Dynamics for holographic codes

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Holographic quantum error-correcting codes: toy models for the bulk/boundary correspondence

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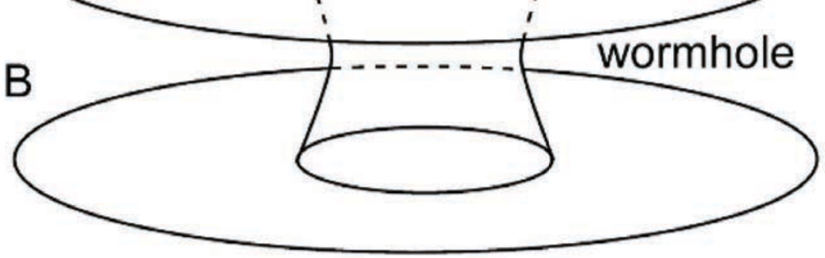
Abstract: We propose a family of exactly solvable toy models for the AdS/CFT correspondence based on a novel construction of quantum error-correcting codes with a tensor network structure. Our building block is a special type of tensor with maximal entanglement along any bipartition, which gives rise to an isometry from the bulk Hilbert space to the boundary Hilbert space. The entire tensor network is an encoder for a quantum error-correcting code, where the bulk and boundary degrees of freedom may be identified as



(b) Wormhole.

hic code, and the corresponding wormhole geometry.

legs at their horizons, as shown in figure 18b. It with recent speculations about how the length of of the tensor network describing the state [52–54], obably need to incorporate dynamics into our model.

n information science and quantum gravity has acular by a vision of quantum entanglement as the e expect this interface area to continue to grow in ommunities struggle to develop a common language by the connection between AdS/CFT and quantum

n invariance in a lattice model. What features in ou /N corrections in the continuum theory? In AdS/C l to the Planck scale when the bulk theory is weakly for example the curvature scale is comparable to th ter bulk geometries we should study more general tes al ones. A particularly serious drawback of our toy r duced any bulk or boundary dynamics. Can holograp sses like the formation and evaporation of a black hol have emphasized that holographic states and codes me aspects of AdS/CFT, but they may also be i nple as models of topological matter. Furthermore ncatenated quantum codes that have been extensively quantum computing [44], and might likewise be app ntum computers against noise. For this application i heory of holographic codes in a variety of direction n rate and distance, formulating efficient schemes an erasure errors, and finding ways to realize a uni ; on the code space.

ents

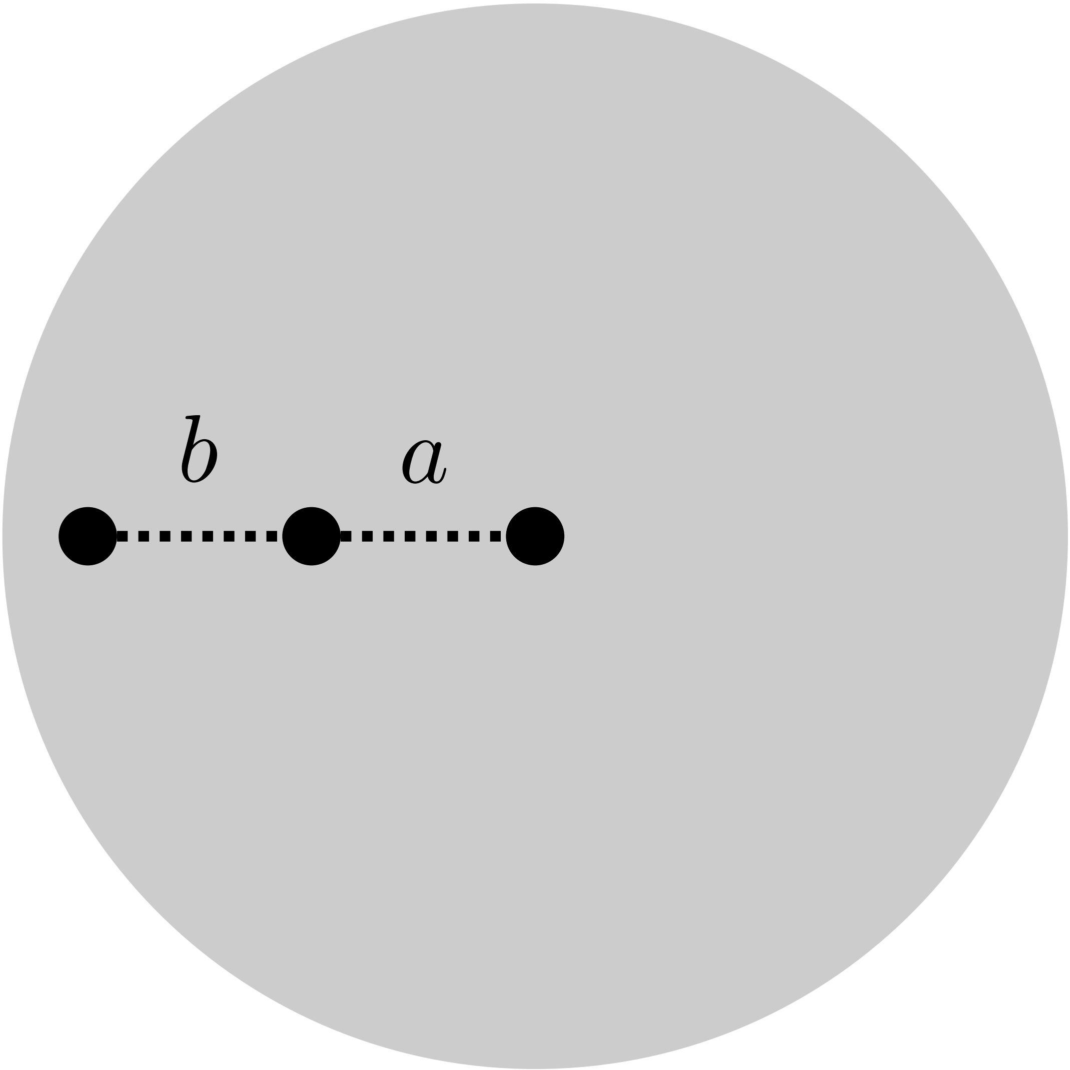
The Poincaré disk

interior of the unit disk

interior of the unit disk

distances are different!

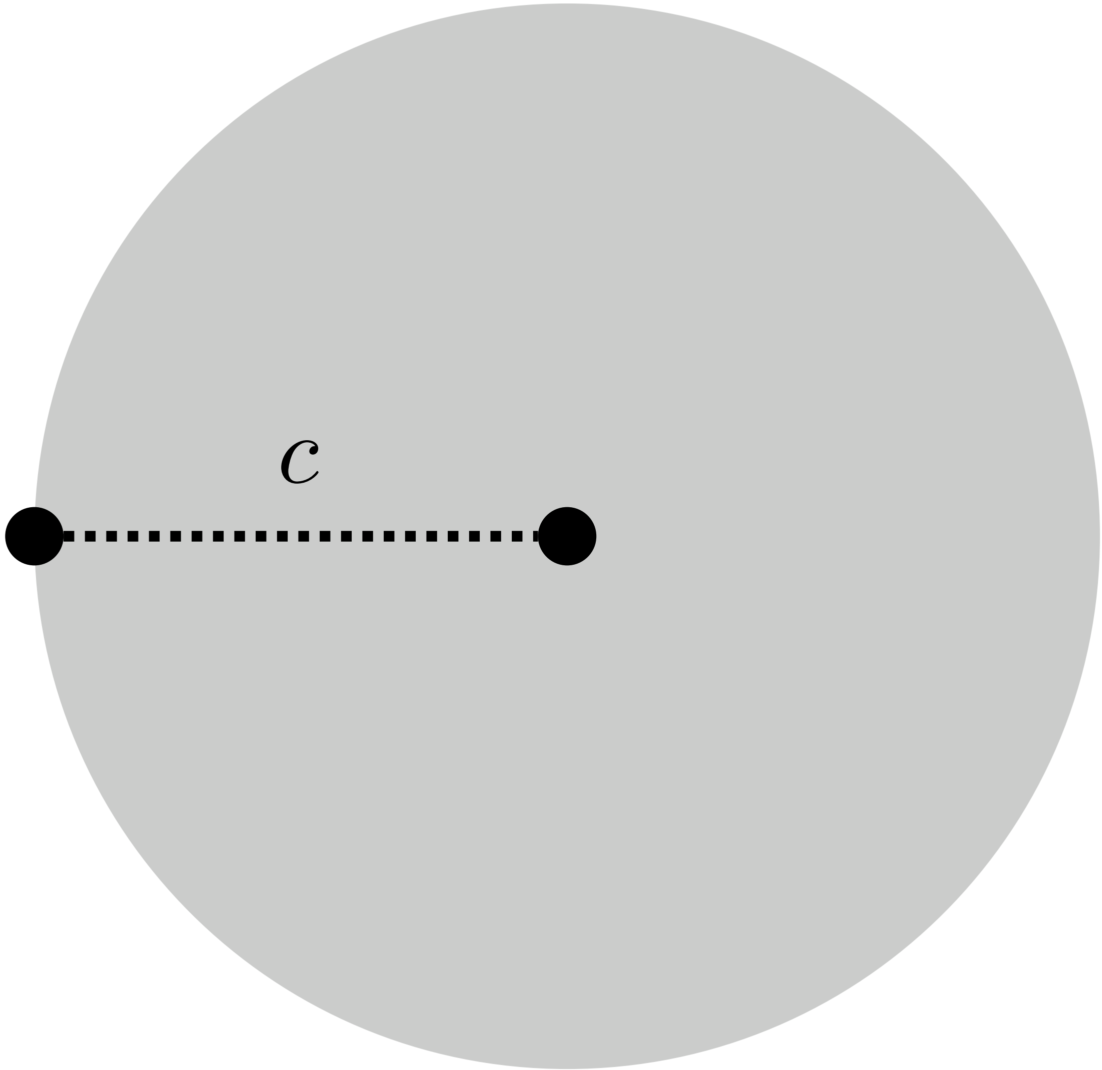
$$|b| > |a|$$



interior of the unit disk

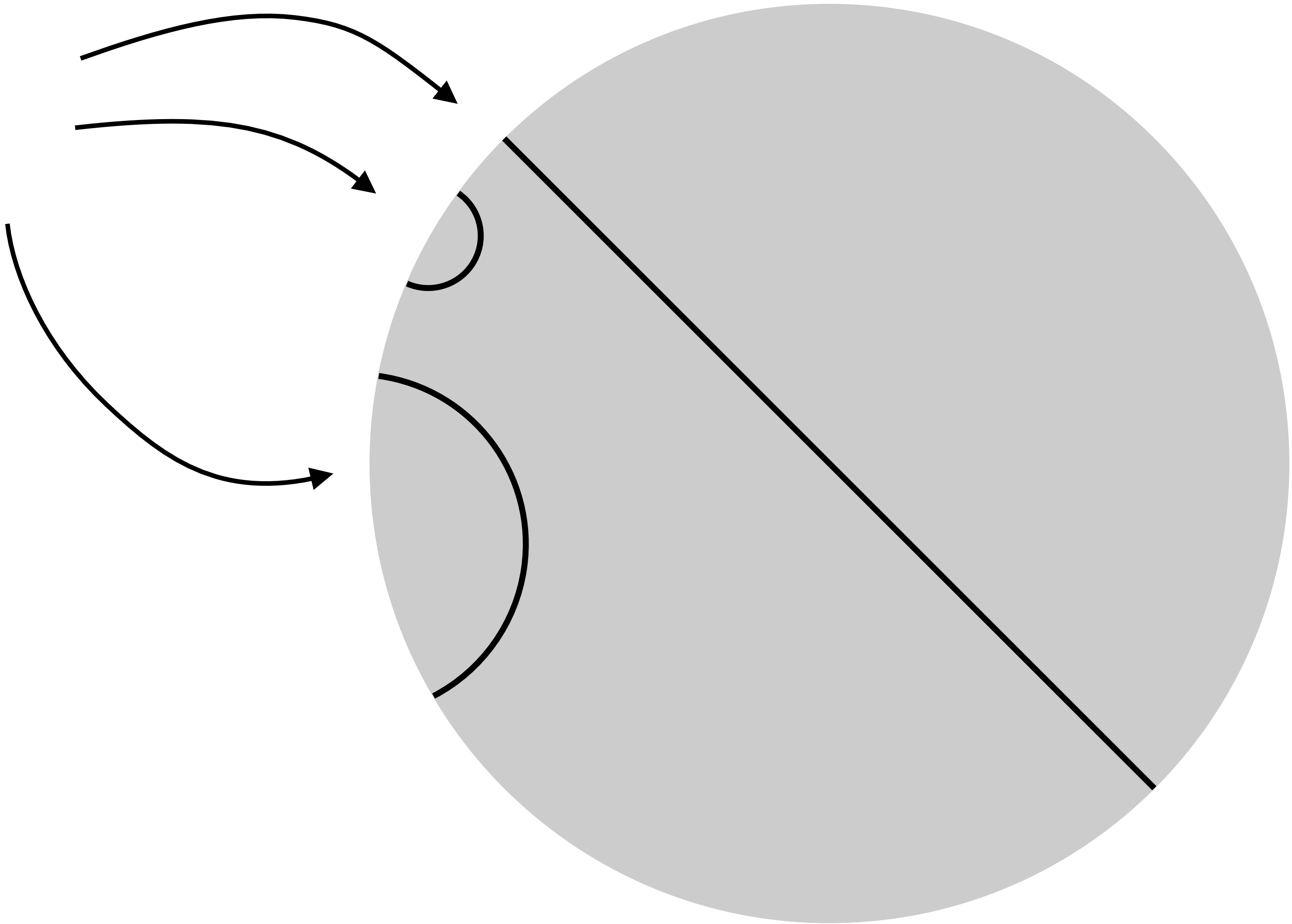
distances are different!

$$|c| = \infty$$

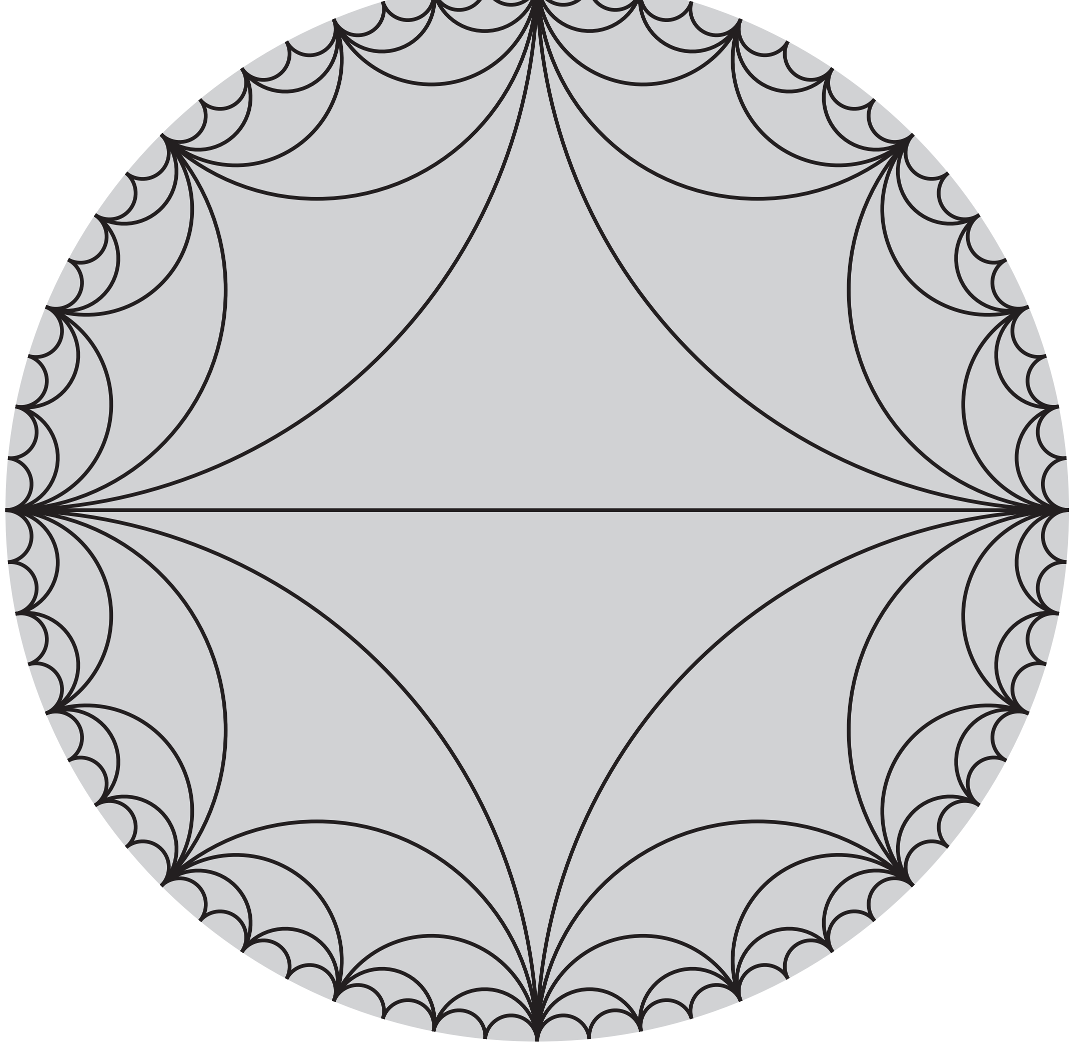


geodesics

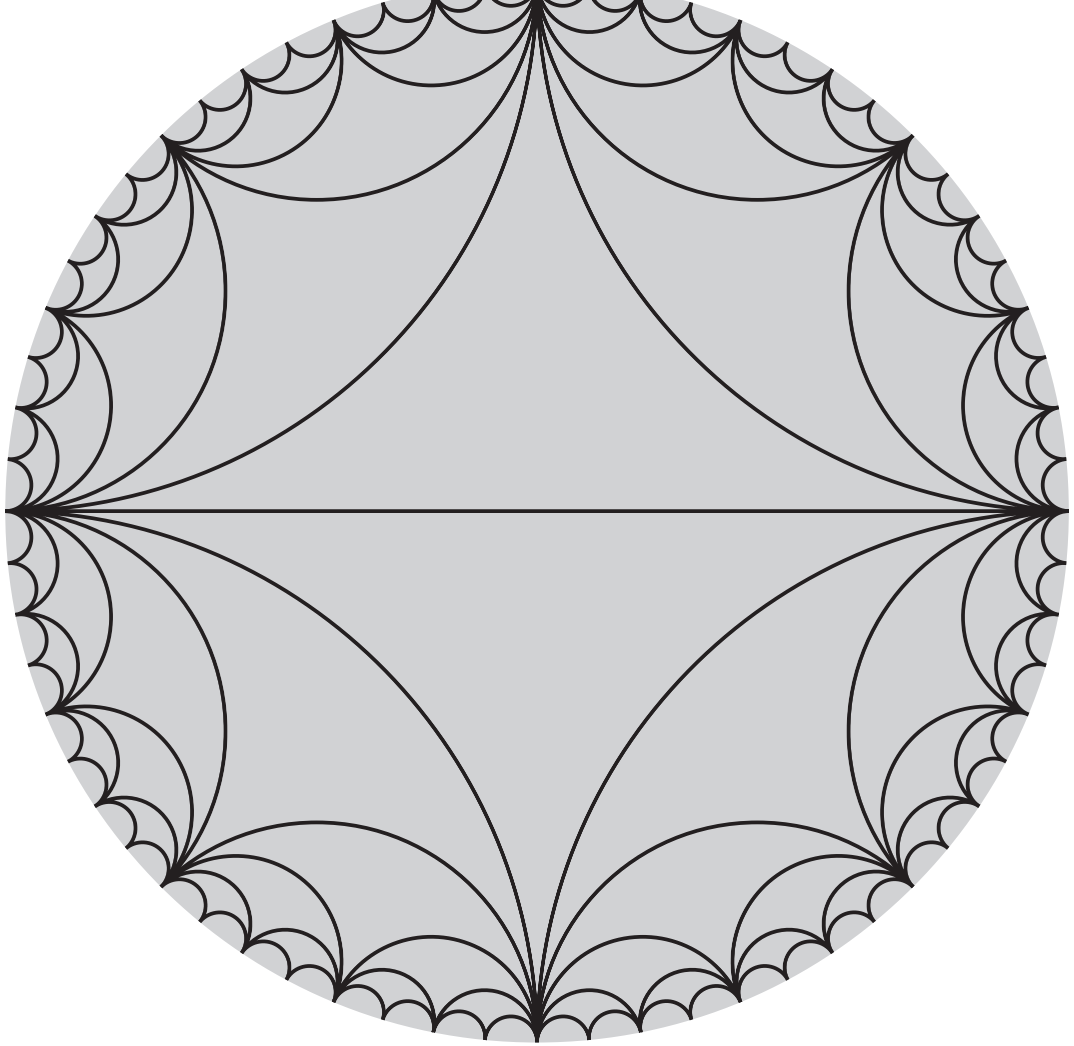
are **diameters**
and **circles** that
meet the
boundary at
right angles



Tessellations

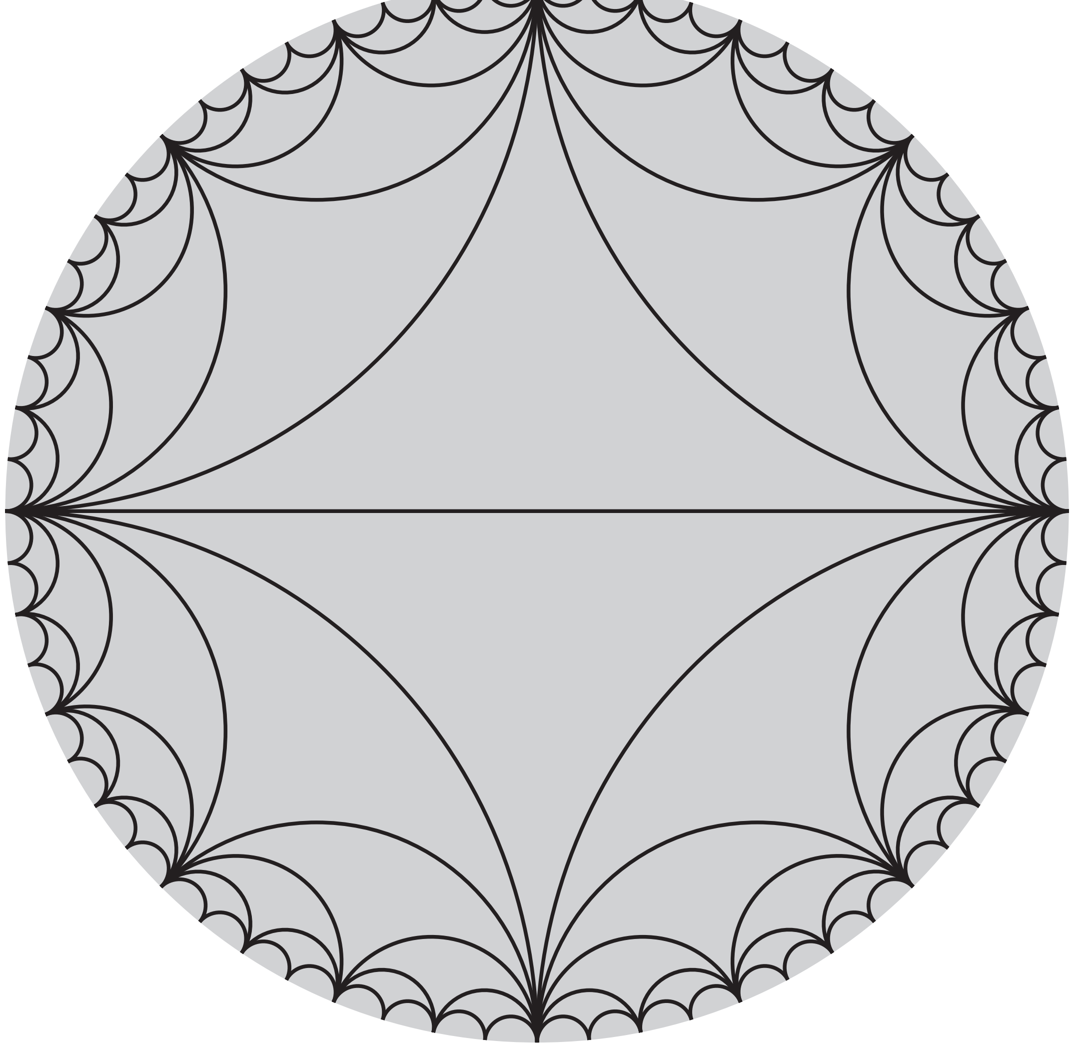


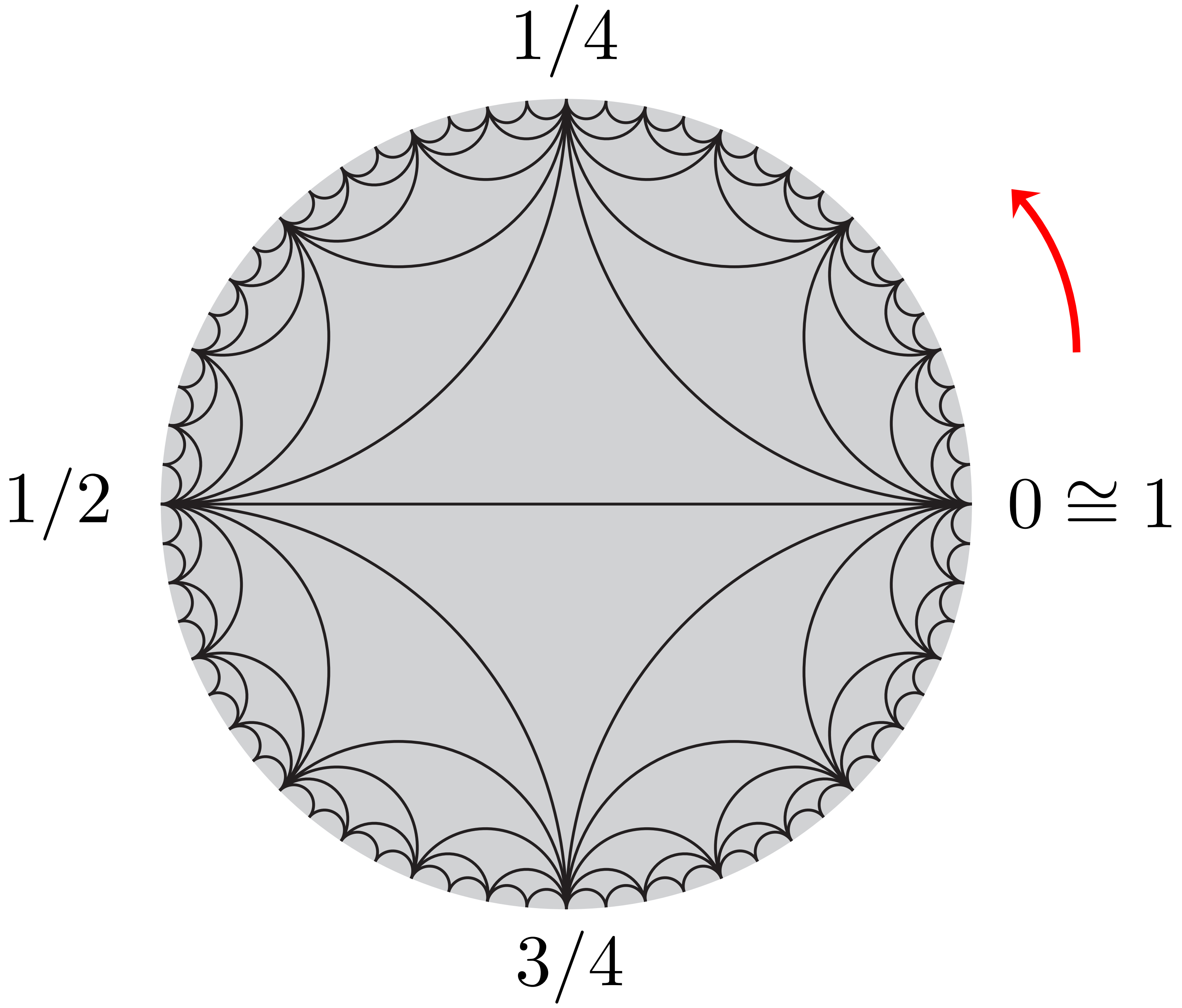
tessellation into **triangles**

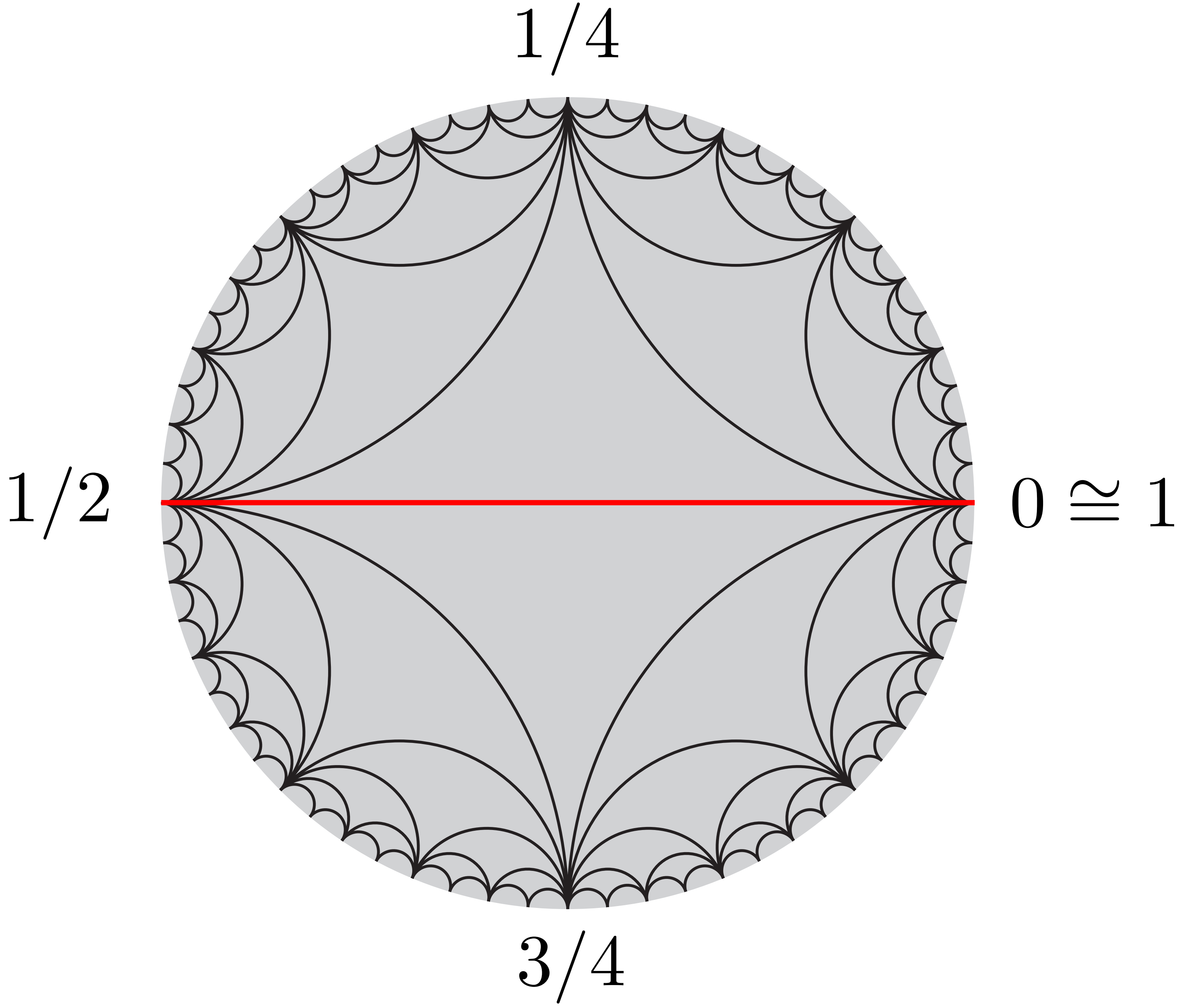


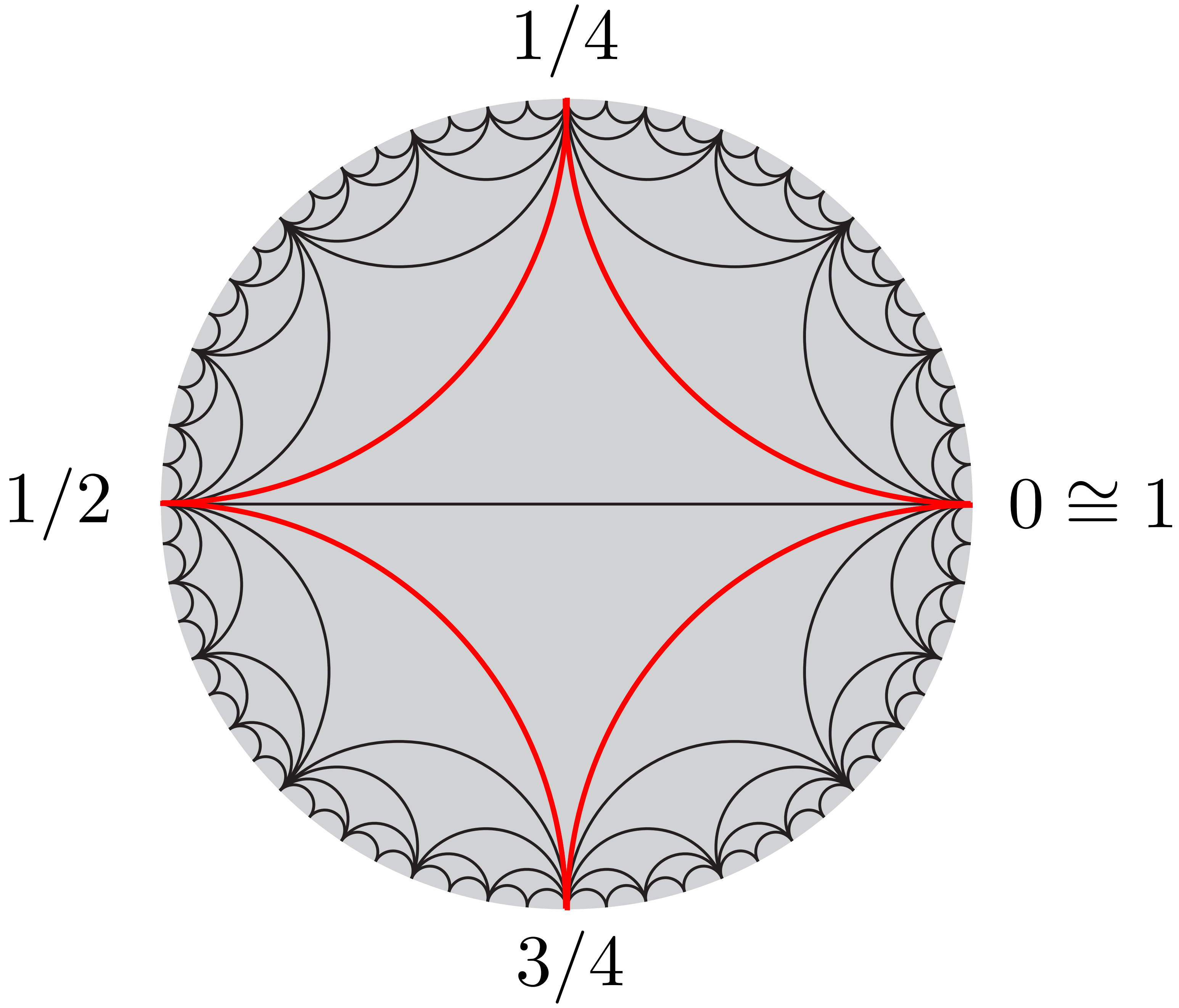
tessellation into **triangles**

all **vertices** of the triangles lie on the **boundary**



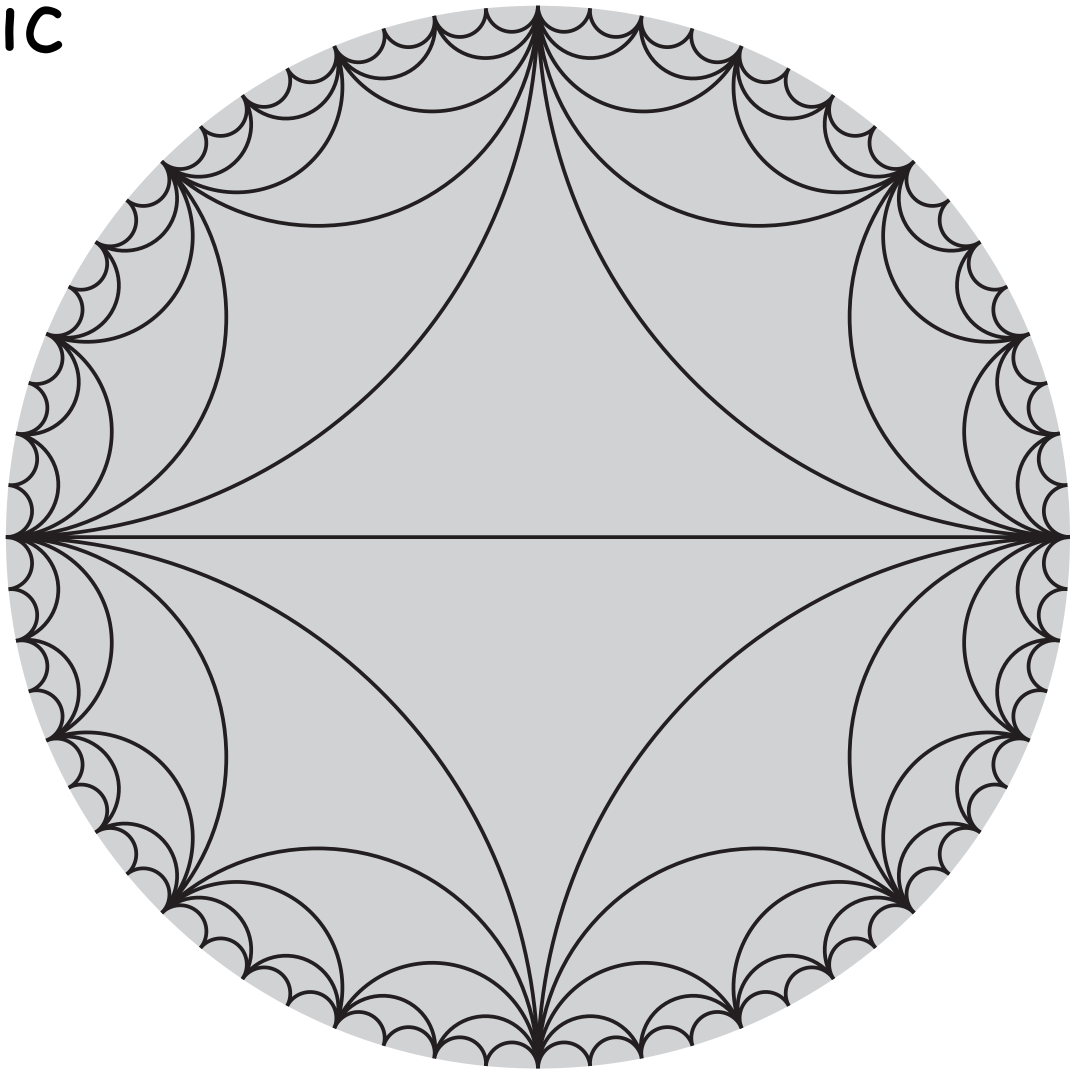

1/4
1/2
0 ≅ 1
3/4


1/4
1/2
$0 \cong 1$
3/4



1/4
1/2
0 ≅ 1
3/4

standard dyadic
tessellation



dyadic rational number

$$\frac{a}{2^n}$$

$$(a, n \in \mathbb{N})$$

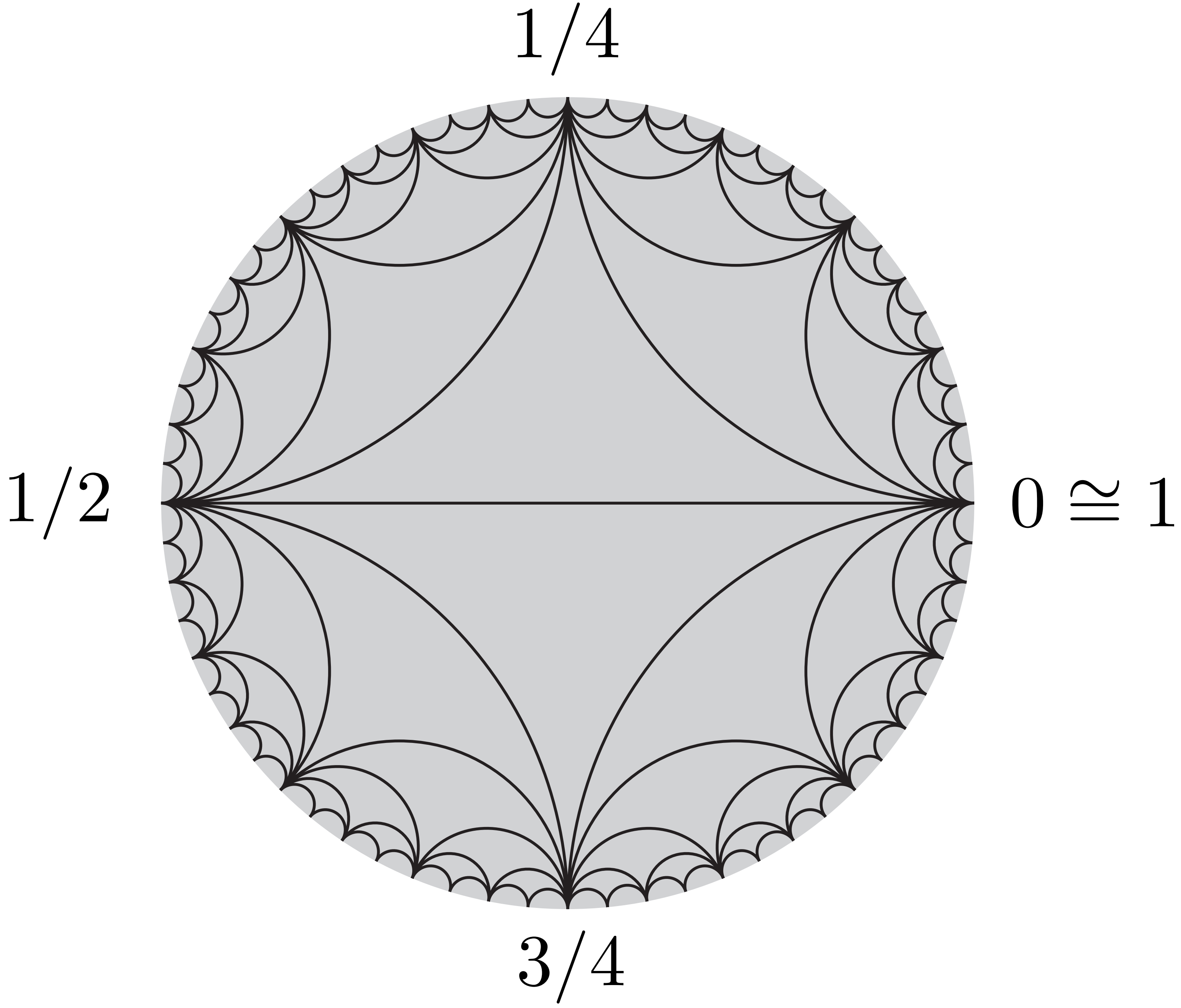
dyadic rational number

$$\frac{a}{2^n}$$

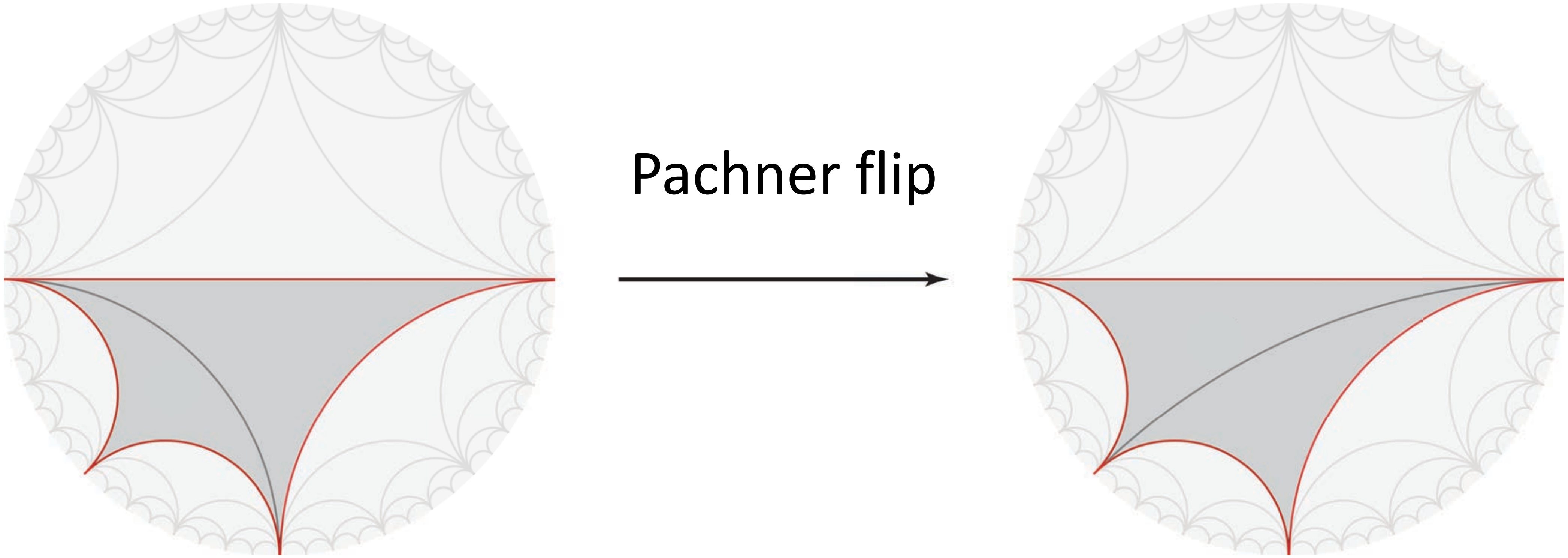
$$(a, n \in \mathbb{N})$$

for example

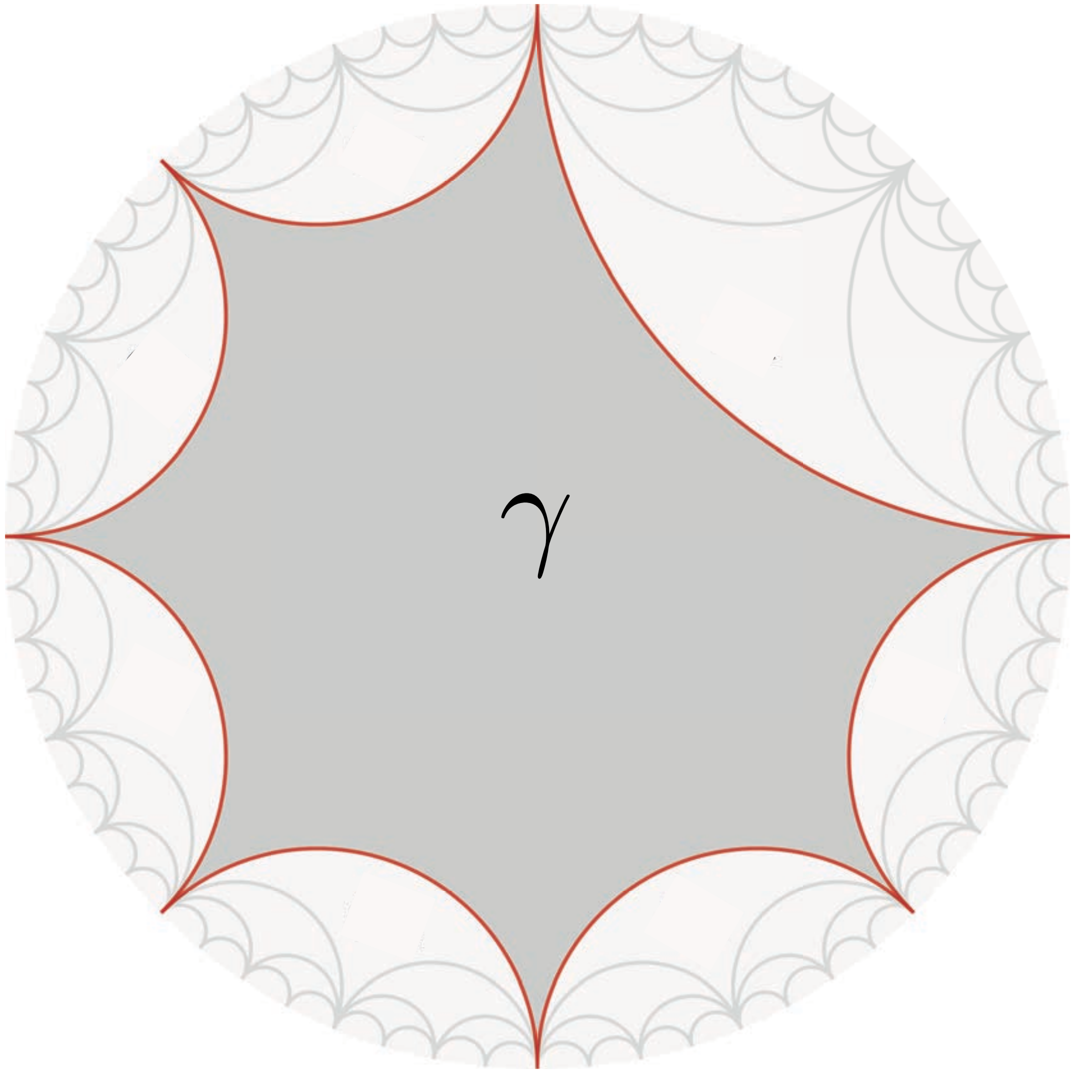
$$\frac{1}{2}, \quad \frac{3}{4}, \quad \frac{7}{8}$$


1/4
1/2
0 ≅ 1
3/4

admissible tessellations:

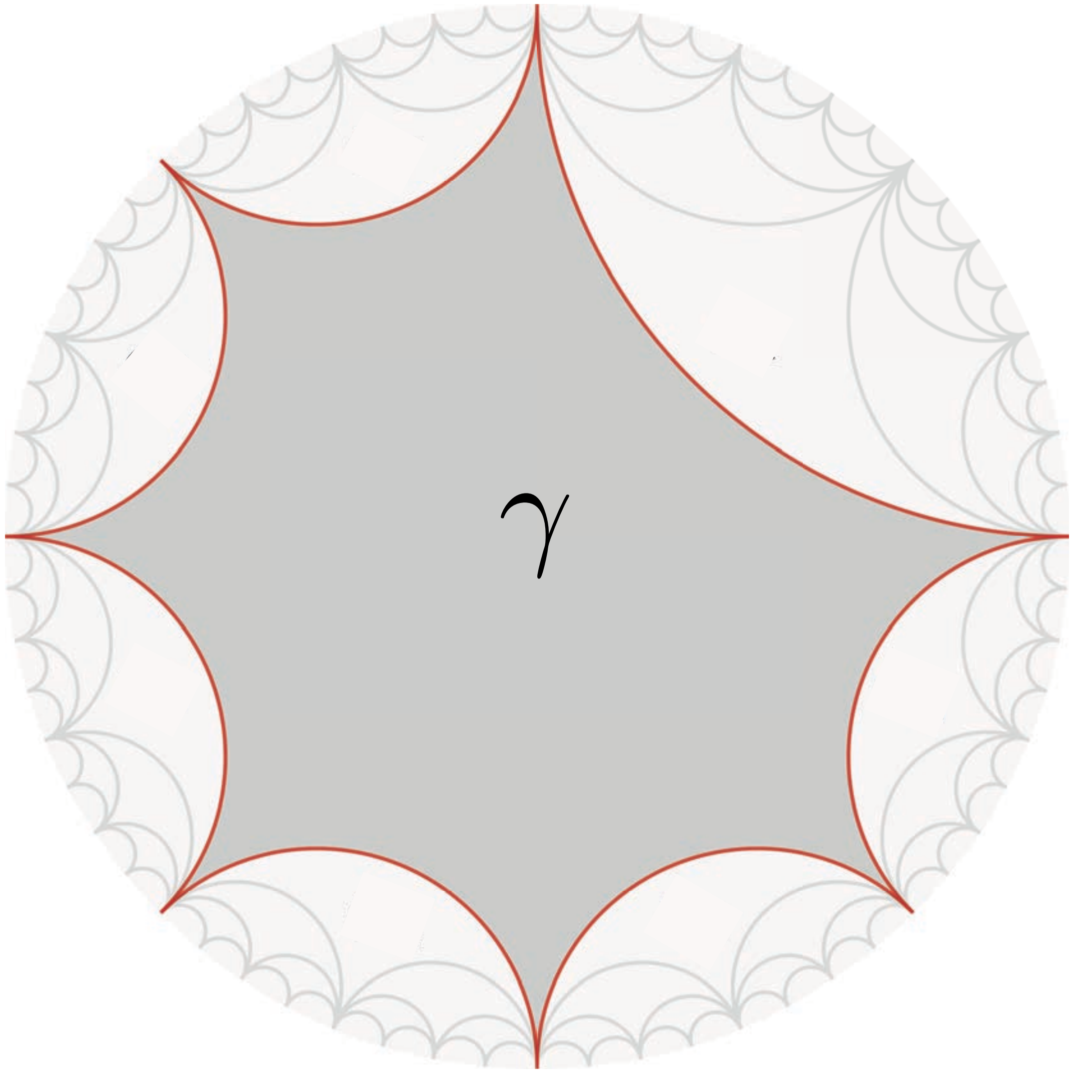


Cutoffs



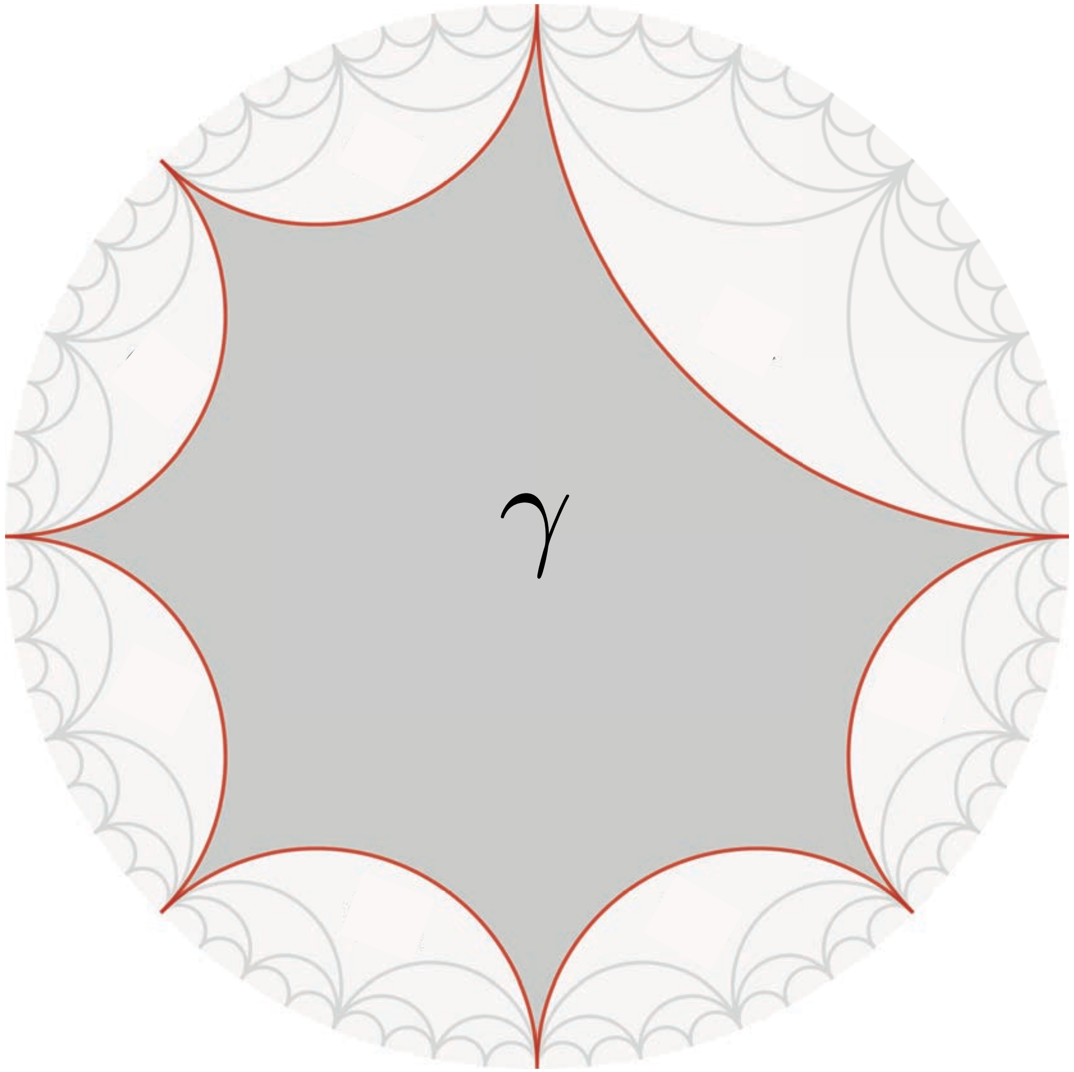
γ

finite volume



finite volume

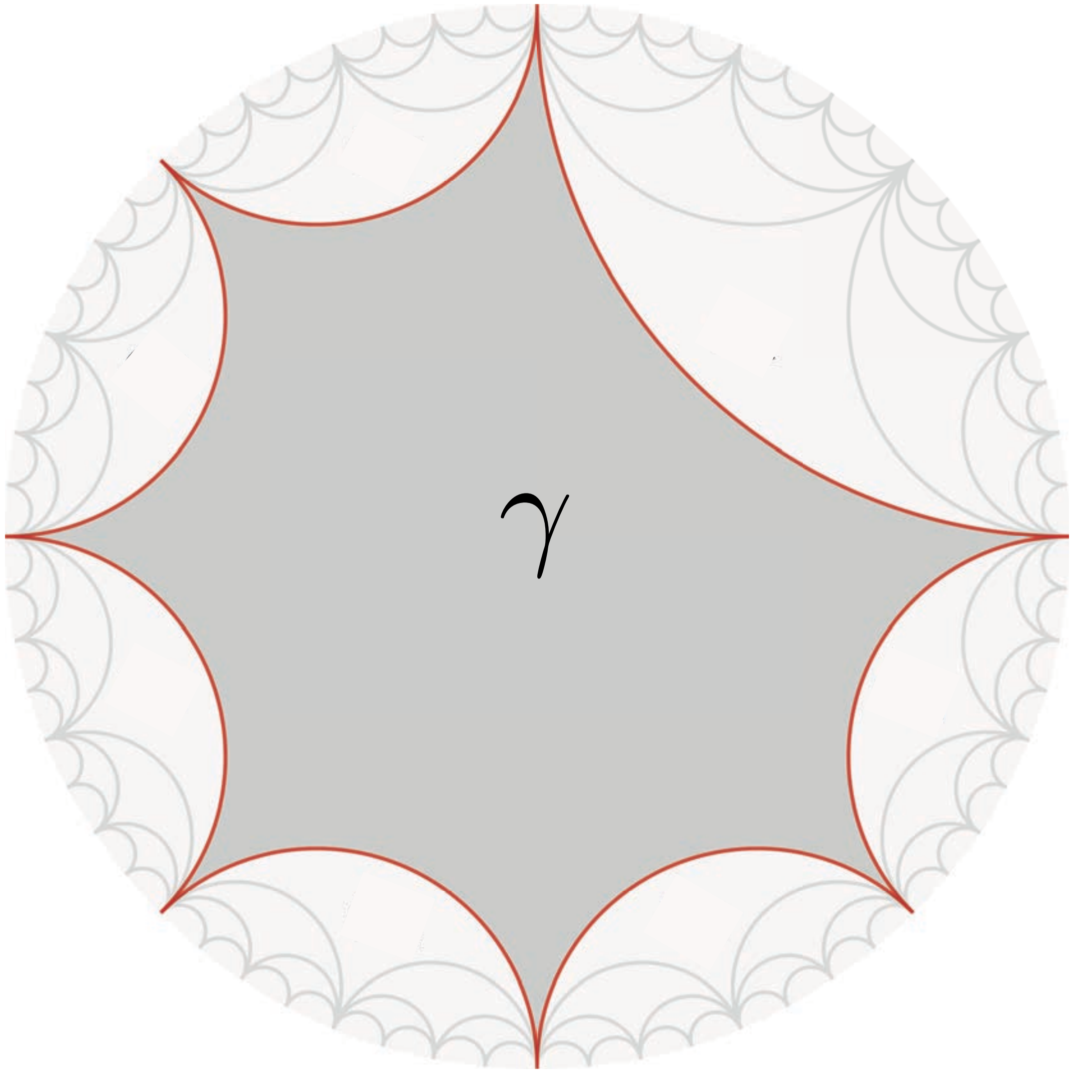
convex region

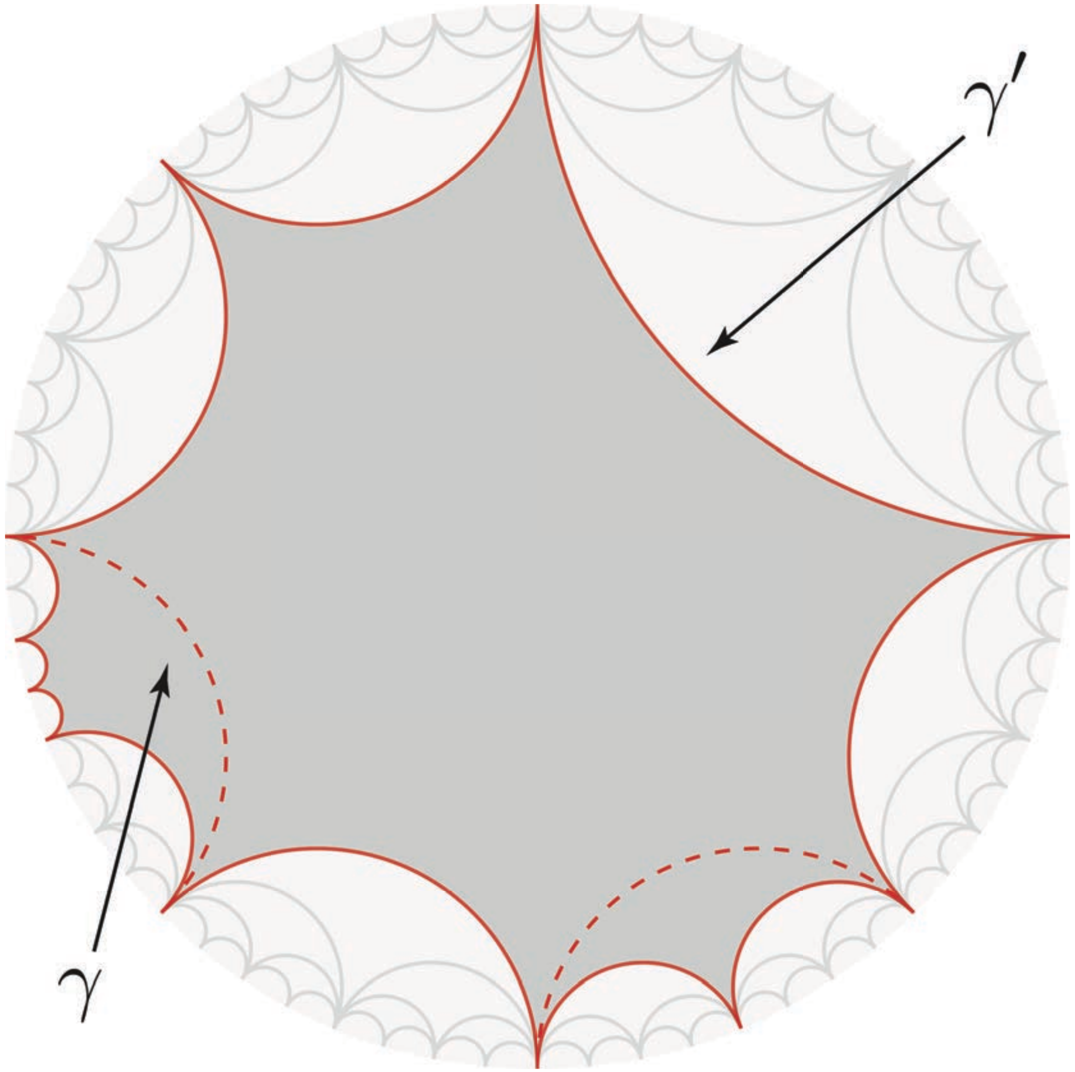


finite volume

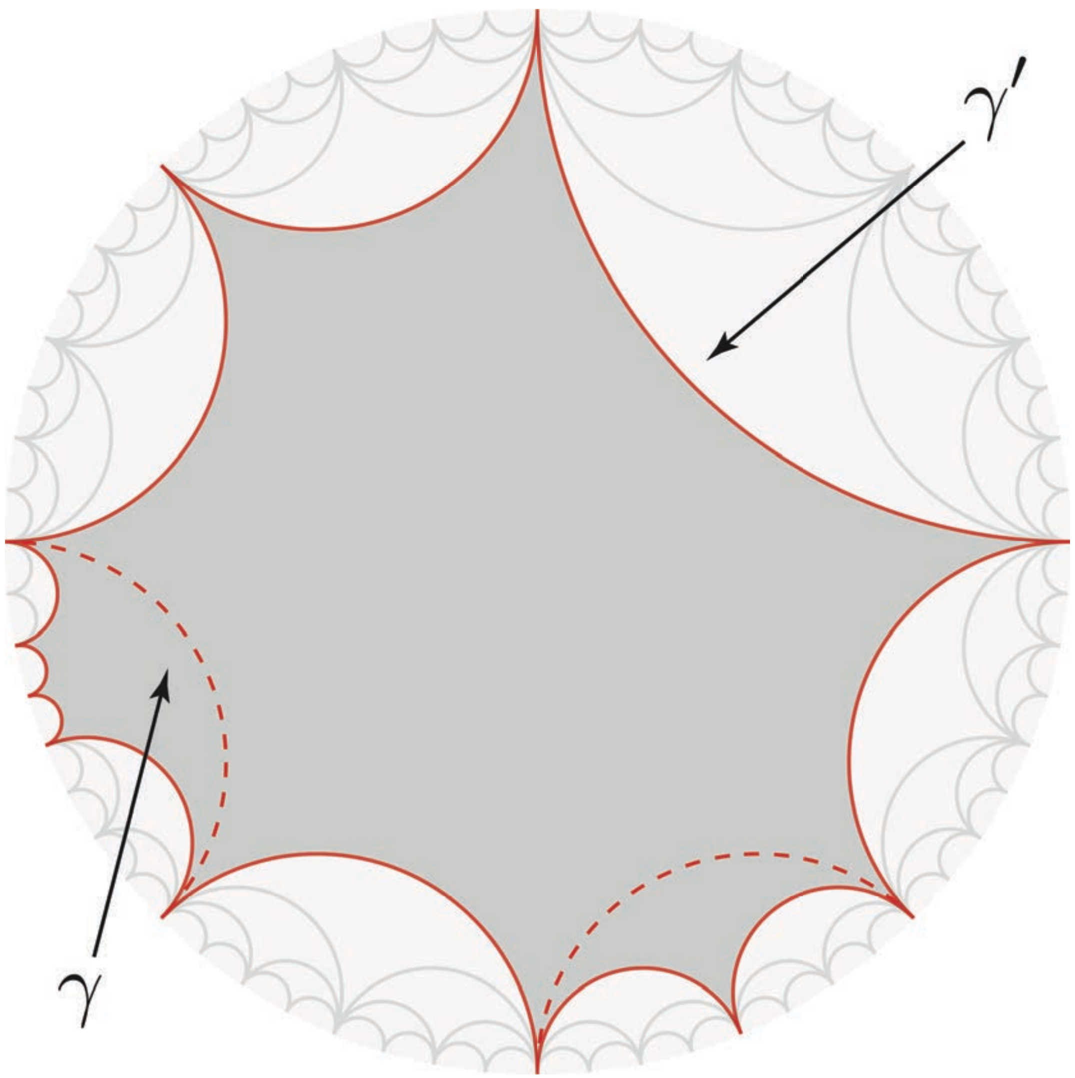
convex region

bounded by
closed curve of
finitely many **geodesics**
that come from a
tessellation

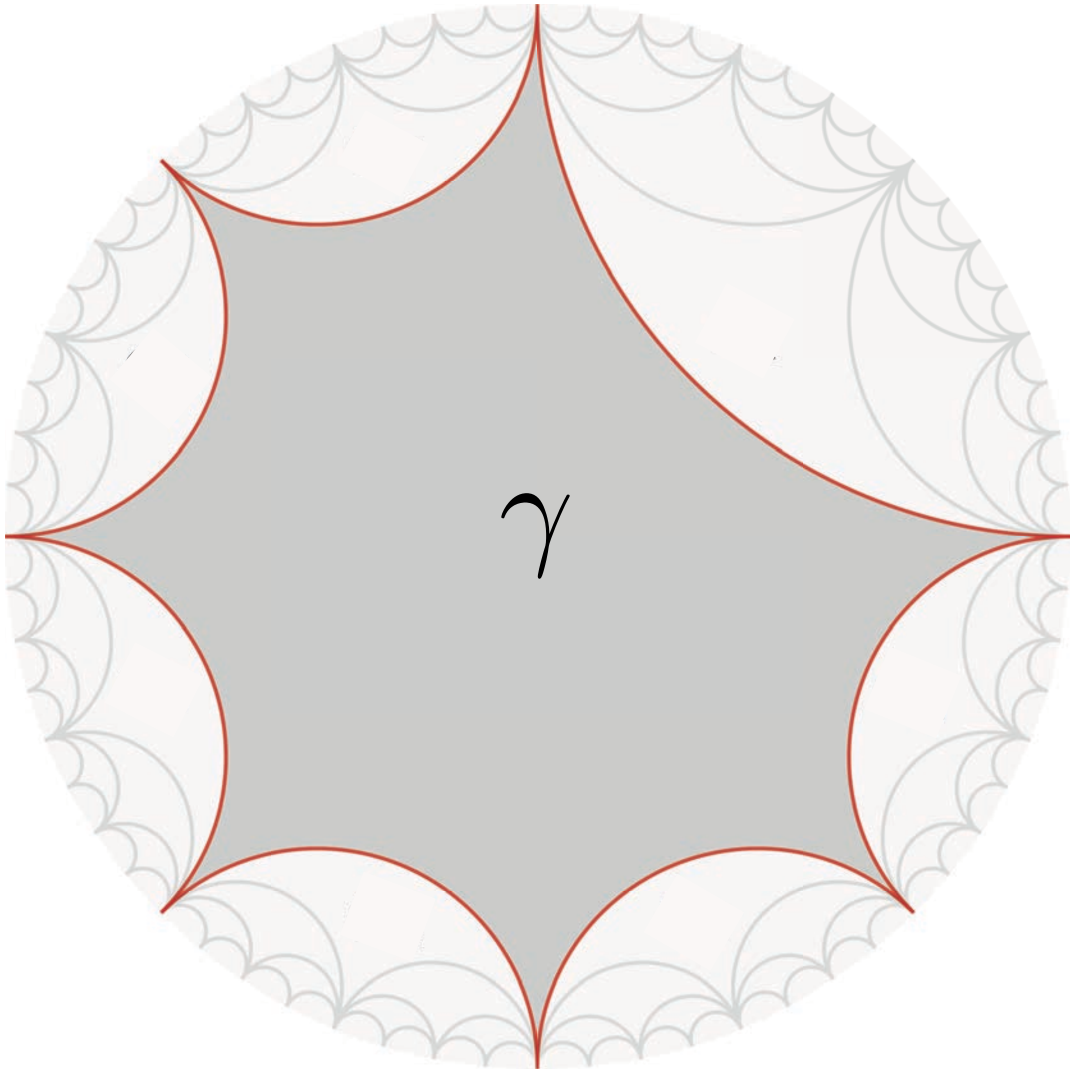




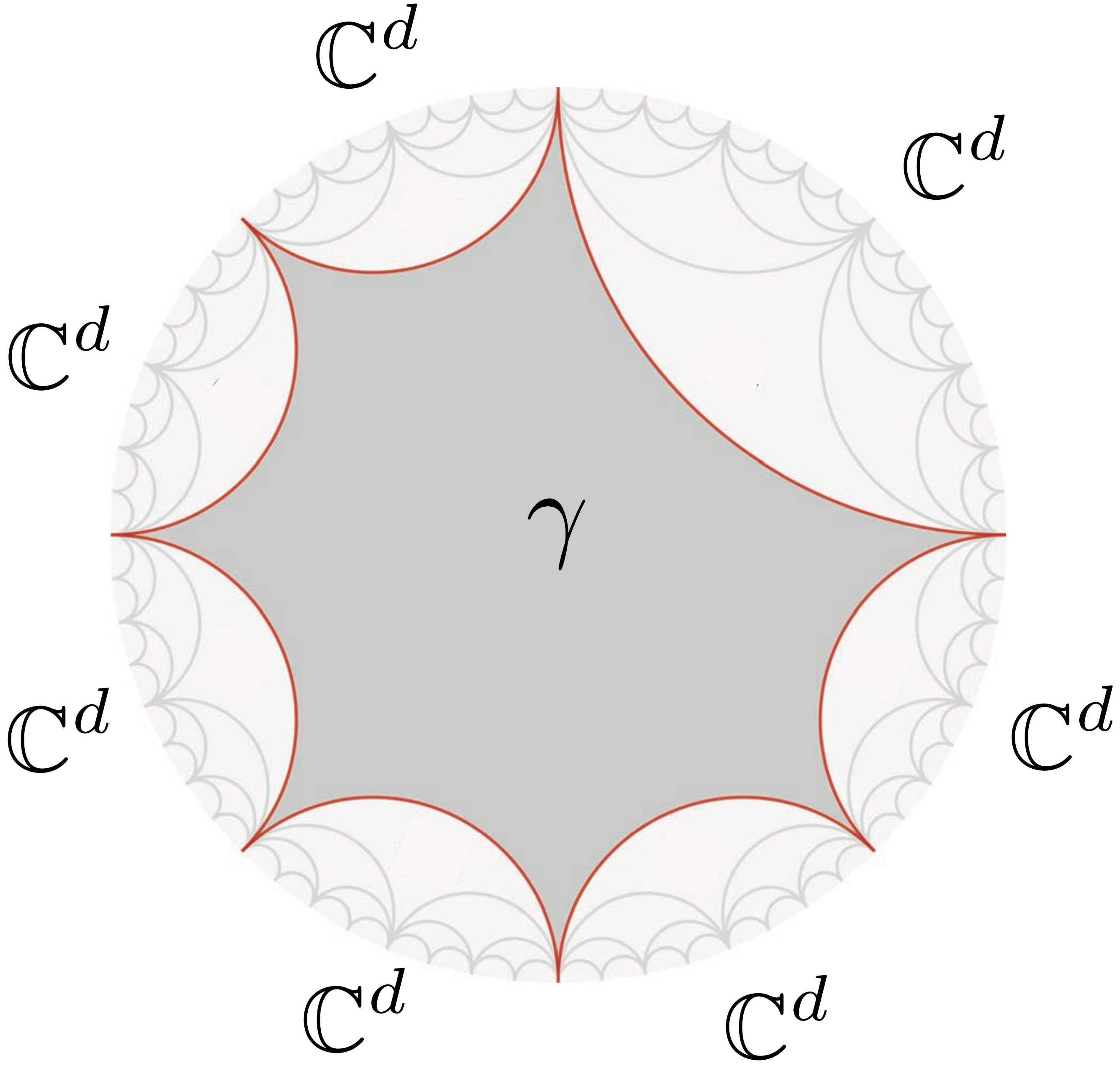
γ′
γ

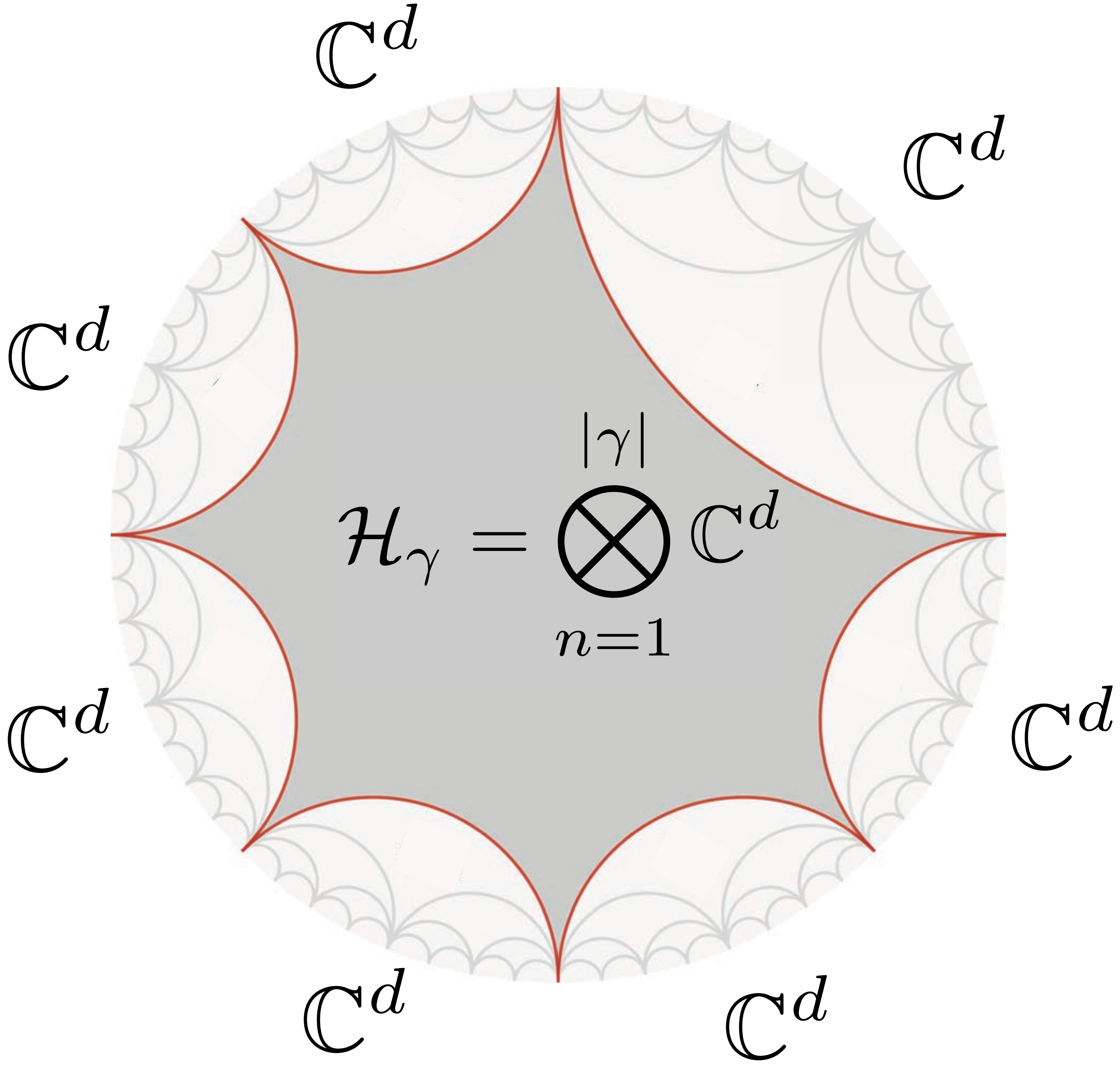


$$\gamma' \geq \gamma$$



γ

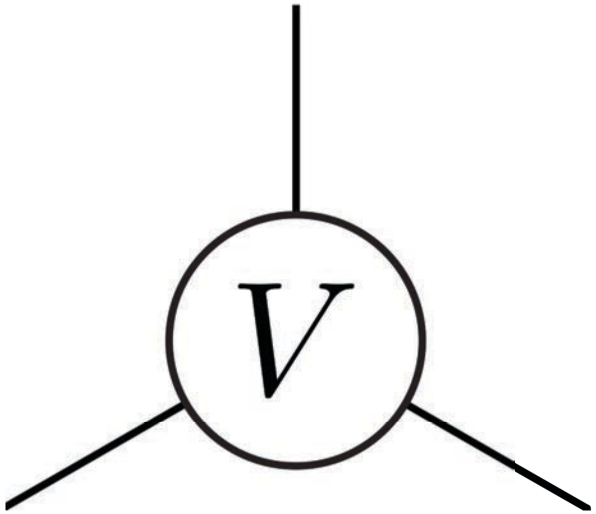

$\mathbb{C}^d$
$\mathbb{C}^d$
$\mathbb{C}^d$
γ
$\mathbb{C}^d$
$\mathbb{C}^d$
$\mathbb{C}^d$
$\mathbb{C}^d$

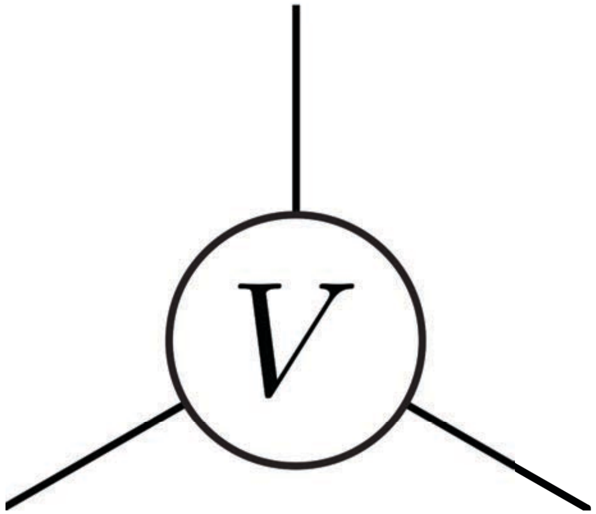


$\mathbb{C}^d$
$\mathbb{C}^d$
$\mathbb{C}^d$
$\mathcal{H}_\gamma = \bigotimes_{n=1}^{|\gamma|} \mathbb{C}^d$
$\mathbb{C}^d$
$\mathbb{C}^d$
$\mathbb{C}^d$
$\mathbb{C}^d$

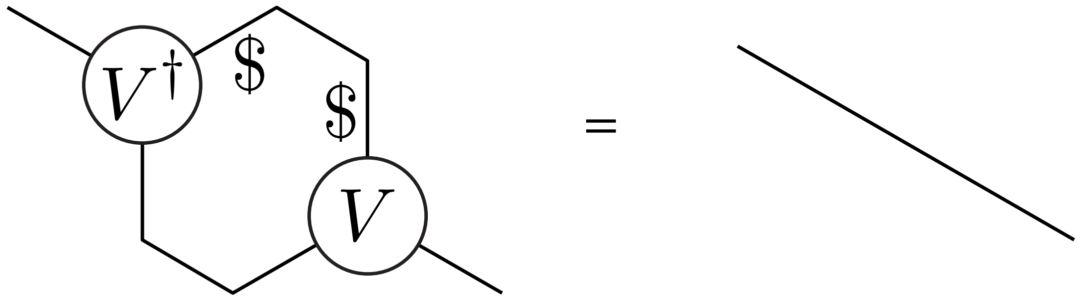
holographic states are elements of these Hilbert spaces

Perfect Tensor

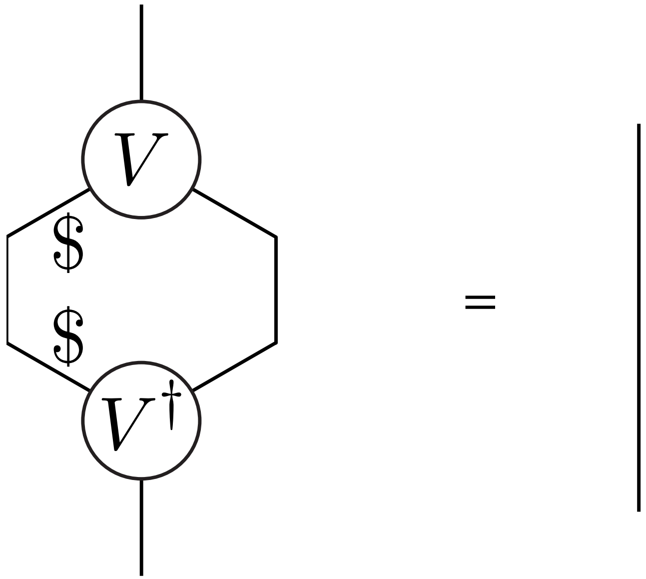

V



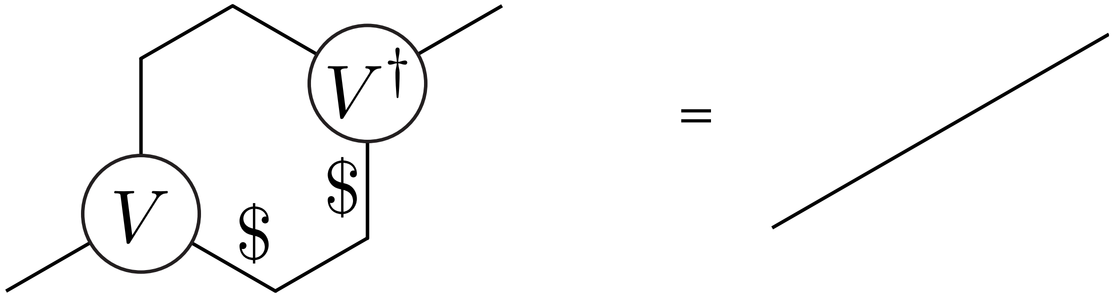
$$V : \mathbb{C}^d \otimes \mathbb{C}^d \to \mathbb{C}^d$$



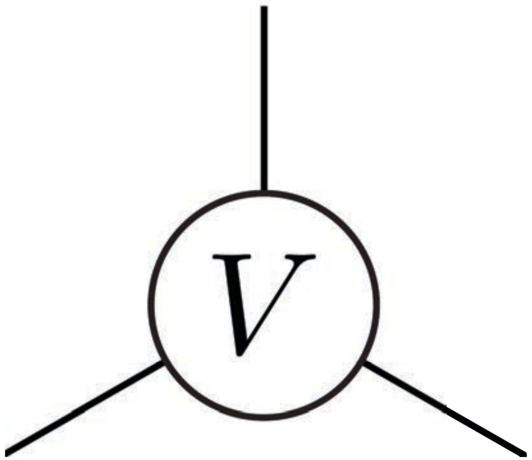
$V^\dagger$
$
$
V
=



V
$
$
$V^\dagger$
=



$V^\dagger$
$
$
V
=

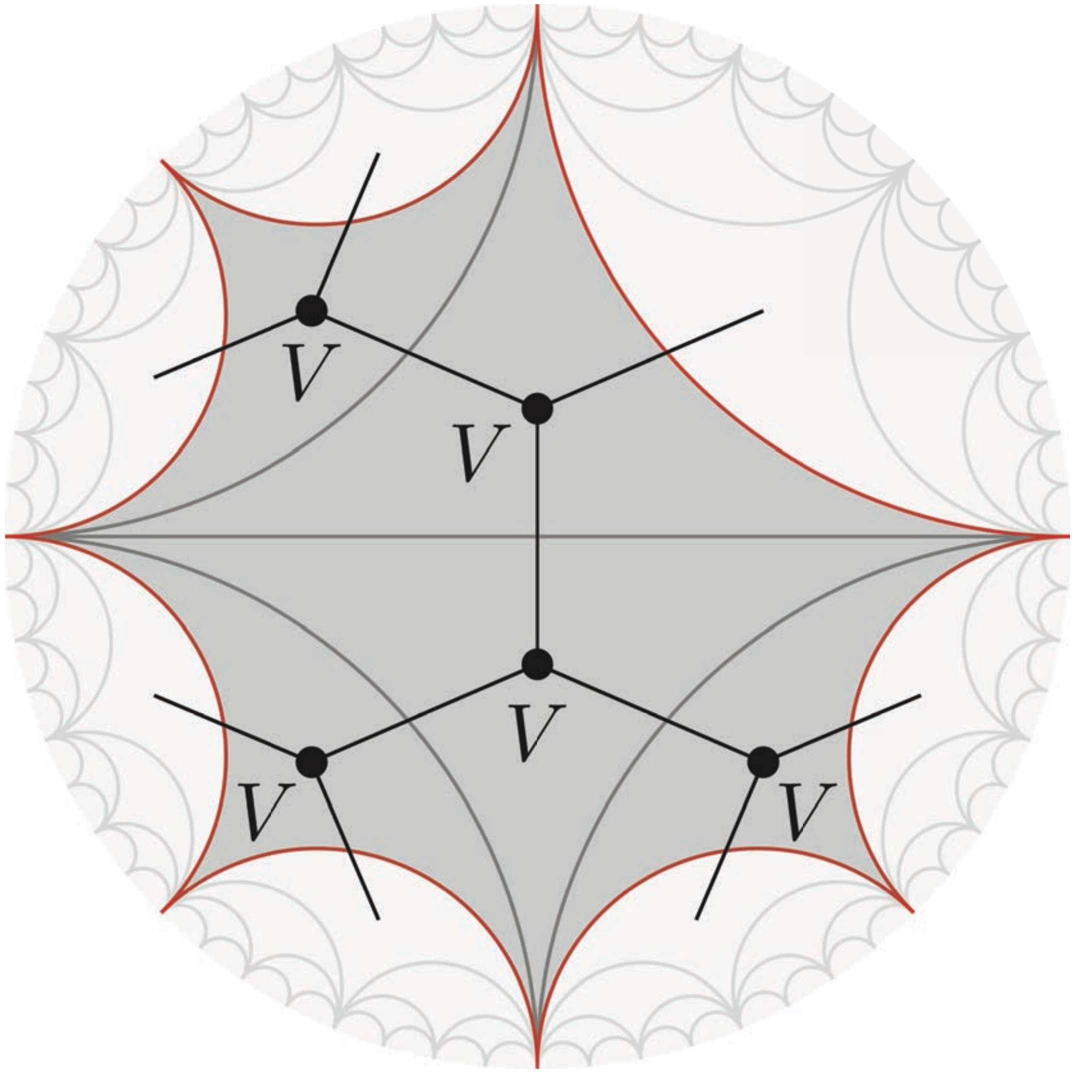


perfect
&
invariant under rotations

for example,

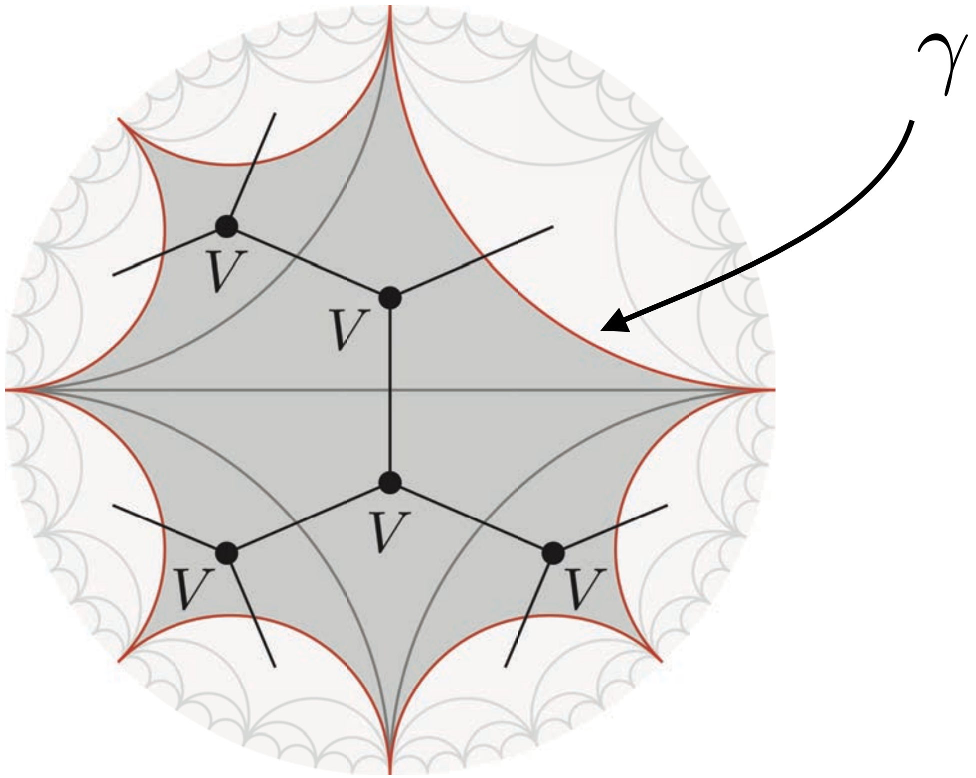
$$V : \mathbb{C}^3 \otimes \mathbb{C}^3 \to \mathbb{C}^3,$$

$$\langle j|V|kl\rangle = \begin{cases} 0 & \text{if } j = k,\ k = l, \text{ or } j = l, \\ 1 & \text{otherwise} \end{cases}$$

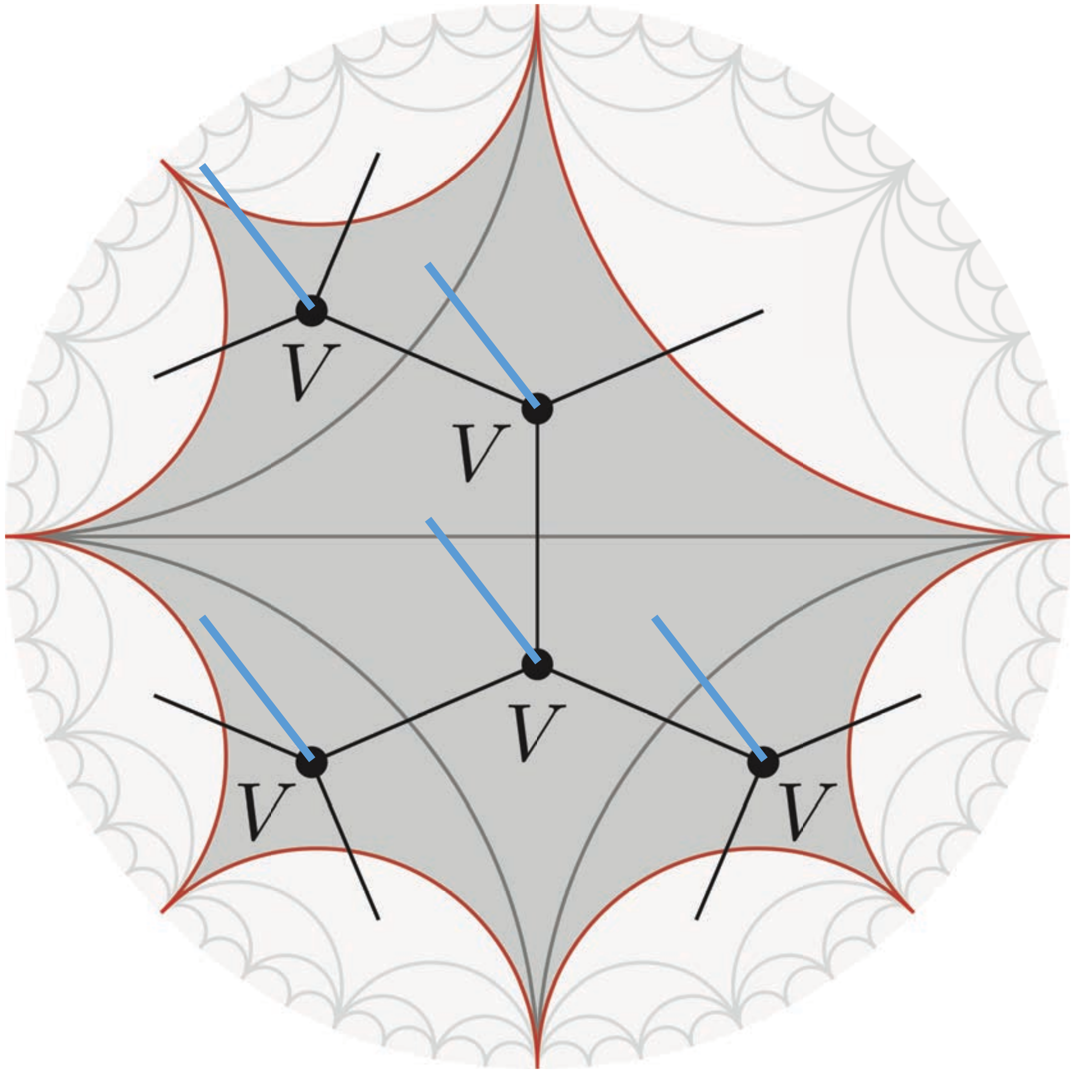

V
V
V
V
V

Holographic state

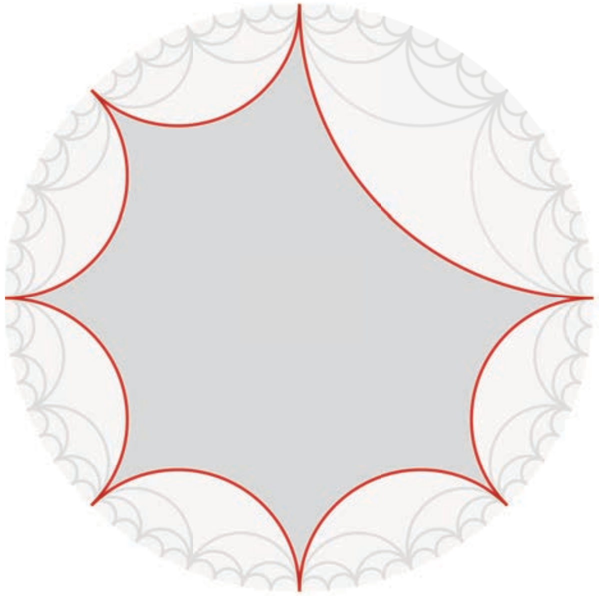
$$|\Omega_\gamma\rangle =$$

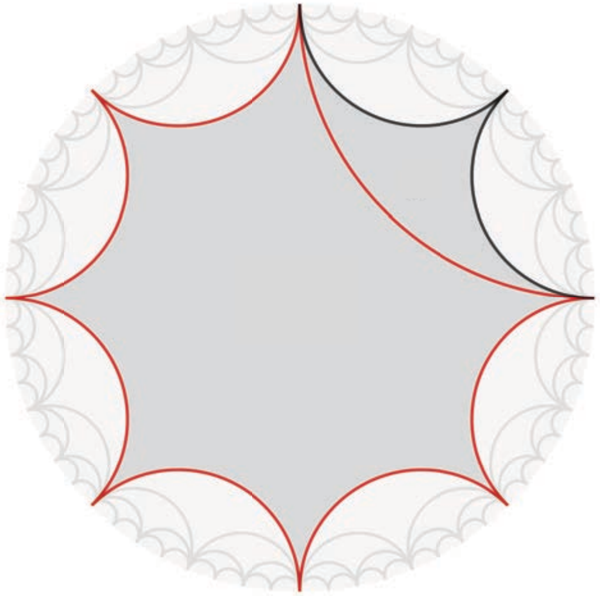


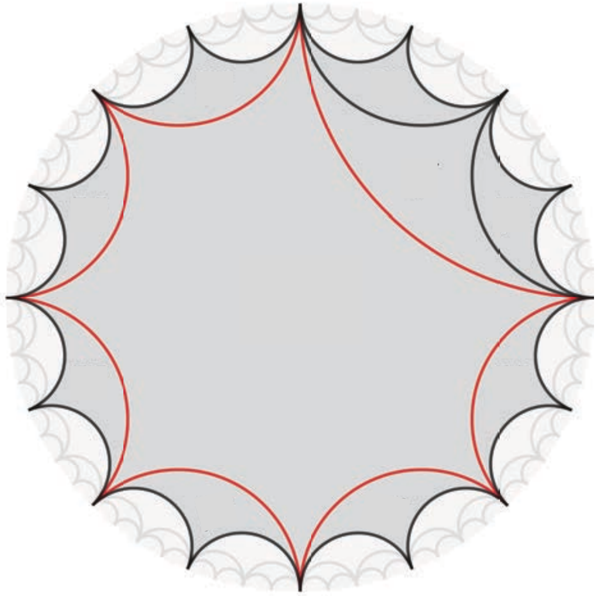
Holographic code



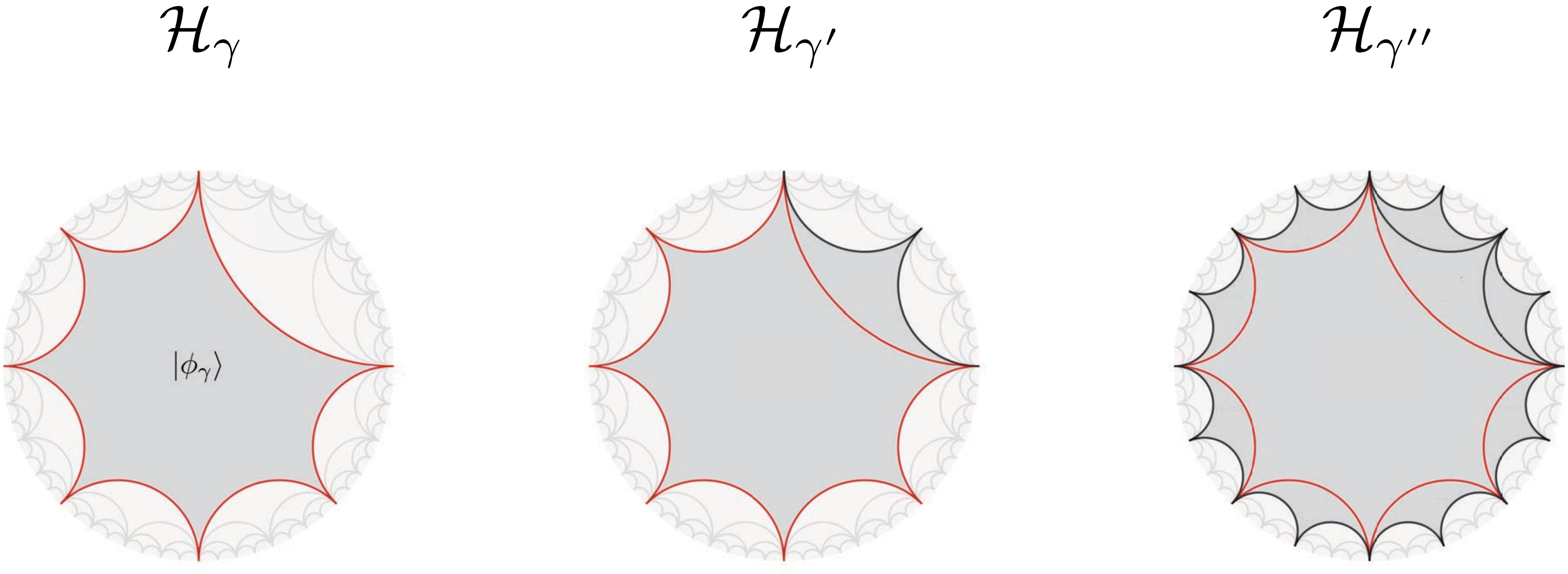
$\mathcal{H}_\gamma$ $\mathcal{H}_{\gamma'}$ $\mathcal{H}_{\gamma''}$



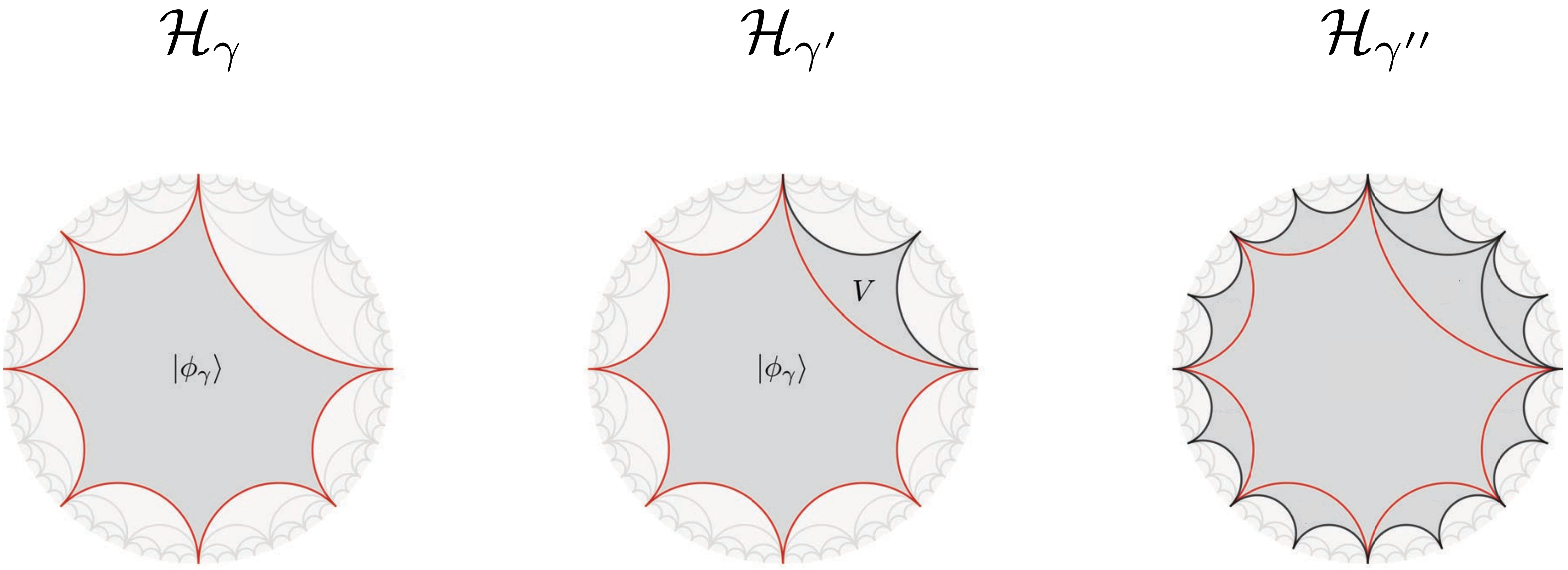




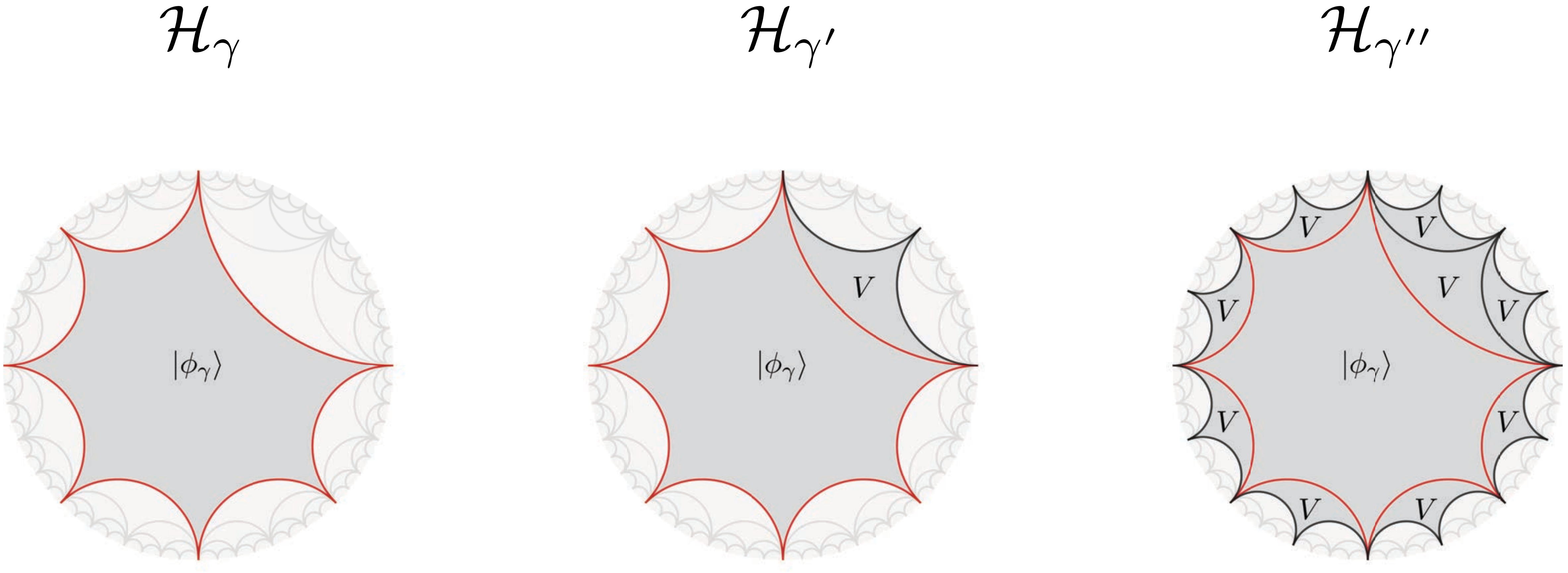
γ γ' γ''



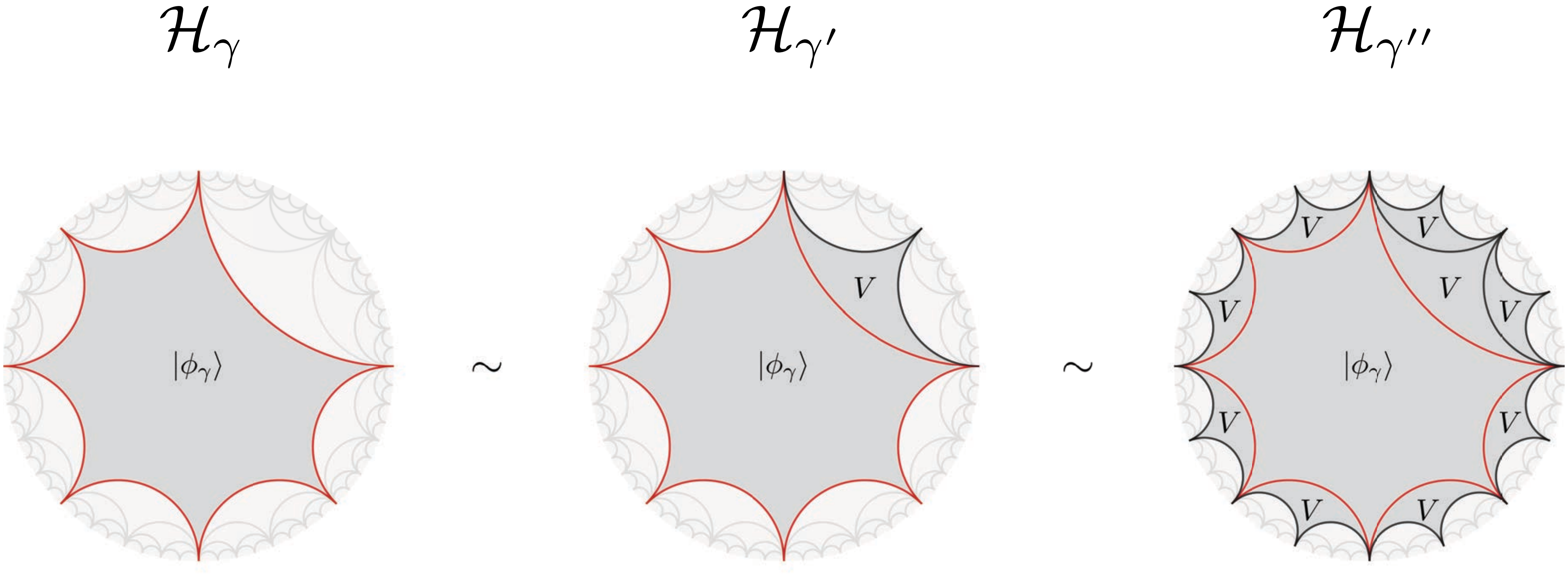
$\mathcal{H}_\gamma$
$\mathcal{H}_{\gamma'}$
$\mathcal{H}_{\gamma''}$
$|\phi_\gamma\rangle$



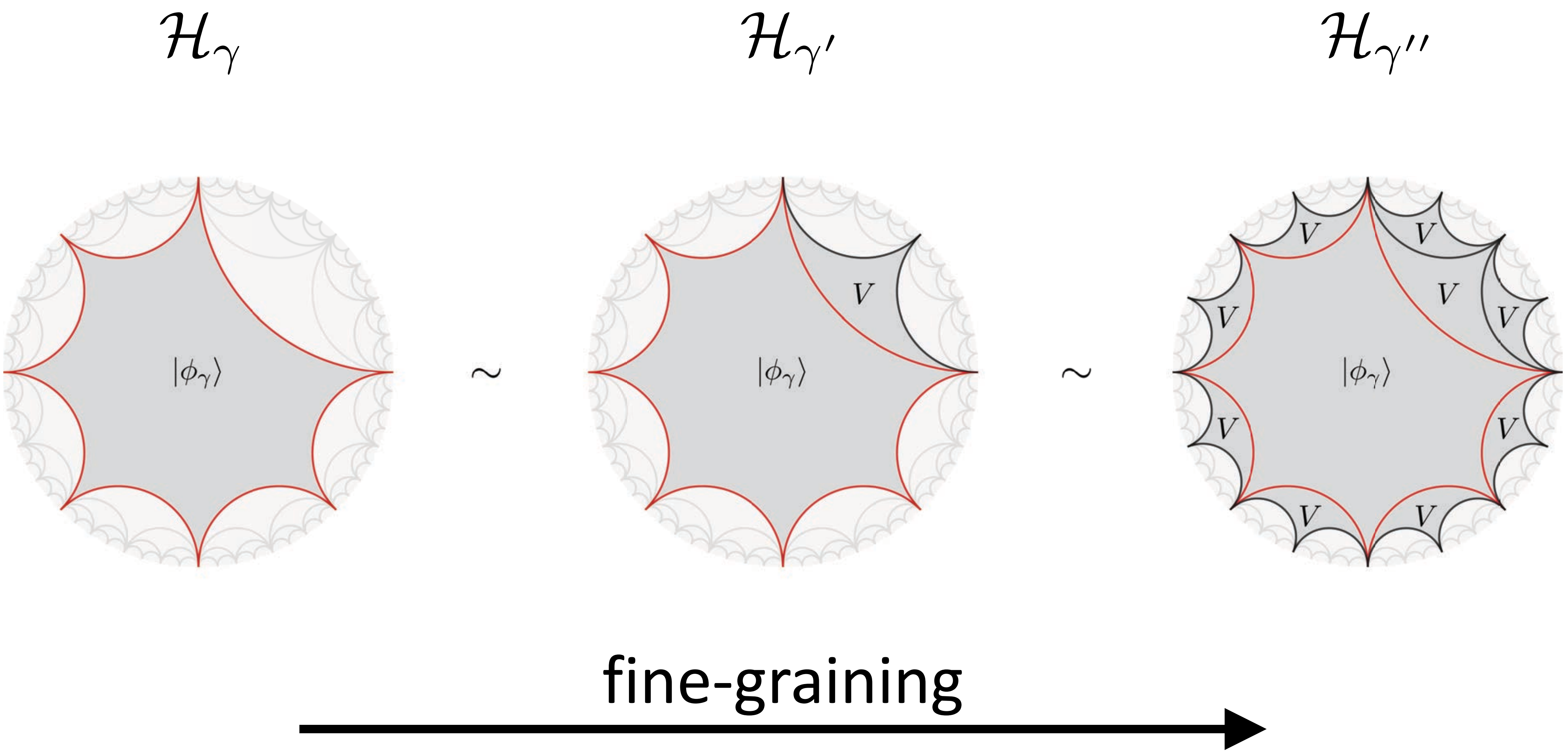
$\mathcal{H}_\gamma$
$\mathcal{H}_{\gamma'}$
$\mathcal{H}_{\gamma''}$
$|\phi_\gamma\rangle$
$|\phi_\gamma\rangle$
V



$\mathcal{H}_\gamma$
$\mathcal{H}_{\gamma'}$
$\mathcal{H}_{\gamma''}$
$|\phi_\gamma\rangle$
$|\phi_\gamma\rangle$
$|\phi_\gamma\rangle$
V
V
V
V
V
V
V
V
V
V

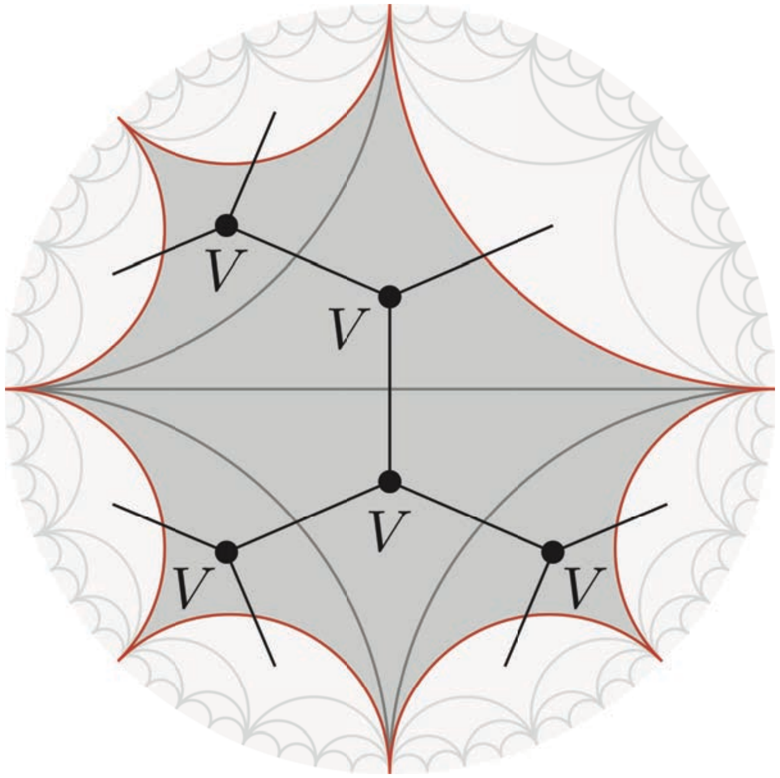


$\mathcal{H}_\gamma$
$\mathcal{H}_{\gamma'}$
$\mathcal{H}_{\gamma''}$
$|\phi_\gamma\rangle$
$\sim$
$|\phi_\gamma\rangle$
V
$\sim$
$|\phi_\gamma\rangle$
V



$\mathcal{H}_\gamma$
$\mathcal{H}_{\gamma'}$
$\mathcal{H}_{\gamma''}$
$|\phi_\gamma\rangle$
$\sim$
$|\phi_\gamma\rangle$
V
$\sim$
$|\phi_\gamma\rangle$
V
fine-graining

exactly one holographic state
for every tessellation



Dynamics

Dynamics ?

Ordinary **quantum mechanics**

$$\boxed{|\psi(t)\rangle = U(t)|\psi_0\rangle}$$

$$U(t_1 + t_2) = U(t_1)U(t_2)$$

$$U(0) = I$$

here:

$$|f\rangle = U(f)|\Omega\rangle$$

$f \in$ a group G

U unitary representation of G

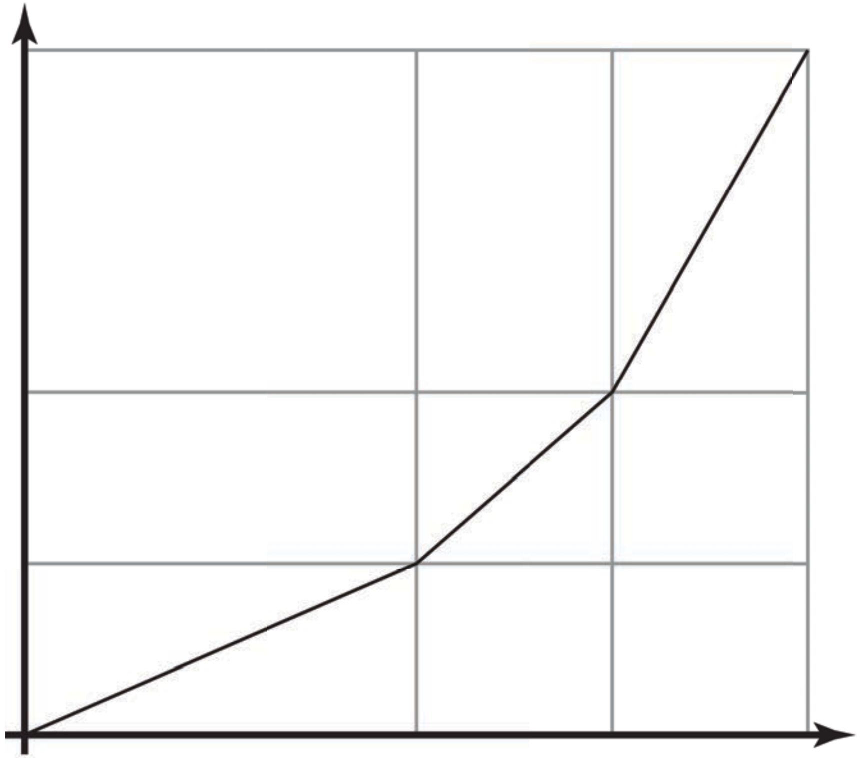
Thompson's group T

Definition. Thompson's group T is the group of piecewise linear homeomorphisms of the circle S^1 (understood as the interval $[0,1]$ with endpoints identified) that

- map dyadic rational numbers to dyadic rational numbers,
- are differentiable except at finitely many dyadic rational numbers

such that

- on intervals of differentiability the slopes are powers of 2.



J. W. Cannon, W. J. Floyd, W. R. Parry, *Introductory Notes on Richard Thompsons's Groups*, Ens. Math. 42 (1996), no. 3–4, 215–256

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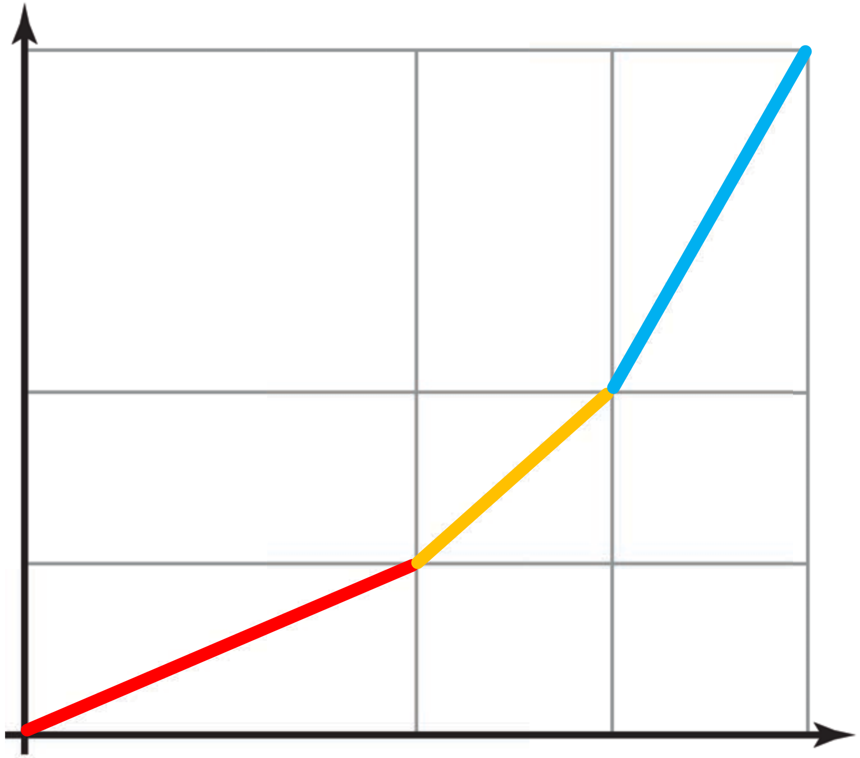

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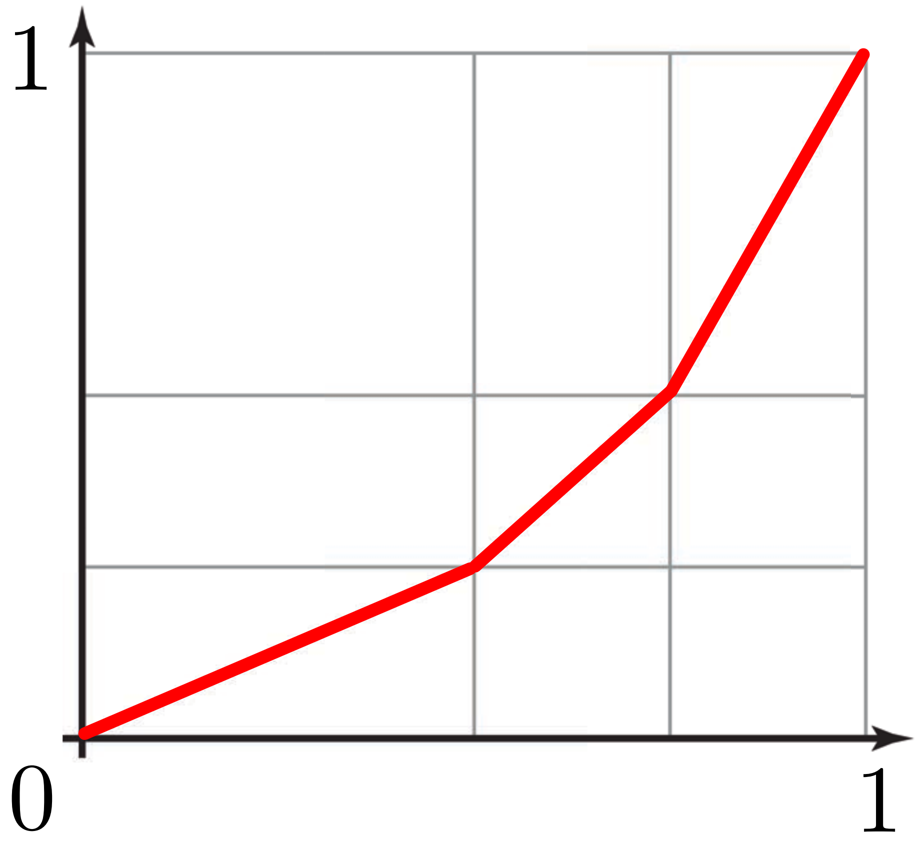

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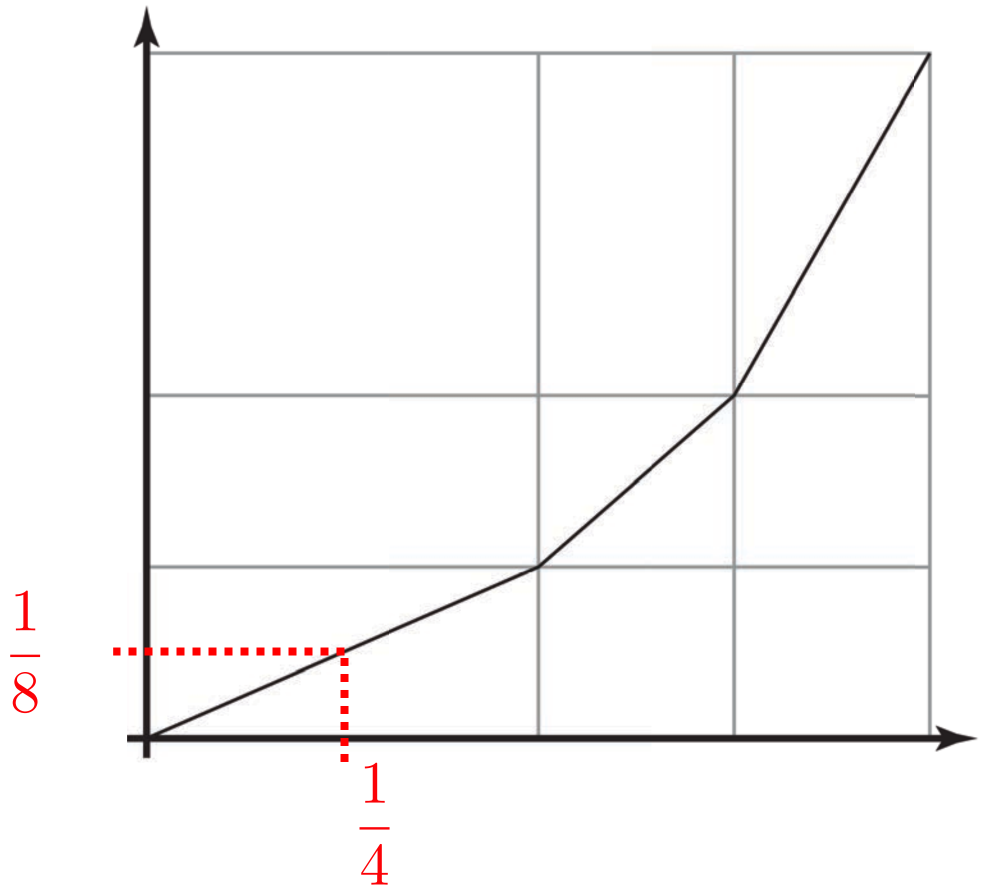

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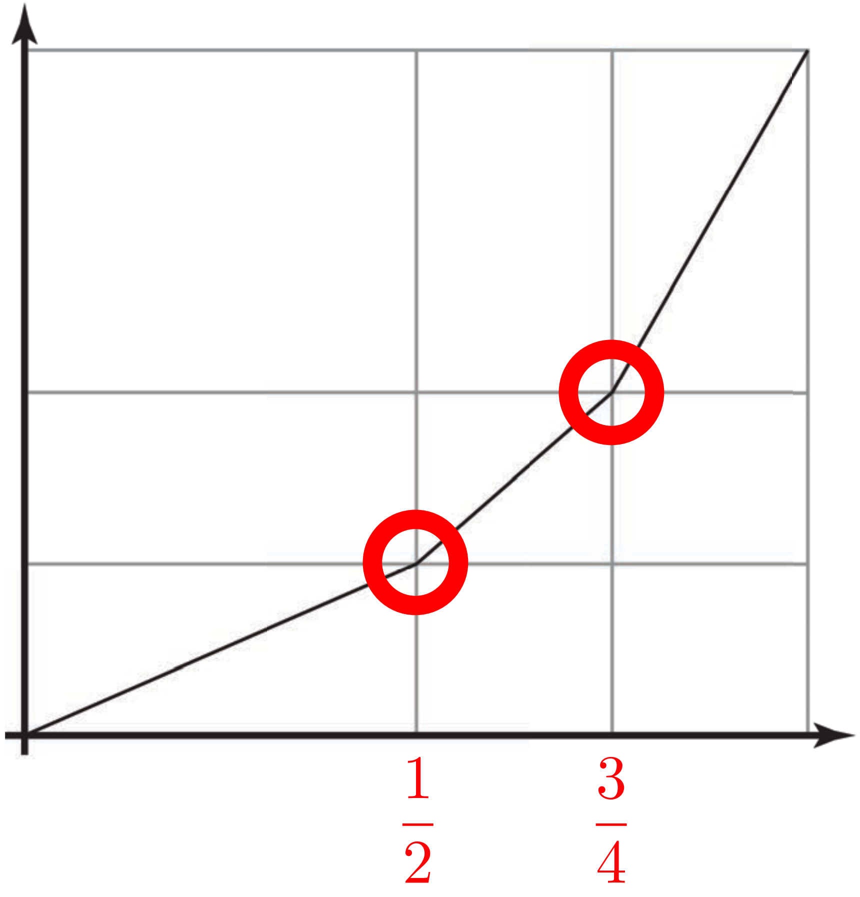

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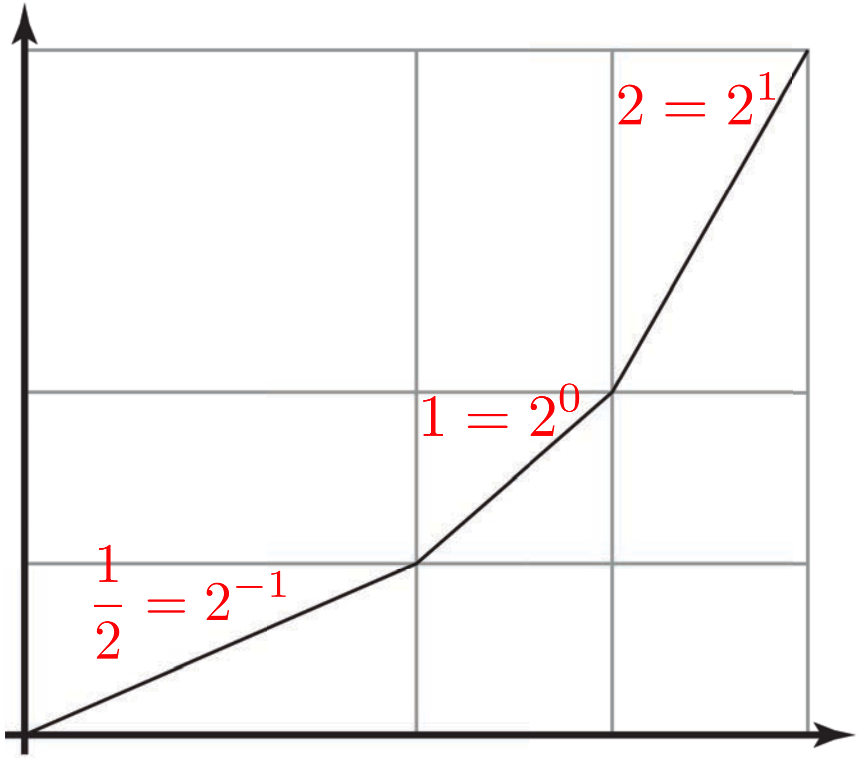
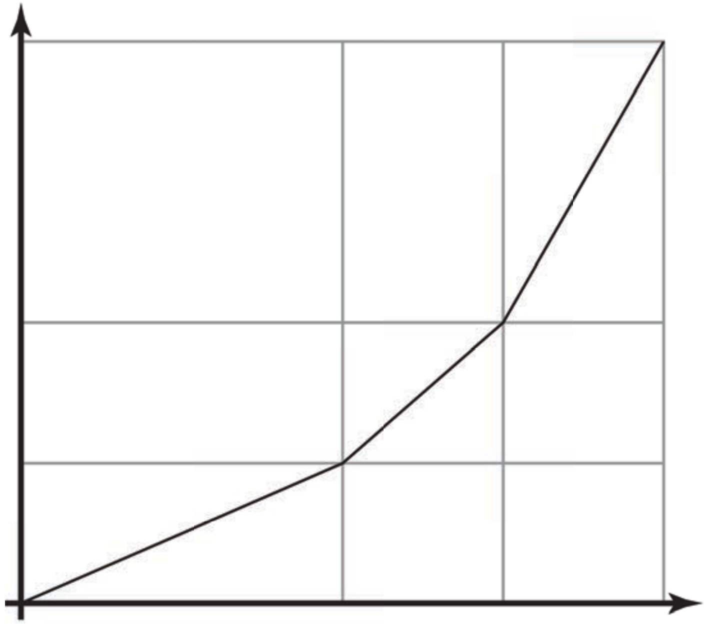
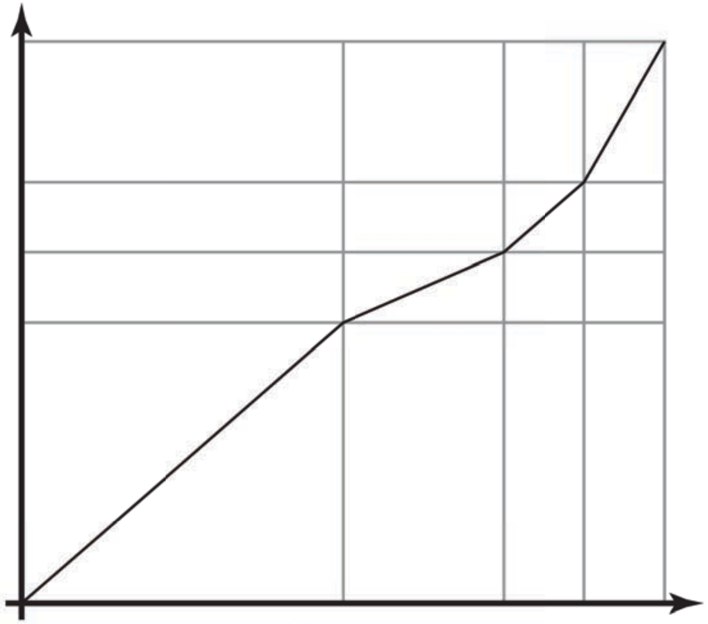


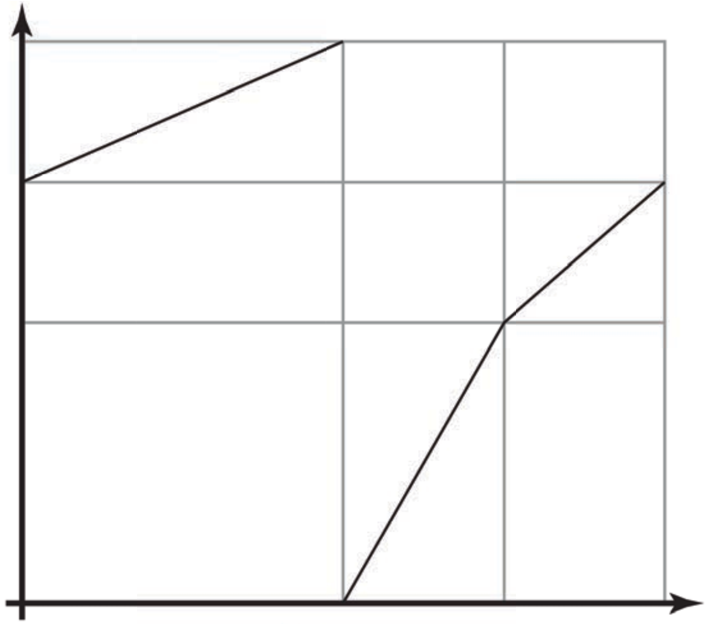
Generators of T



$A(x)$



$B(x)$



$C(x)$

J. W. Cannon, W. J. Floyd, W. R. Parry, *Introductory Notes on Richard Thompsons's Groups*, Ens. Math. 42 (1996), no. 3–4, 215–256

How does Thompson’s group T act

on **tessellations**,

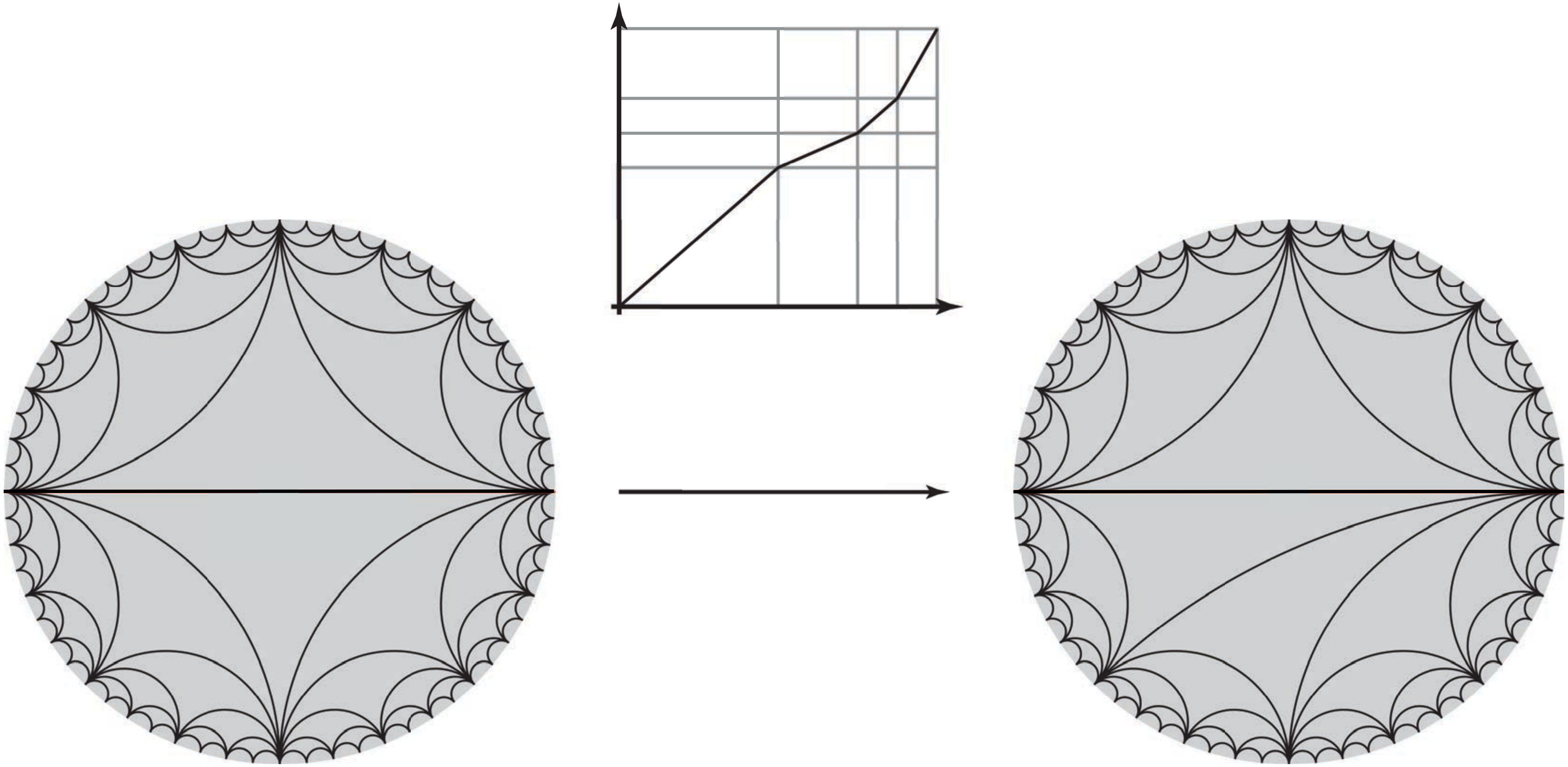
cutoffs, and

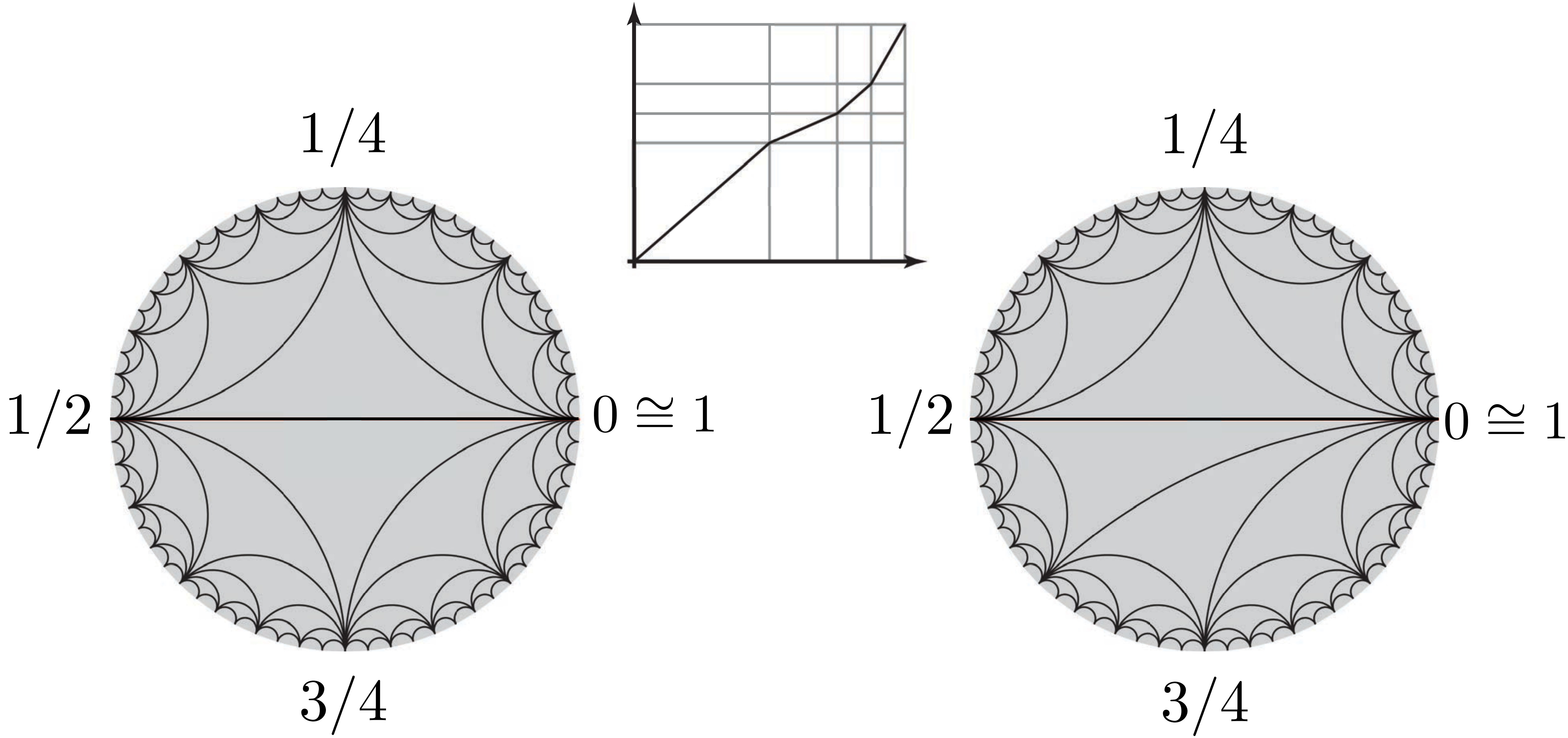
holographic states?

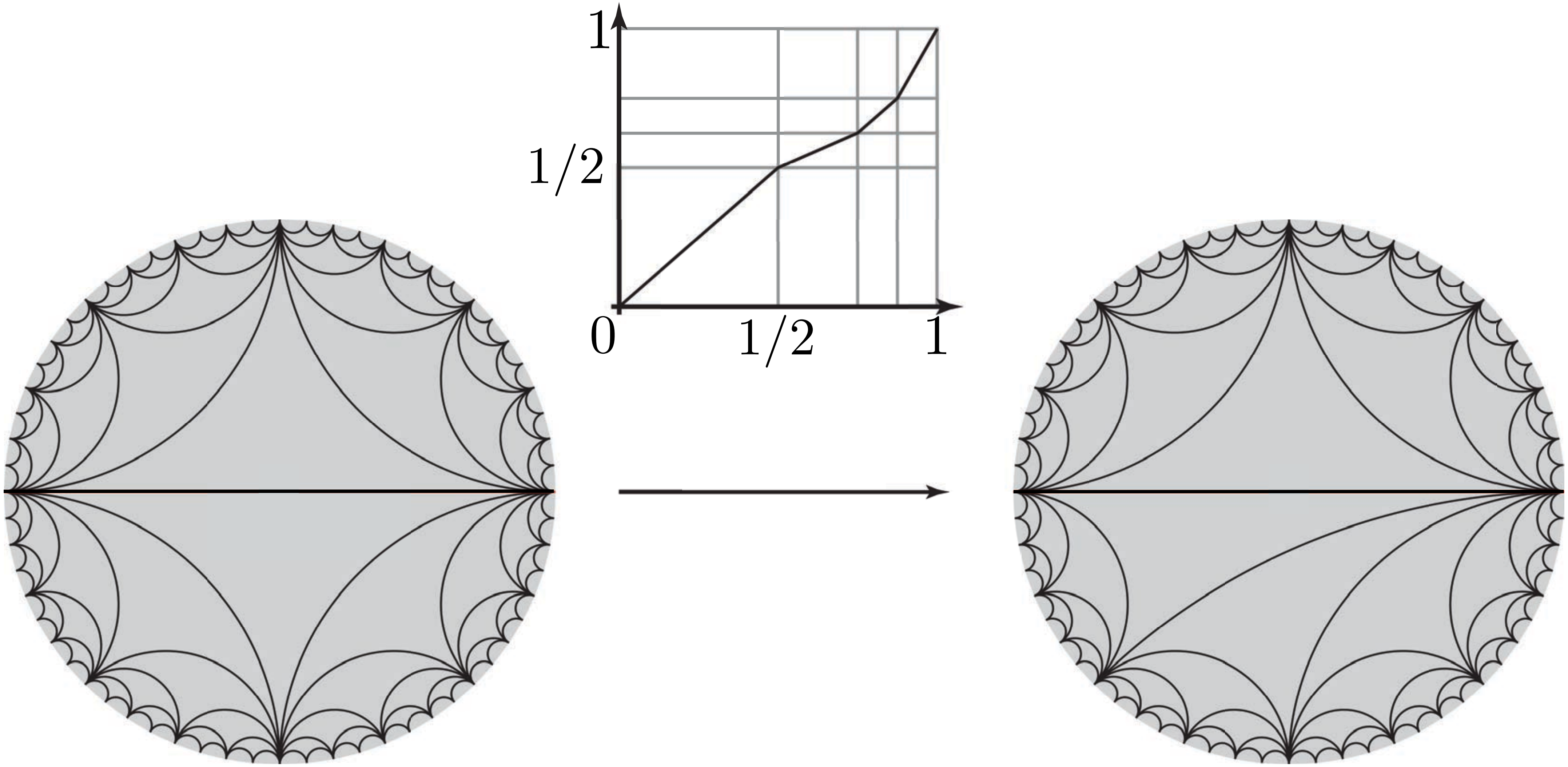
T acts on the **boundary** of the disk.

T acts on the **boundary** of the disk.

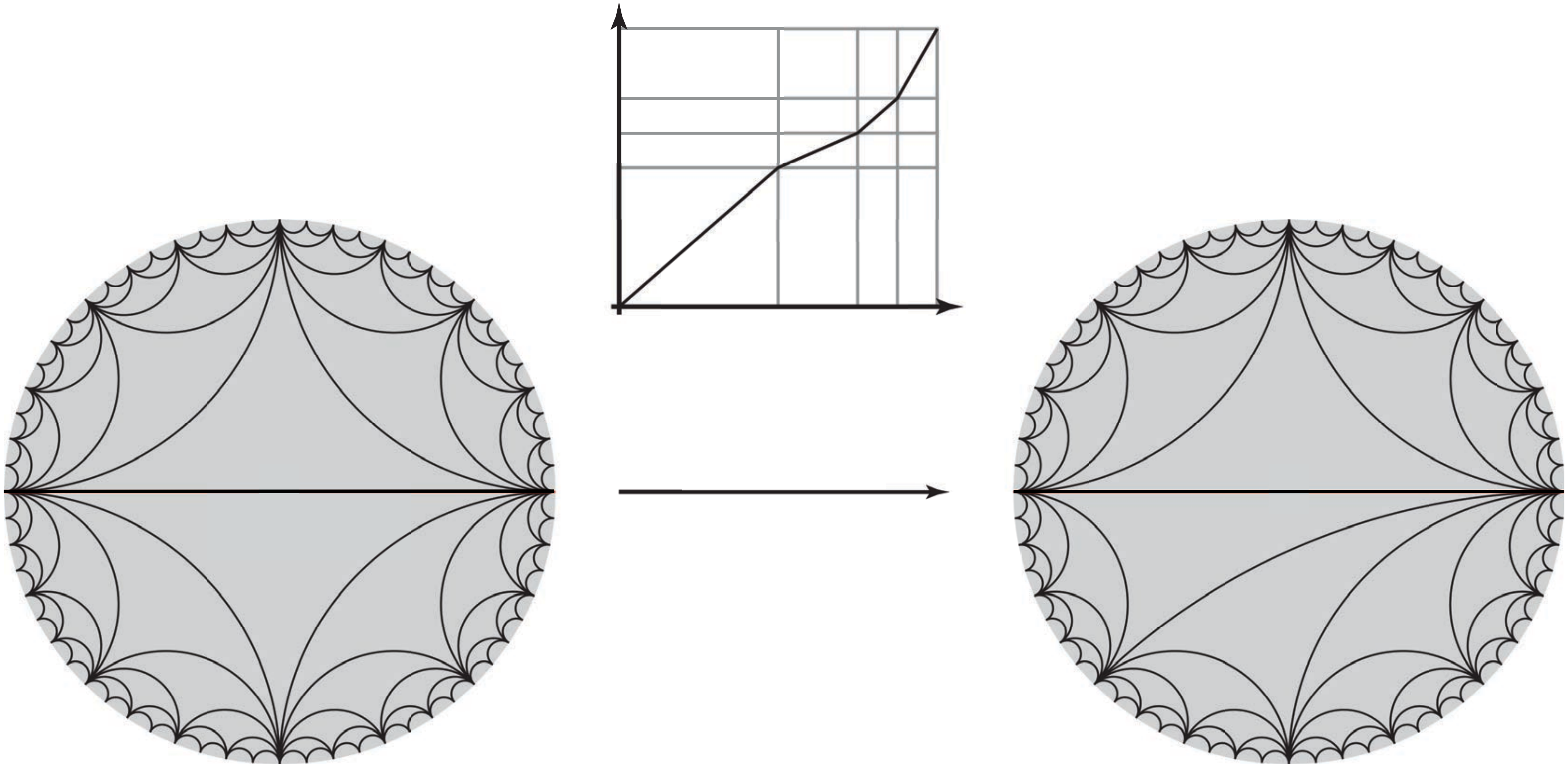
(All **vertices** of the tessellations
lie on the **boundary**.)

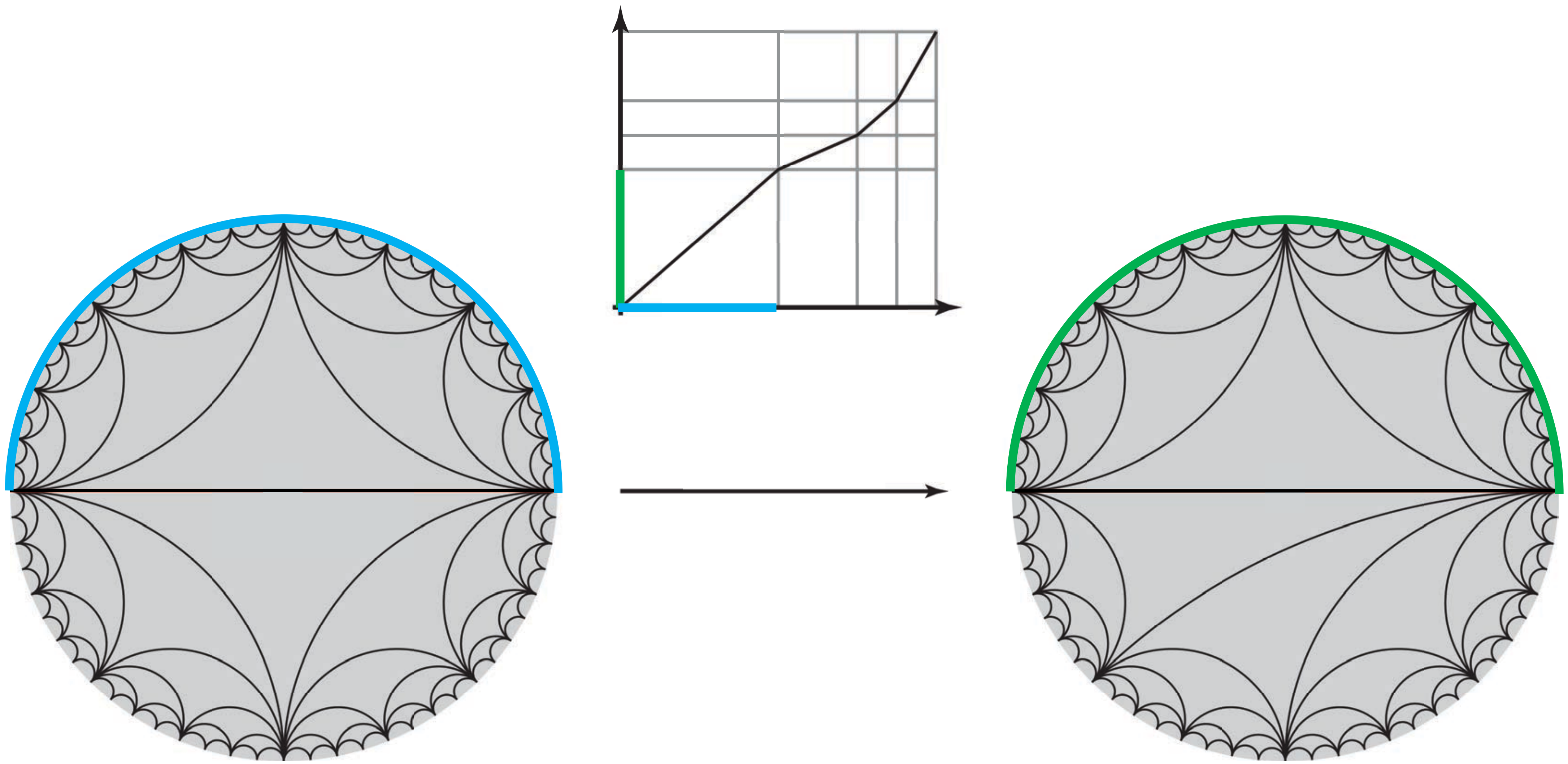


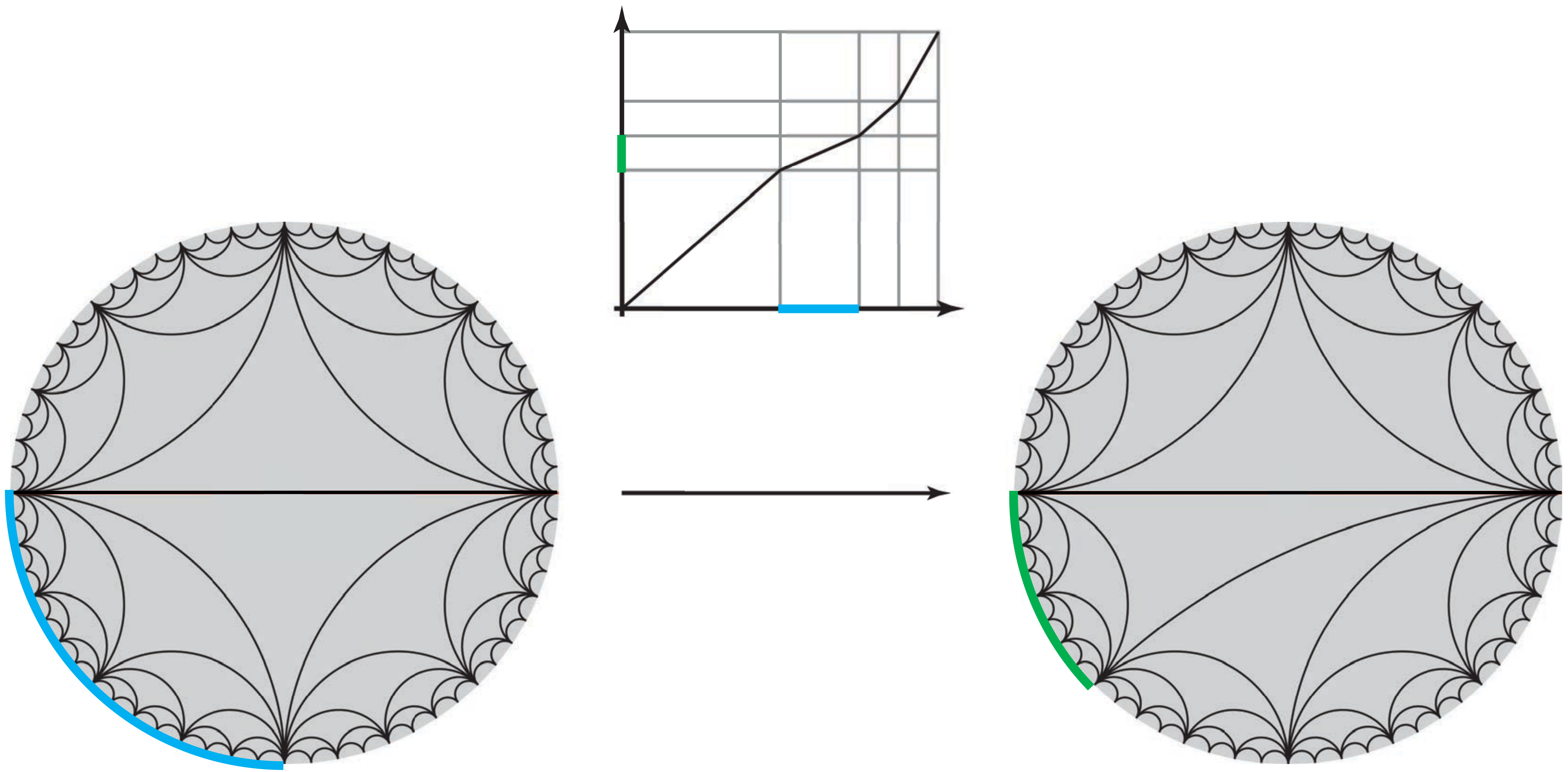

1/4
1/2
0 ≅ 1
3/4
1/4
1/2
0 ≅ 1
3/4

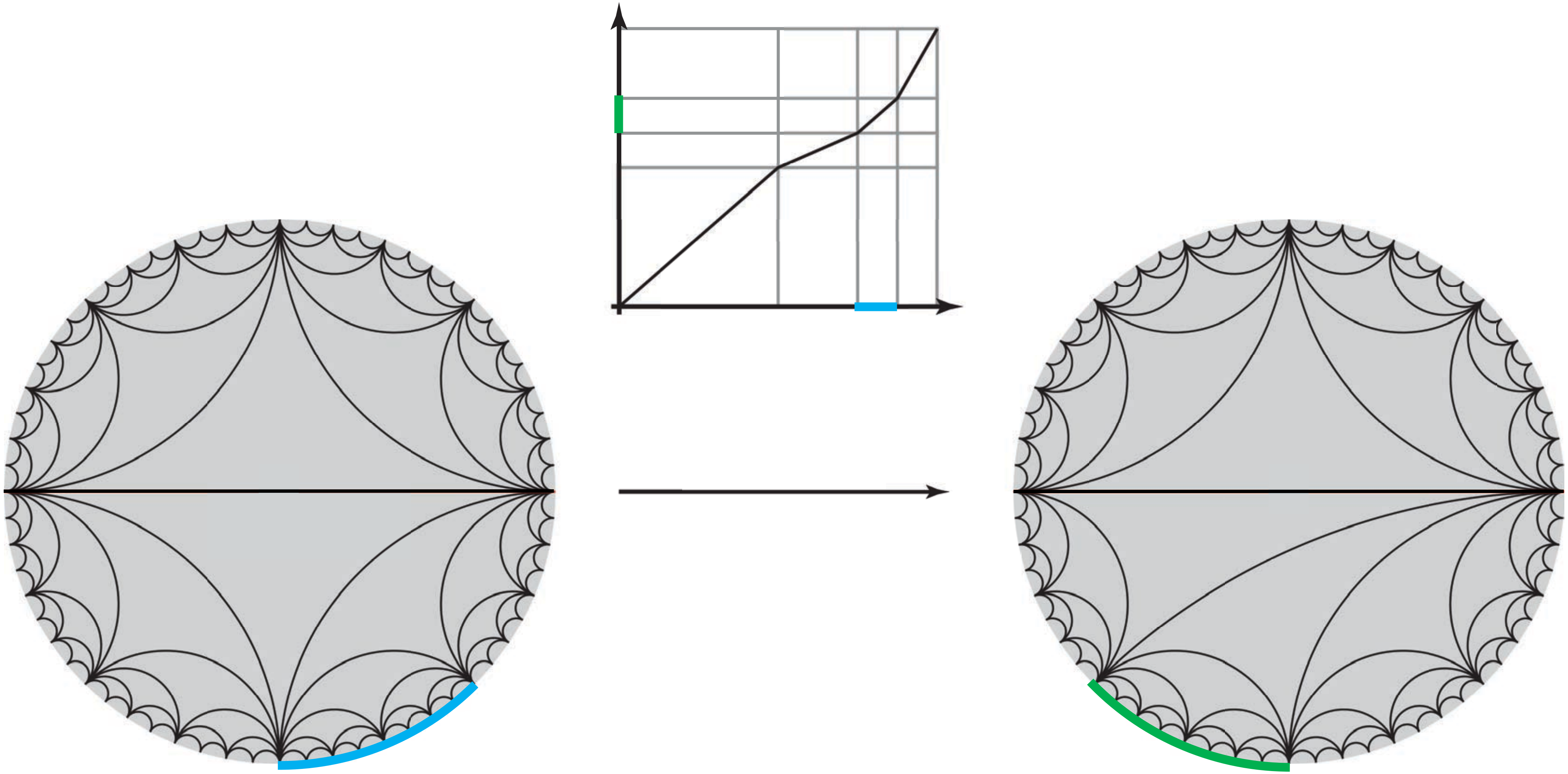


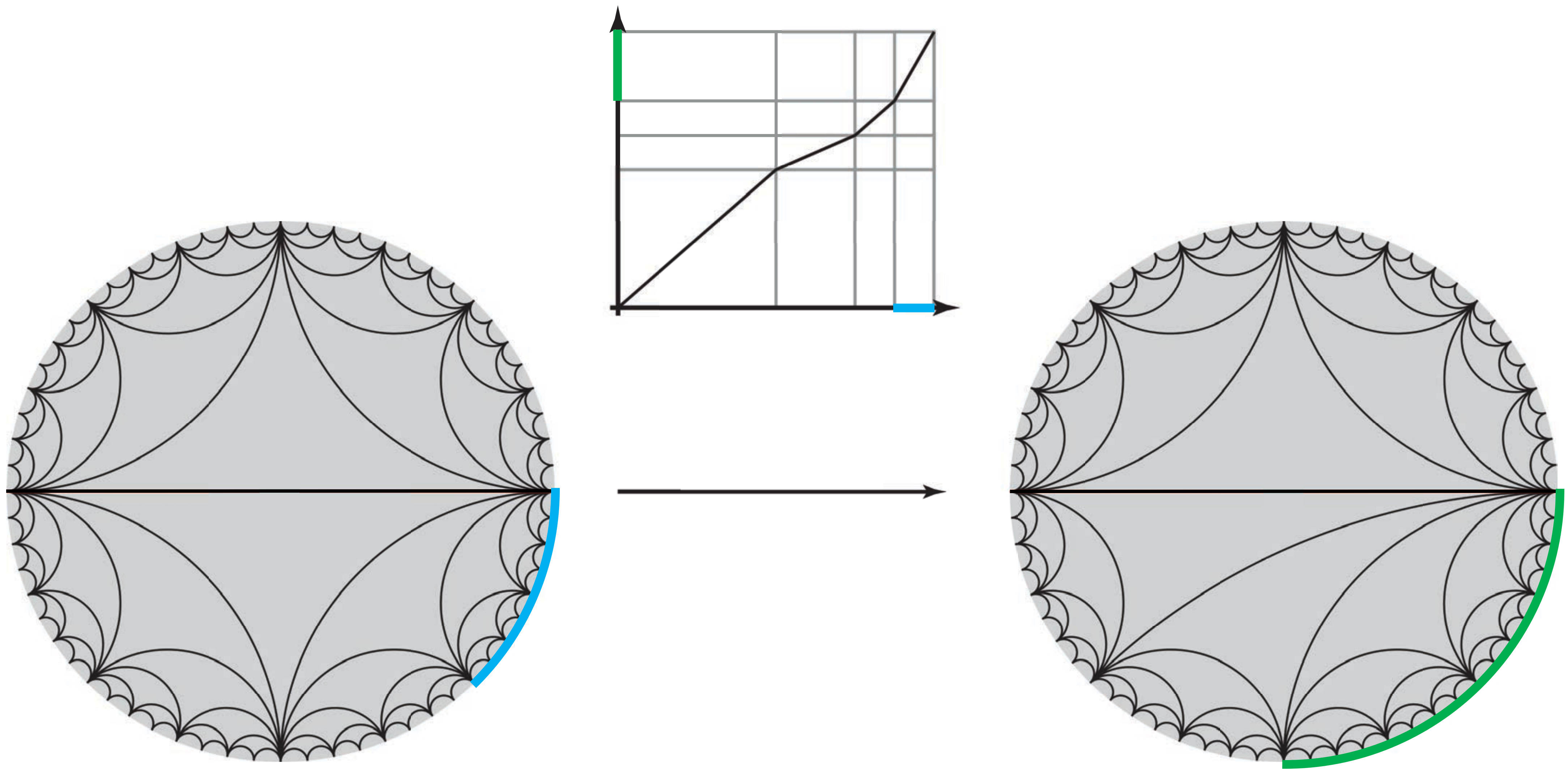
1
1/2
0
1/2
1

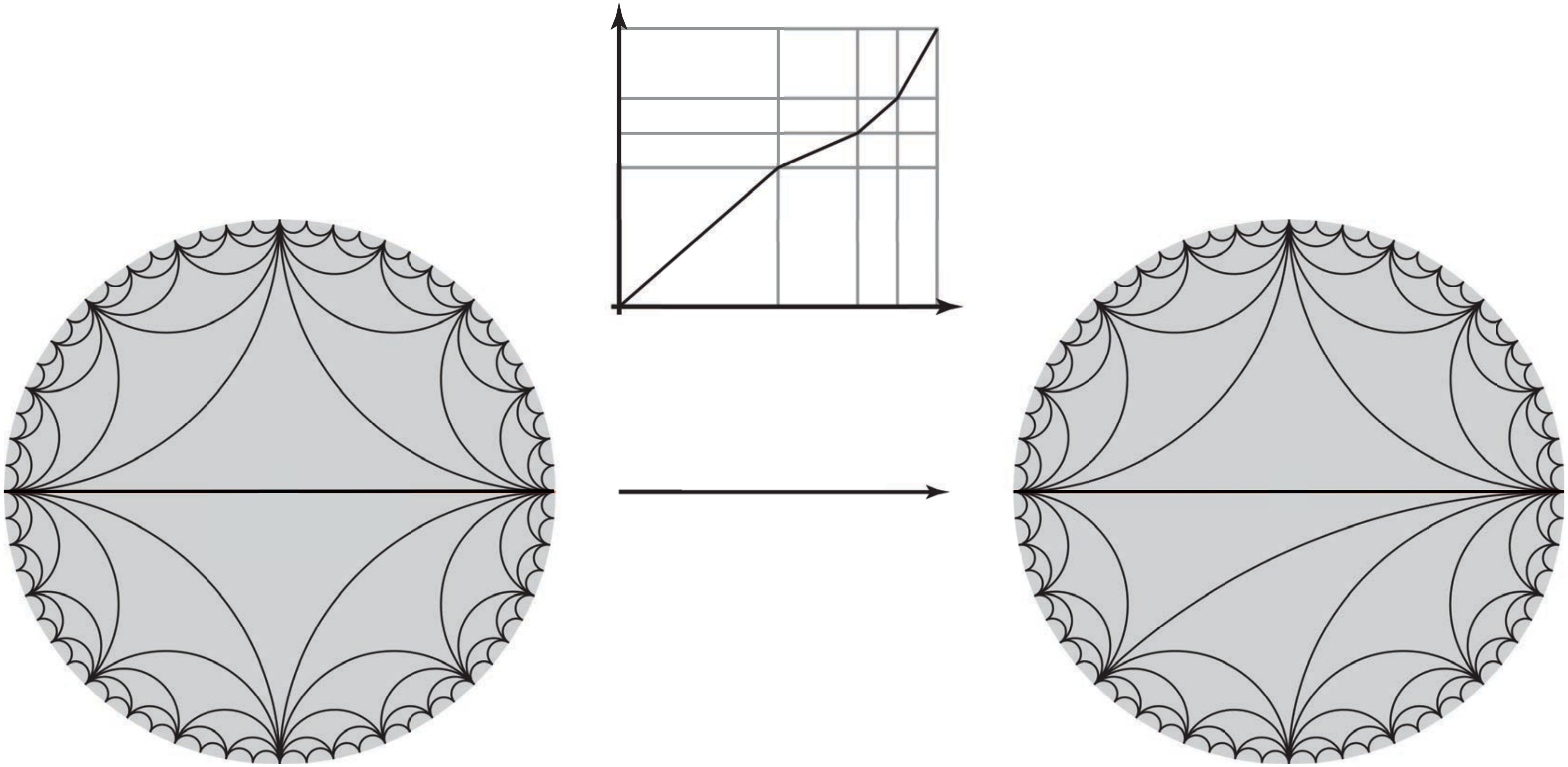








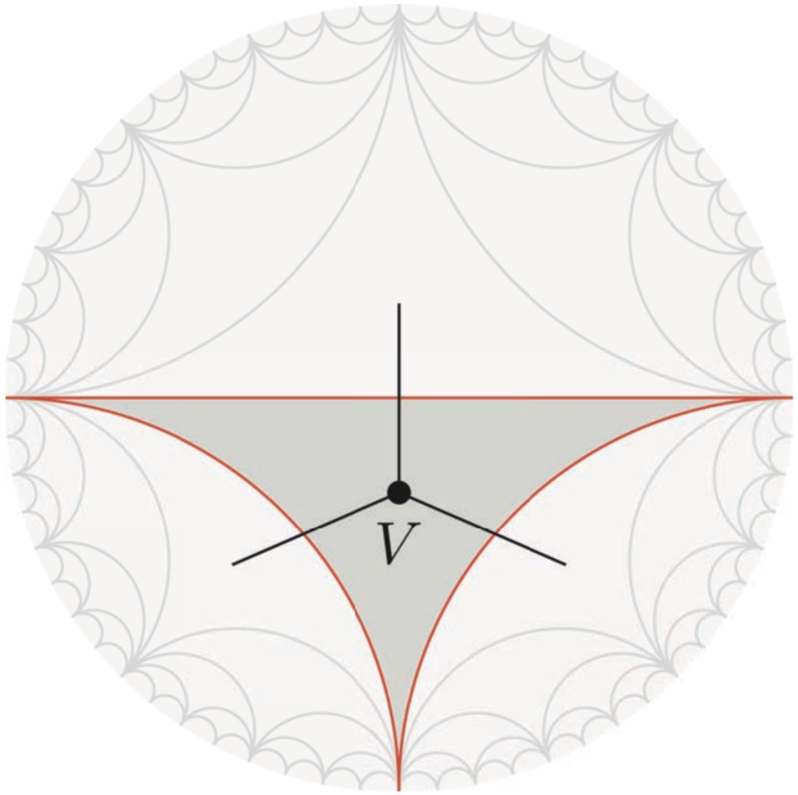




admissible

admissible

R. Penner, M. Imbert, P. Lochak and L. Schneps in *Geometric Galois Actions: Volume 2, The Inverse Galois Problem, Moduli Spaces and Mapping Class Groups*. London Mathematical Society Lecture Note Series, Cambridge University Press 1997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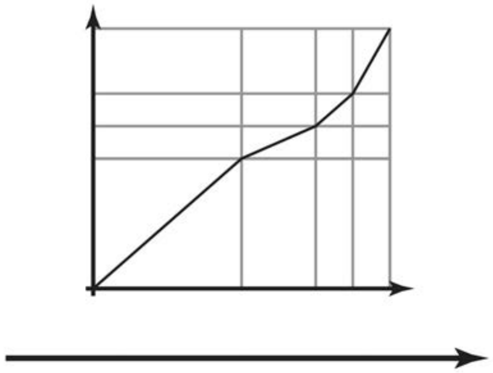

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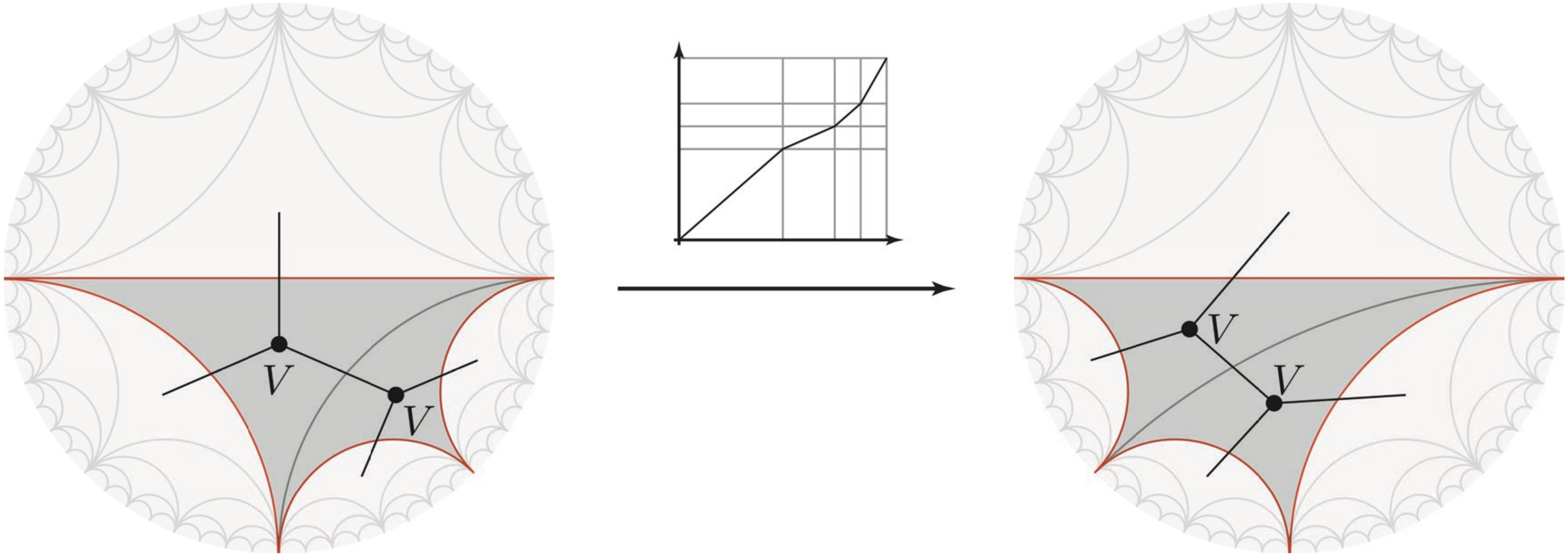
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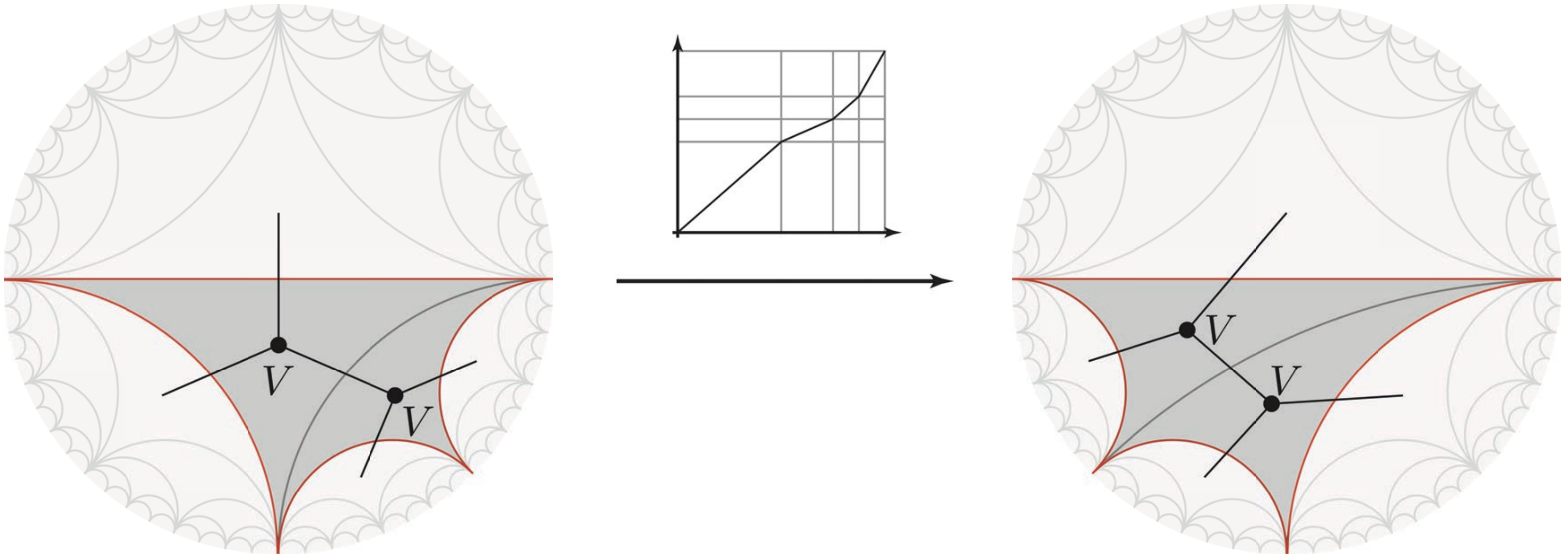

$$|\Omega_\gamma\rangle \quad \mapsto \quad |f\rangle \equiv \pi(f)|\Omega_\gamma\rangle$$

Theorem (Jones)**.** The action

$$\pi(f)|g\rangle \equiv |fg\rangle$$

defines a unitary representation of Thompson's group T on the Hilbert space spanned by all states of the form

$$|f\rangle \equiv \pi(f)|\Omega_\gamma\rangle.$$

V. F. R. Jones. arXiv:1607.08769

We have found a group that

✓ matches our choice of tessellations;

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✓ has a unitary representation on our choice of Hilbert spaces;

We have found a group that

- ✓ matches our choice of tessellations;
- ✓ has a unitary representation on our choice of Hilbert spaces;
- ✓ can be understood as a discrete version of $\mathrm{diff}_+(S^1)$.

Future Work

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field operators for Thompson’s group

Future Work

field operators for Thompson’s group

MERA instead of trees

Future Work

field operators for Thompson’s group

MERA instead of trees

other geometries & groups

The dynamics for these holographic states is given by Thompson's group T.

In September 2018 I'll be looking for postdoc positions.