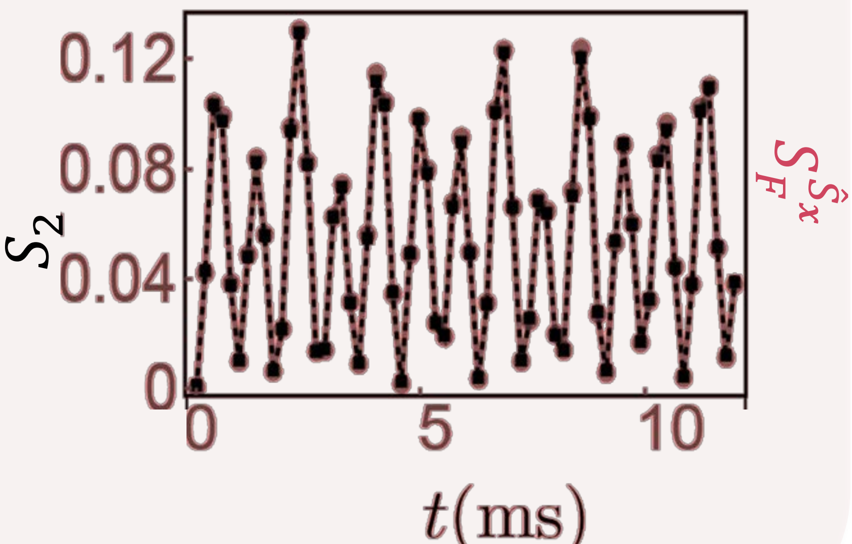
FOTOCS can give use access to Renyi entropy

$$|\Psi_0^c\rangle = |\rightarrow \cdots \rightarrow\rangle \otimes |0\rangle$$ N=40

B>B$_c$: Integrable case

$$S_F^{\hat{S}_x} = -\log\left(I_0^{\hat{S}_x}\right)$$

$|C_{\text{off}}^{\hat{S}_x,\hat{n}}| \ll 0$ $\quad I_0^{\hat{n}} \approx D_{\text{diag}}^{\hat{S}_r,\hat{n}}$



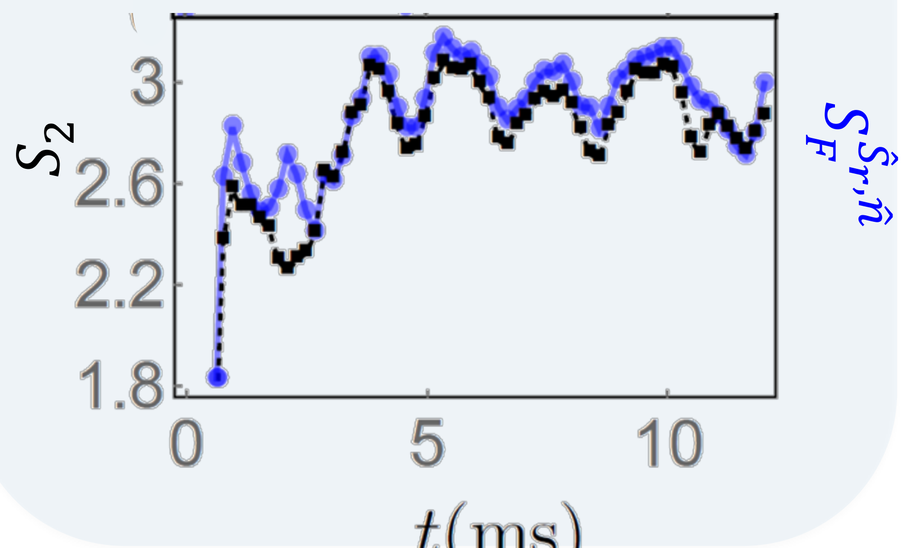
B<B$_c$: Chaotic case

$$S_F^{\hat{S}_r,\hat{n}} = -\log\left(I_0^{\hat{S}_r} + I_0^{\hat{n}}\right)$$

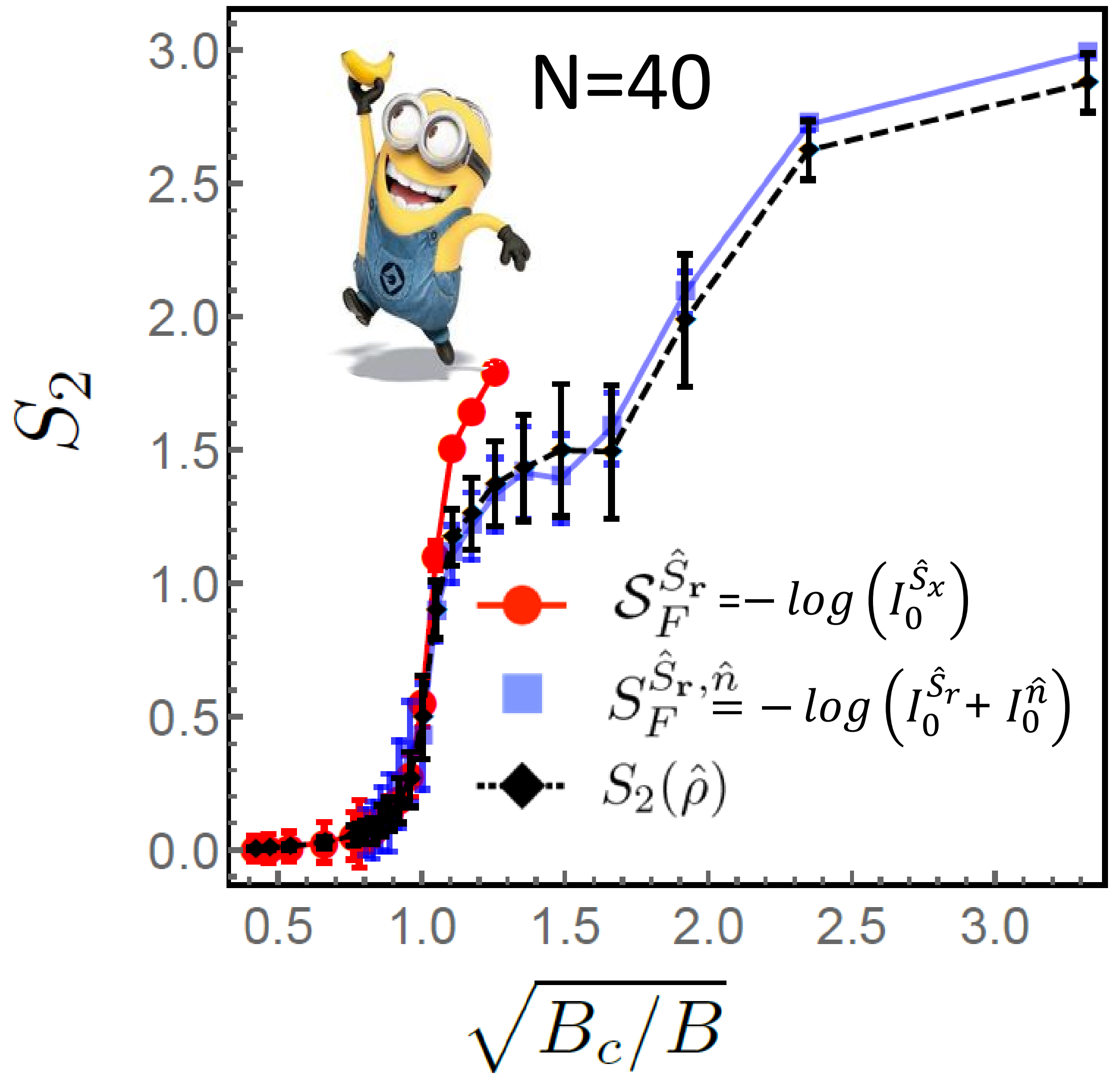
$|C_{\text{off}}^{\hat{S}_x,\hat{n}}(t < \lambda_Q^{-1})| \ll 0$ Initial condition

$|C_{\text{off}}^{\hat{S}_r,\hat{n}}(t > \lambda_Q^{-1})| \ll 0$ Scrambling

$D_{\text{diag}}^{\hat{S}_r,\hat{n}}$<<1



FOTOCs & Renyi Entropy

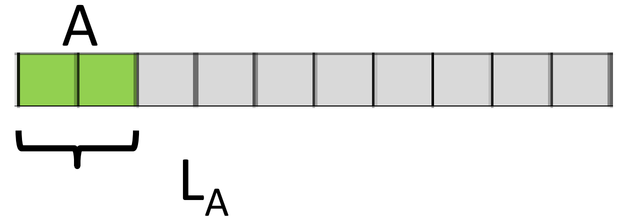


$$|\Psi_0^c\rangle = |\rightarrow \cdots \rightarrow\rangle \otimes |0\rangle$$

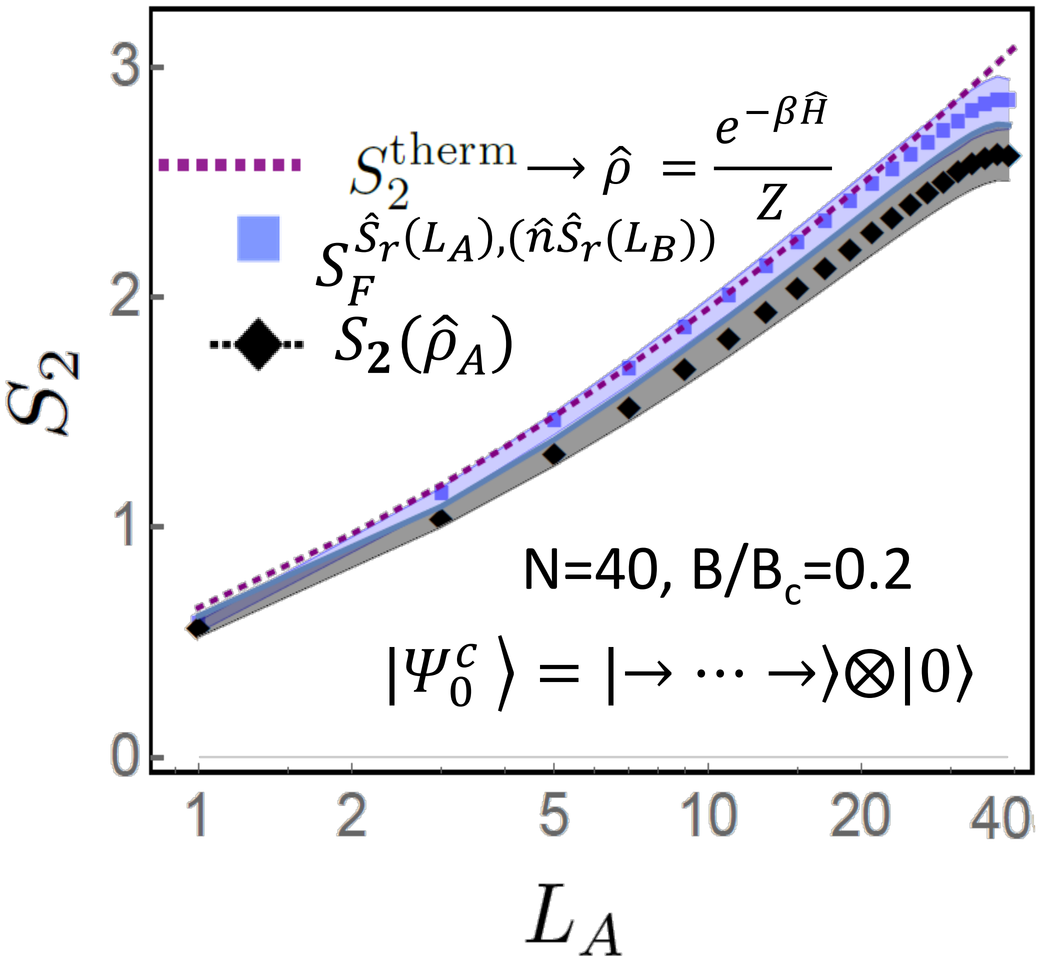
- Growth of entanglement entropy $B>B_c$
- System explore large part of Hilbert space: Ergodicity

FOTOCs and Thermalization

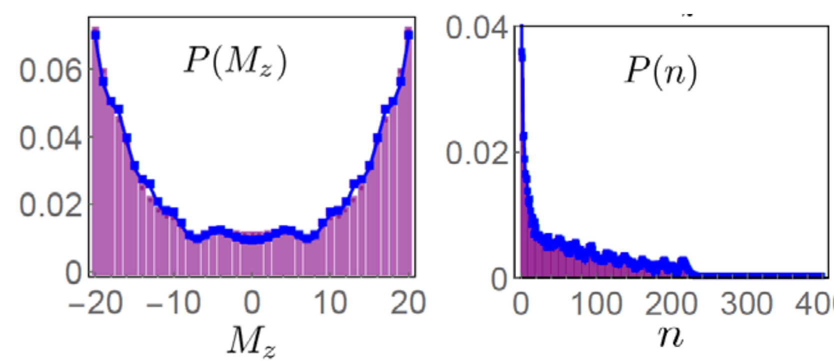
Applying $\widehat{W}_G$ only to part of the spin system



$$S_F^{\hat{S}_r(L_A),(\hat{n}\hat{S}_r(L_B))} = I_0^{\hat{S}_r(L_A)} + I_0^{\hat{n},\hat{S}_r(L_B)}$$



- **Volume law:** $S_2(\hat{\rho}_A) \propto L_A$
- **Thermalization**



Experimental Status

1. **Measured FOTOCs (B=0)**

   Garttner *et al* Nature Physics(2017)

2. **Benchmarked the Dicke Model**

   Safavi-Naini *et al* Phys. Rev. Lett. (2018)

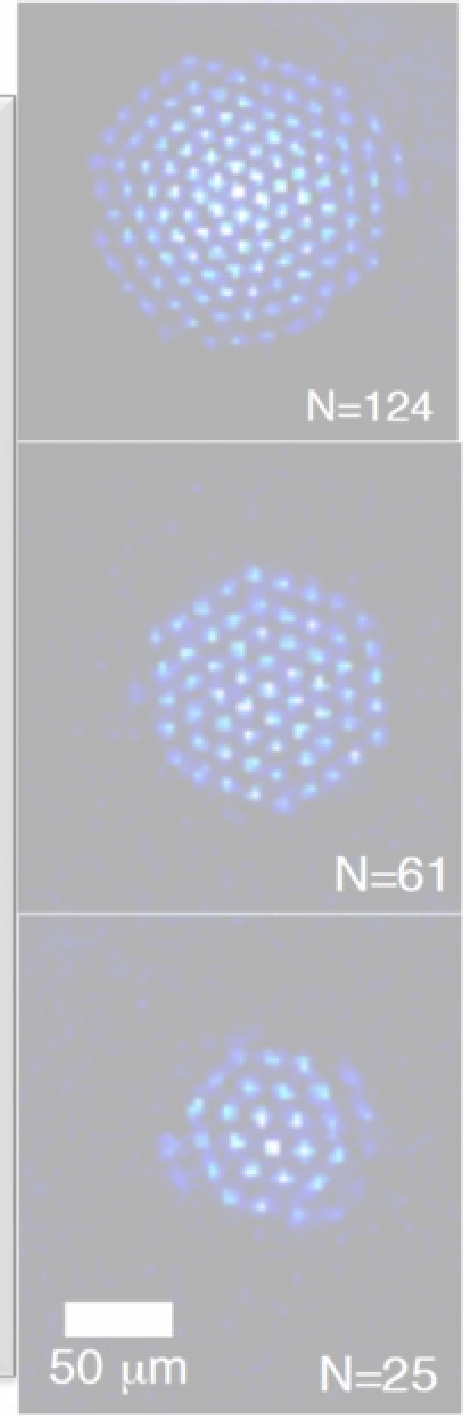
3. **Implemented EIT Cooling** $(\bar{n}\sim 0)$

   In preparation

4. **FOTOCS in Dicke Molel**

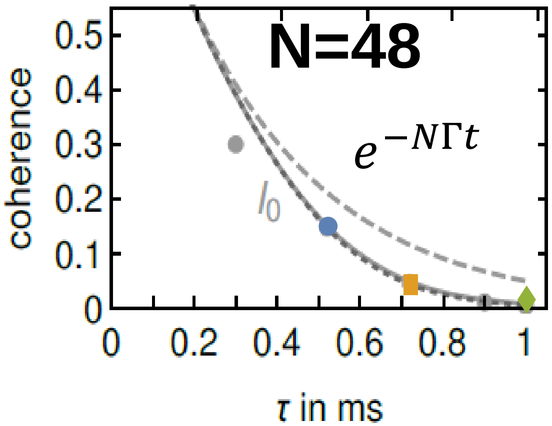
5. **Control of Decoherence (Parametric drive)**

   Wenchao Ge *et al:* arXiv:1807.00924



Experimental Status

Measured FOTOCs in the Collective Ising model: $\delta \rightarrow -\delta$



Garttner et *al Nature Physics (2017)*

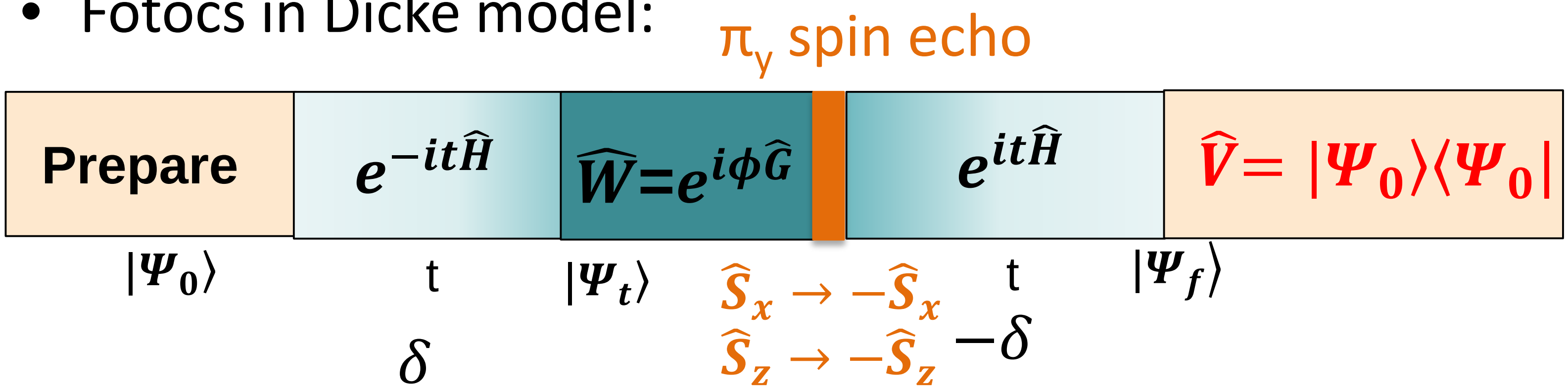
Light scattering

$$I_0(t) = e^{-\Gamma N t} I_0^{pure}$$

Issue: slow measurements

Wanted to decoupled from phonons

- Fotocs in Dicke model:



- Need to measure $|-N/2\rangle\langle -N/2| \otimes |0\rangle\langle 0|$
- Possible: Two steps:

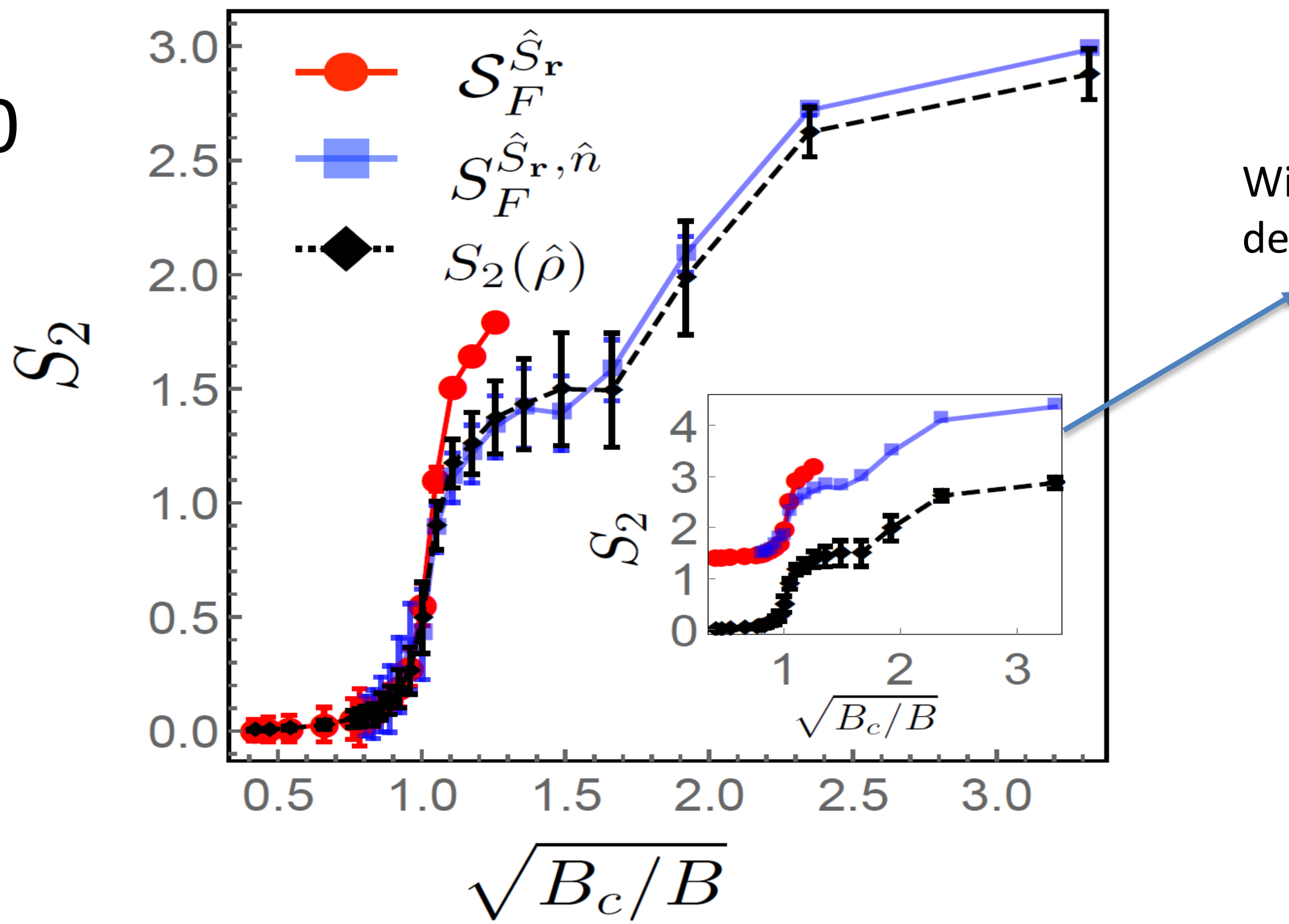
Probability to be dark (no affect motion): DONE

Probability to be in ground motional state (STIRAP): Gebert et al NJP. 18 013037(2016)

Experimental Status

- Gain: No need to decoupled from phonons (faster dynamics)
- Increase $\frac{B_C}{\Gamma}$ by an order of magnitude (parametric drive, Wenchao Ge *et al:* arXiv:1807.00924)

N=40



Theory

R. Lewis-Swan

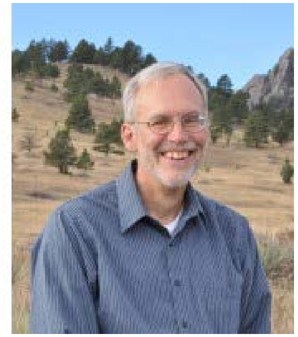
A. Safavi-Naini

M. Gärttner

M. Wall

M. Foss-Feig

Experiment



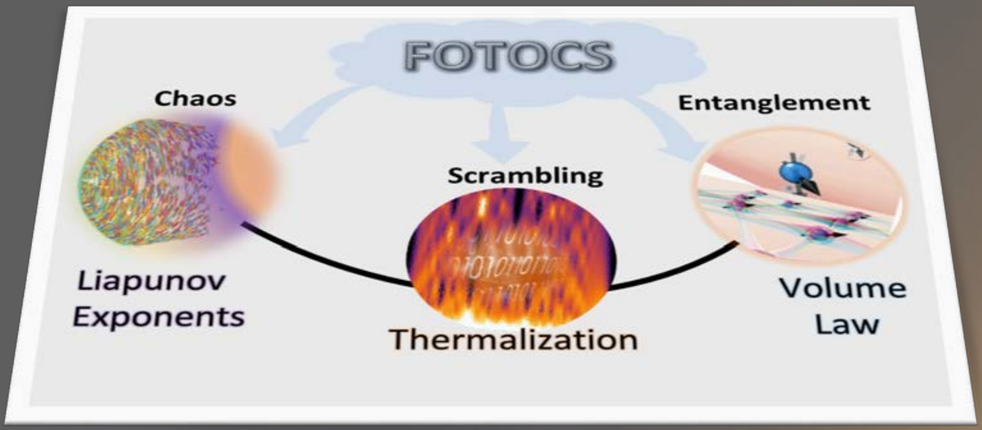
J. Bollinger

J. Bohnet

K. Gilmore

JILA pfc an NSF physics frontier center

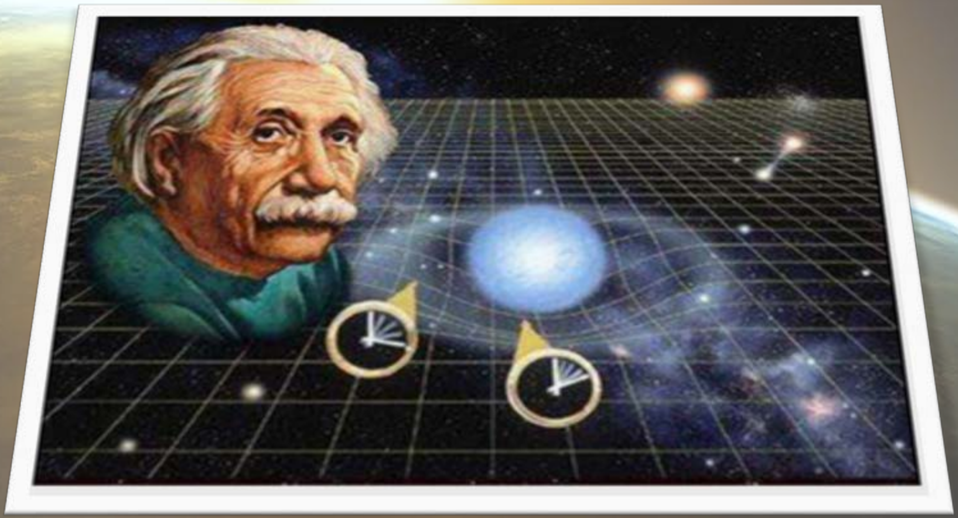
Only the beginning: Bright vista ahead



- Bounds on scrambling. Pure states?
- Quantum chaos. (away from semi-classical limit)?
- Error correction / information hiding?
- Design of duals of black hole.
- …..

Thank You!

R. Lewis-Swan *et al*, arXiv:1808.07134



Thanks for your attention