Low-degree testing for quantum states
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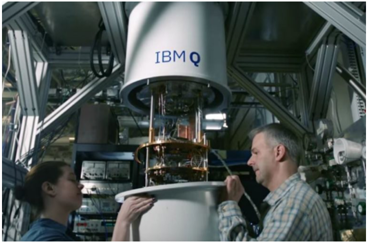

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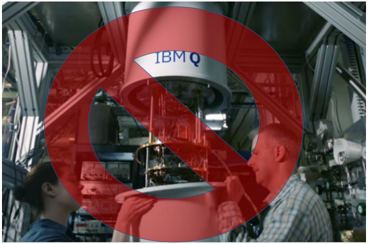

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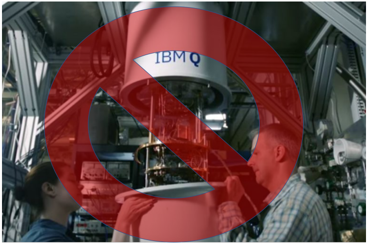

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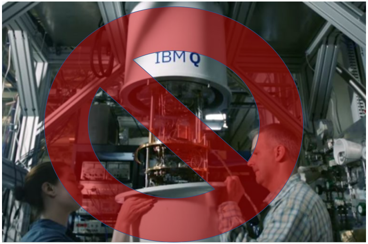

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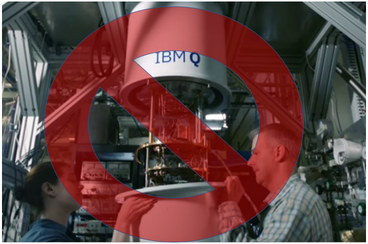

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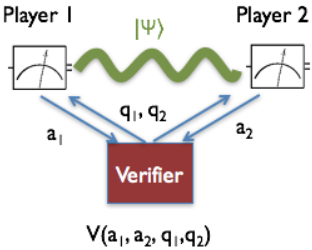

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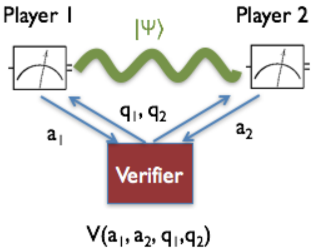

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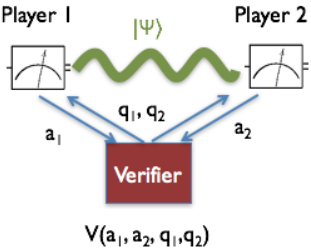

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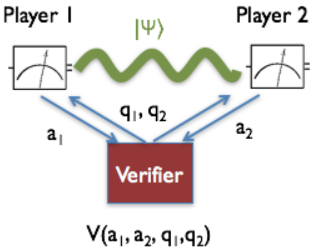
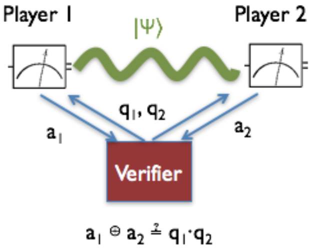
- Test strategies up to local isometry :

$$|\psi\rangle \mapsto (U_1 \otimes U_2)|\psi\rangle$$
$$M^a_{q,i} \mapsto U_i M^a_{q,1} U_i^\dagger$$



Example: CHSH

CHSH test: 1 round, 2 players, 1-bit messages



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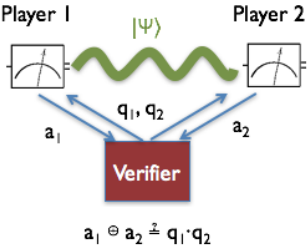

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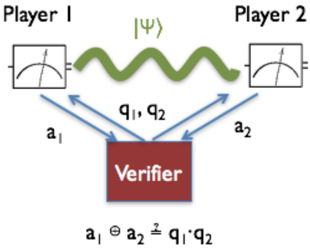

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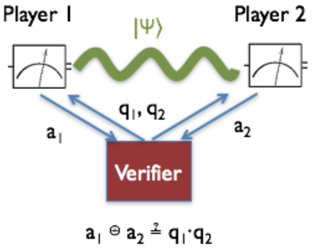

Theorem (SW88, MYS12)

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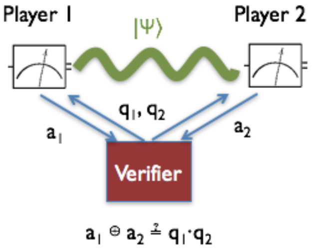

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CHSH game is a "**self-test**" for the state $|\text{EPR}\rangle$ with **robustness** $\delta(\varepsilon)$.

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  - $\langle\psi|\frac{1}{2}(XX + ZZ)|\psi\rangle \approx 1 \implies |\psi\rangle \approx |\mathrm{EPR}\rangle$.

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    - Expectation values $\langle\psi|X(a)Z(b)|\psi\rangle$ determine the shared state $|\psi\rangle$.

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$$M_X(a)M_Z(b) \approx (-1)^{a\cdot b} M_Z(b)M_X(a) \Longrightarrow$$
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Test for $|\mathrm{EPR}\rangle^{\otimes n}$ with ✗robustness $\delta = O(n^{5/2}\varepsilon)$ and ✓$O(\log(n))$ bits of communication.

More qubits: Our result

Theorem (Quantum low-degree test)

There exists a 1-round, 2-player protocol with $O(\text{poly}\log(n))$-bit questions and answers such that any players succeeding with probability $1-\varepsilon$ must share a state that is $\delta(\varepsilon)$-close to $|\text{EPR}\rangle^{\otimes n}$, where δ is **independent** *of n.*

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- Test certifies $n^{\mathrm{poly}\log(n)}$-size subset of Pauli operators, arising from **low degree polynomials**.

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  - Conjecture ("games QPCP"): it is QMA-hard to approximate entangled value up to constant error
- To show QMA-hardness of entangled value, design **self-test** for a QMA witness state $|\psi\rangle$ (e.g. ground state of local Hamiltonian)

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Theorem ("Hamiltonian QPCP $\Longrightarrow$ Games QPCP")

If it is QMA*-hard to estimate ground energy of local H up to constant fraction, then previous theorem holds under deterministic reductions.*

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- Take C to be Reed-Muller code, based on multivariate polynomials over finite fields. Locally testable by low-degree test [RS97]

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Lemma (Main)

Suppose players' operators $M_X(a), M_Z(b)$ acting on $|\psi\rangle$ pass test with prob $1 - \varepsilon$. Then $\exists$ local isometry V s.t.

$$M_X(a)|\psi\rangle \approx V^\dagger X(a) V|\psi\rangle \quad M_Z(b)|\psi\rangle \approx V^\dagger Z(b) V|\psi\rangle$$

for $a, b \in S$.

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  - Need guarantees even when success probability is far from optimal, as in [AFY17]

Thank You!

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