Verifier on a Leash

Andrea Coladangelo

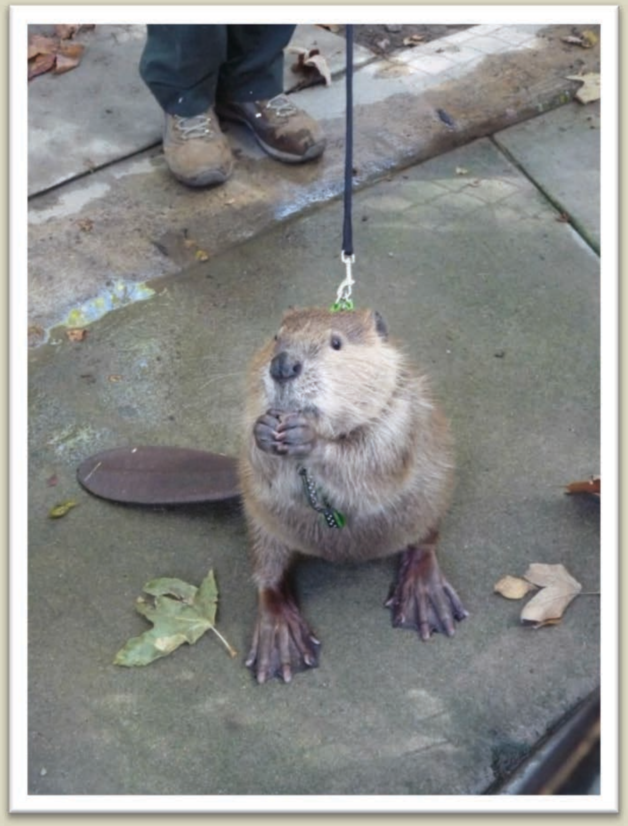
Based on joint work with Alex Grilo, Stacey Jeffery and Thomas Vidick

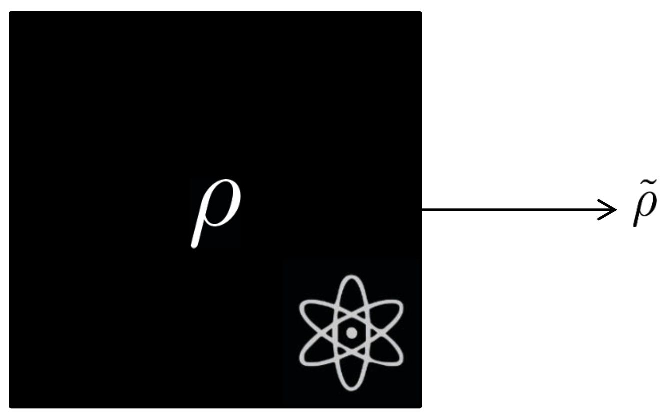
QuSoft
Research Center for Quantum Software

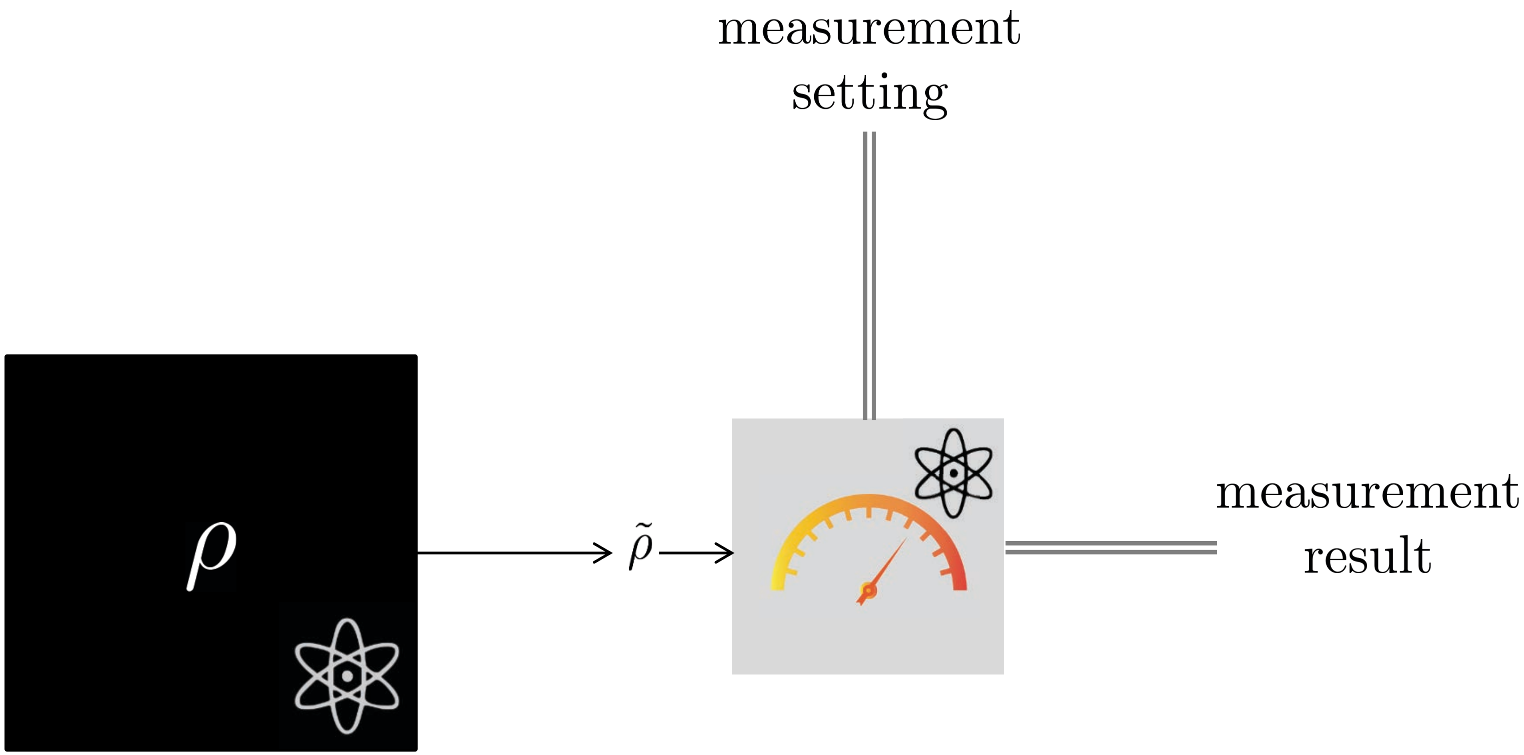
Caltech

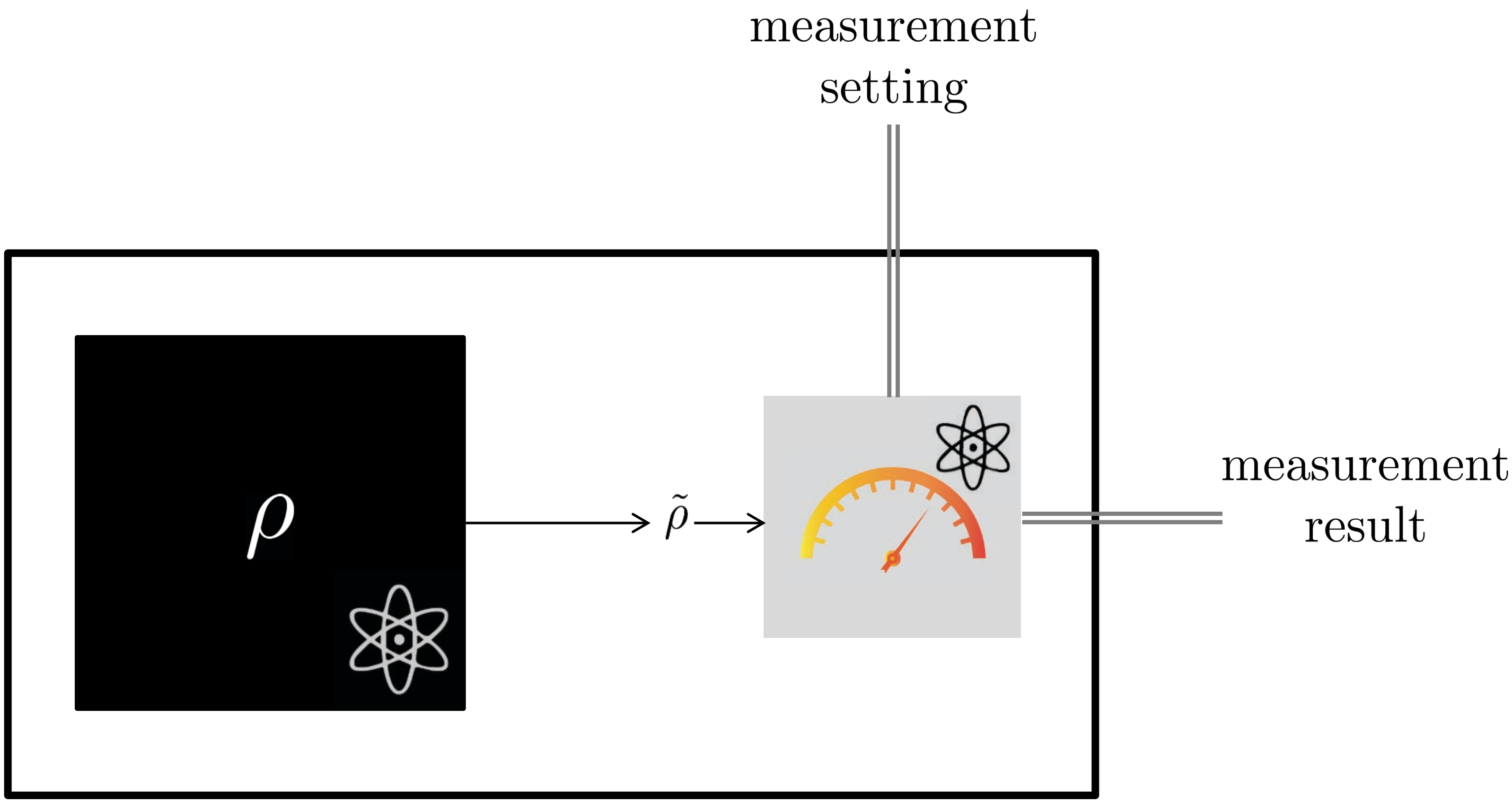
université
PARIS DIDEROT
PARIS 7

Testing a Quantum Device




ρ
$\tilde{\rho}$


measurement
setting
ρ
$\tilde{\rho}$
measurement
result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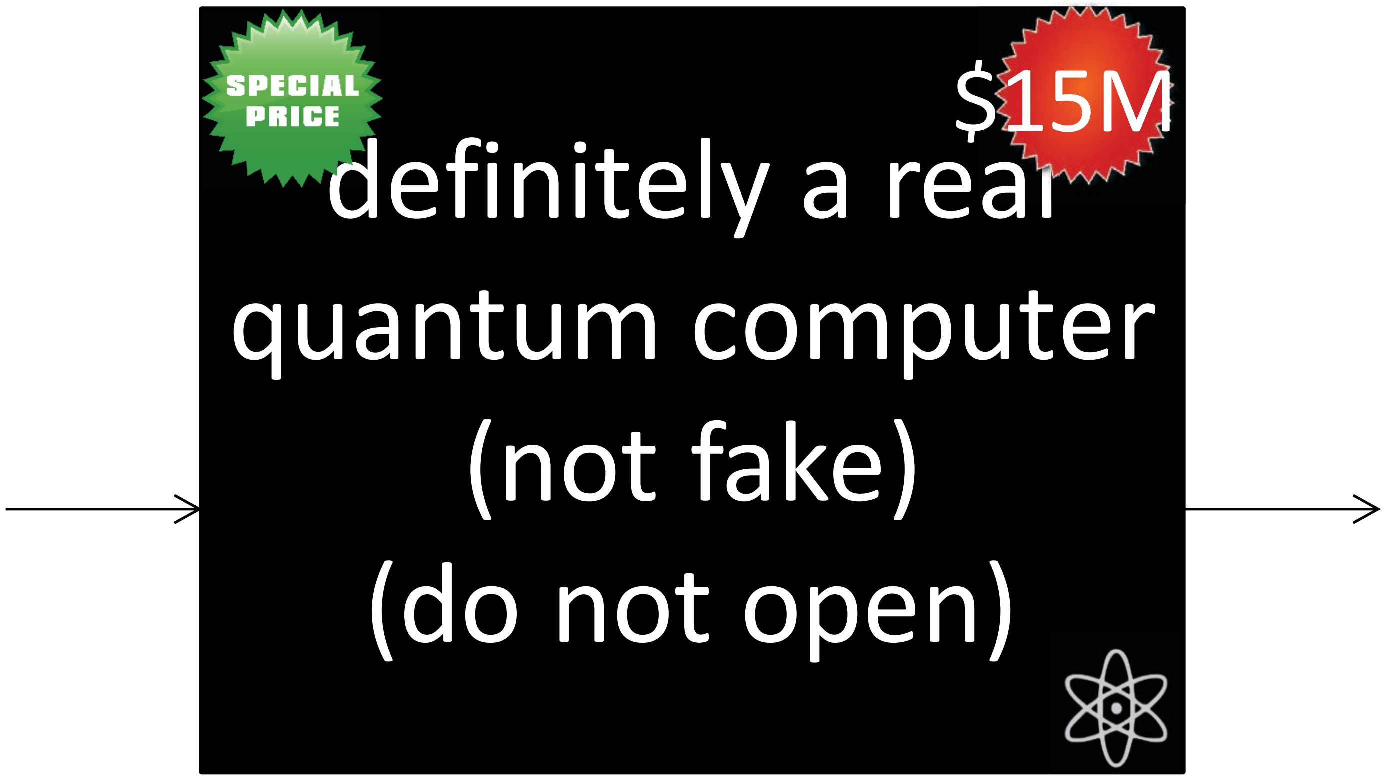
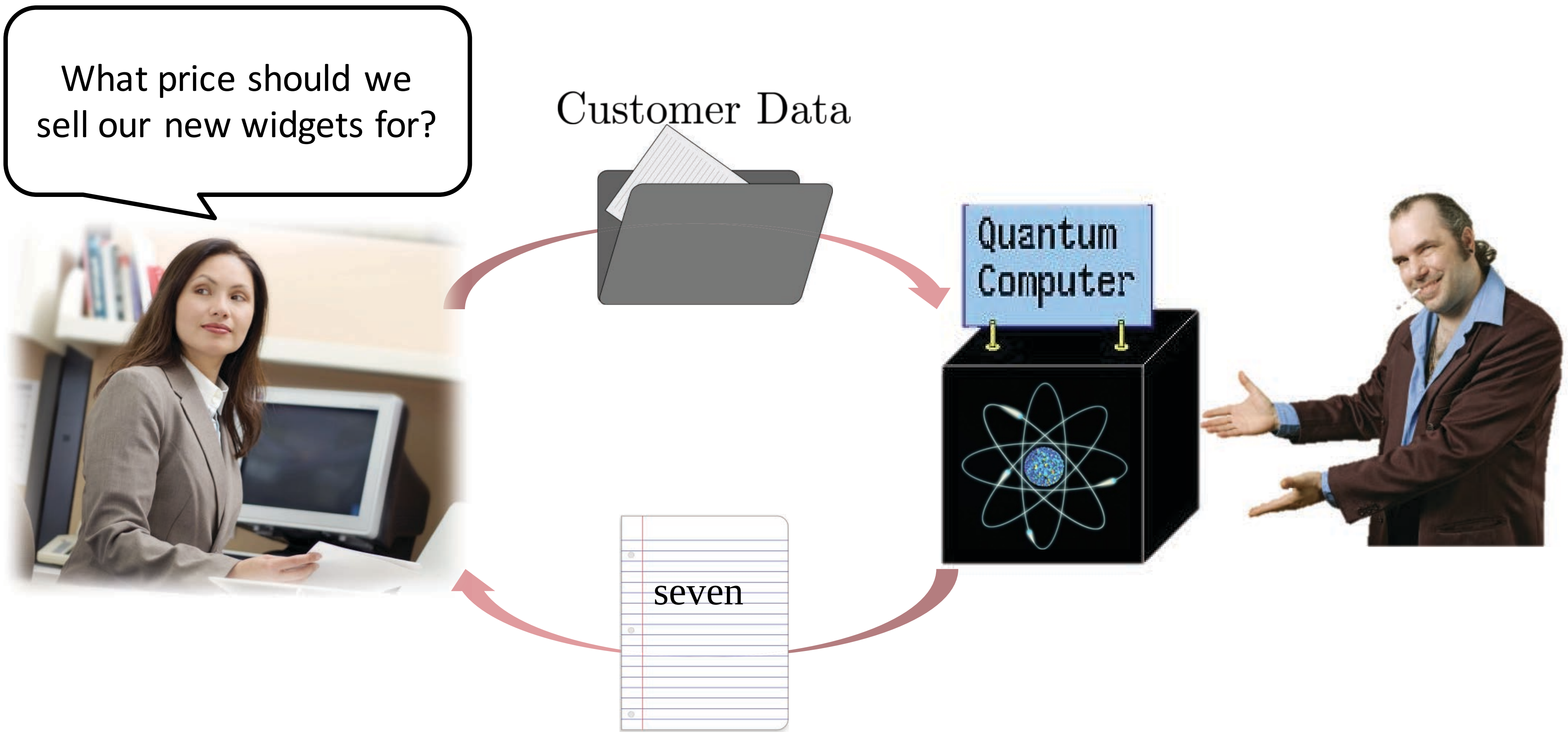
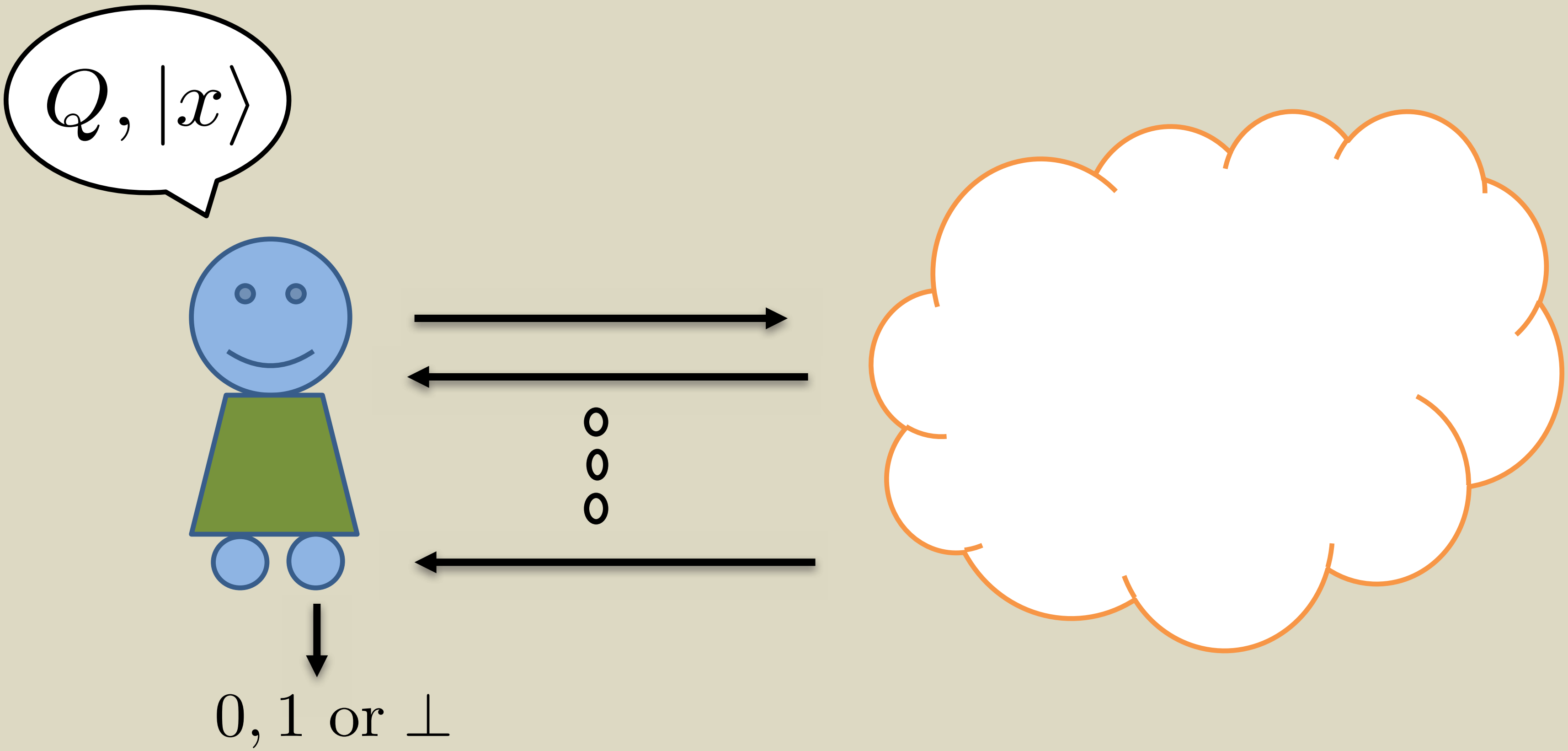

SPECIAL PRICE
$15M
definitely a real quantum computer
(not fake)
(do not open)


What price should we sell our new widgets for?
Customer Data
Quantum Computer
seven

Delegating a Quantum Computation



Desired properties

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Verifiability:

Either the verifier outputs $\perp$,
OR she is outputting the correct outcome of the computation (with very high probability).

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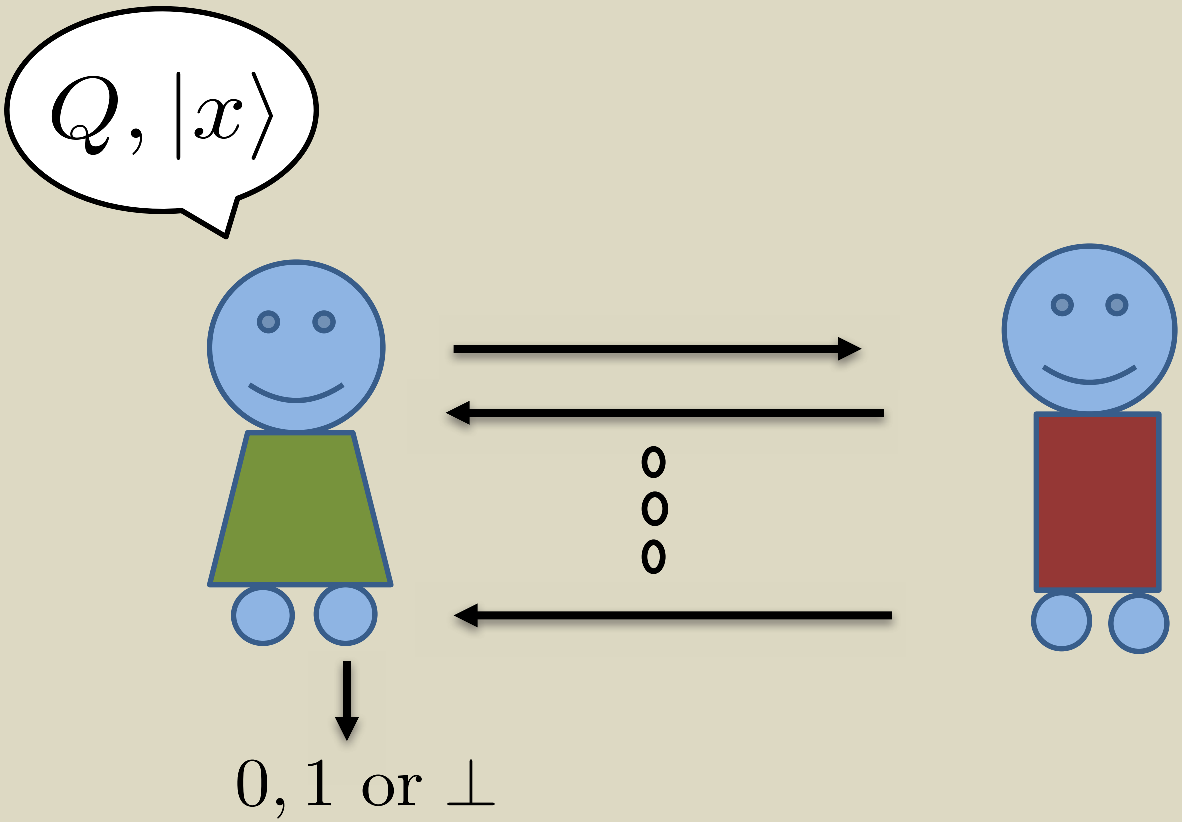
Blindness:

The final state of the server and his view of the transcript don't depend on the verifier's input to the protocol.

Single-Prover Delegation



Single-Prover delegation

Single-Prover delegation

- (Slightly) Quantum verifier,
- Single prover bound by quantum mechanics,
- Verifier interacts (quantumly) with provers

[Aharonov, Ben-Or, Eban 2010]

[Broadbent 2015]

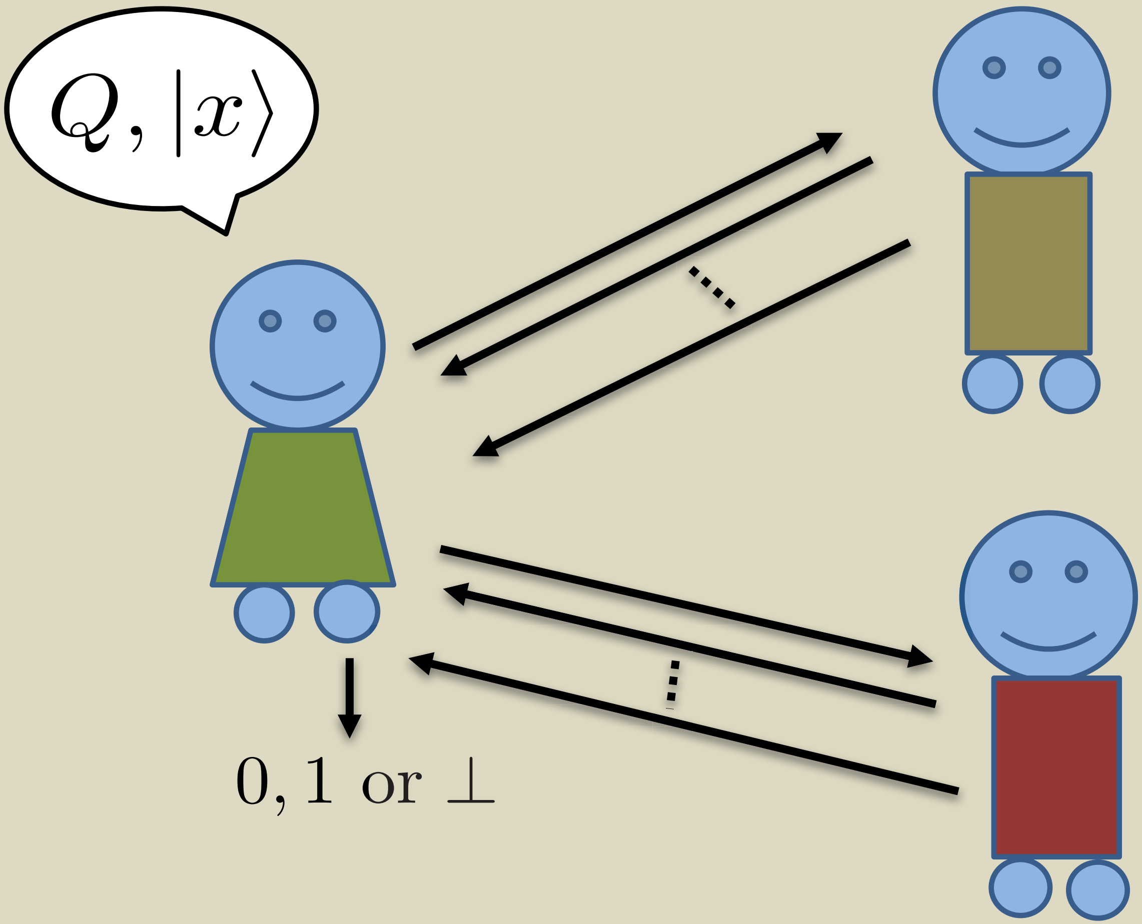
[Fitzsimons, Kashefi 2017]

[Morimae 2014]

[Morimae, Fitzsimons 2016]

Complexity of delegating m-gate circuit: $O(m)$

Two-Provers Delegation



Two-provers delegation

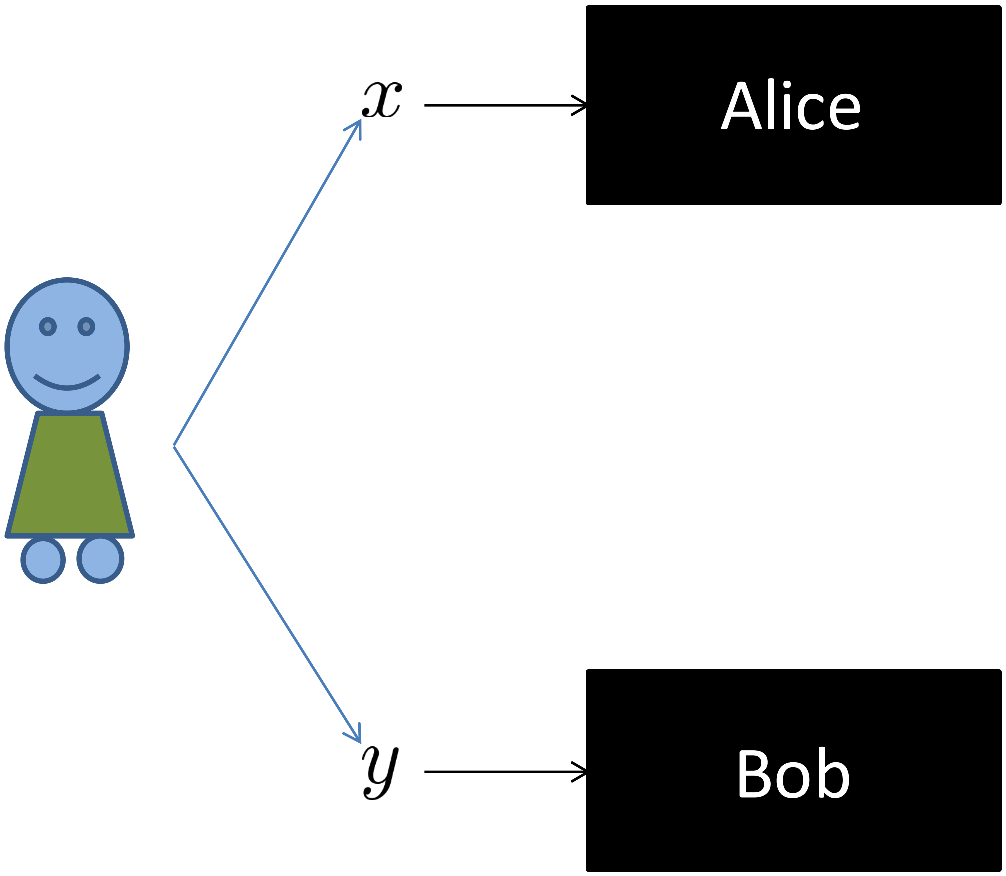
Two-provers delegation

- Classical verifier
- Two provers bound by quantum mechanics, and non-communicating.
- Verifier interacts (classically) with provers.

Bell Inequalities

CHSH Game

CHSH Game



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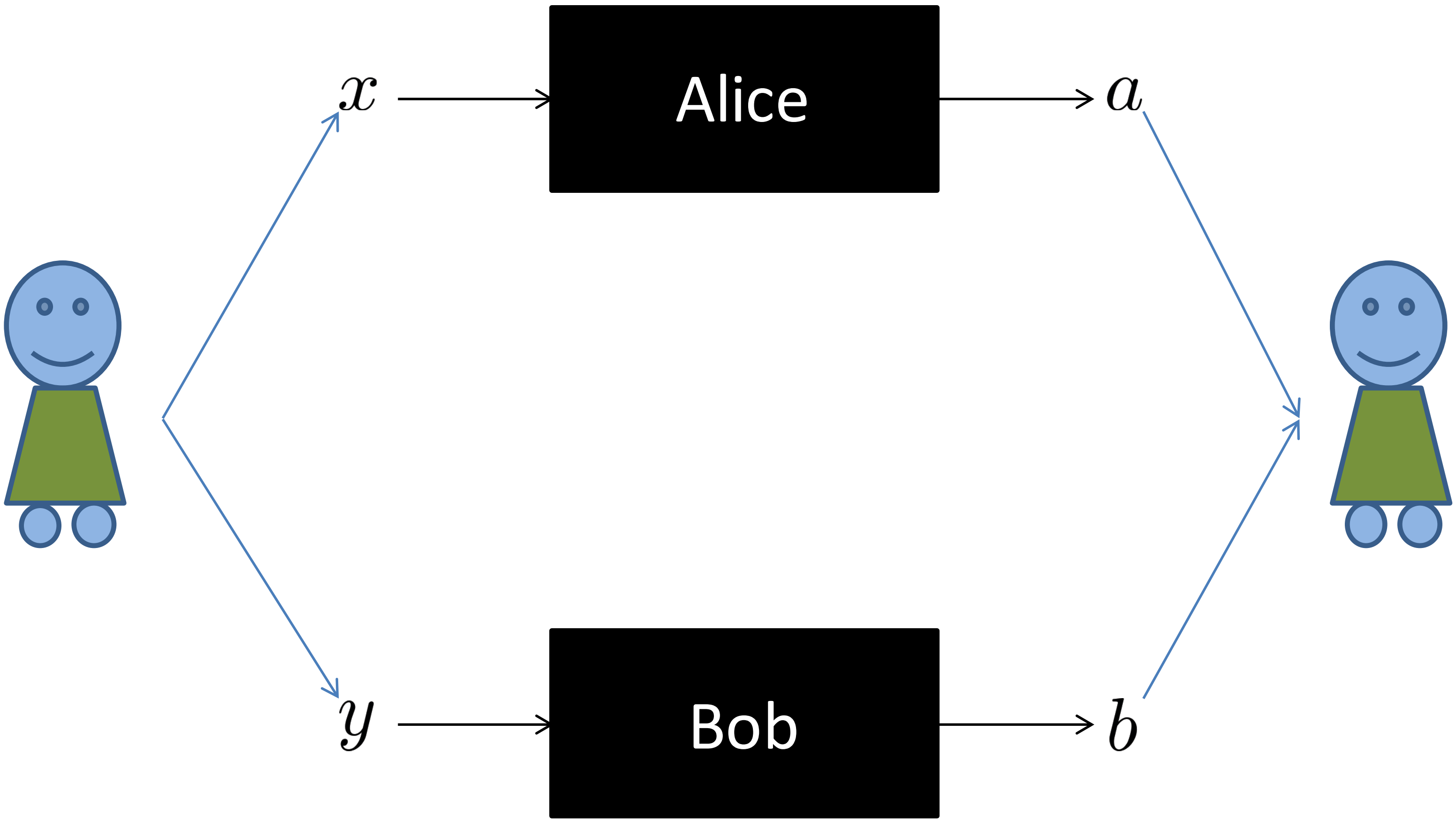

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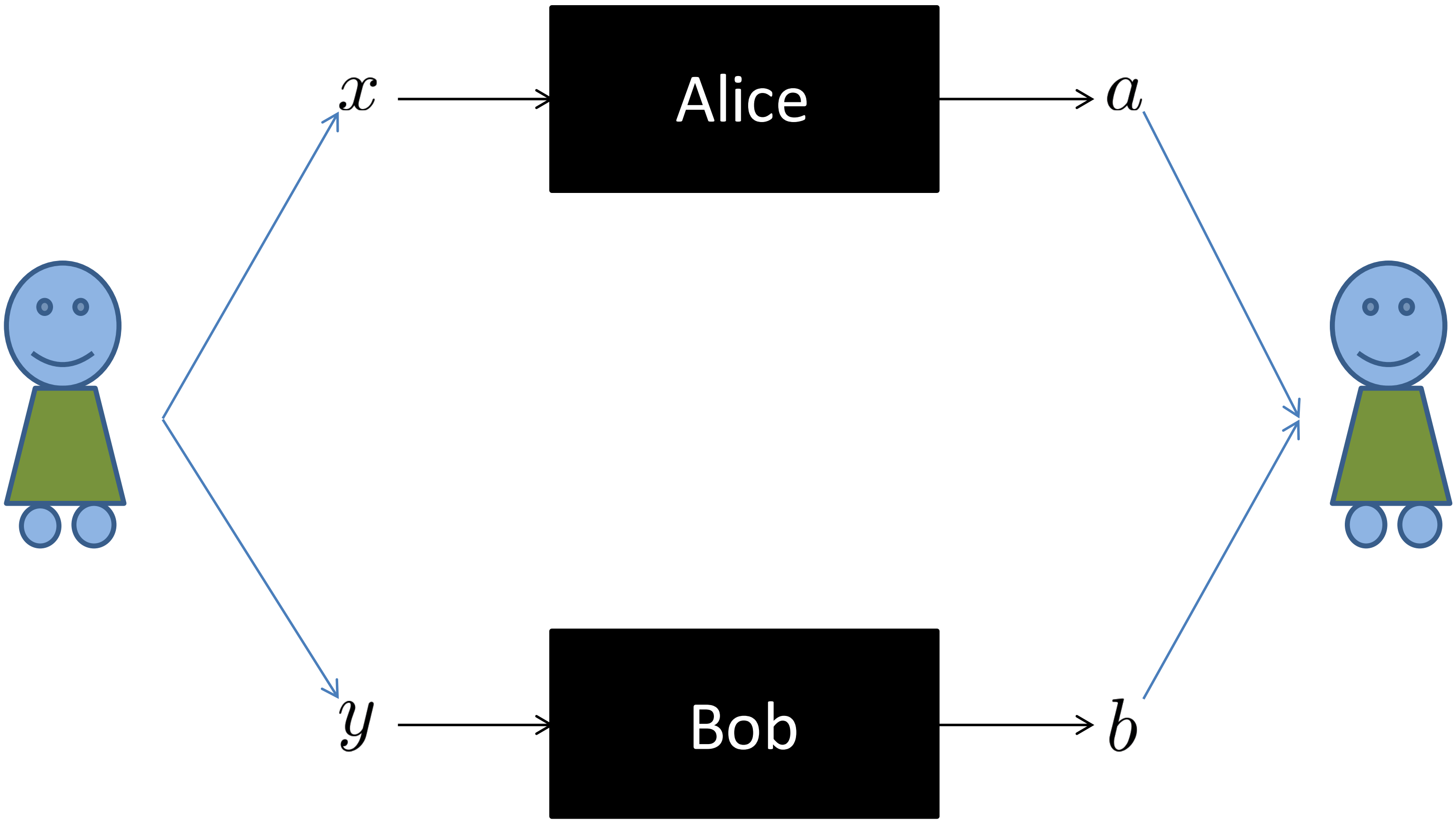

Alice and Bob win if $a \oplus b = xy$

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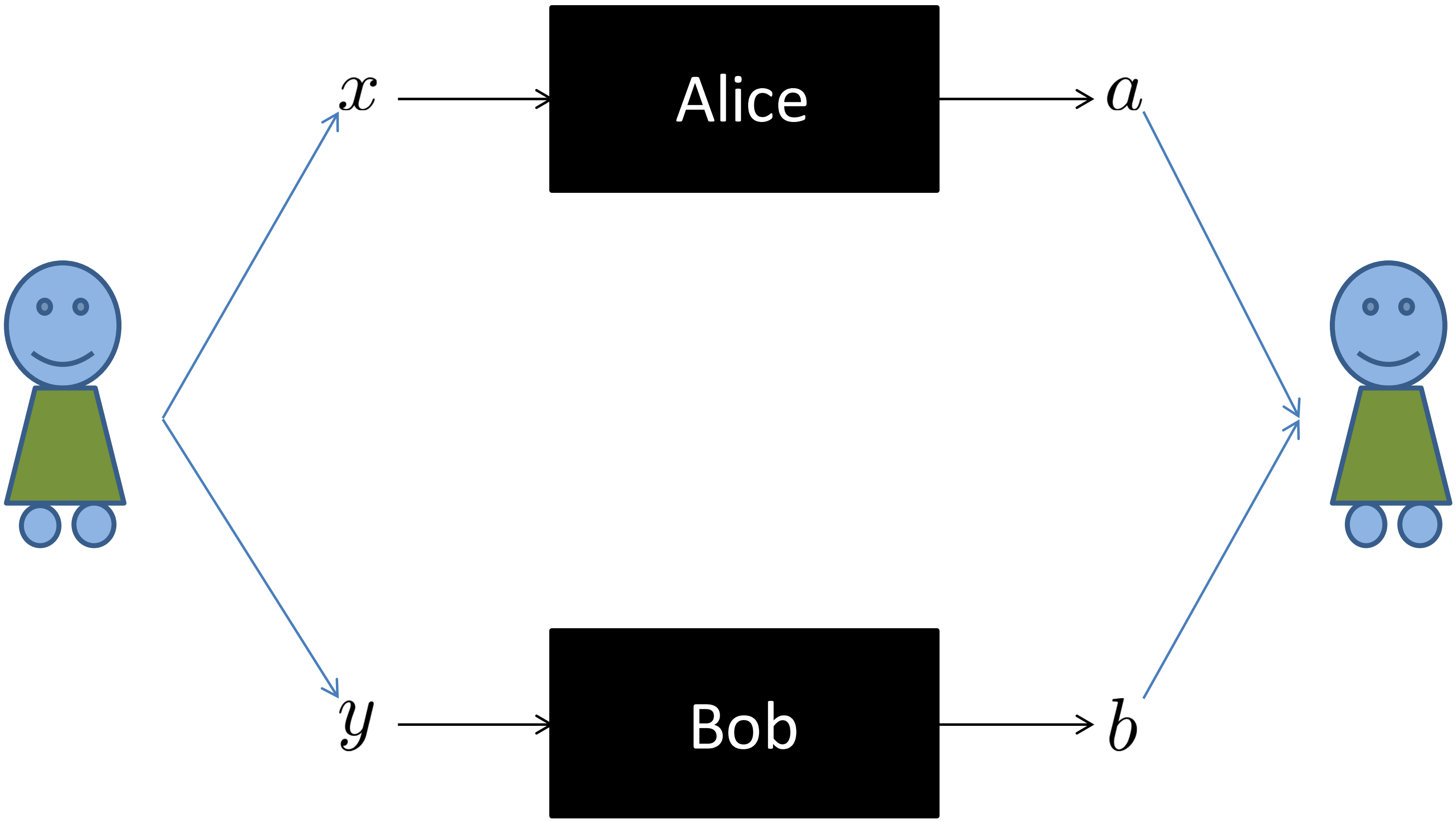

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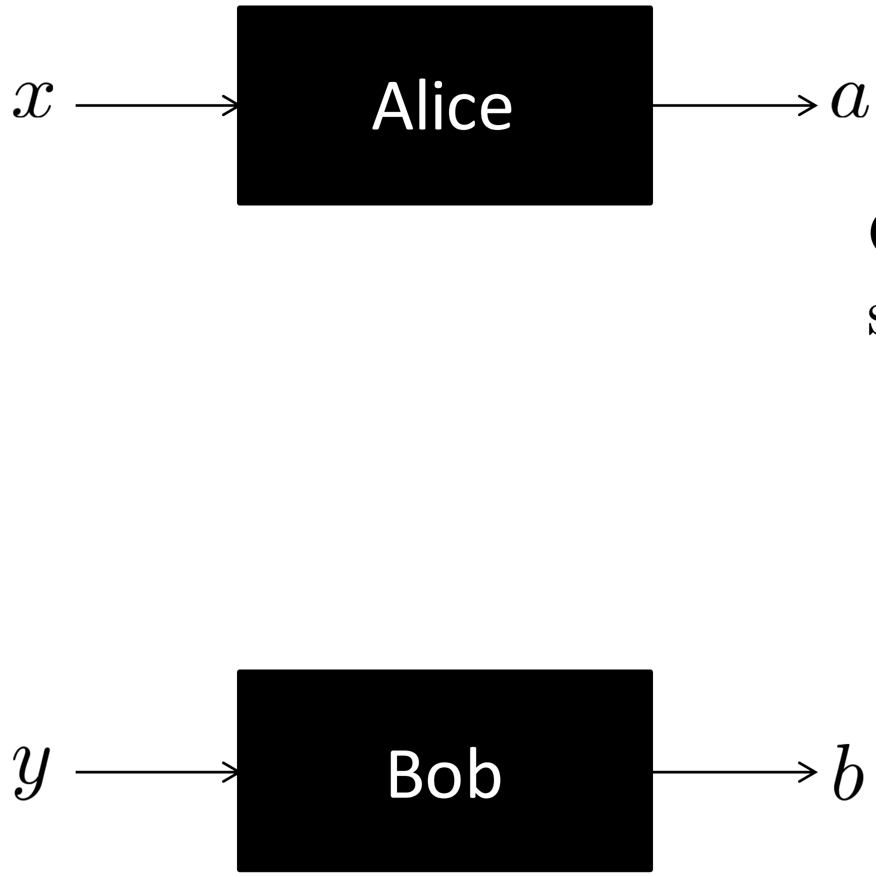

Optimal classical success prob.: $\frac{3}{4}$

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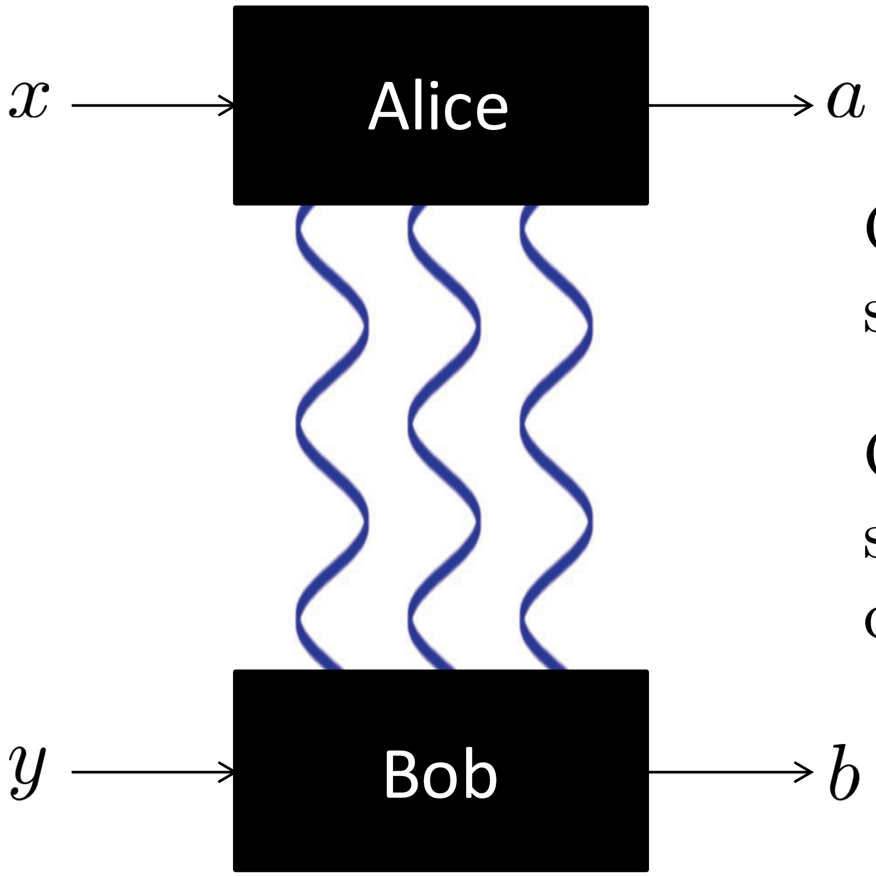

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$$|EPR\rangle = \frac{1}{\sqrt{2}}|0\rangle_A|0\rangle_B + \frac{1}{\sqrt{2}}|1\rangle_A|1\rangle_B$$

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$$A_0 = \sigma_Z, \quad A_1 = \sigma_X$$

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$$B_0 = \frac{\sigma_Z + \sigma_X}{2}, \quad B_1 = \frac{\sigma_Z - \sigma_X}{2}$$

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If Alice and Bob play n rounds of CHSH and win an $\text{opt} - \epsilon$ fraction of the games, their strategy must be within $\delta(\epsilon, n)$ of the n-fold tensor product of optimal single-round strategies.

[Reichardt, Unger, Vazirani 2012]

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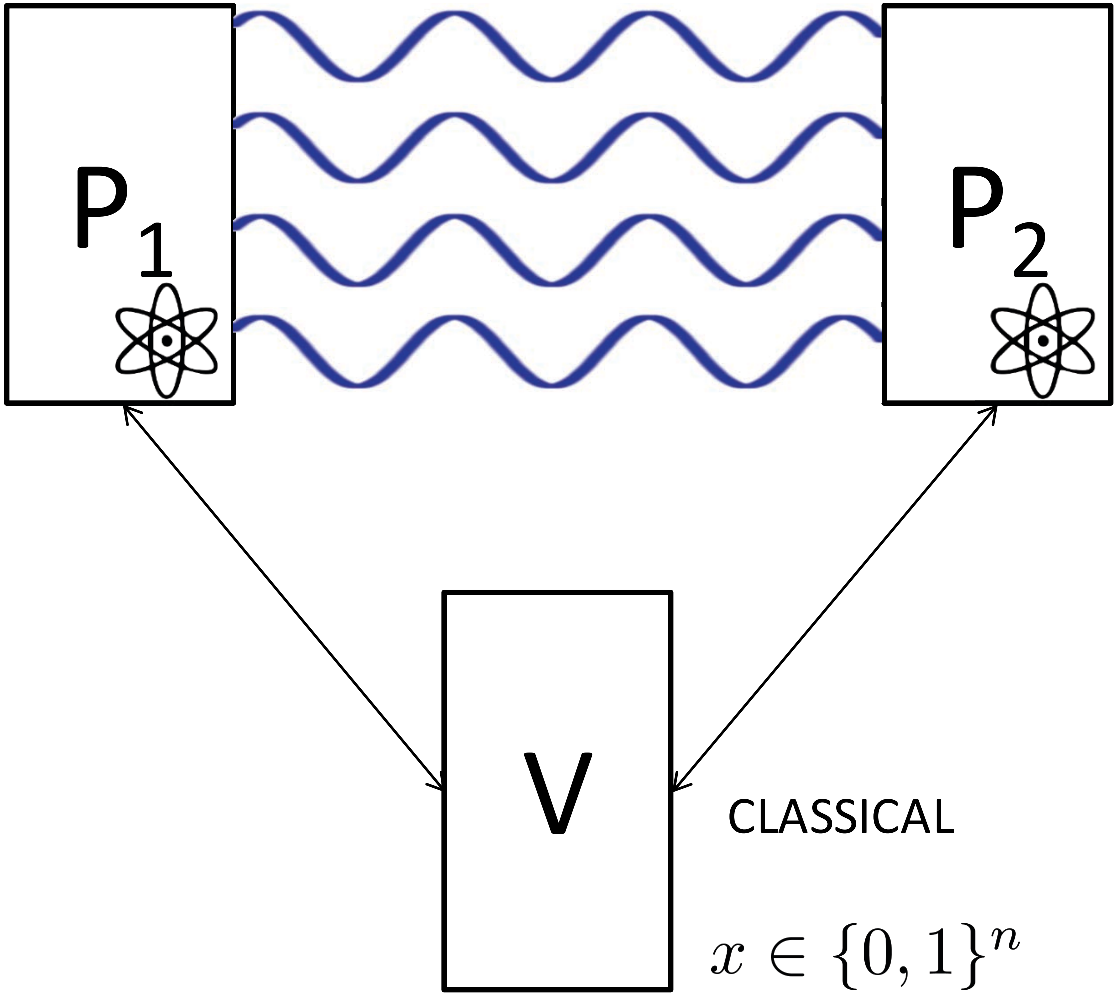
If Alice and Bob play n rounds of CHSH and win an $\text{opt} - \epsilon$ fraction of the games, their strategy must be within $\delta(\epsilon, n)$ of the n-fold tensor product of optimal single-round strategies.

This property is called RIGIDITY or SELF-TESTING

[Reichardt, Unger, Vazirani 2012]

RUV Protocol



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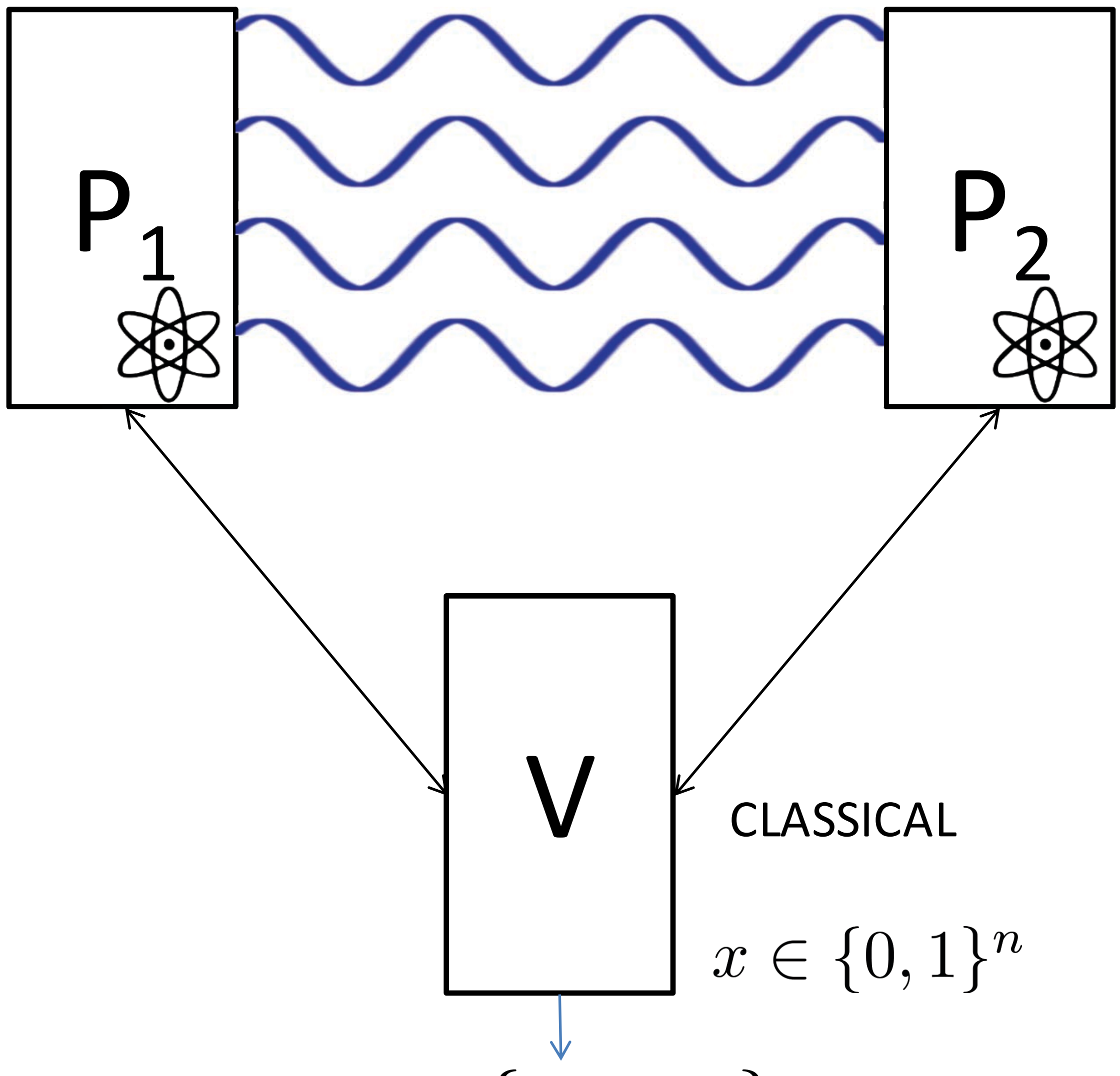
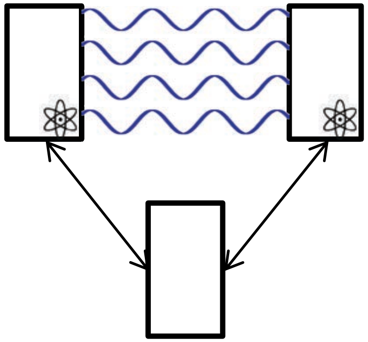


$$c \in \{0, 1, \perp\}$$

[Reichardt, Unger, Vazirani 2012]

RUV Protocol



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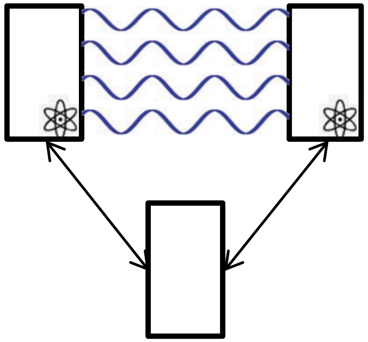
RUV Protocol

- Key insight: alternate Rigidity test with actual computation.



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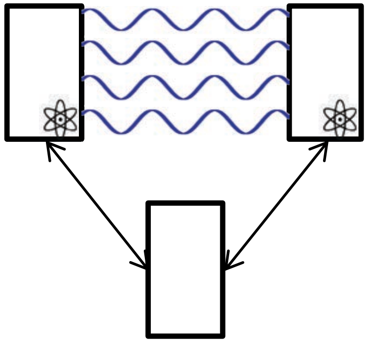
- Enforce that Prover 2 prepares certain resource states on Prover 1’s side.



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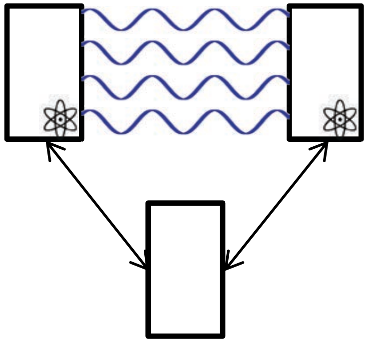

- Key insight: alternate Rigidity test with actual computation.
- Enforce that Prover 2 prepares certain resource states on Prover 1's side.
- Have Prover 1 use these resource states to perform computation by teleportation.

[Reichardt, Unger, Vazirani 2012]

Complexity of delegating m-gate circuit:

[Reichardt, Unger, Vazirani 2012]

Complexity of delegating m-gate circuit: $O(m^{8192})$

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[McKague 2013]

[Gheorghiu, Kashefi, Wallden 2015]

[Hajdušek, Perez-Delgado, Fitzsimons 2015]

[Fitzsimons, Hajdušek 2015]

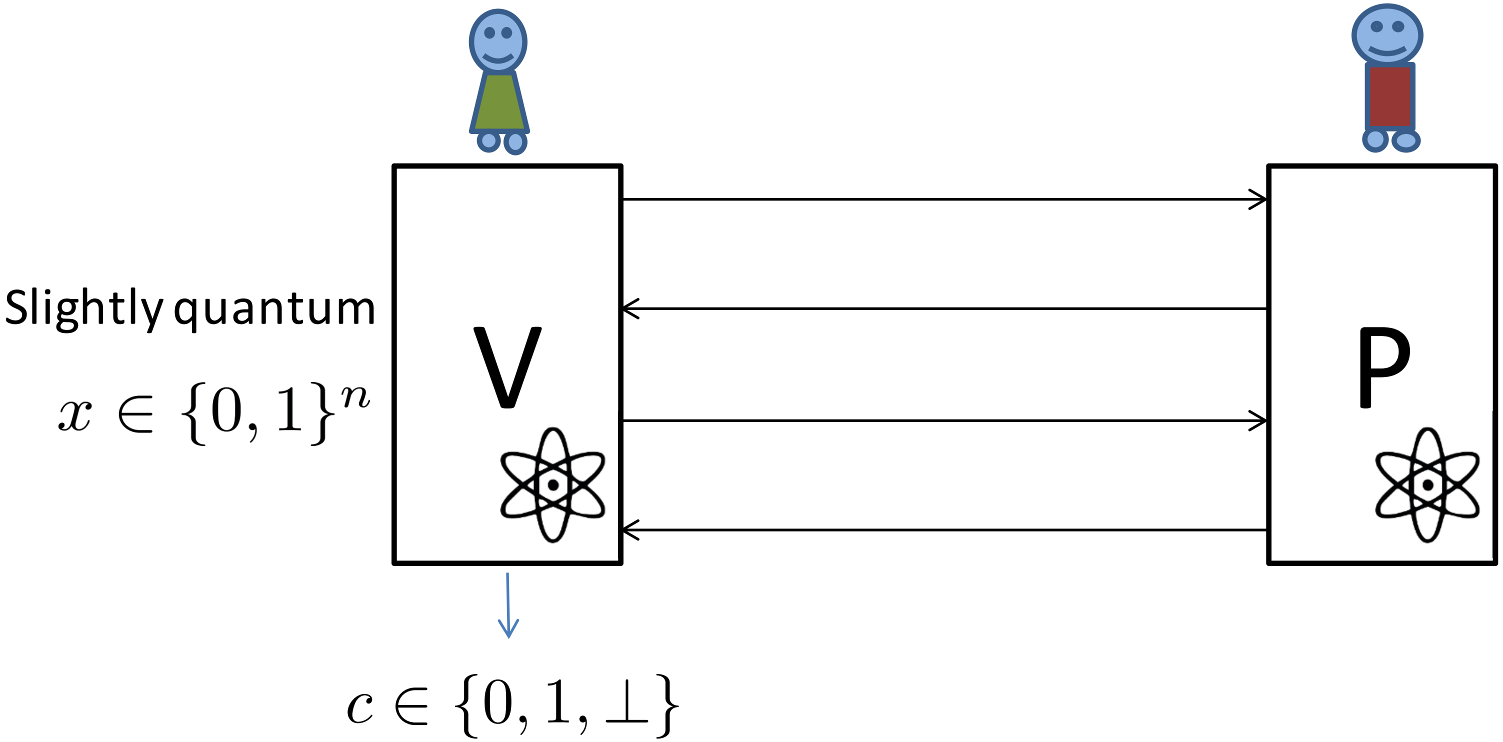
[Natarajan, Vidick 2016]

$\Omega(m^4)$

Our result in one sentence:

We develop new rigidity results, and use them to turn a single-prover delegation protocol into a two prover protocol with overall complexity scaling as $O(m \log m)$.

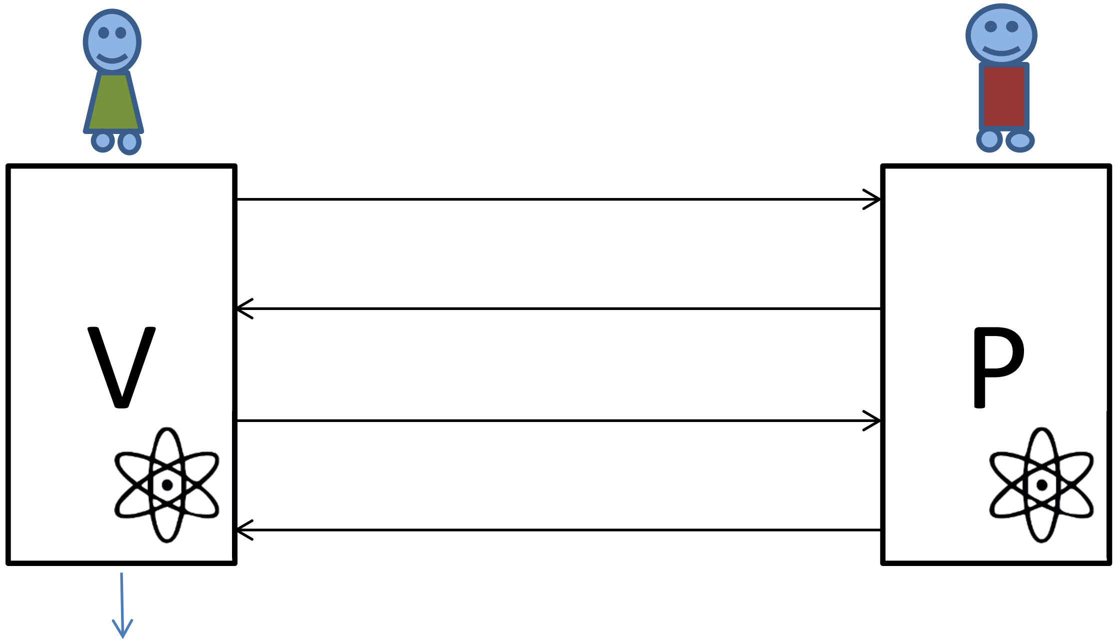
EPR Protocol [Broadbent 2016]



EPR Protocol [Broadbent 2016]

Slightly quantum

$x \in \{0,1\}^n$



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Complexity of delegating m-gate circuit: $O(m)$

EPR Protocol

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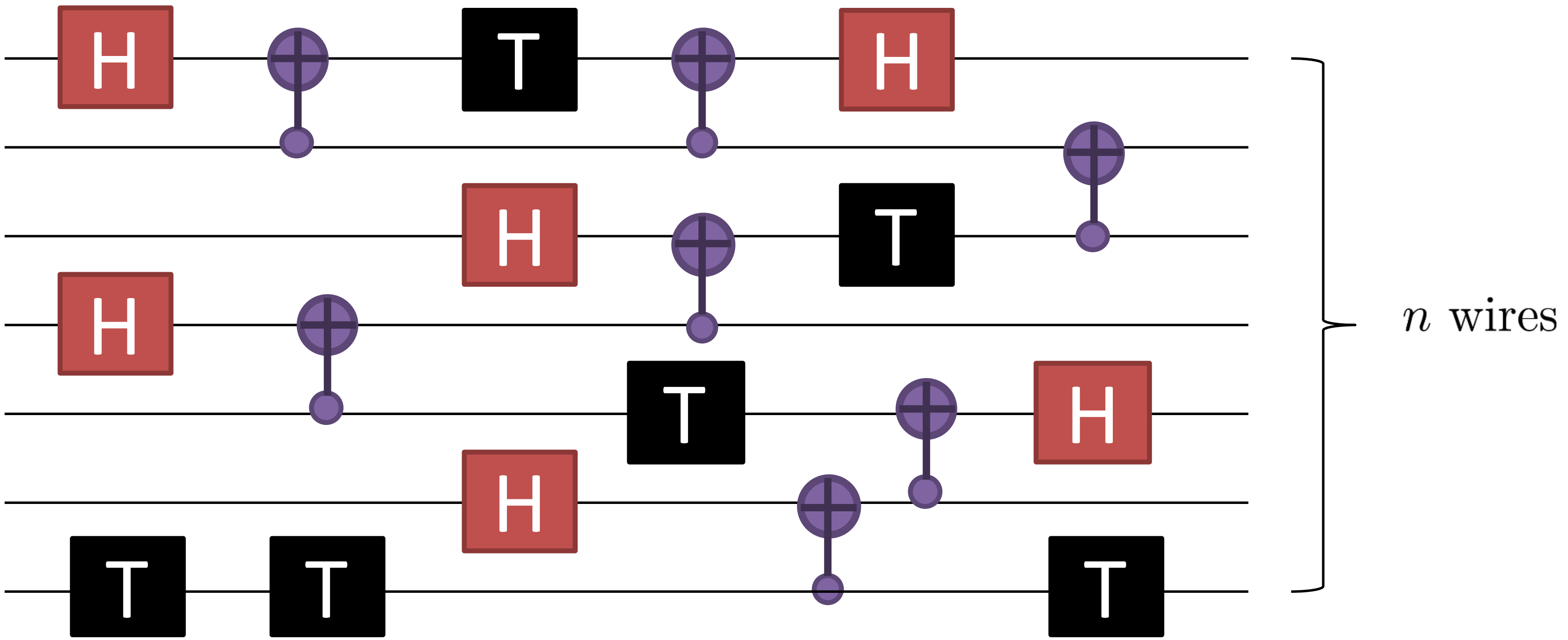

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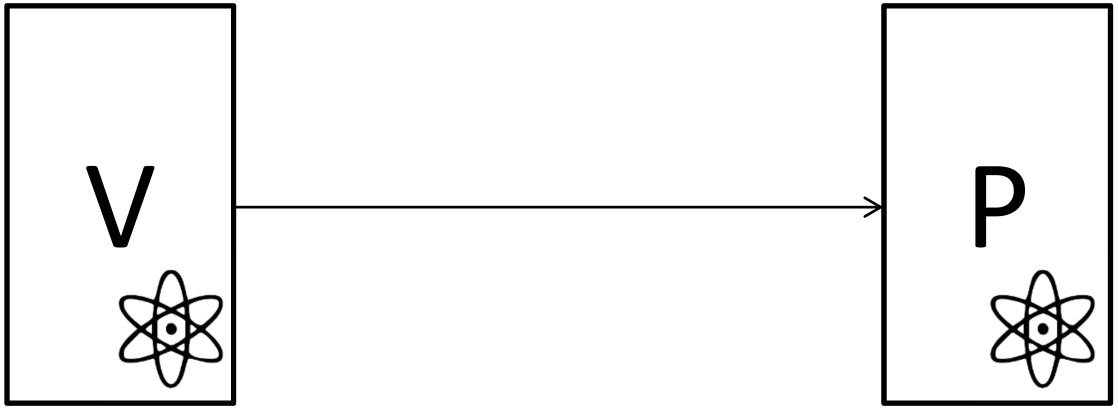

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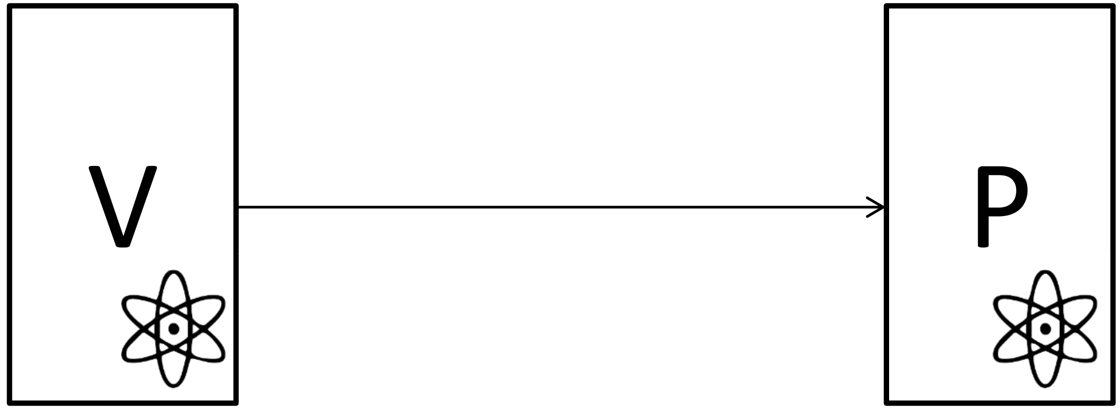

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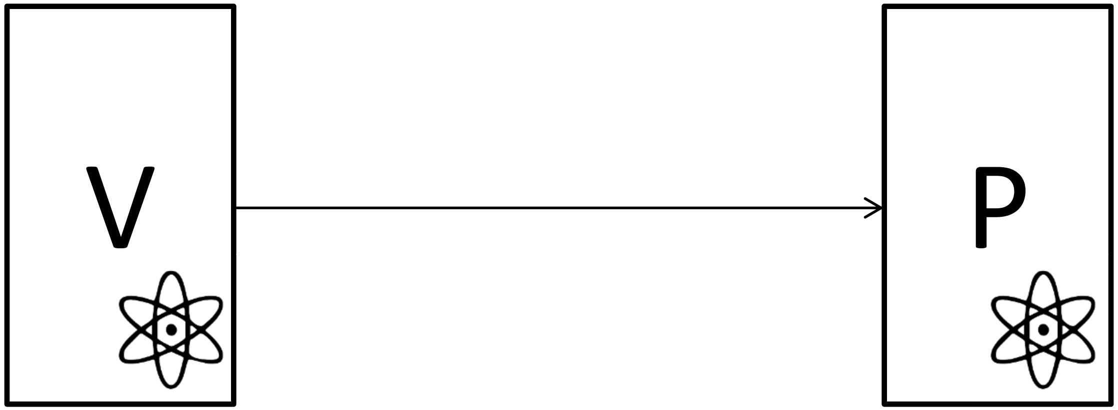

- V selects encrypting key at random: $a \in \{0,1\}^n$
- Sends encrypted input to P: $X^{a_1}\,|x_1\rangle \otimes \cdots \otimes X^{a_n}\,|x_n\rangle$

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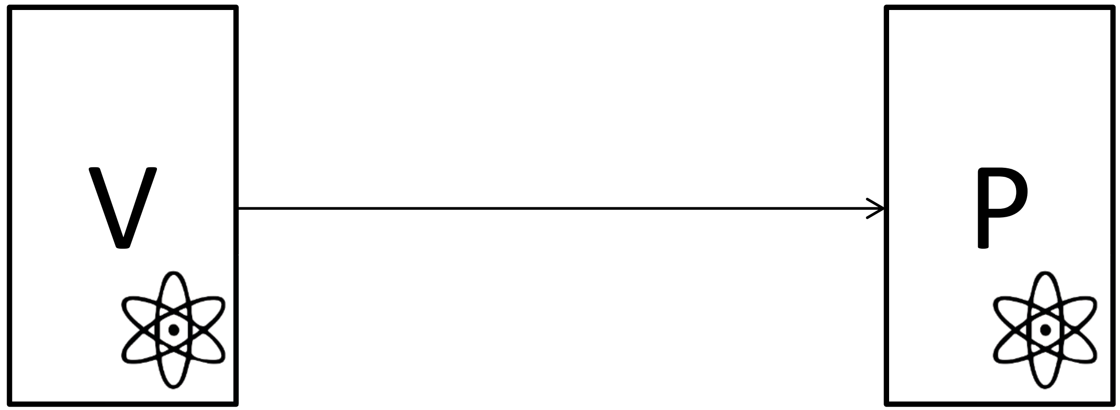

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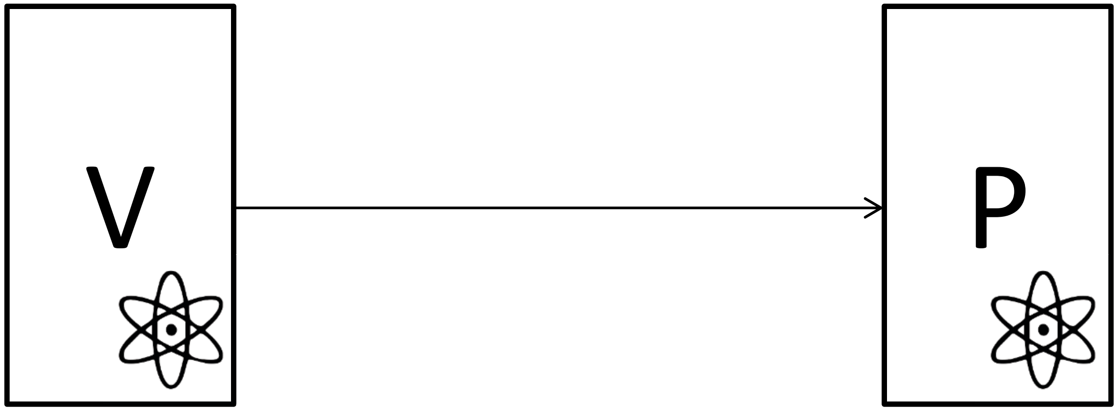

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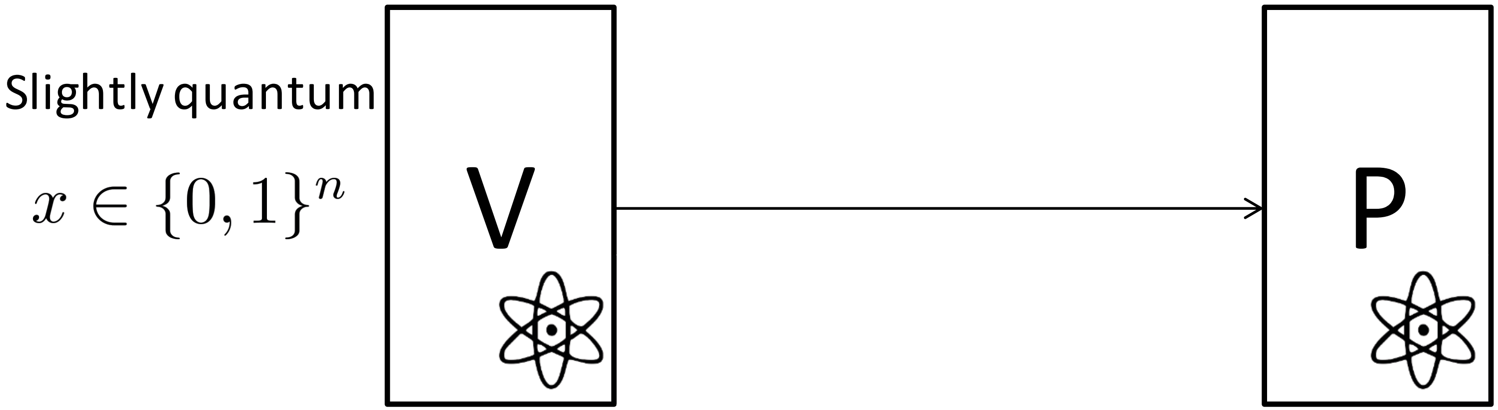

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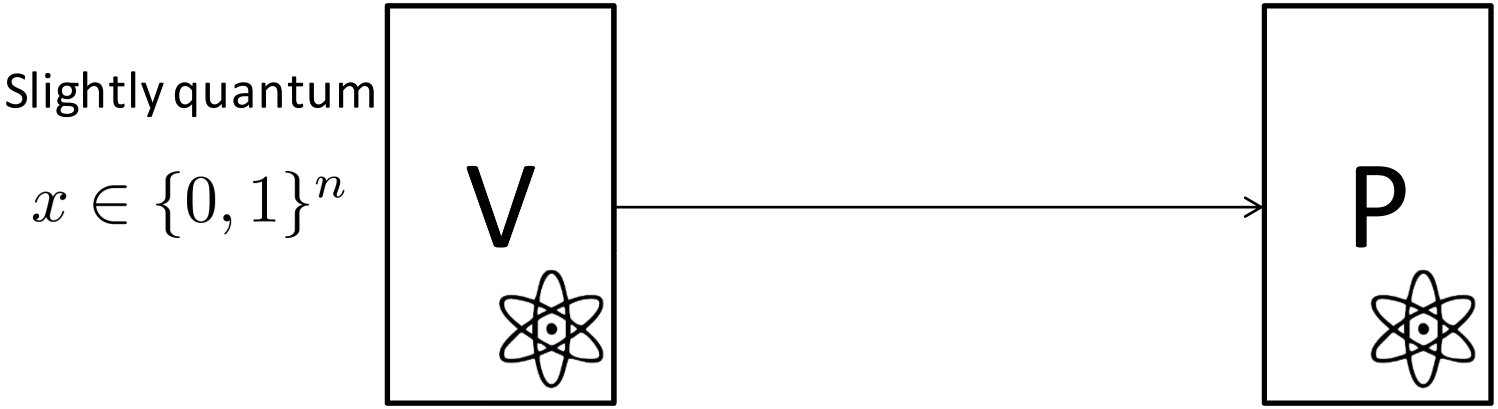
  - V sets the encrypting keys to $a_i := e_i \oplus x_i$

[Broadbent 2016]

EPR Protocol



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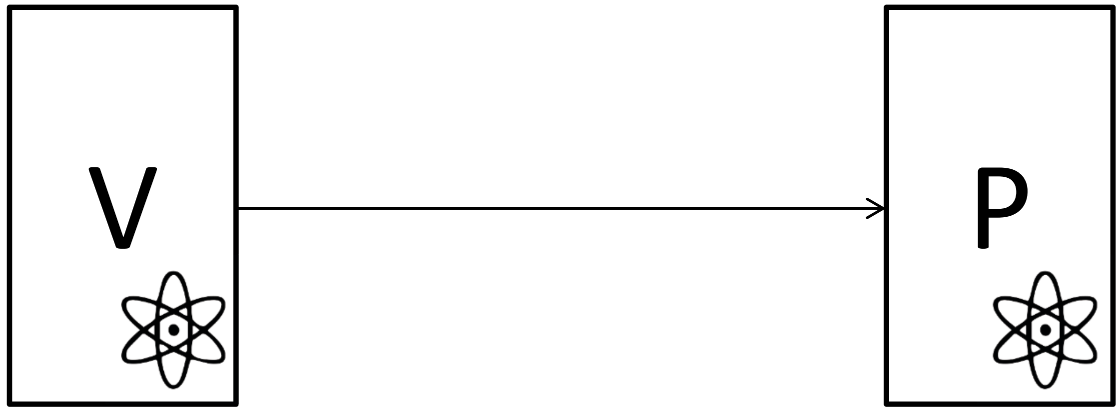

- More generally, the state of a wire is:

$$X^a Z^b \left|\psi\right\rangle, \quad a, b \in \{0,1\}$$

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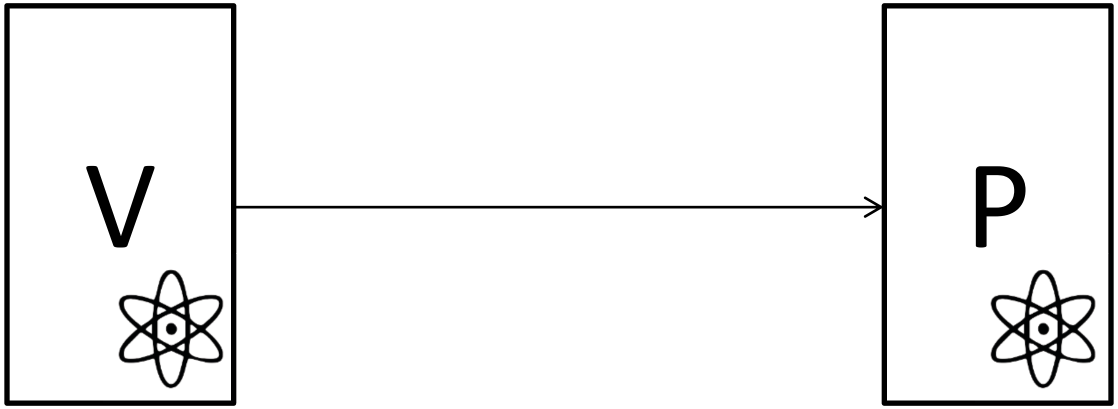

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- For example, P applies Hadamard gate:

$$H X^a Z^b \left|\psi\right\rangle = X^b Z^a H \left|\psi\right\rangle$$

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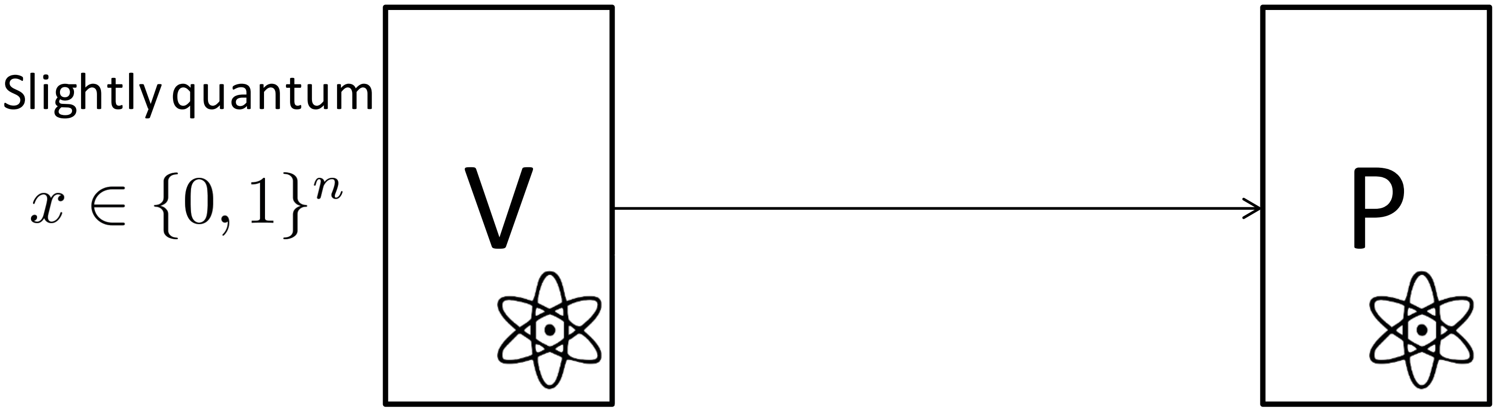

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- For example, P applies Hadamard gate:

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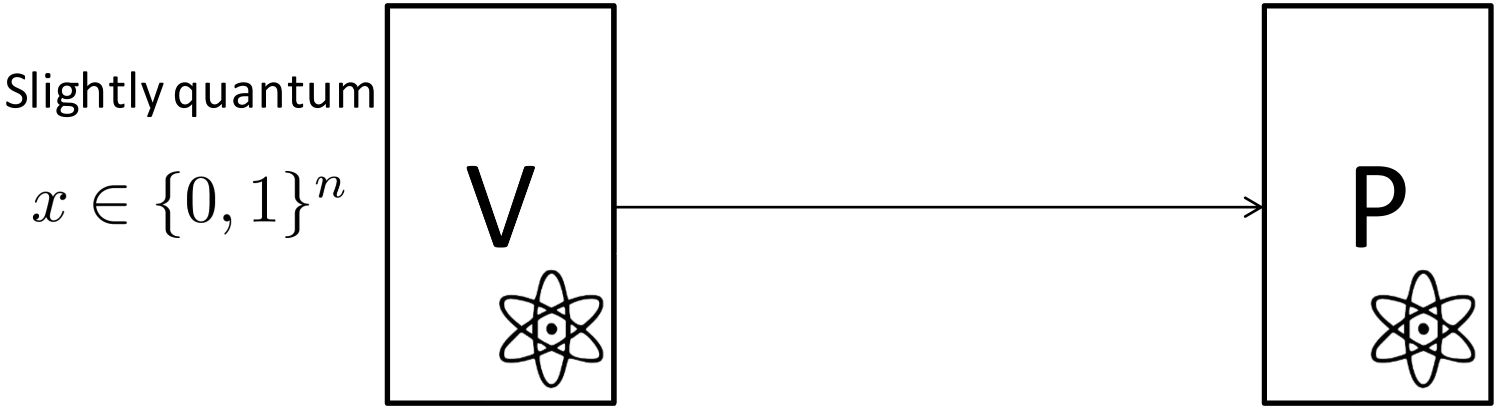
- V updates encrypting keys:

$$a' \leftarrow b, \quad b' \leftarrow a$$

[Broadbent 2016]

EPR Protocol



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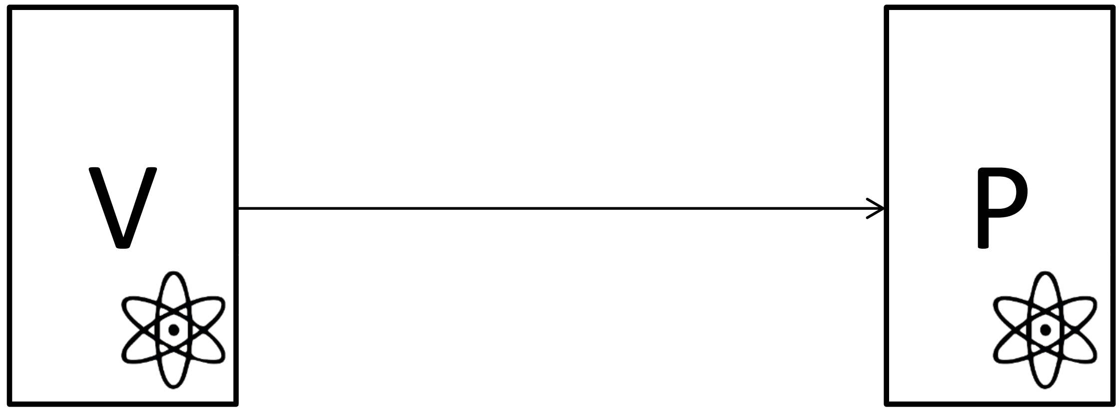

- Clifford gates (H, CNOT) are easy to implement!

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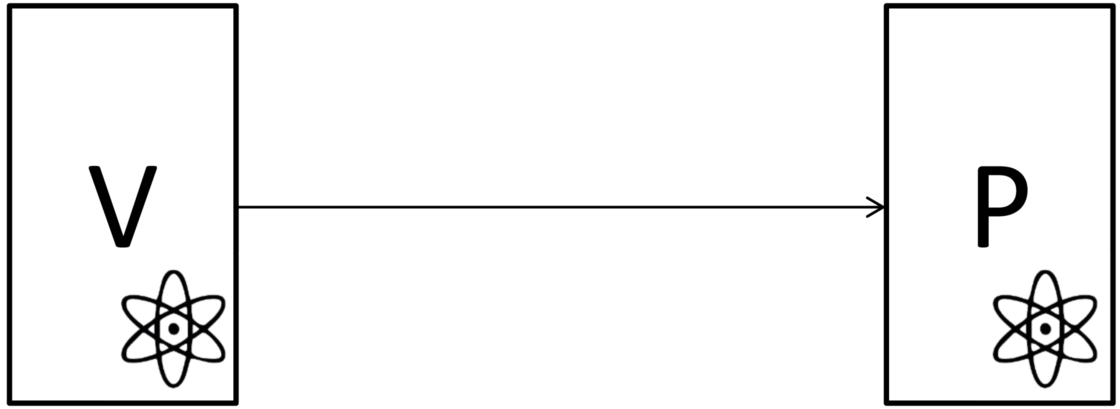

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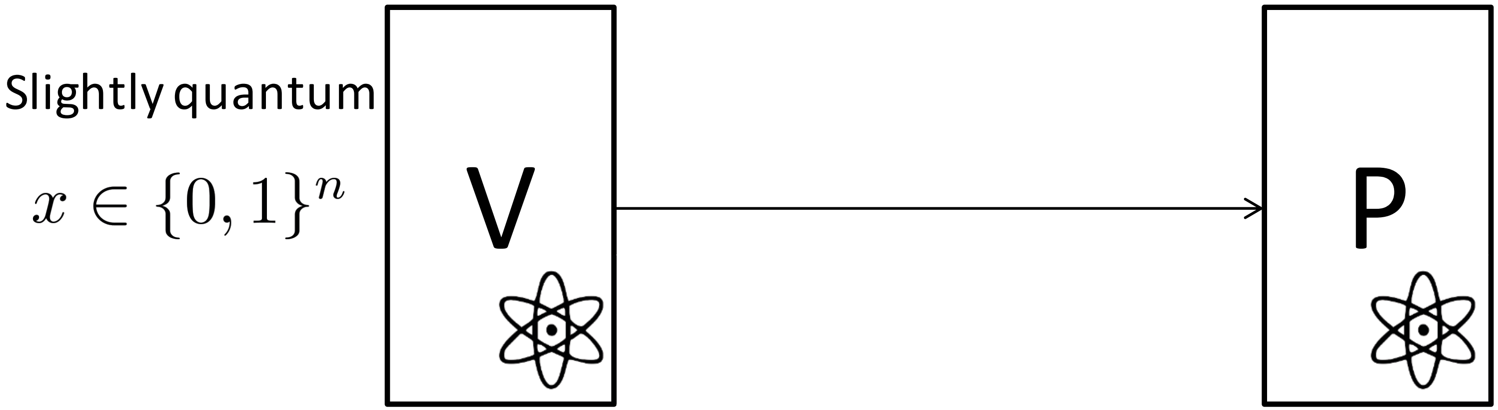

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$$TX^aZ^b\left|\psi\right\rangle = X^aZ^{a\oplus b}\left(P^aT\left|\psi\right\rangle\right)$$

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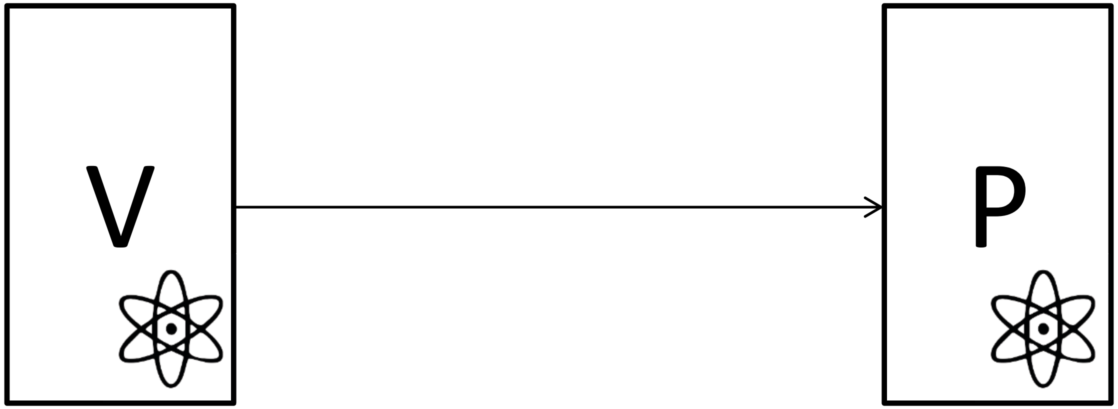

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[Broadbent 2016]

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[Broadbent 2016]

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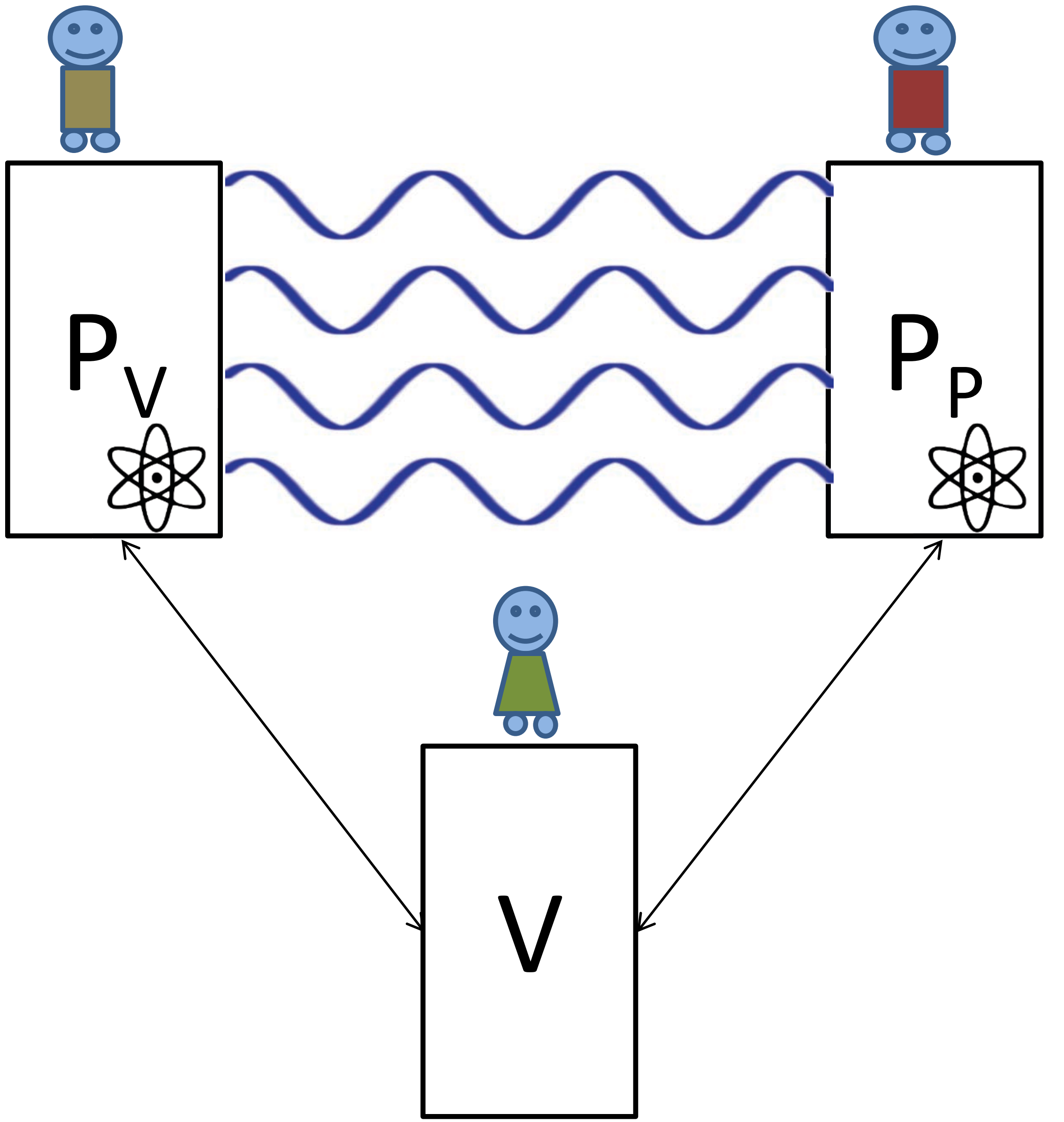
What about verifiability?

Verifier randomly chooses between a computation run and test runs.

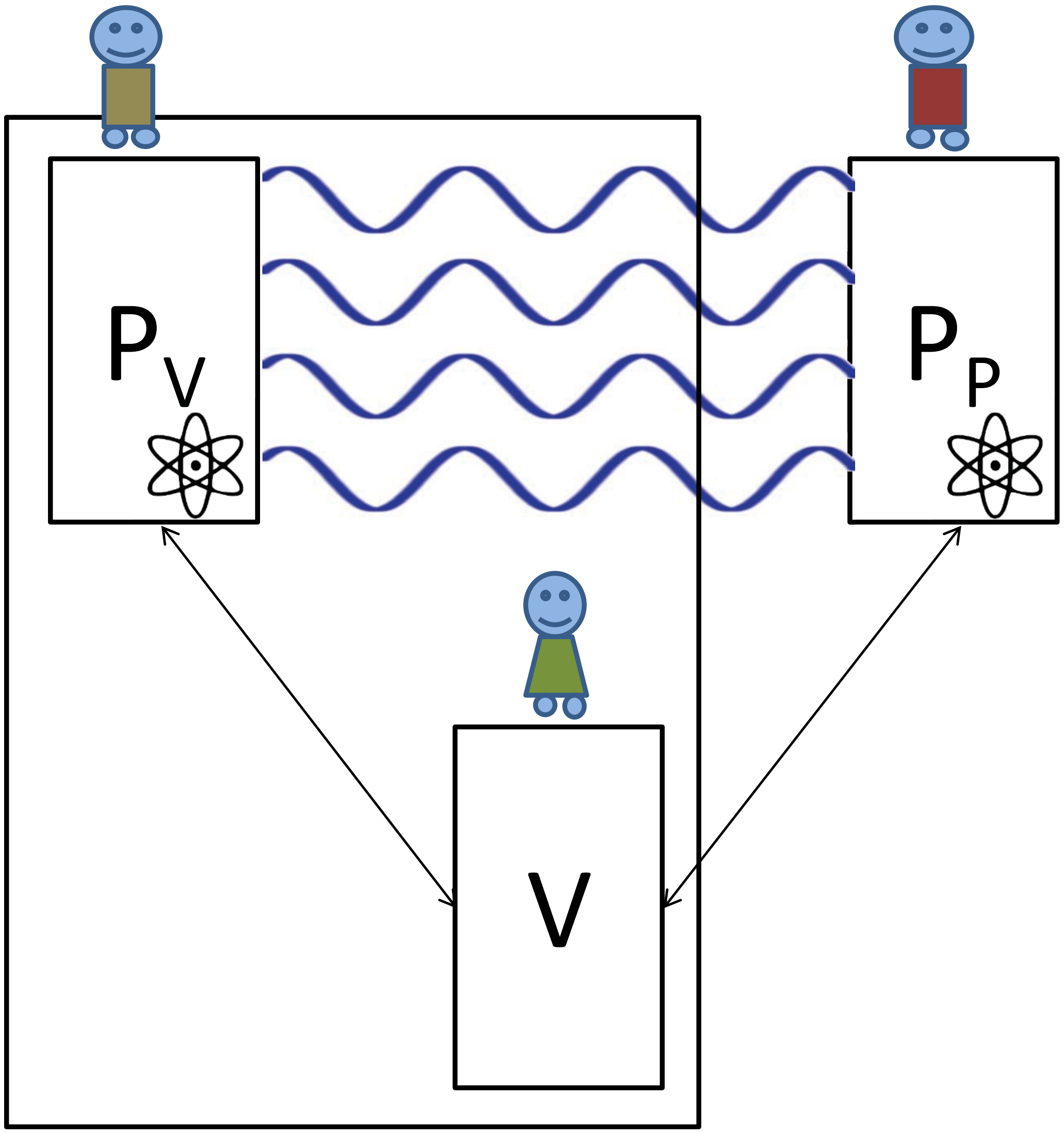
Test runs are indistinguishable from computation runs from the Prover's perspective!
But the verifier's input is fixed, and she expects deterministic replies.

[Broadbent 2016]

Putting the Verifier on a Leash



P_V
P_P
V


P_V
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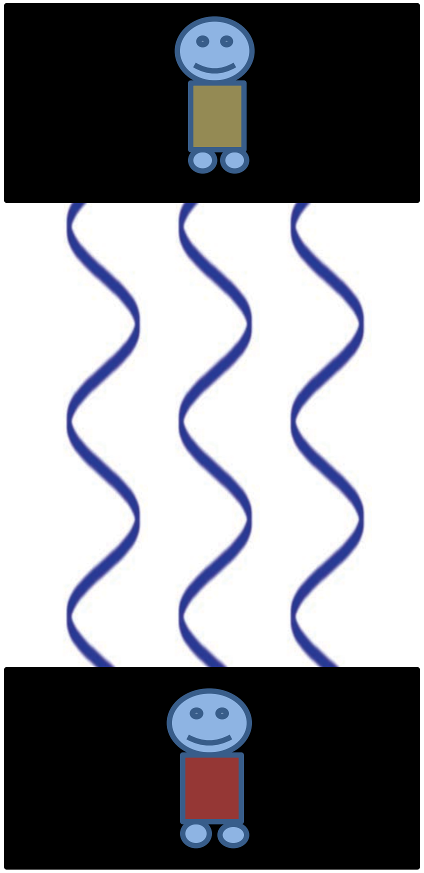
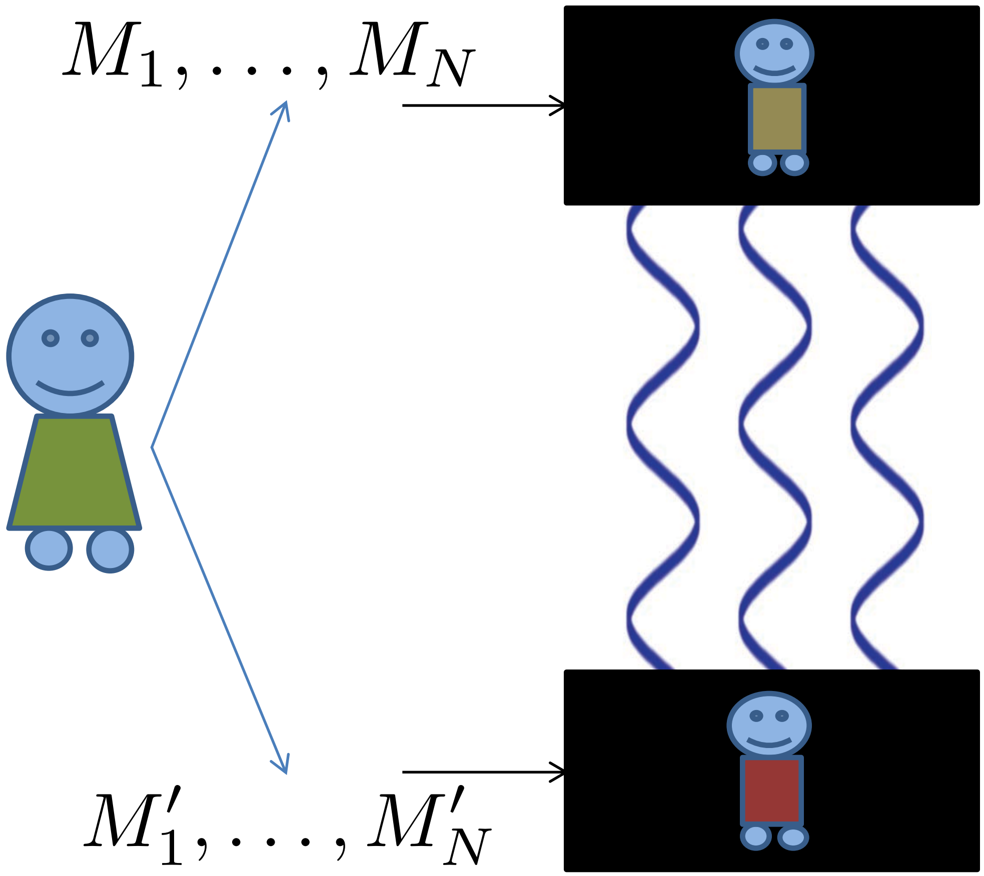
V

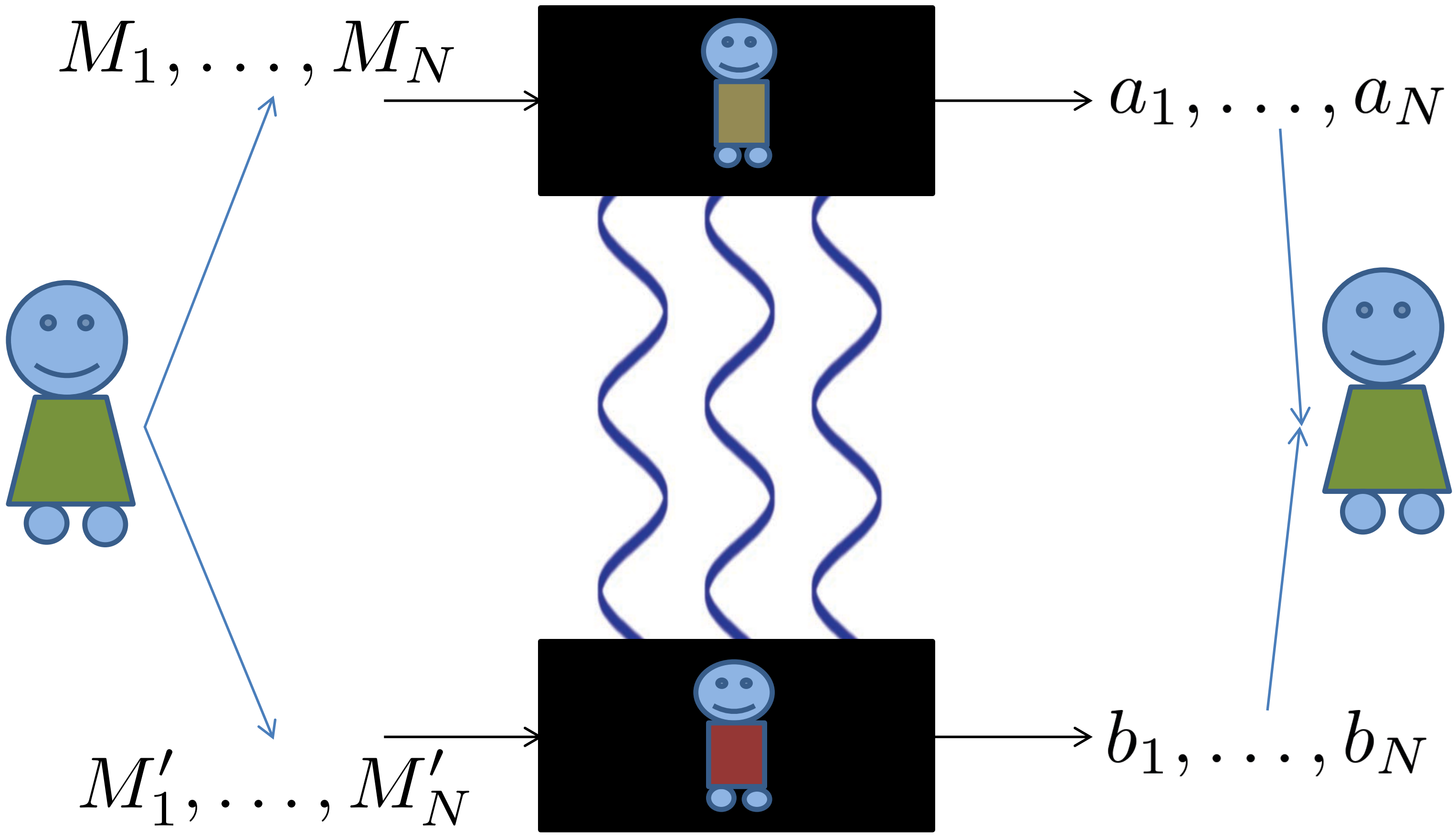
Rigidity Game



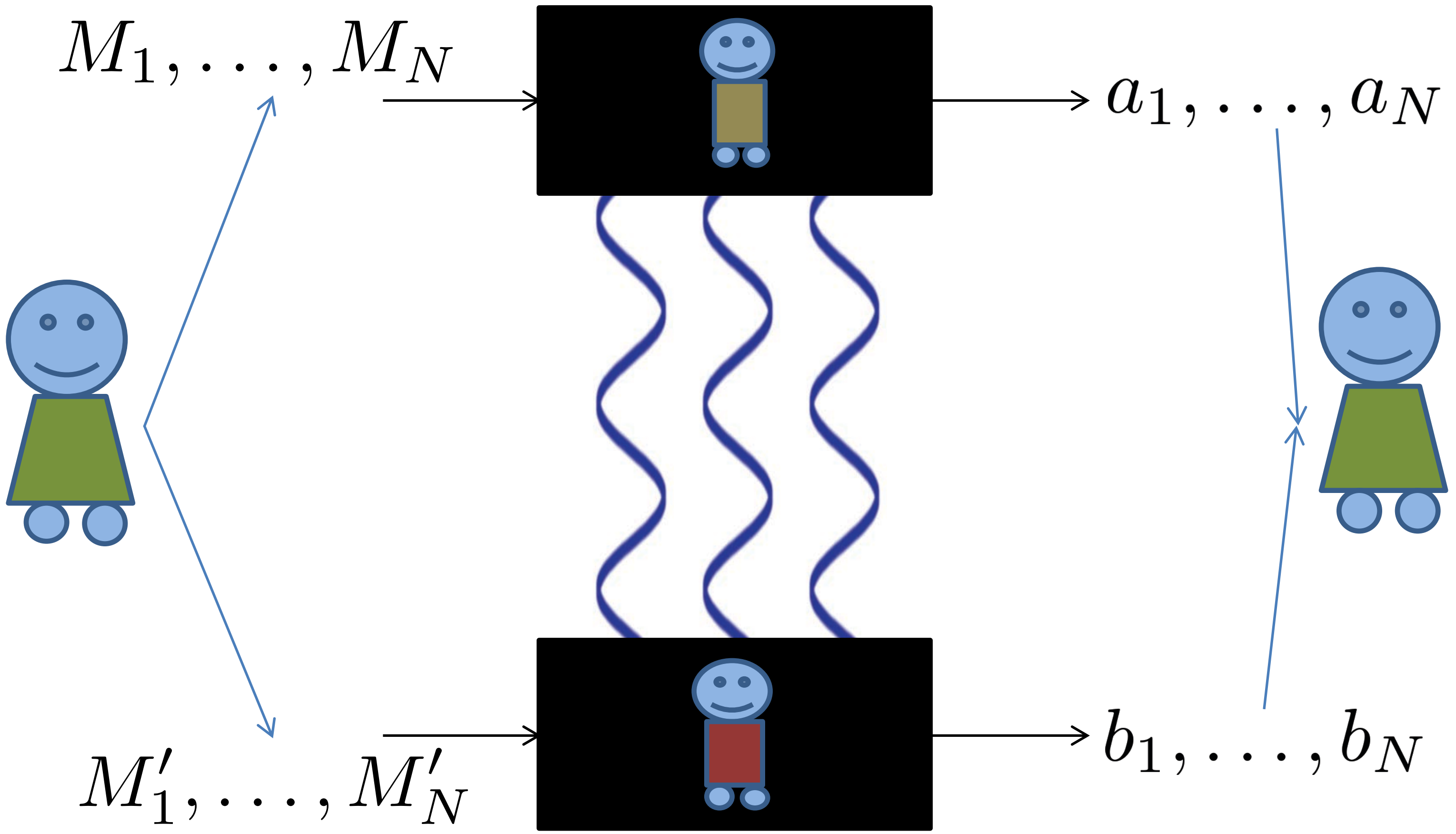
Rigidity Game



Rigidity Game

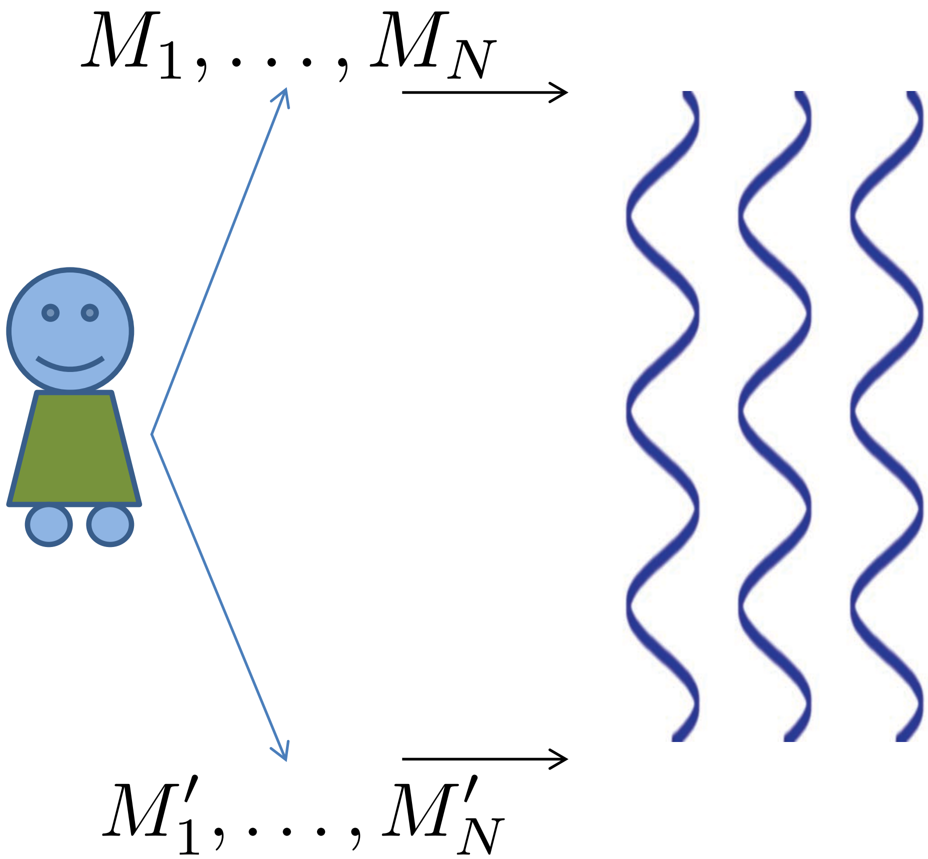


Rigidity Game

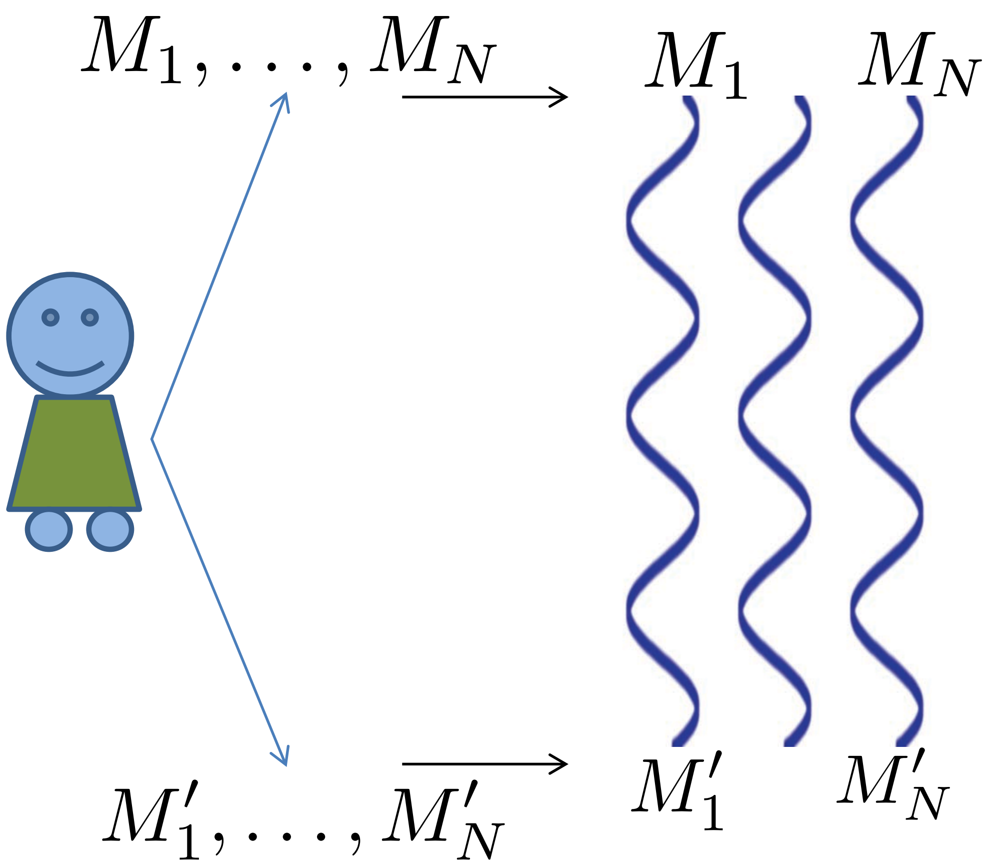


Provers win if $(\{M_i\}, \{M'_i\}, a, b) \in S_{\text{rigid}}$

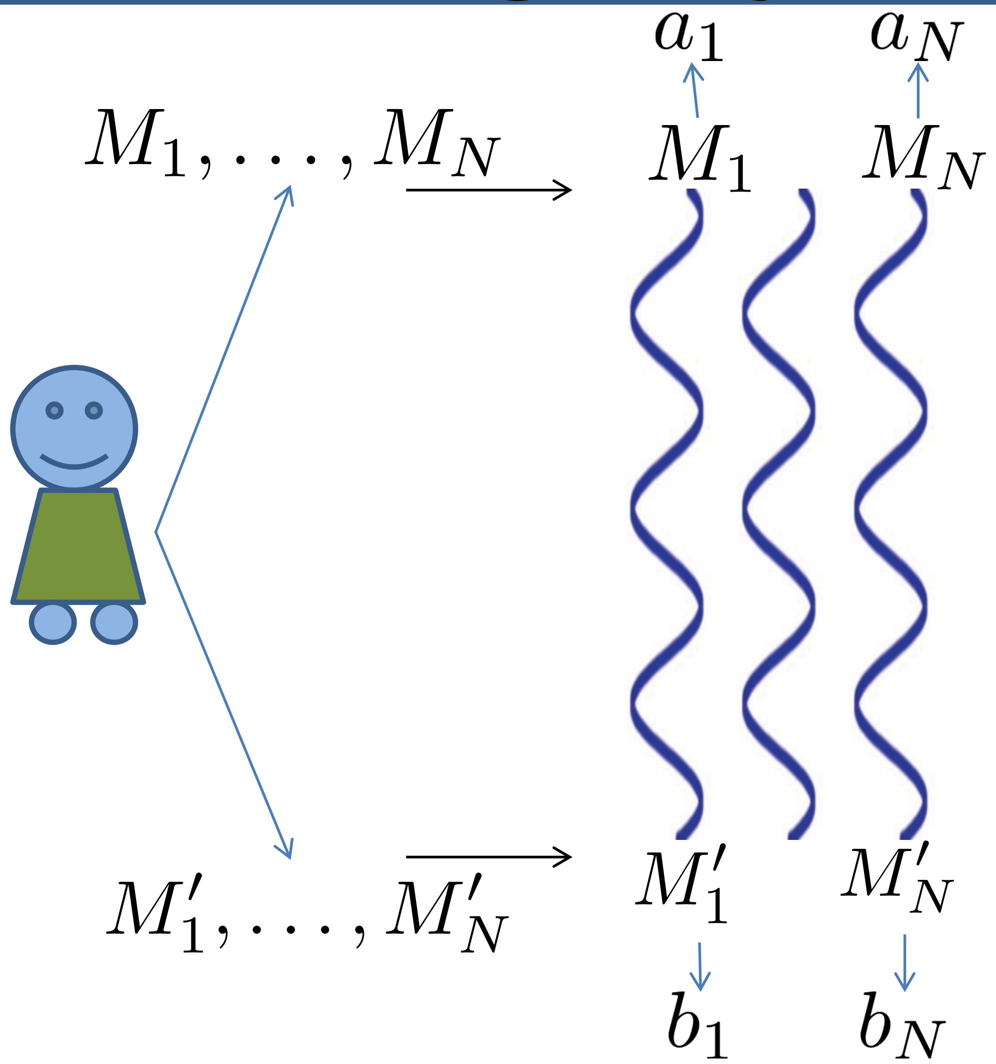
Rigidity Game



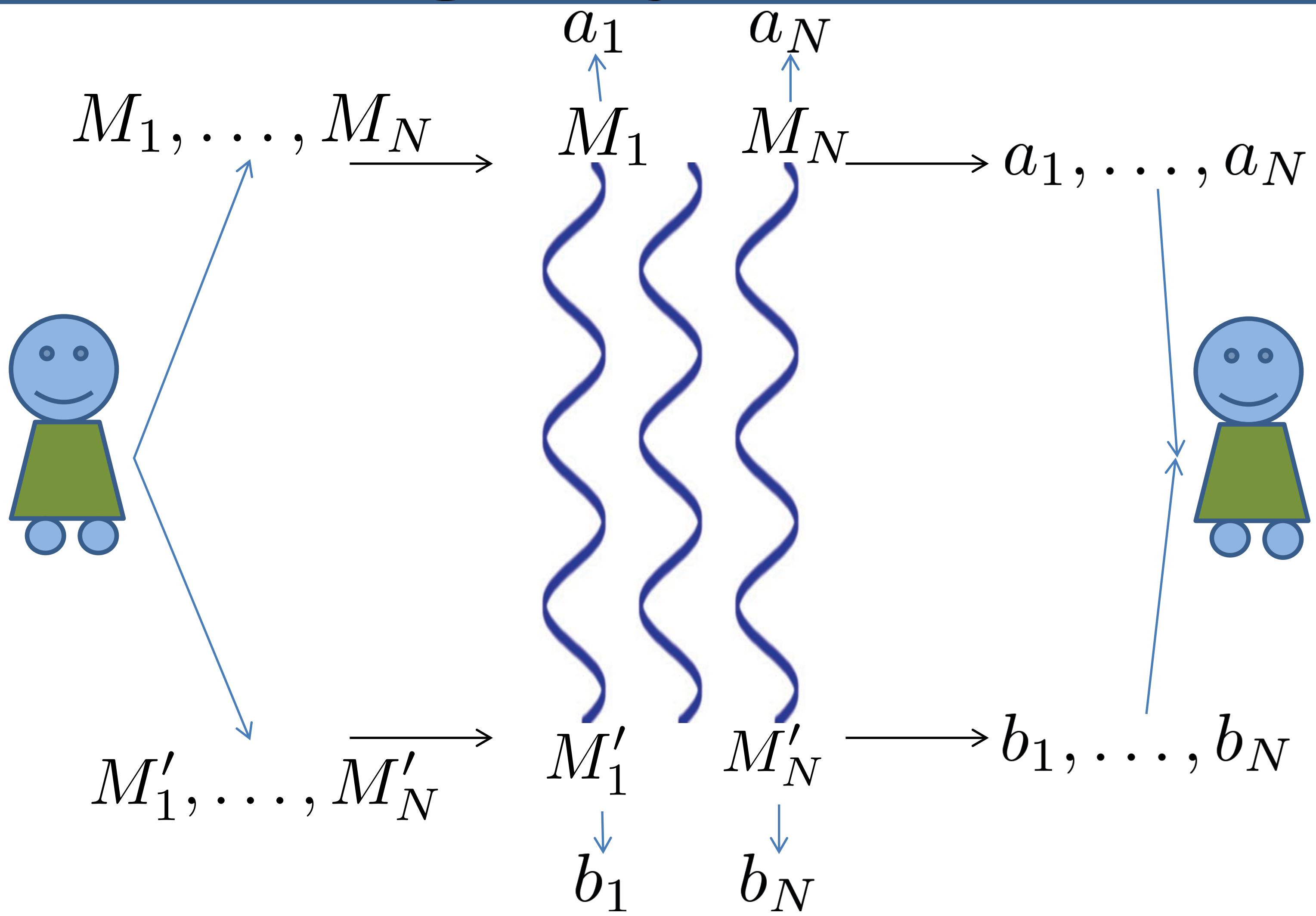
Rigidity Game



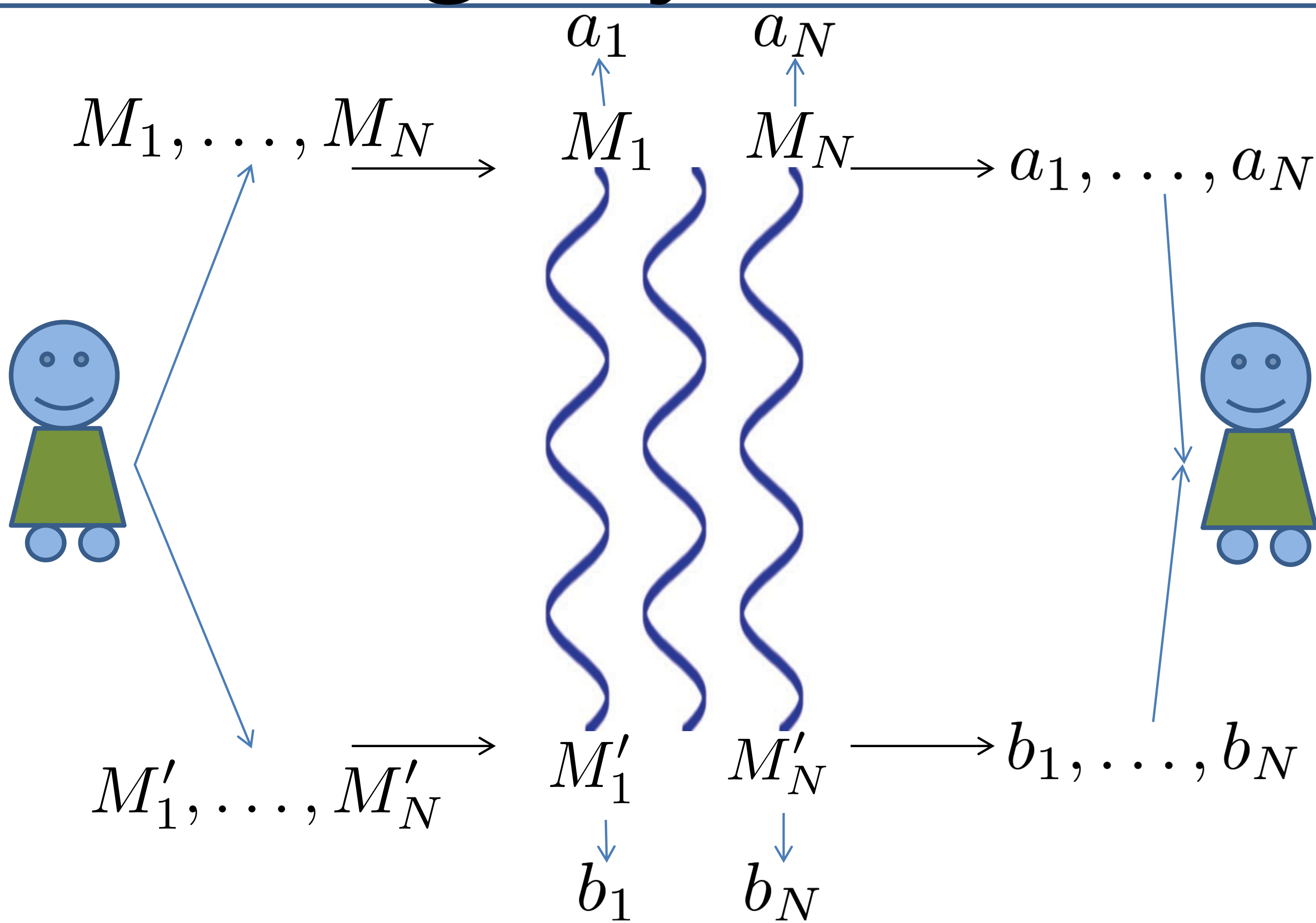
Rigidity Game



Rigidity Game



Rigidity Game



Completeness: Honest Alice and Bob win with probability 1.

Rigidity Game

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Soundness: If Alice and Bob win with probability $1-\epsilon$, their strategy must be within $O(\epsilon^c)$ of the honest strategy.

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Features:

- Can test that provers measure any product of single-qubit Clifford observables.
- Robustness independent of number of EPR pairs tested.

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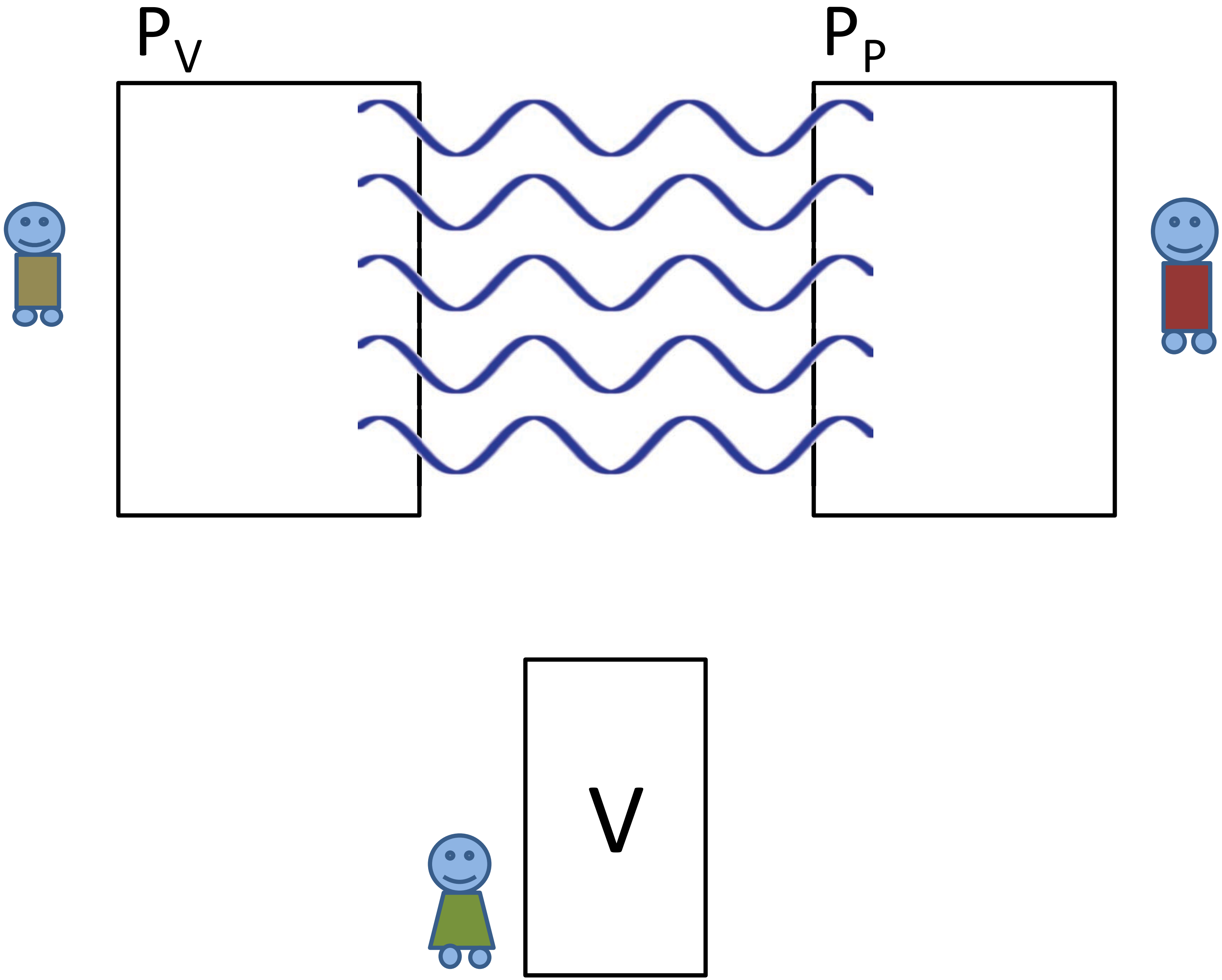
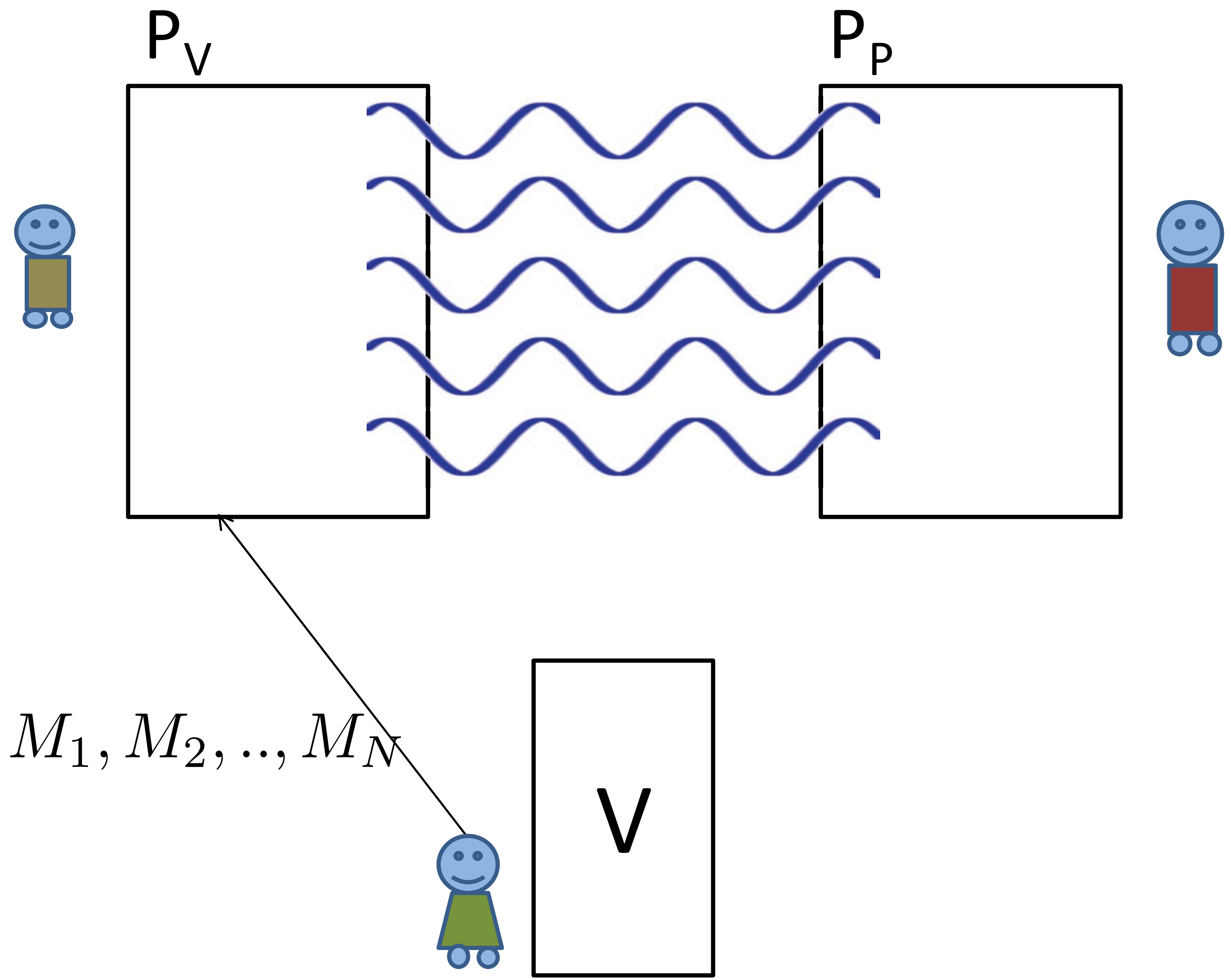
Features:

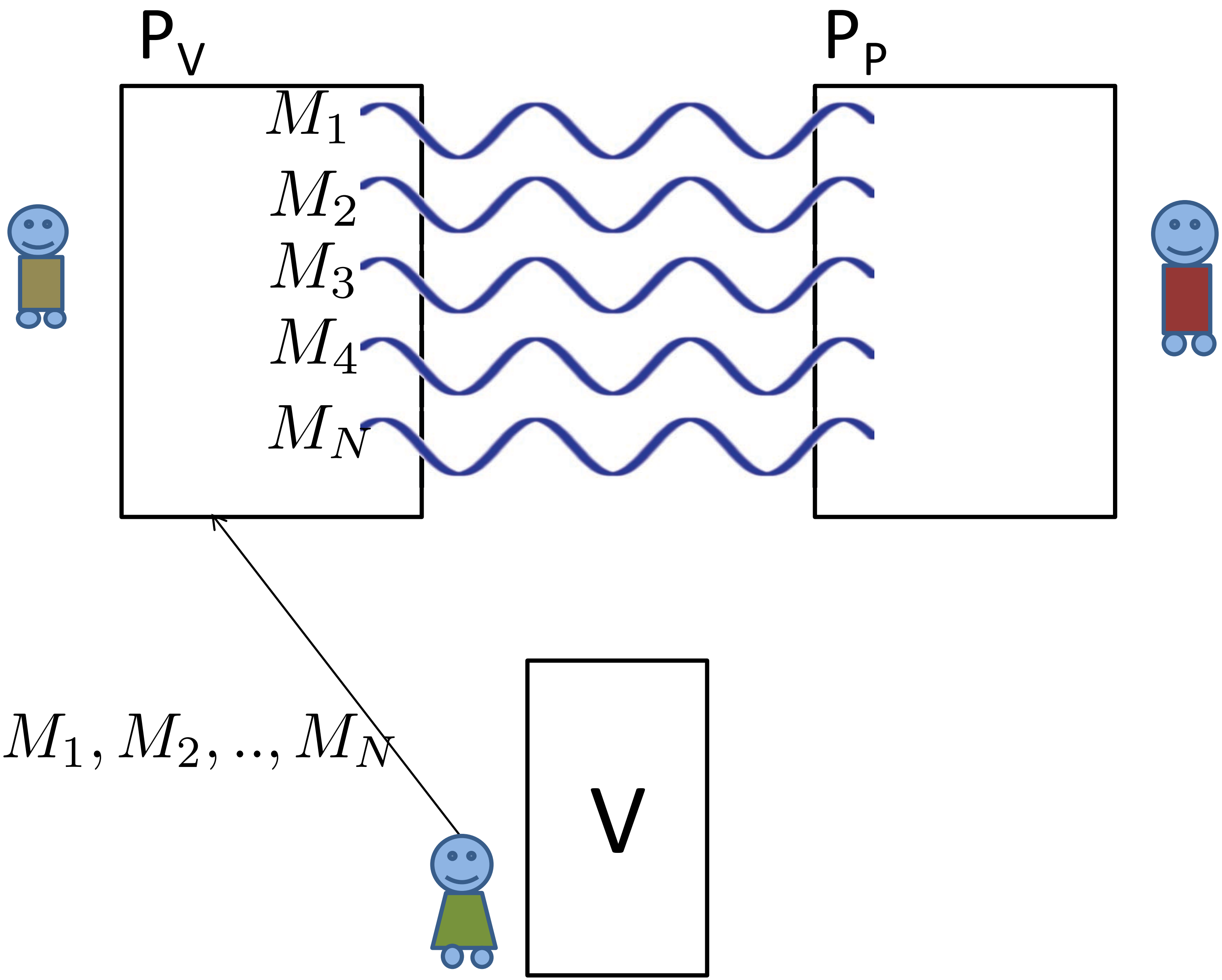
- Can test that provers measure any product of single-qubit Clifford observables.
- Robustness independent of number of EPR pairs tested. ([Natarajan, Vidick 2016] tests for Pauli X and Z measurements)



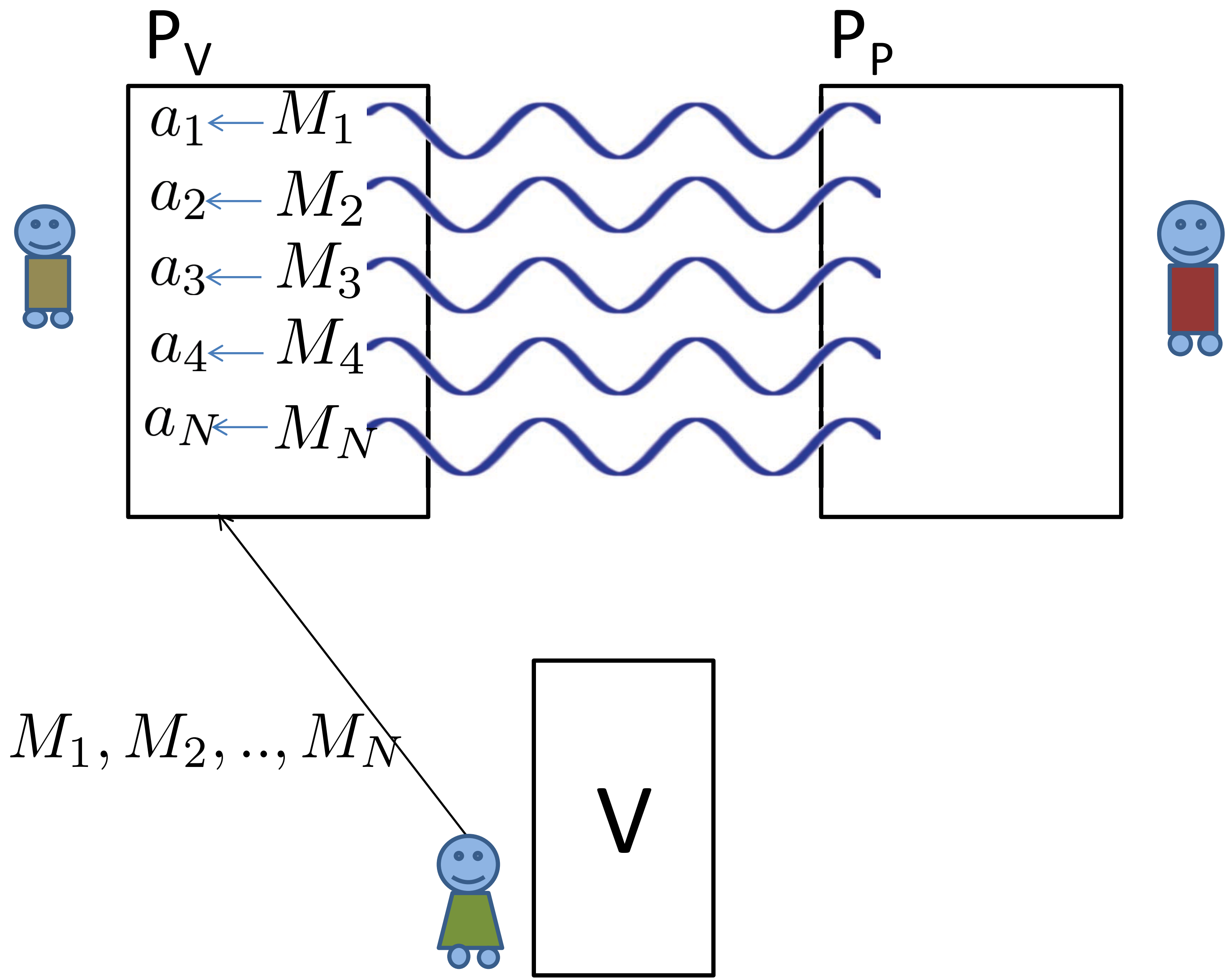
P_V
P_P
V



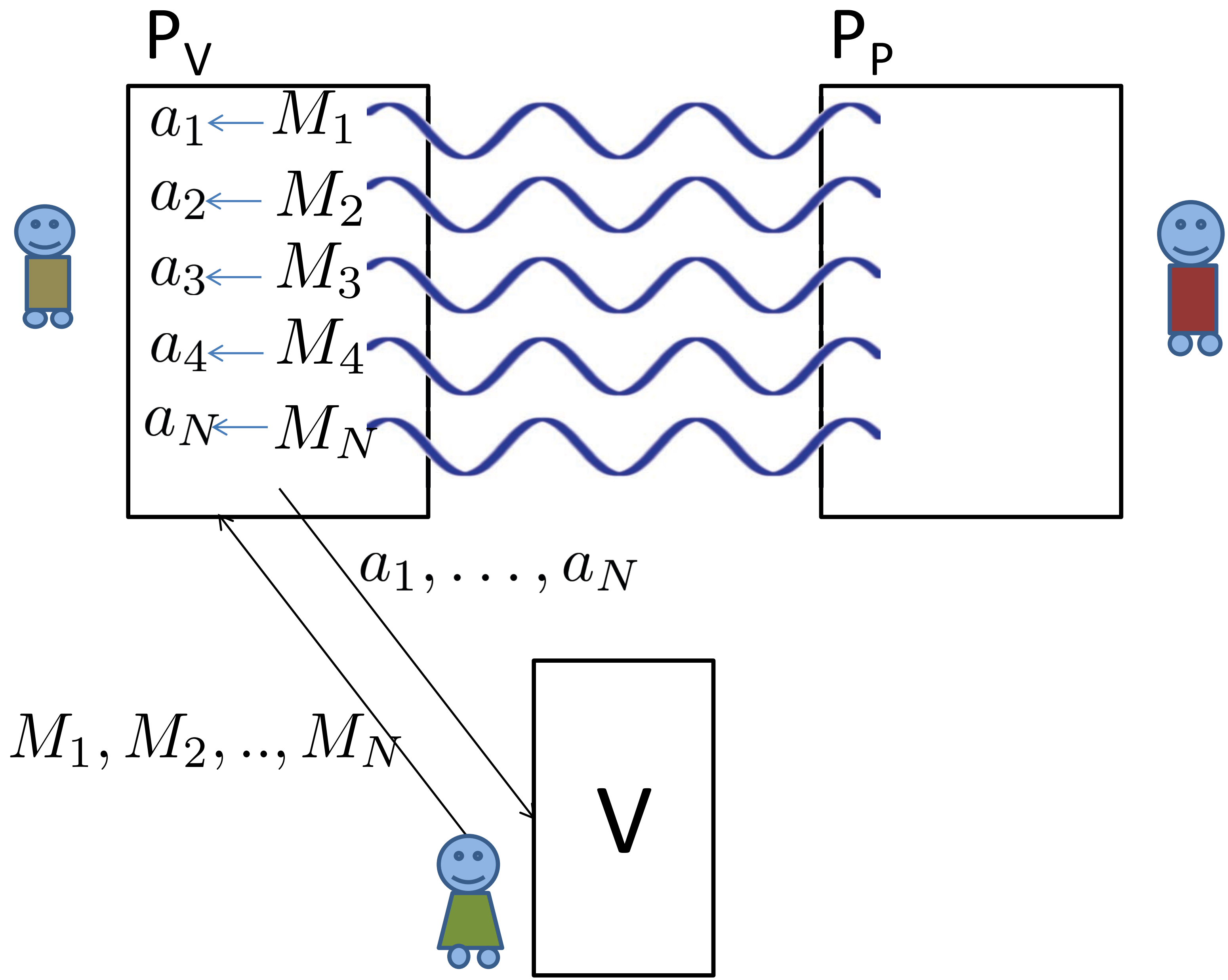
P_V
P_P
$M_1, M_2, .., M_N$
V



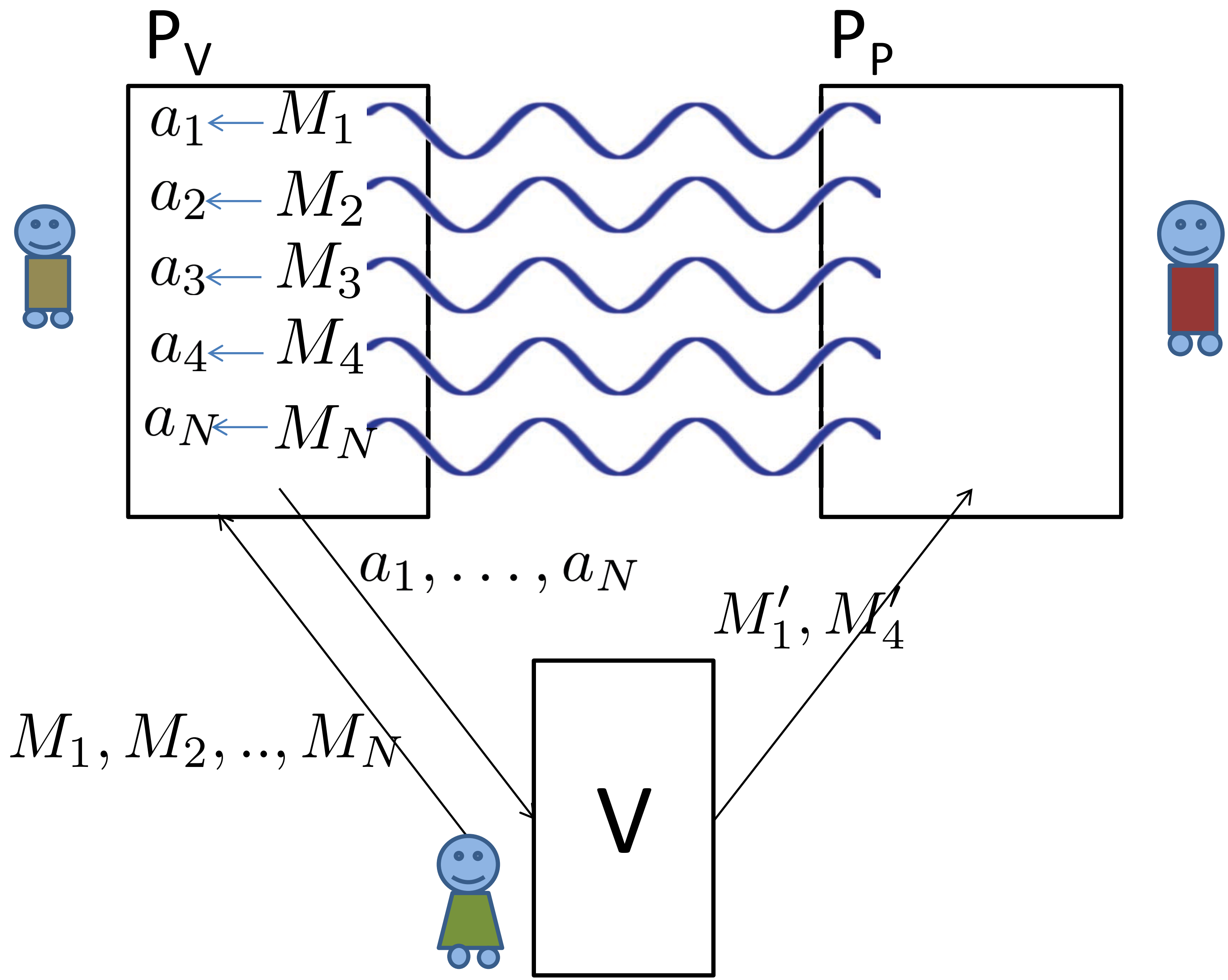
P_V
P_P
M_1
M_2
M_3
M_4
M_N
$M_1, M_2, .., M_N$
V



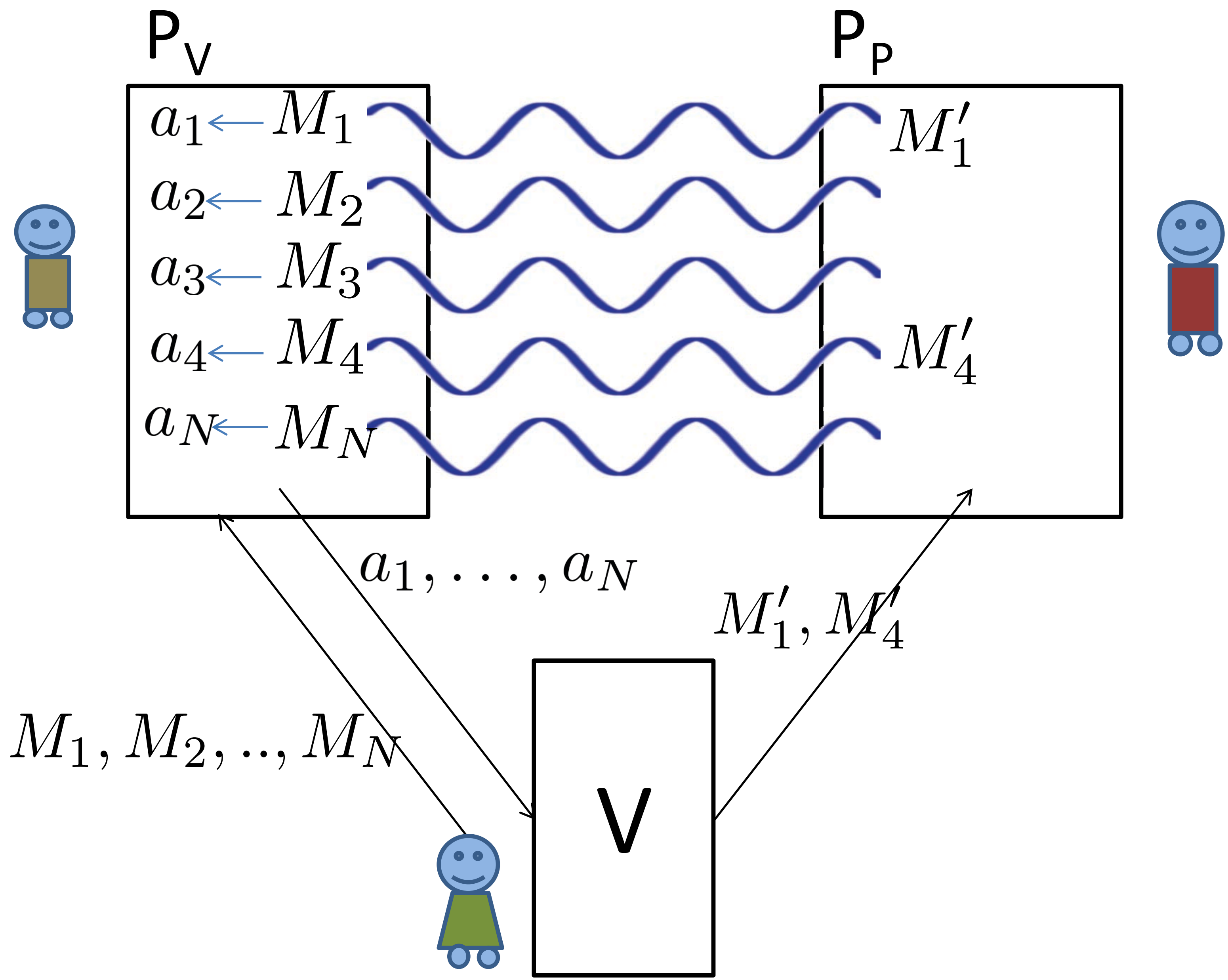
P_V
P_P
$a_1 \leftarrow M_1$
$a_2 \leftarrow M_2$
$a_3 \leftarrow M_3$
$a_4 \leftarrow M_4$
$a_N \leftarrow M_N$
$M_1, M_2, .., M_N$
V



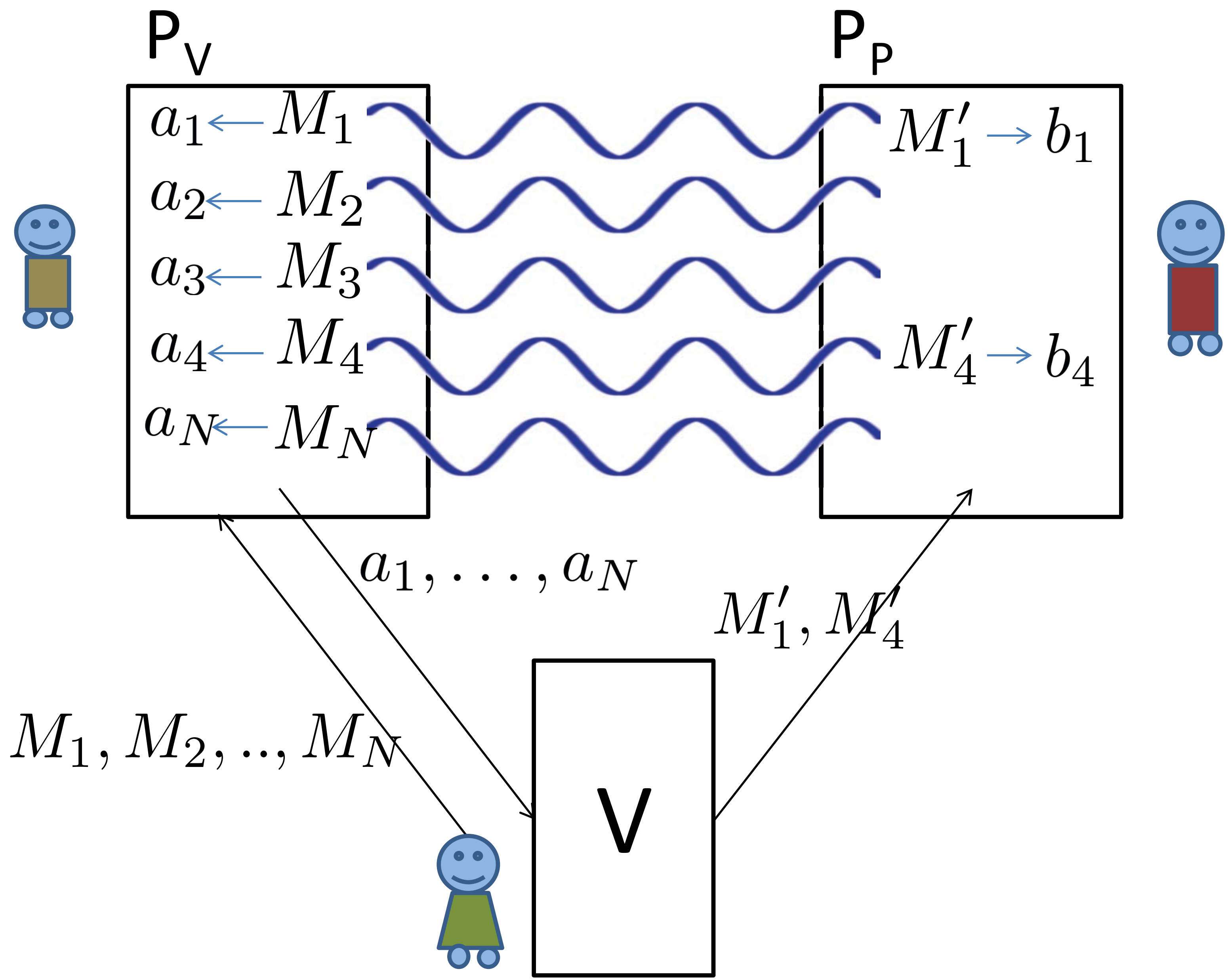
P_V
P_P
a_1 M_1
a_2 M_2
a_3 M_3
a_4 M_4
a_N M_N
$a_1, \ldots, a_N$
$M_1, M_2, .., M_N$
V



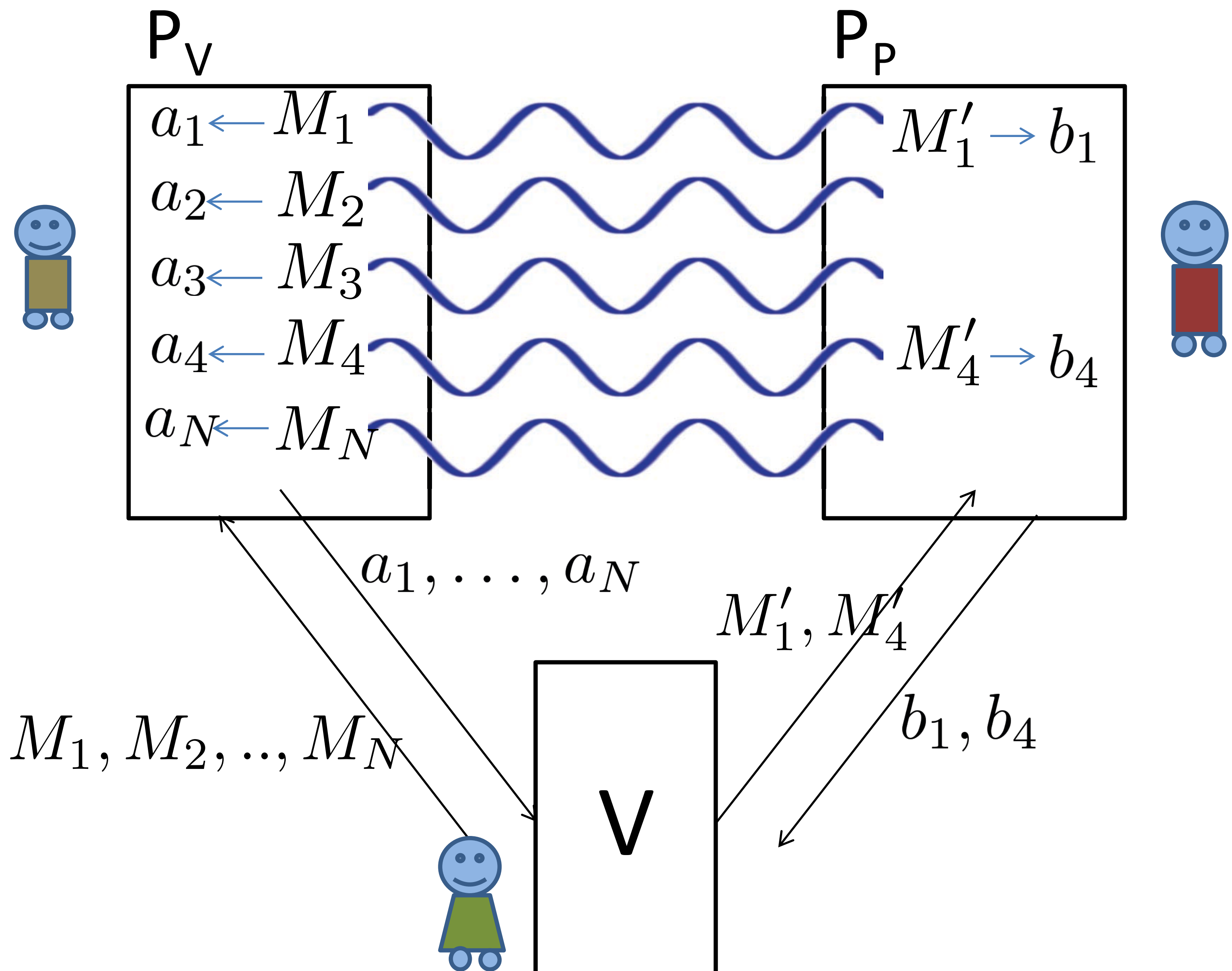
P_V
P_P
a_1 M_1
a_2 M_2
a_3 M_3
a_4 M_4
a_N M_N
$a_1, \ldots, a_N$
M'_1, M'_4
$M_1, M_2, .., M_N$
V



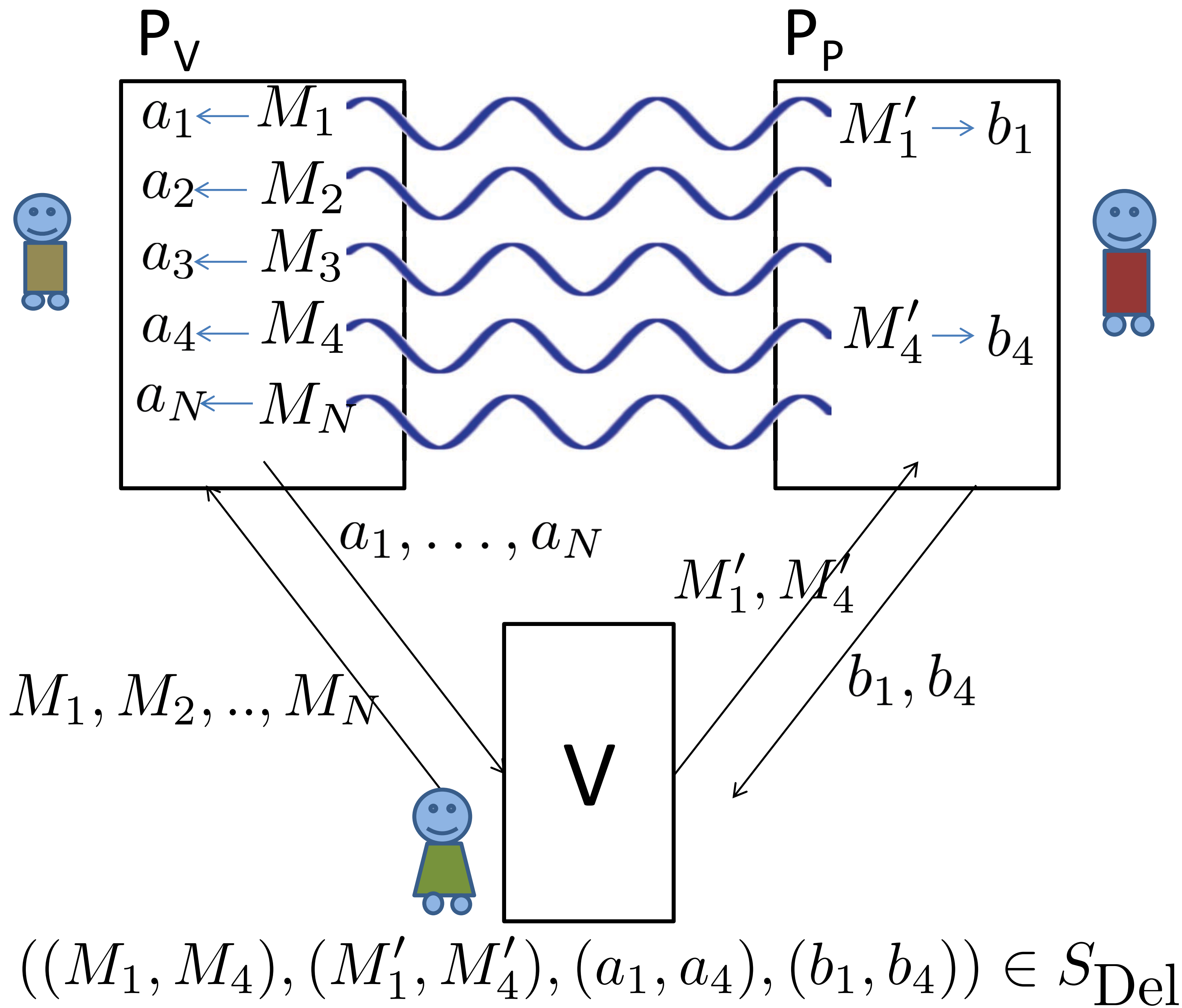
P_V
P_P
$a_1 \leftarrow M_1$
$a_2 \leftarrow M_2$
$a_3 \leftarrow M_3$
$a_4 \leftarrow M_4$
$a_N \leftarrow M_N$
M'_1
M'_4
$a_1, \ldots, a_N$
M'_1, M'_4
$M_1, M_2, .., M_N$
V



P_V
P_P
$a_1 \leftarrow M_1$
$a_2 \leftarrow M_2$
$a_3 \leftarrow M_3$
$a_4 \leftarrow M_4$
$a_N \leftarrow M_N$
$M'_1 \rightarrow b_1$
$M'_4 \rightarrow b_4$
$a_1, \ldots, a_N$
M'_1, M'_4
$M_1, M_2, .., M_N$
V



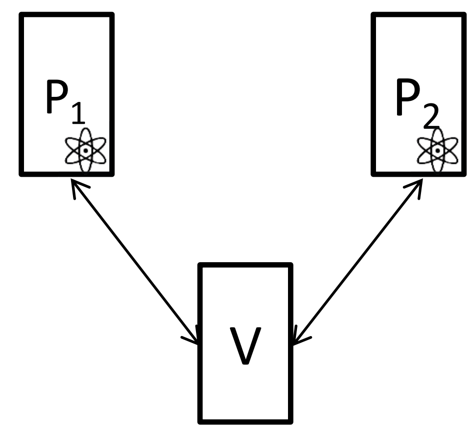
P_V
P_P
$a_1 \leftarrow M_1$
$a_2 \leftarrow M_2$
$a_3 \leftarrow M_3$
$a_4 \leftarrow M_4$
$a_N \leftarrow M_N$
$M_1' \rightarrow b_1$
$M_4' \rightarrow b_4$
$a_1, \ldots, a_N$
M_1', M_4'
b_1, b_4
$M_1, M_2, .., M_N$
V



P_V
P_P
$a_1 \leftarrow M_1$
$a_2 \leftarrow M_2$
$a_3 \leftarrow M_3$
$a_4 \leftarrow M_4$
$a_N \leftarrow M_N$
$M'_1 \rightarrow b_1$
$M'_4 \rightarrow b_4$
$a_1, \ldots, a_N$
M'_1, M'_4
b_1, b_4
$M_1, M_2, .., M_N$
V
$((M_1, M_4), (M'_1, M'_4), (a_1, a_4), (b_1, b_4)) \in S_{\mathrm{Del}}$

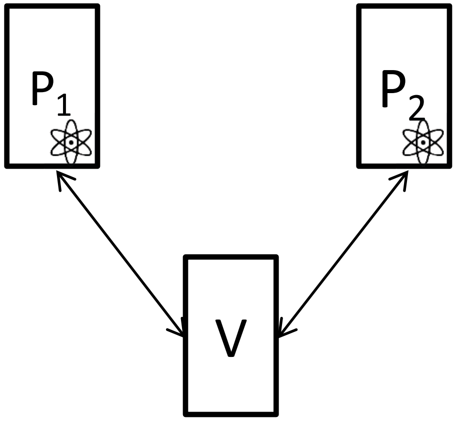
Verifier on a Leash Protocol

- Classical verifier, two quantum provers
- Total complexity: $O(m \log m)$
- Round complexity: $O(T\text{-depth})$



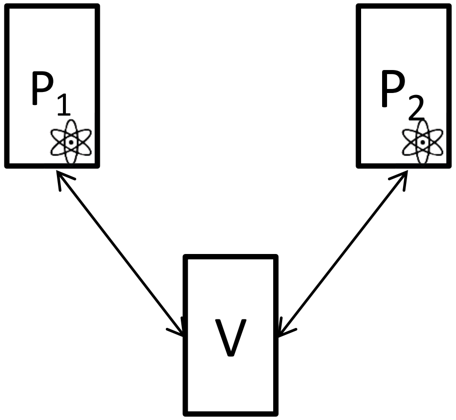
Verifier on a Leash Protocol

- Classical verifier, two quantum provers
- Total complexity: $O(m \log m)$
- Round complexity: $O(T\text{-depth})$
- Encrypted input



Dog-Walker Protocol

- Classical verifier, two quantum provers
- Total complexity: $O(m \log m)$
- Round complexity: $O(1)$
- Input is known to P_V



Summary

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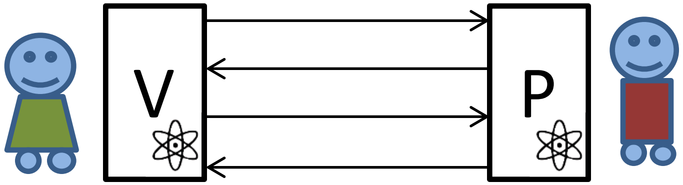
Previously:

- $O(m)$ protocol for slightly-quantum verifier to delegate m-gate circuit to quantum prover



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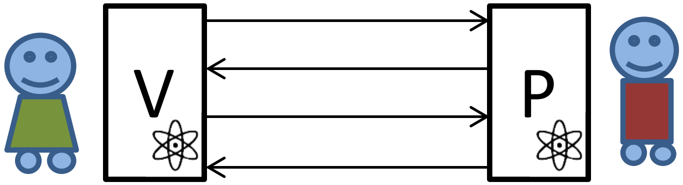
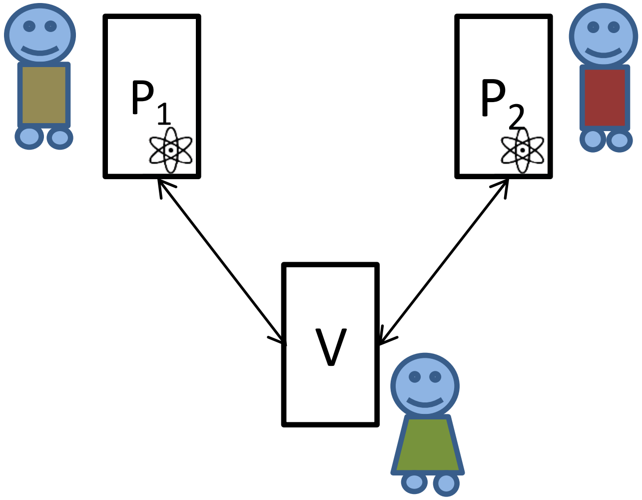


- from $O(m^4)$ to $O(m^{8192})$ protocols for classical verifier to delegate m-gate circuit to two quantum provers



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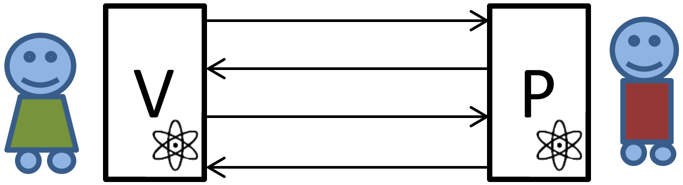

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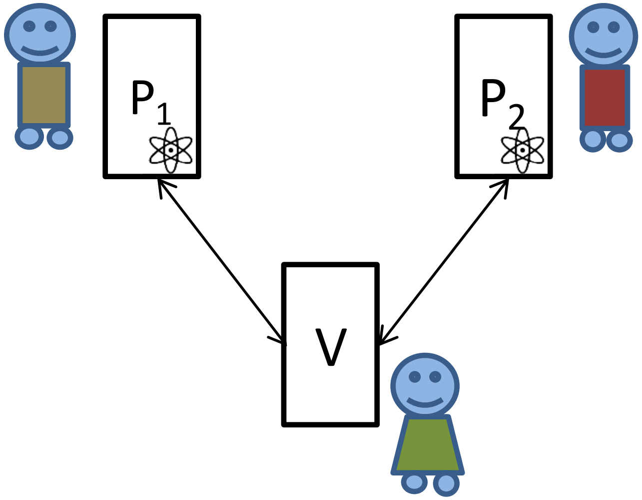

New:

Summary

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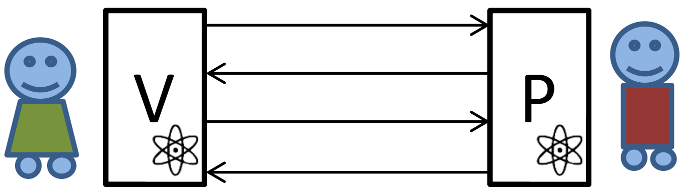
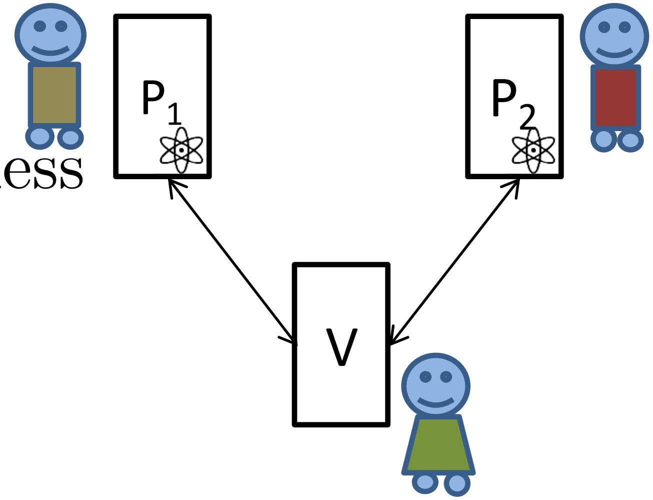


- from $O(m^4)$ to $O(m^{8192})$ protocols for classical verifier to delegate m-gate circuit to two quantum provers

New:

- new rigidity theorems with robustness independent of number of EPR pairs tested.



Summary

Previously:

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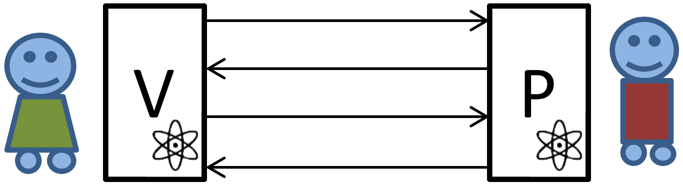
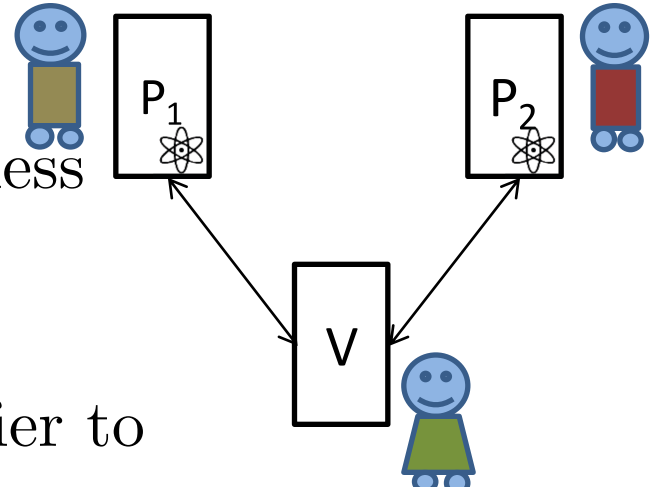


- from $O(m^4)$ to $O(m^{8192})$ protocols for classical verifier to delegate m-gate circuit to two quantum provers

New:

- new rigidity theorems with robustness independent of number of EPR pairs tested.
- $O(m \log m)$ protocols for classical verifier to delegate m-gate circuit to two quantum provers.



Open Questions

- Avoiding the non-communication assumption (while keeping the client classical)?

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  - Single-Round Protocols: [Grilo 2017]

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  - Single-server Protocols (with classical client): [Mahadev 2017]

- Noise tolerance?
  - [Arnon-Friedman, Yuen 2017]