START 2017

Hauser-Raspe Foundation

Quantum Simulation

(…as a tool for computation)

Wolfgang Lechner

QIP 2018 lectures

universität innsbruck

Der Wissenschaftsfonds.
START 2017

Hauser-Raspe Foundation

IQOQI Innsbruck

1. Introduction
2. Adiabatic Theorem
3. Quantum Annealing
4. Entanglement in adiabatic processes

---

5. Parity constraints
6. QRAM
7. Implementations

QIP 2018 lectures

1. Introduction

Quantum Computing

- Entanglement
- Superposition
- Measurement

… as a resource.

Universal Quantum Computer

Gate Models

One-Way Quantum Computing

Raussendorf, Briegel PRL (2001).

P. Walther et. al Nature (2005).

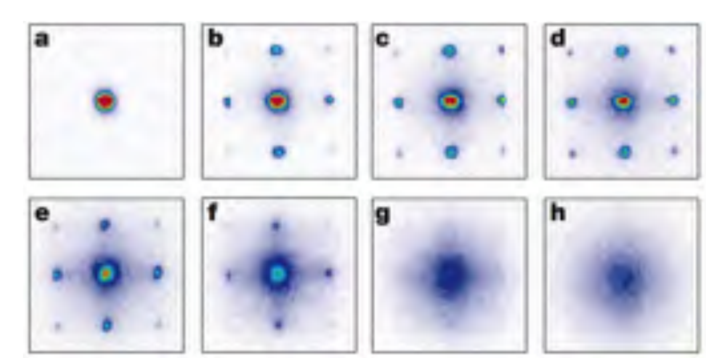
Adiabatic Quantum Computing

Fahri et. al., Science (2001).

Quantum Simulation

e.g. Hubbard Model

$$H = -t \sum_{\langle i,j \rangle} a_i^\dagger a_j + \frac{U}{2} \sum_i \hat{n}_i \left(\hat{n}_i - 1 \right) - \mu \sum_i \hat{n}_i$$



D Jaksch, et. al. PRL 81, 3108 (1998).

M. Greiner, et. al. Nature, 415 (2002).

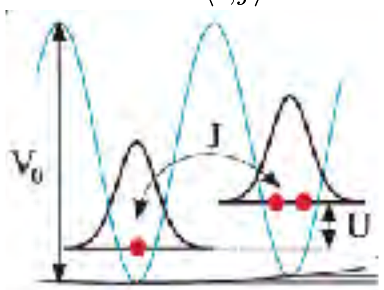
Quantum Simulation

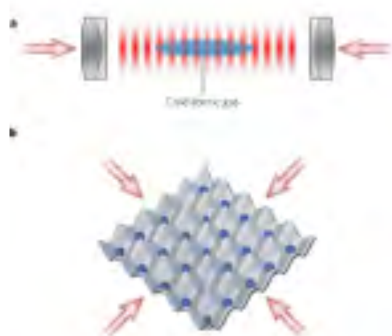
Principle: Mimic a quantum system with another controllable quantum system.

Hubbard Models

Theoretical Proposal

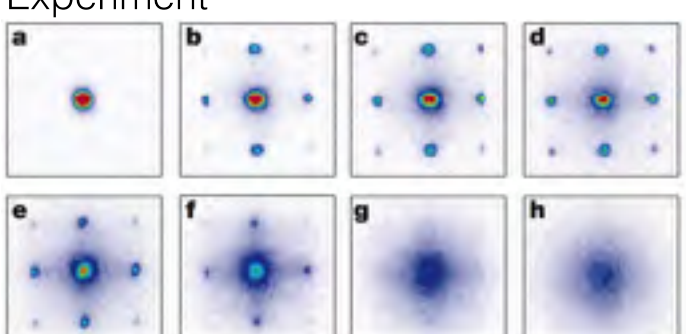
$$H = -t \sum_{\langle i,j \rangle} a_i^\dagger a_j + \frac{U}{2} \sum_i \hat{n}_i \left(\hat{n}_i - 1 \right) - \mu \sum_i \hat{n}_i$$





D Jaksch, et. al. PRL 81, 3108 (1998).

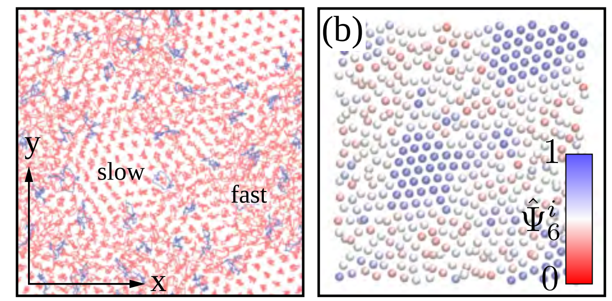
Experiment



M. Greiner, et. al. Nature, 415 (2002).

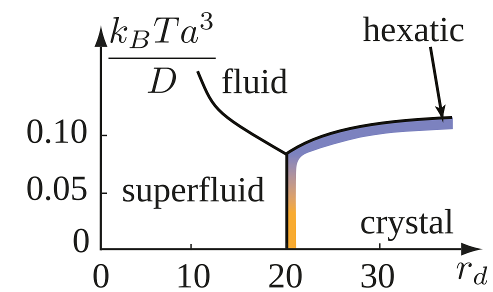
Non-equilbirium dynamics, Quantum Soft Matter …

Quantum Glass



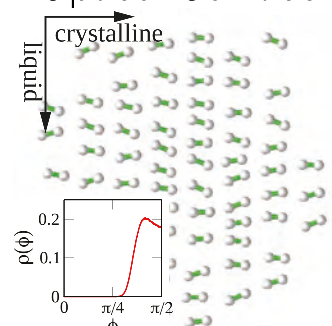
WL and P. Zoller, PRL (2013).

Quantum Hexatic



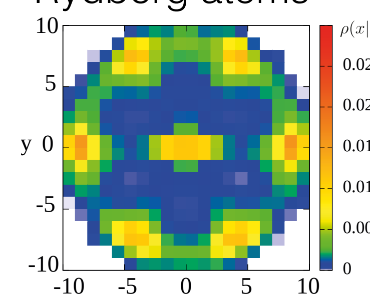
WL , Büchler, P. Zoller, PRL (2014).

Liquid Crystals in Optical Cavities



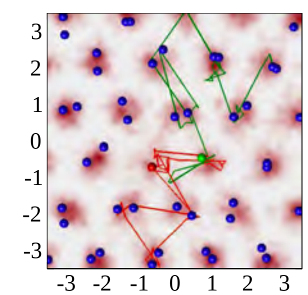
WL , Habraken, Kiesel, Aspelmeyer, P. Zoller, PRL (2013).

Logarithmic potentials in Rydberg atoms



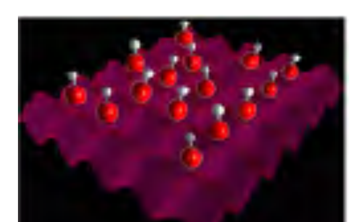
WL and P. Zoller, PRL (2015).

Supersolid from ultrasoft interactions



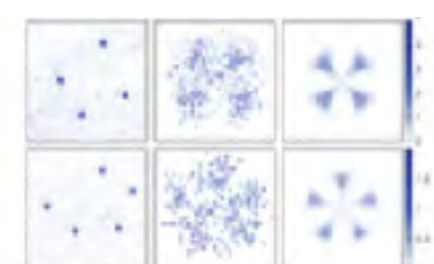
Nature Comm. (2015).

Ultracold Molecules



Jun Ye, Group

Rydberg Atoms



Bloch group Munich
Greiner group Harvard
Saffman group Madison

Quantum Computing

- Entanglement
- Superposition
- Measurement

… as a resource.

Universal Quantum Computer

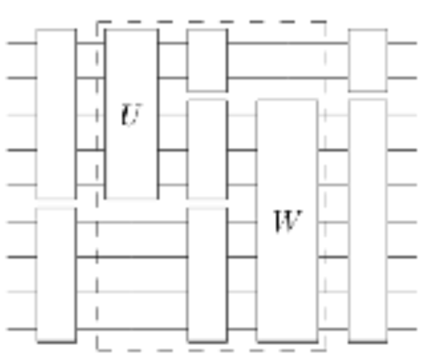
Quantum Simulation

e.g. Hubbard Model

$$H = -t\sum_{\langle i,j\rangle} a_i^\dagger a_j + \frac{U}{2}\sum_i \hat{n}_i\left(\hat{n}_i - 1\right) - \mu\sum_i \hat{n}_i$$

D Jaksch, et. al. PRL 81, 3108 (1998).
M. Greiner, et. al. Nature, 415 (2002).

Gate Models



One-Way Quantum Computing

Raussendorf, Briegel PRL (2001).
P. Walther et. al Nature (2005).

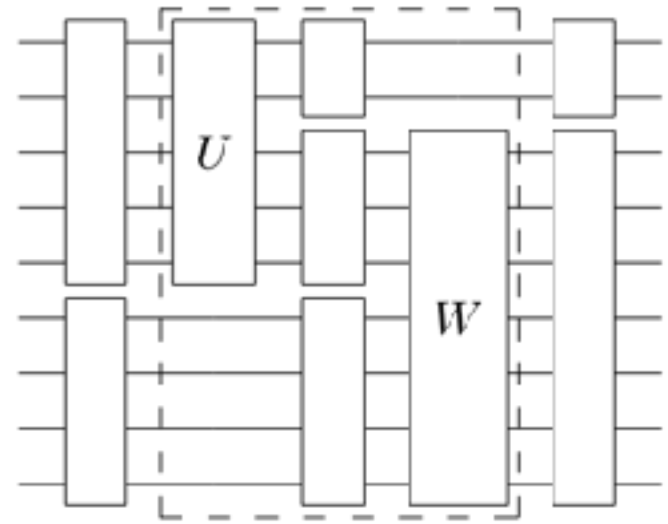
Adiabatic Quantum Computing

Fahri et. al., Science (2001).

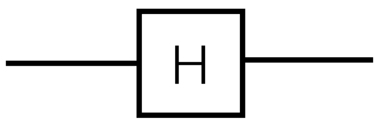
Universal Quantum Computing - Gate Models

Principle: Universal gates from single qubit unitary and CNOT.

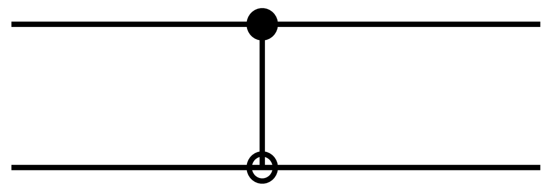
$$|\psi\rangle = U|\psi_0\rangle = U_1 U_2 U_3 ... |\psi\rangle$$



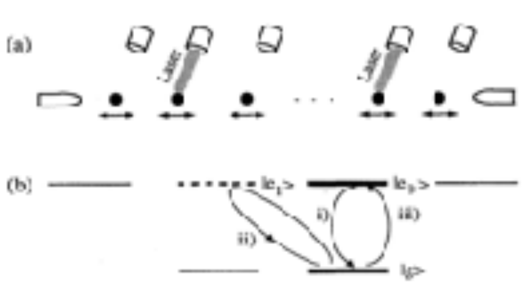
Single qubit gate



CNOT Gate

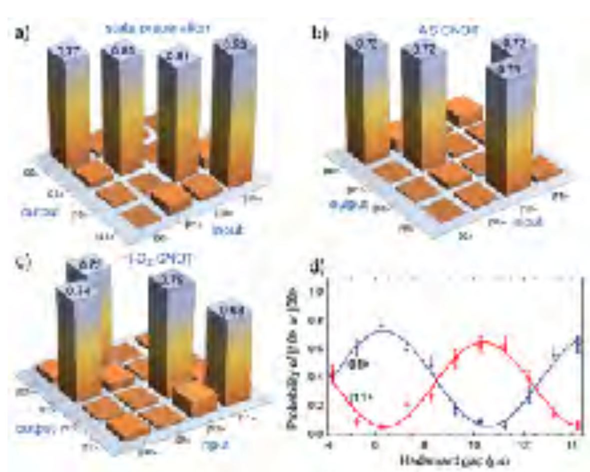


Ions



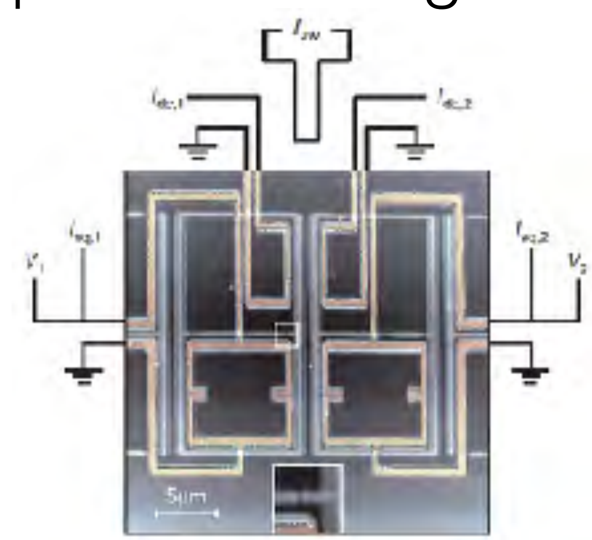
Cirac Zoller PRL 74, 4091 (1995).
Exp: R. Blatt. (2003).

Rydberg Atoms



Saffman group PRL 104, 010503 (2010)

Superconducting Qubits



Mooij group Delft Nature (2007).
J. Martinis, Santa Barbara

Quantum Computing

- Entanglement
- Superposition
- Measurement

… as a resource.

Universal Quantum Computer

Gate Models

One-Way Quantum Computing

Raussendorf, Briegel PRL (2001).

P. Walther et. al Nature (2005).

Adiabatic Quantum Computing

Fahri et. al., Science (2001).

Quantum Simulation

e.g. Hubbard Model

$$H = -t \sum_{\langle i,j \rangle} a_i^\dagger a_j + \frac{U}{2} \sum_i \hat{n}_i (\hat{n}_i - 1) - \mu \sum_i \hat{n}_i$$

D Jaksch, et. al. PRL 81, 3108 (1998).

M. Greiner, et. al. Nature, 415 (2002).

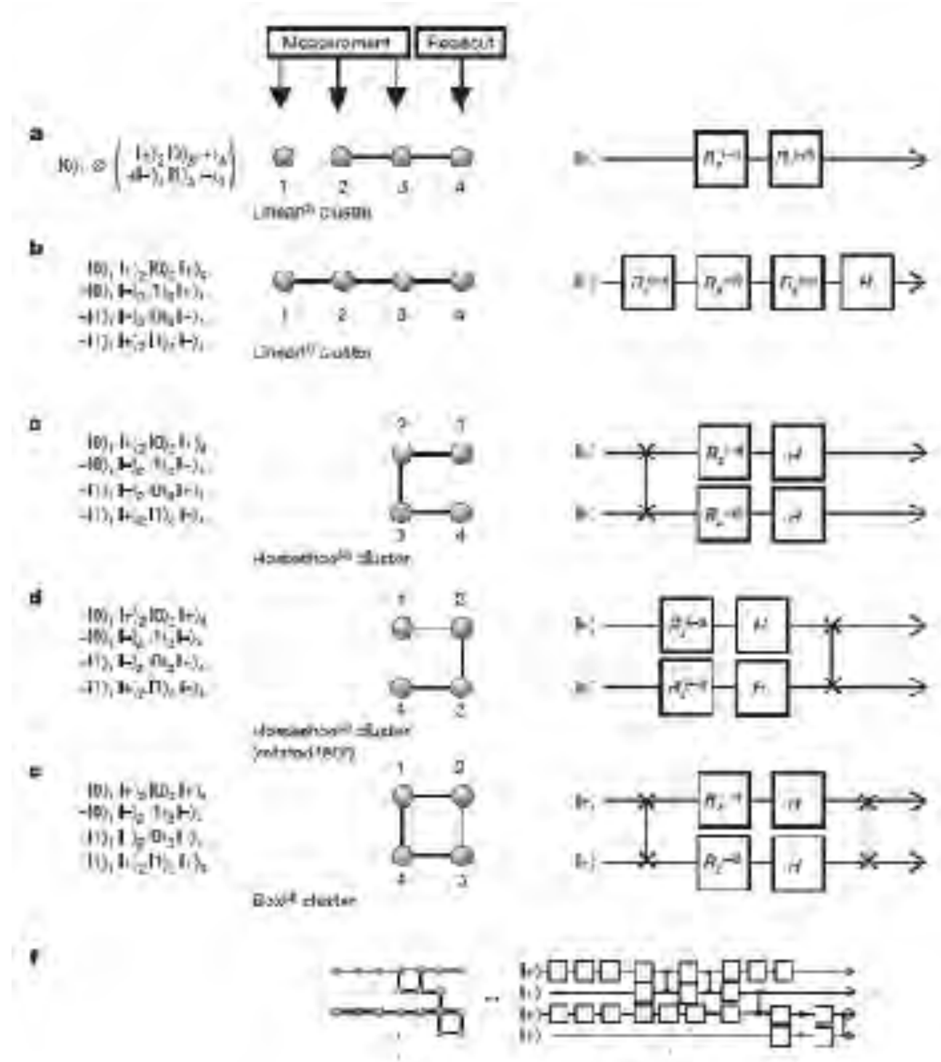
Universal Quantum Computing - One Way QC

Principle: Start in a entangled state, get result form consecutive measurements.

Theory:

R .Raussendorf and H. Briegel, PRL 86, 5188 (2001).

Experiment:



P. Walther, Resch, Rudolph, Schenck, Weinfurter, V. Vedral,
M. Aspelmeyer and A. Zeilinger Nature 434, 176 (2005).

Quantum Computing

- Entanglement
- Superposition
- Measurement

… as a resource.

Universal Quantum Computer

Gate Models

One-Way Quantum Computing

Raussendorf, Briegel PRL (2001).
P. Walther et. al Nature (2005).

Adiabatic Quantum Computing

Fahri et. al., Science (2001).

Quantum Simulation

e.g. Hubbard Model

$$H = -t \sum_{\langle i,j \rangle} a_i^\dagger a_j + \frac{U}{2} \sum_i \hat{n}_i (\hat{n}_i - 1) - \mu \sum_i \hat{n}_i$$

D Jaksch, et. al. PRL 81, 3108 (1998).
M. Greiner, et. al. Nature, 415 (2002).

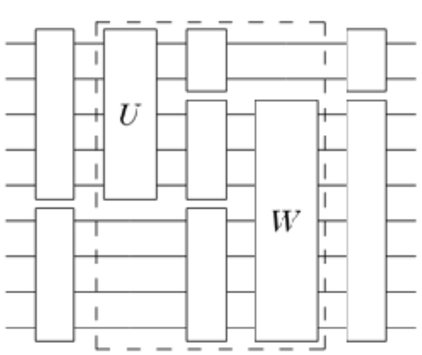
Quantum Computing

- Entanglement
- Superposition
- Measurement

… as a resource.

Universal Quantum Computer

Gate Models



One-Way Quantum Computing

Raussendorf, Briegel PRL (2001).

P. Walther et. al Nature (2005).

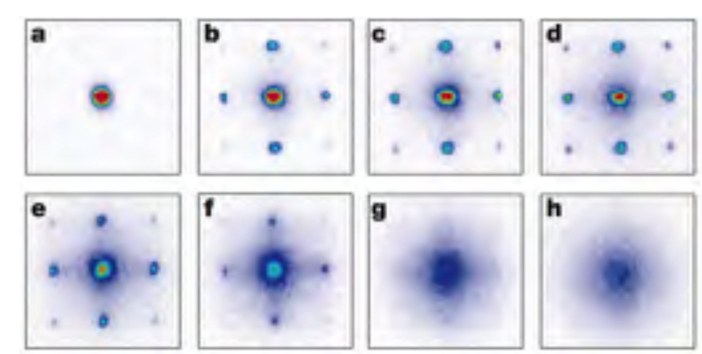
Adiabatic Quantum Computing

Farhi et. al., Science (2001).

Quantum Simulation

e.g. Hubbard Model

$$H = -t \sum_{\langle i,j \rangle} a_i^\dagger a_j + \frac{U}{2} \sum_i \hat{n}_i \left(\hat{n}_i - 1 \right) - \mu \sum_i \hat{n}_i$$



D Jaksch, et. al. PRL 81, 3108 (1998).

M. Greiner, et. al. Nature, 415 (2002).

2. The Adiabatic Theorem

Adiabatic Theorem

A physical system **remains** in its **instantaneous eigenstate** if a given perturbation is acting on it **slowly enough** and if **there is a gap** between the eigenvalue and the rest of the Hamiltonian's spectrum.

M. Born and V. A. Fock "Beweis des Adiabatensatzes" Zeitschrift für Physik A. **51** 165 (1928).

Avoided Level Crossings

Two-level system

$$H = \begin{pmatrix} E_1 & 0 \\ 0 & E_2 \end{pmatrix}$$

Eigenvalues

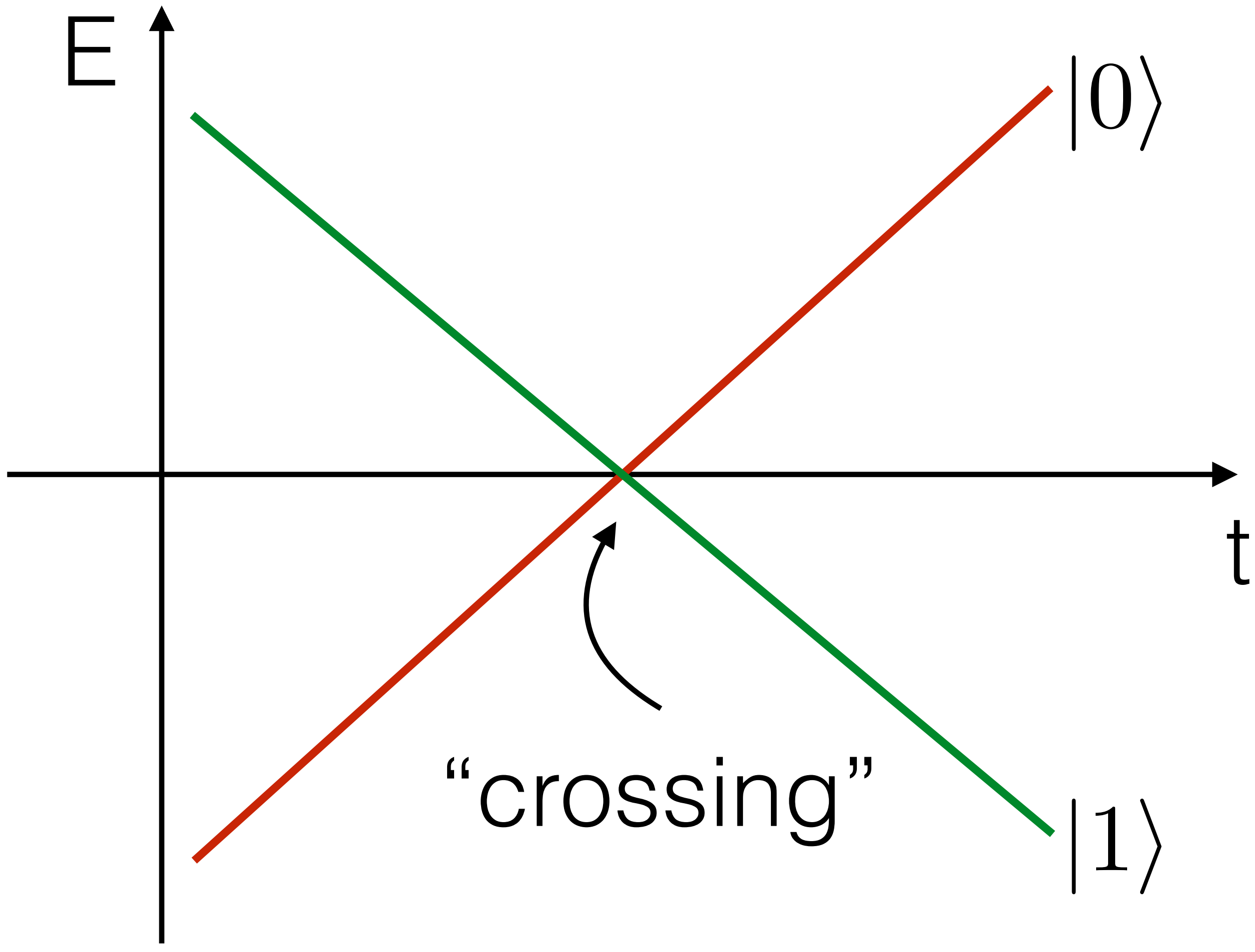
E_1 and E_2

Eigenvectors

$\begin{pmatrix} 0 \\ 1 \end{pmatrix}$ and $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$

Time-dependent

$$H(t) = \begin{pmatrix} E_1(t) & 0 \\ 0 & E_2(t) \end{pmatrix}$$



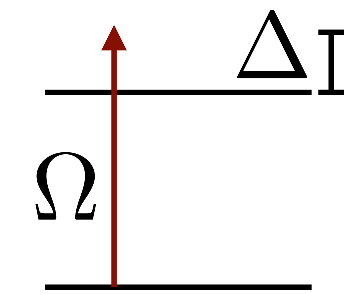
Avoided Level Crossings

Two-level system with coupling

$$H(t) = \begin{pmatrix} E_1 & W \\ W^* & E_2 \end{pmatrix}$$

e.g. Rabi Problem

$$\hbar \begin{pmatrix} -\Delta & \frac{\Omega}{2} e^{-i\varphi} \\ \frac{\Omega}{2} e^{-i\varphi} & 0 \end{pmatrix}$$



Eigenvalues

$$E_+ = \frac{1}{2}(E_1 + E_2) + \frac{1}{2}\sqrt{(E_1 - E_2)^2 + 4|W|^2}$$

$$E_- = \frac{1}{2}(E_1 + E_2) - \frac{1}{2}\sqrt{(E_1 - E_2)^2 + 4|W|^2}$$

Eigenstates ("dressed states")

$$|+\rangle = \frac{1}{\sqrt{2}}(e^{i\phi}|0\rangle + |1\rangle)$$

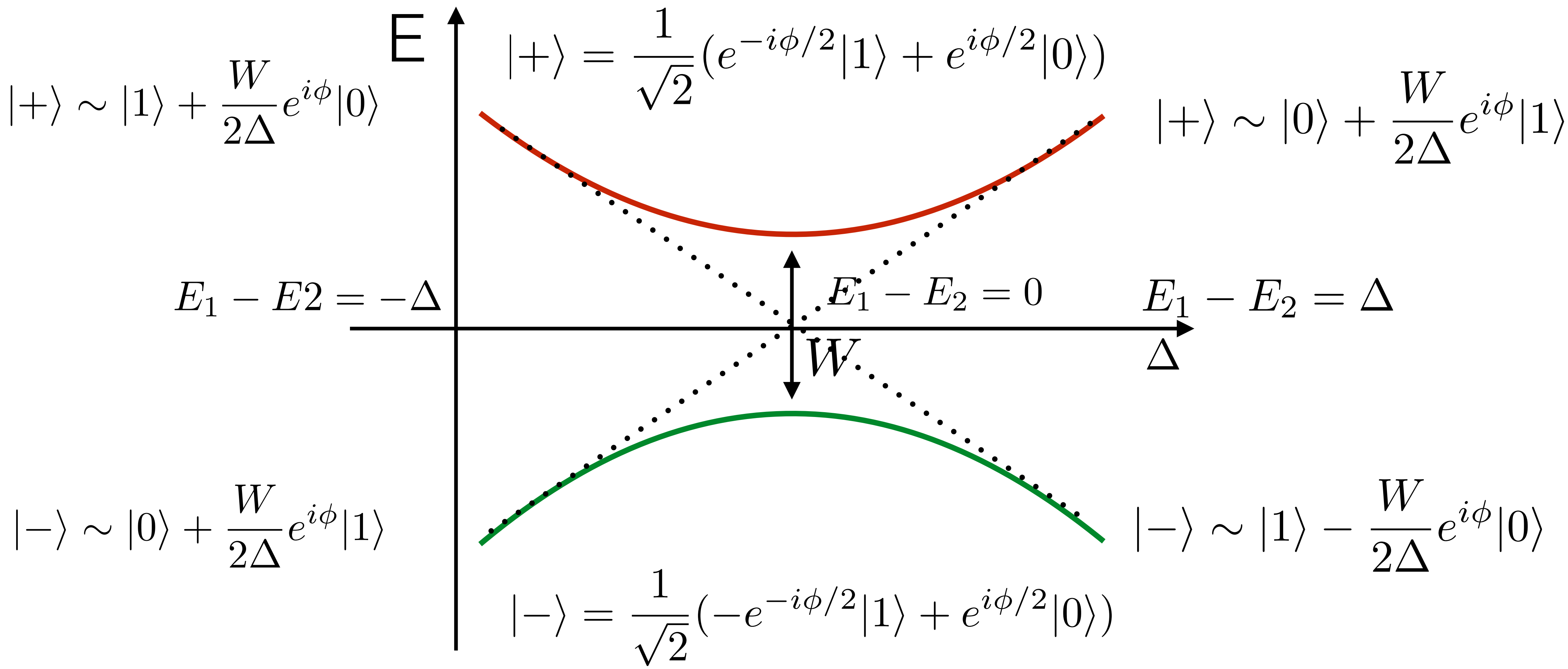
$$|-\rangle = \frac{1}{\sqrt{2}}(-e^{i\phi}|0\rangle + |1\rangle)$$

with $\tan\phi = -\dfrac{\Omega}{\Delta}$

Avoided Level Crossings

Two-level system with coupling

$$H(t) = \begin{pmatrix} E_1 & W \\ W^* & E_2 \end{pmatrix}$$



"avoided crossing"

Adiabatic Theorem

Time-dependent Schrödinger Equation

$$i\hbar\frac{\partial}{\partial t}\Psi(x,t) = H\Psi(x,t)$$

General Solution as a function of instantaneous eigenstates

$$\Psi(x,t) = \sum_n c_n \Psi_n(x,t) = \sum_n c_n \psi_n(x) e^{-iE_n t/\hbar}$$

with $\quad \Psi_n(x,t) = \psi_n(x) e^{-iE_n t/\hbar}$

For each instantaneous time slice we have

$H(t)\psi_n(x,t) = E_n(t)\psi_n(x,t)$ with $\langle\psi_n(t)|\psi_m(t')\rangle = \delta_{nm}\delta(t-t')$

time-independent Schrödinger equation with time as parameter

Adiabatic Theorem

Time-dependent Schrödinger Equation

$$i\hbar \frac{\partial}{\partial t}\Psi(x,t) = H\Psi(x,t)$$

with "dynamical Phase factor"

$$\Psi(t) = \sum_n c_n(t)\psi_n(t)e^{i\theta_n(t)} \qquad \theta_n(t) = -\frac{1}{\hbar}\int_0^t E_n(t')dt'$$

Plug this into Schrödinger equation

$$i\hbar \sum_n (\dot{c}_n\psi_n + c_n\dot{\psi}_n + \cancel{ic_n\psi_n\dot{\theta}_n})e^{i\theta_n} = \sum_n \cancel{c_n H\psi_n}e^{i\theta_n}$$

$$\dot{\theta}_n = -E_n/\hbar$$

$$=> \quad \sum_n \dot{c}_n\psi_n e^{i\theta_n} = -\sum_n c_n\dot{\psi}_n e^{i\theta_n}$$

Adiabatic Theorem

$$\sum_n \dot{c}_n \psi_n e^{i\theta_n} = -\sum_n c_n \dot{\psi}_n e^{i\theta_n}$$

multiply with $\langle\psi_m|$ from both sides

$$\dot{c}_m(t) = -\sum_n c_n \langle\psi_m|\dot{\psi}_n\rangle e^{i(\theta_n-\theta_m)}$$ (with $\langle\psi_m|\psi_n\rangle = \delta_{nm}$)

evaluating $\dot{\psi}_n$ from the Schrödinger equation

$$\dot{c}_m(t) = -c_m\langle\psi_m|\dot{\psi}_m\rangle - \sum_{n\neq m} \cancel{c_n \frac{\langle\psi_m|\dot{H}|\psi_n\rangle}{E_n - E_m}} e^{i(\theta_n-\theta_m)}$$

This is the adiabatic approximation

Consequence: $\dot{c}_m(t) = -c_m\langle\psi_m|\dot{\psi}_m\rangle$

Adiabatic Theorem

As a consequence of the approximation, matrix elements from m to n vanish.

$$\dot{c}_m(t) = -c_m \langle \psi_m | \dot{\psi}_m \rangle$$

with the formal solution:

$$c_m(t) = c_m(0) \exp[-\int_0^t \langle \psi_m(t') | \dot{\psi}_m(t') \rangle dt'] = c_m(0) e^{i\gamma_m(t)}$$

where $\gamma_m(t)$ is the geometric phase factor

$$\gamma_m(t) = i \int_0^t \langle \psi_m(t') | \dot{\psi}_m(t') \rangle dt'$$

plugging $c_m(t)$ into the solution of the Schrödinger equation

$$\boxed{\Psi_n(t) = c_n(t) \psi_n(t) e^{i\gamma_n(t)} = \psi_n(t) e^{i\theta_n(t)} e^{i\gamma_n(t)}}$$

Berry Phase

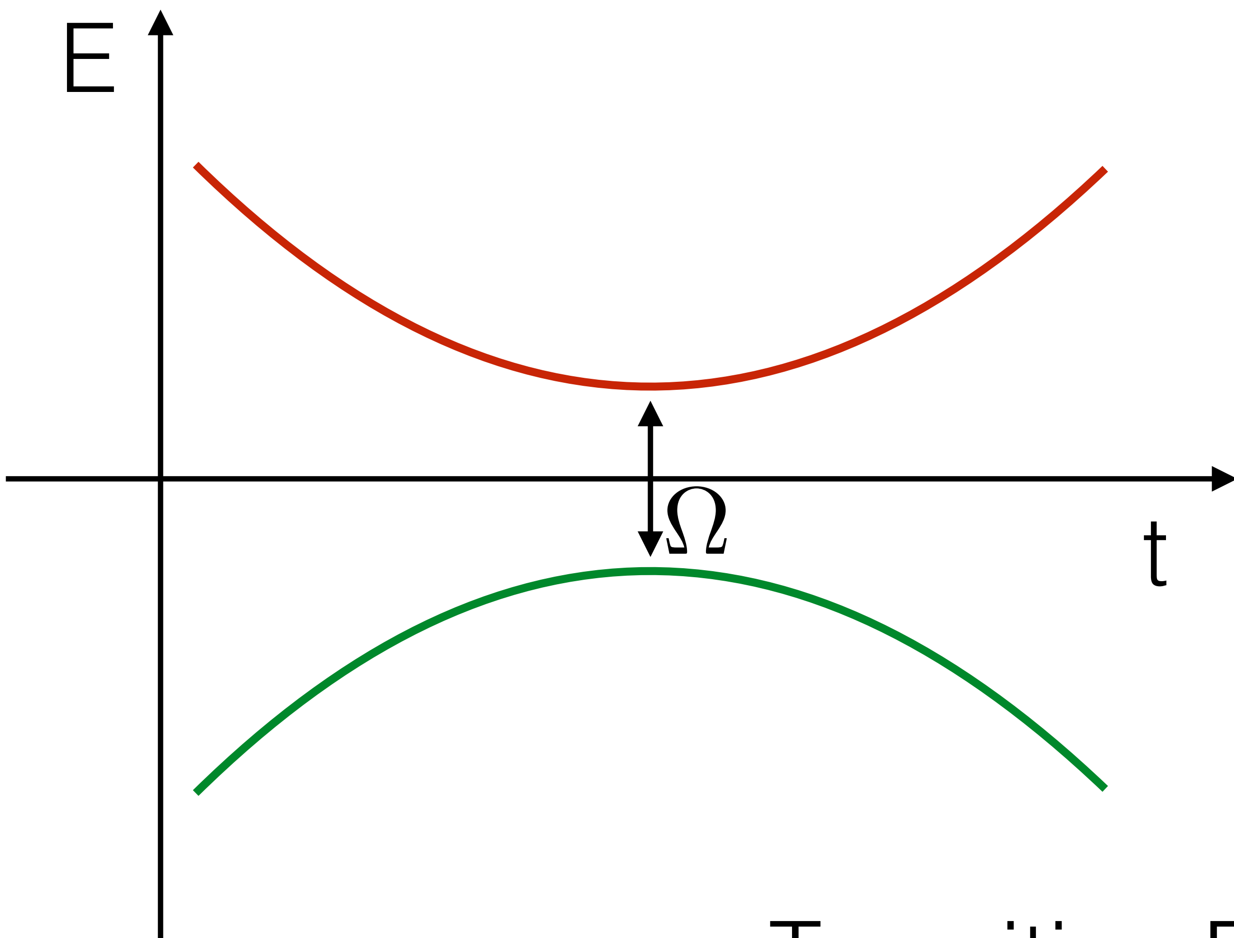
Landau Zener

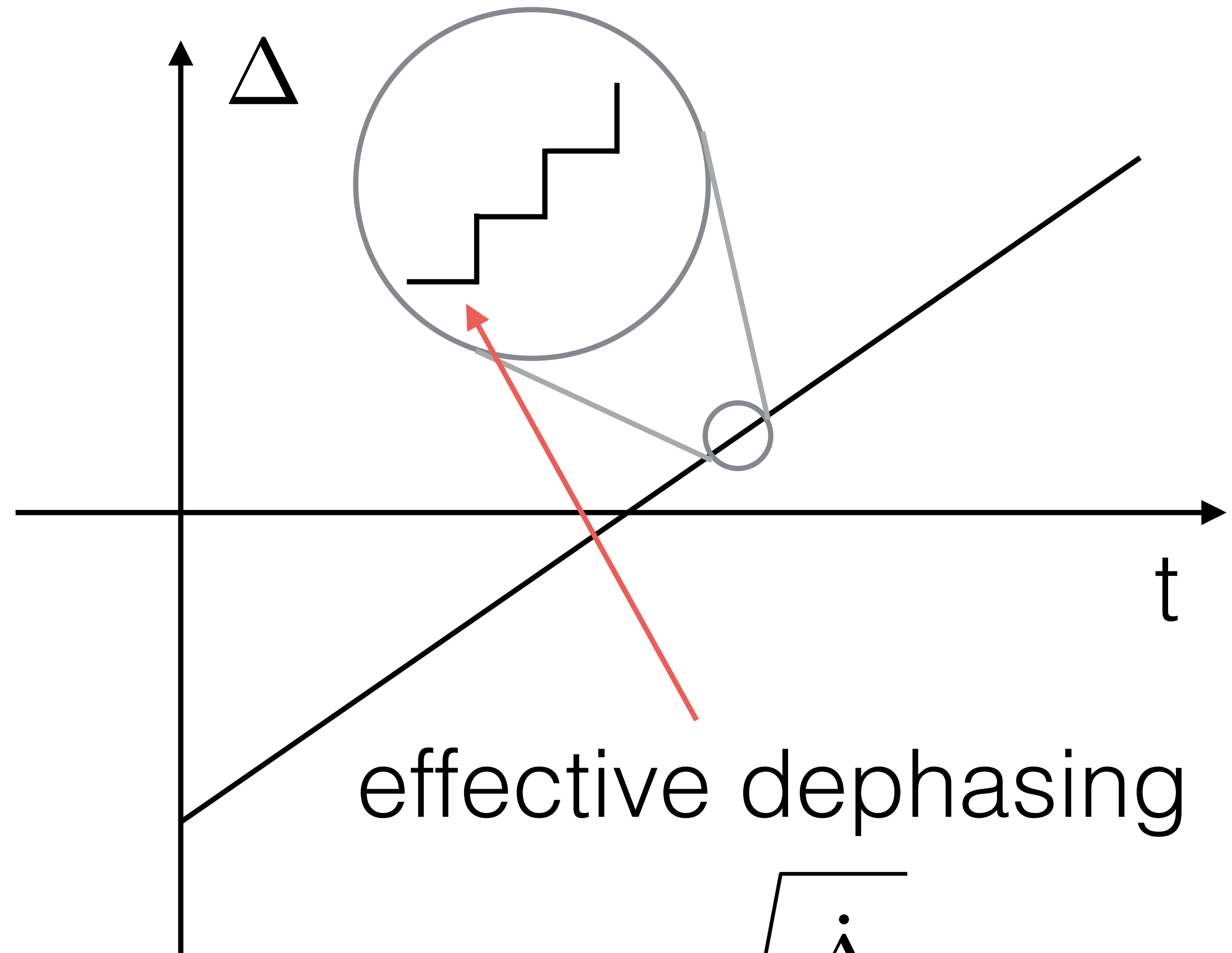
C. Zener, Proc. Royal Soc. A 137 696(1923). A Vutar, arXiv:1001.3322 (2010).

$$\mathbf{H} = \begin{pmatrix} 0 & \Omega^\dagger e^{i\omega t} \\ \Omega e^{-i\omega t} & \omega_0 \end{pmatrix}$$

$$\Delta = \omega - \omega_0$$

$$\dot{\Delta} = \mathrm{const}$$





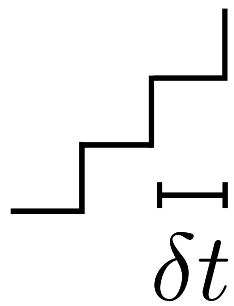
Transition Rate

$$\Gamma = \Omega^2 \frac{\gamma}{\Delta^2 + \gamma^2/4}$$

$$\gamma \sim \sqrt{\frac{\dot{\Delta}}{4\pi}}$$

Landau Zener

$$P_0(t+\delta t) = [1-\Gamma(t)\delta t]P_0(t) \approx e^{-\Gamma(t)\delta t}P_0(t)$$



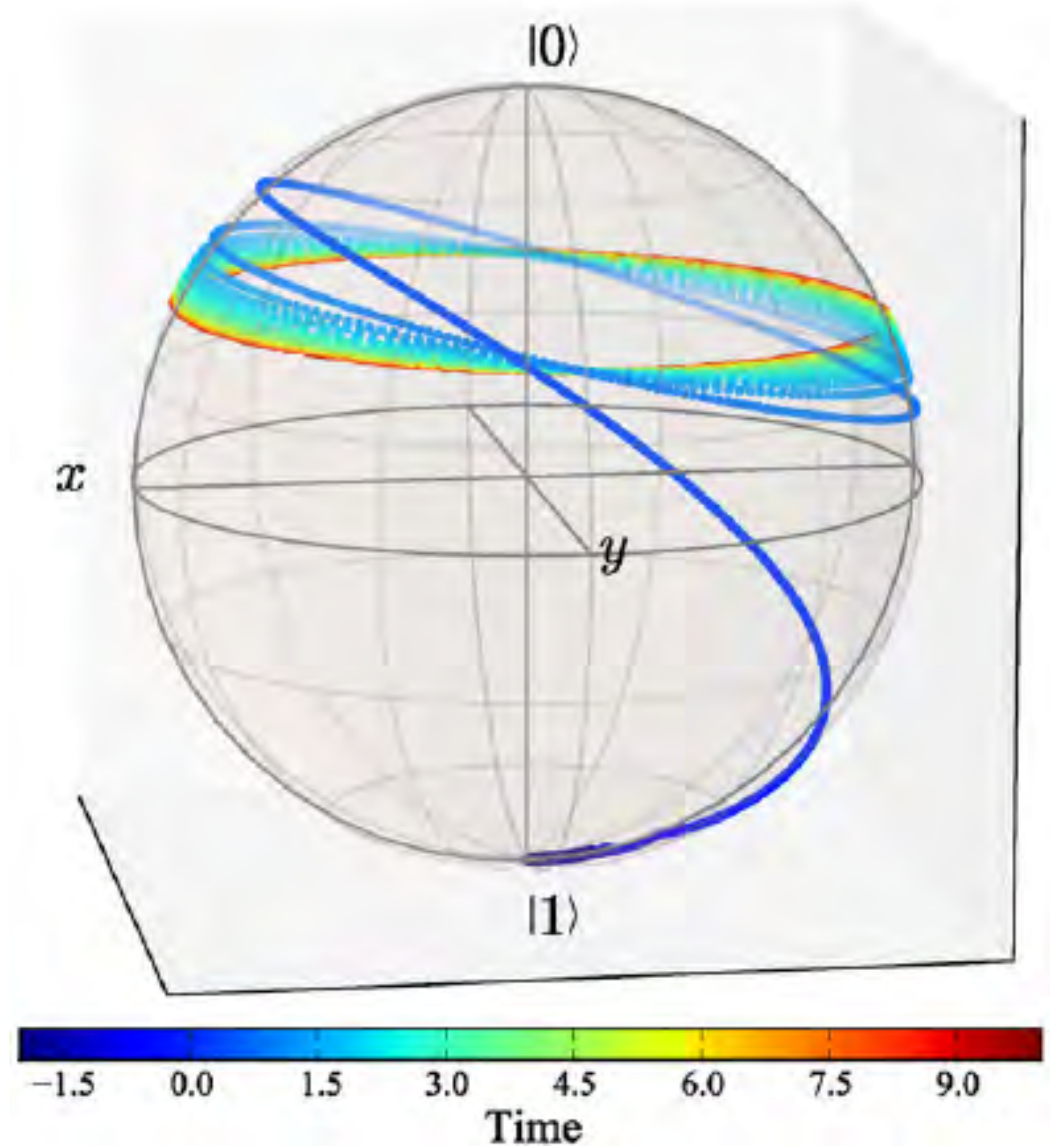
Integrating from initial to final time

$$P(t_f) = \exp[-\int_{t_i}^{t_f} \Gamma(t)dt]$$
$$= \exp\left\{-\frac{2\Omega^2}{\dot{\Delta}}\left[\mathrm{atan}\left(\frac{\Delta(t_f)}{\gamma/2}\right) - \mathrm{atan}\left(\frac{\Delta(t_i)}{\gamma/2}\right)\right]\right\}$$

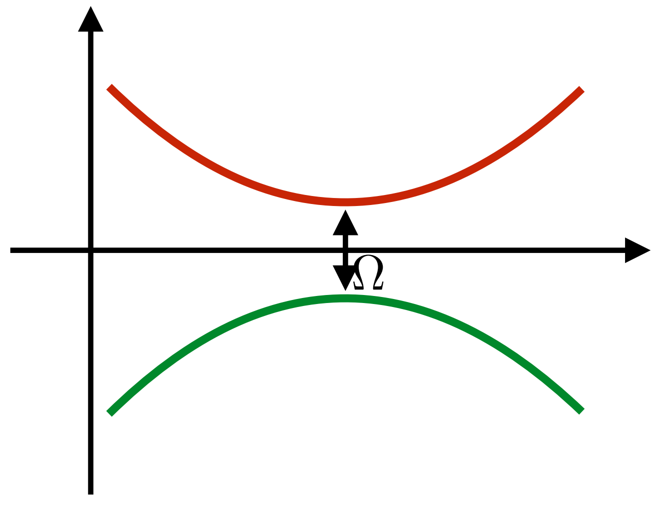
At the level crossing $\Delta \to 0$

$$P(t_f) = e^{-\frac{2\pi\Omega^2}{\dot{\Delta}}}$$ Landau Zener Formula

Landau Zener



Picture taken from J. R Johnson et. al. QuTip Package



Landau Zener

Dimensionless schedule s from $0 \to 1$

$$i\hbar \frac{d}{dt}|\psi(t)\rangle = H(t)|\psi(t)\rangle \Rightarrow \frac{d}{ds}|\psi(s)\rangle = -i\tau(s)H(s)|\psi(s)\rangle$$

introducing the "delay factor" $\tau(s)$

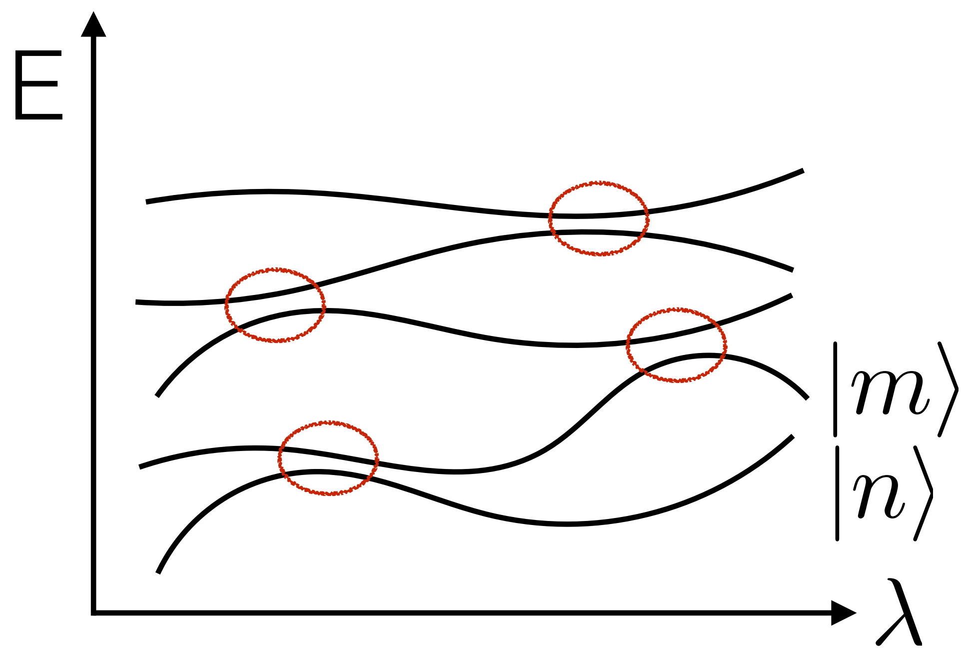
$$\tau(s) \gg \frac{|\frac{d}{ds}H(s)|}{g(s)^2}$$ Landau Zener condition with gap $g(s)$

Total time of the calculation

$$T = \int_{s=0}^{1} \tau(s)ds$$

Landau Zener

Many-Body Landau Zener



Approximation:
sequence of LZ transitions

$$P_{LZ} = e^{-\frac{|G|^2}{v}}$$

Gap is estimated from off-diagonal matrix elements:

$$|G| \sim |\langle n|H|m\rangle|$$

3. Quantum Annealing

Adiabatic Quantum Computing

Nishimori Hidetoshi Phys. Rev. B (1998).

E. Farhi et. al., Science 292, 472 (2001).

$$H(t) = \left(1 - \frac{t}{T}\right) H(0) + \frac{t}{T} H_p$$

Trivial initial Hamiltonian

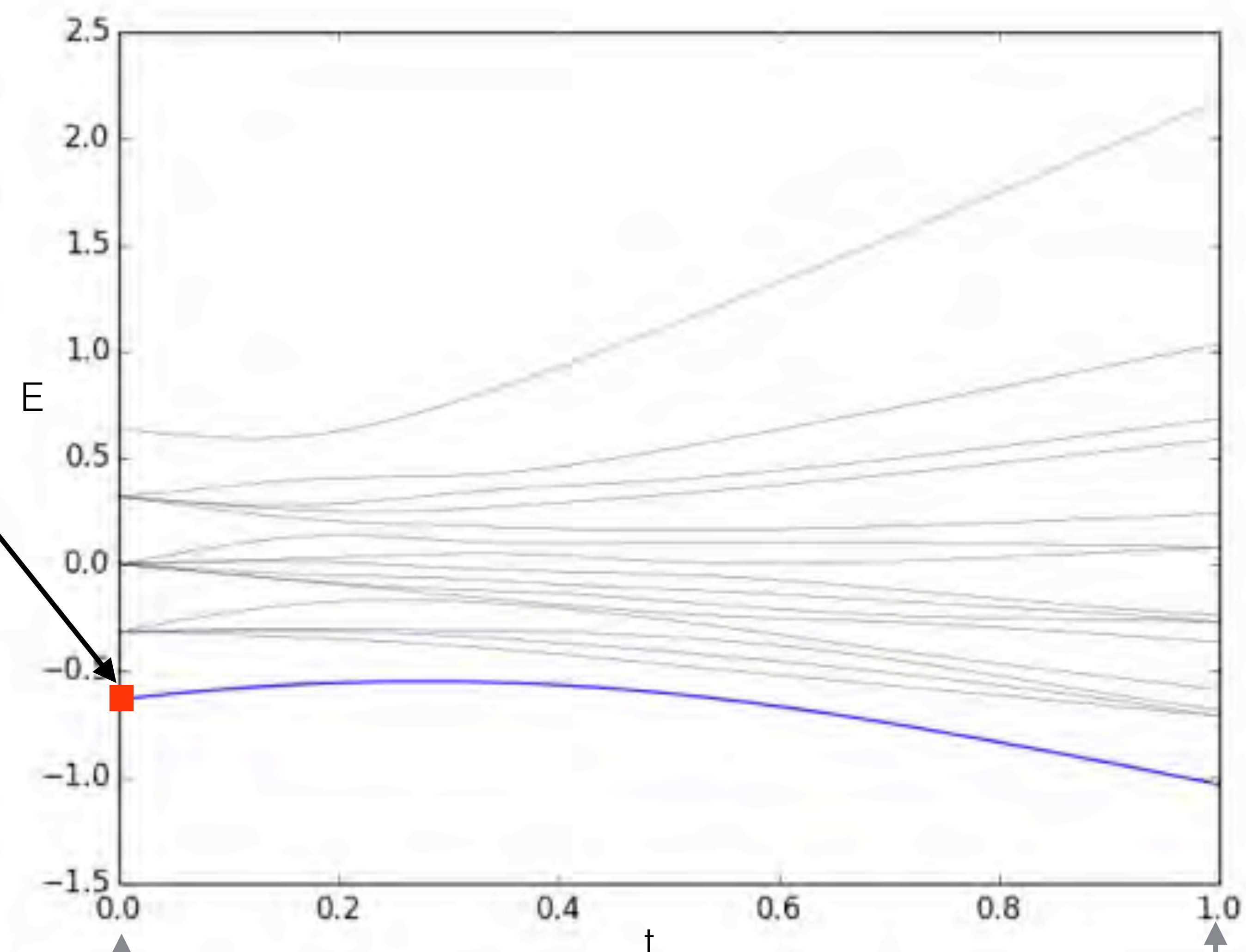
Problem Hamiltonian

1. Prepare System in $|\psi_0\rangle$ which is the ground state of H(0)

2. Slowly evolve the system with

$$|\psi(t)\rangle = e^{-i\int_0^t H(t)dt}|\psi_0\rangle$$

3. Due to the adiabatic theorem, the final state is the ground state of H_p



$$H_0 = \sum_i^N \sigma_x^{(i)}$$

$$H_p = \sum_{i<j} J_{ij}\sigma_z^{(i)}\sigma_z^{(j)}$$

Adiabatic Quantum Computing

Nishimori Hidetoshi Phys. Rev. B (1998).
E. Farhi et. al., Science 292, 472 (2001).

$$H(t) = \left(1 - \frac{t}{T}\right) H(0) + \frac{t}{T} H_p$$

Trivial initial Hamiltonian

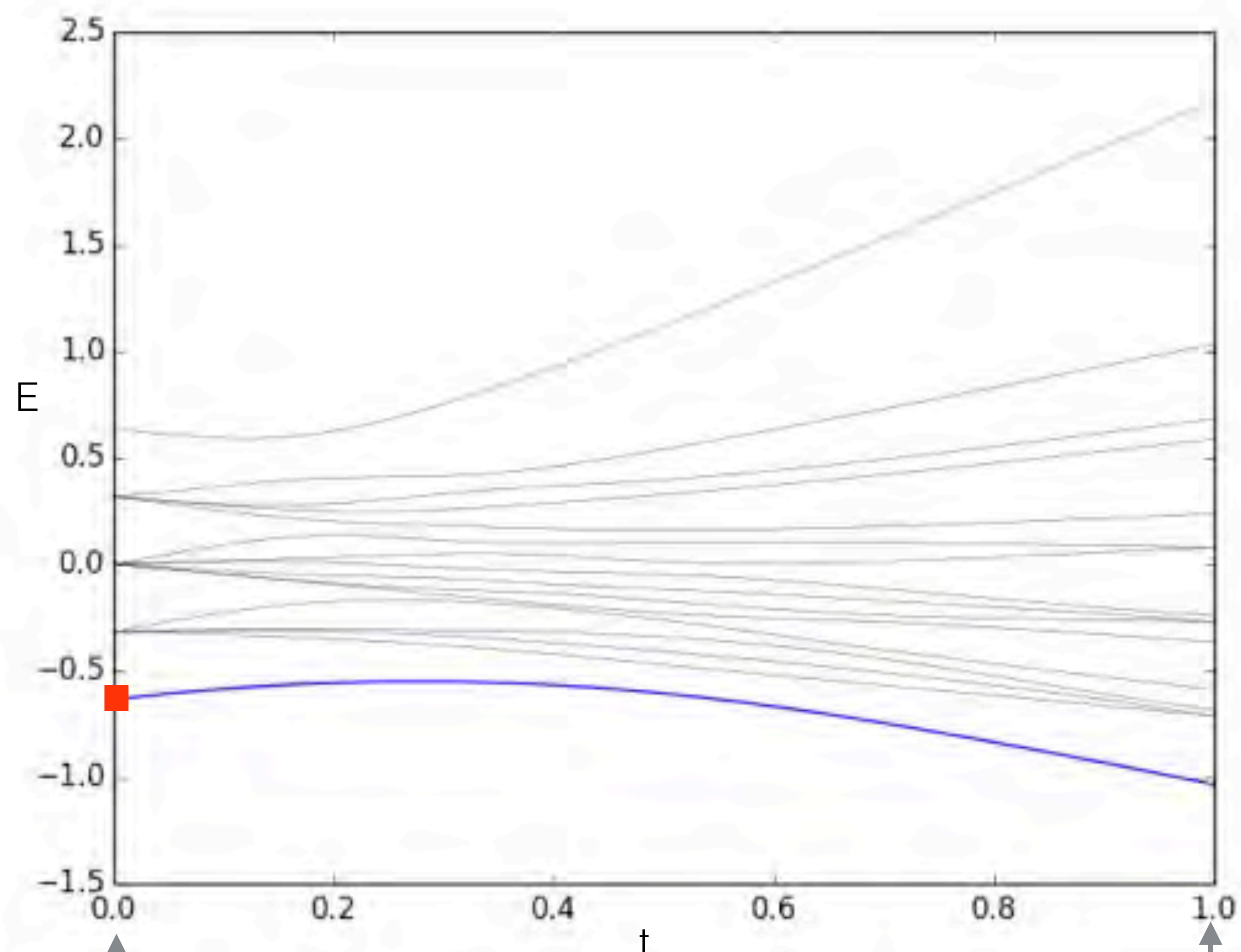
Problem Hamiltonian

1. Prepare System in $|\psi_0\rangle$ which is the ground state of H(0)

2. Slowly evolve the system with

$$|\psi(t)\rangle = e^{-i\int_0^t H(t)dt}|\psi_0\rangle$$

3. Due to the adiabatic theorem, the final state is the ground state of H_p



$$H_0 = \sum_i^N \sigma_x^{(i)}$$

$$H_p = \sum_{i<j} J_{ij}\sigma_z^{(i)}\sigma_z^{(j)}$$

Adiabatic Quantum Computing

Nishimori Hidetoshi Phys. Rev. B (1998).

E. Farhi et. al., Science 292, 472 (2001).

$$H(t) = \left(1 - \frac{t}{T}\right) H(0) + \frac{t}{T} H_p$$

Trivial initial Hamiltonian

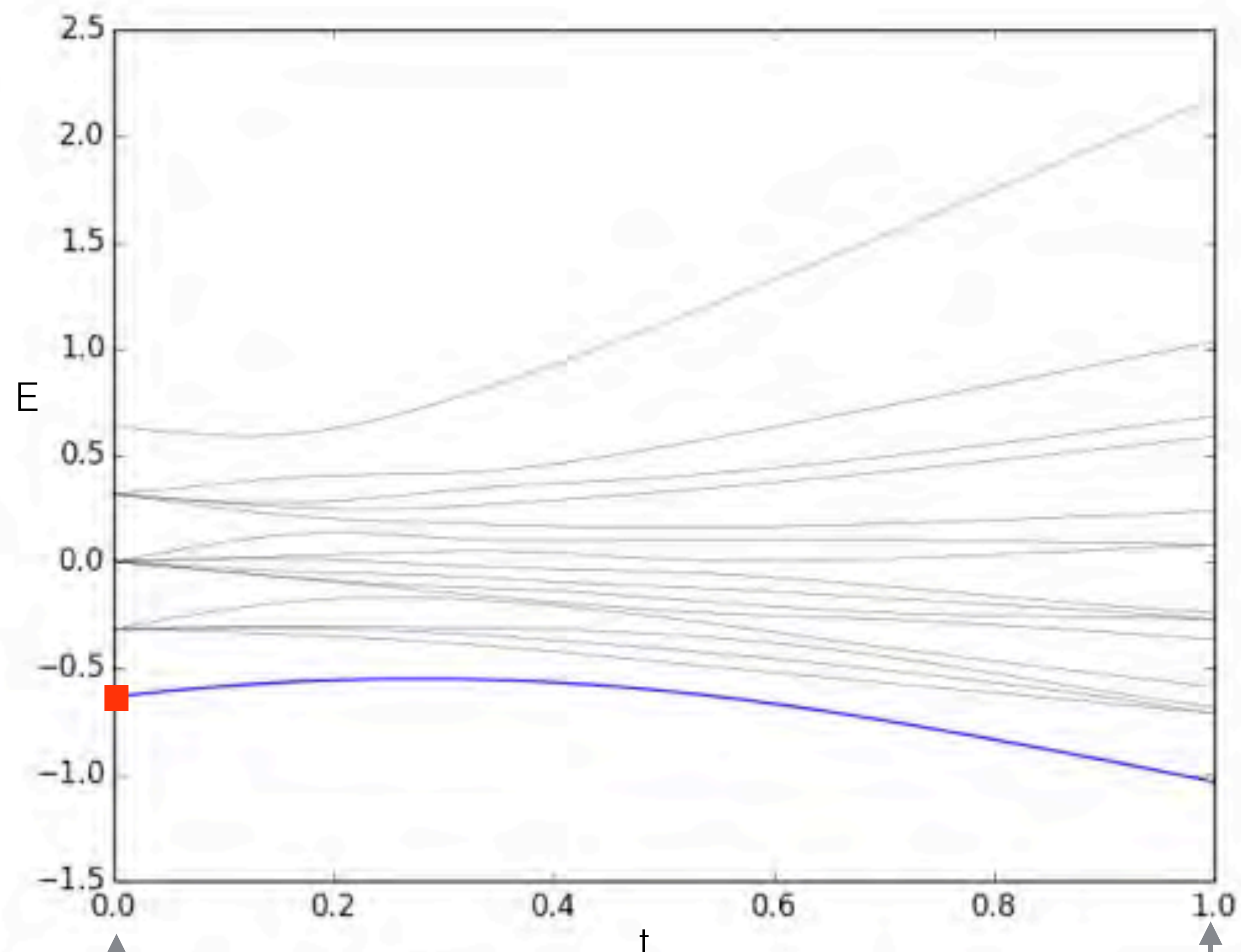
Problem Hamiltonian

1. Prepare System in $|\psi_0\rangle$
which is the ground state of H(0)

2. Slowly evolve the system with

$$|\psi(t)\rangle = e^{-i\int_0^t H(t)dt}|\psi_0\rangle$$

3. Due to the adiabatic theorem, the final state is the ground state of H_p



$$H_0 = \sum_i^N \sigma_x^{(i)}$$

$$H_p = \sum_{i<j} J_{ij}\sigma_z^{(i)}\sigma_z^{(j)}$$

Example: Quantum Adiabatic Search

J. Roland and N. J. Cerf Phys. Rev. A 65, 042308 (2001).
W. Dam, et. al. arxiv:quant-ph/0206003 (2008).

$$f : \{0,1\}^n \to \mathbb{R}$$

$$f(u) = \begin{cases} 0, & \text{if } u \in \{0,1\}^n. \\ 1, & \text{otherwise.} \end{cases}$$

where u is the search string.

where u is the search string.

Hamiltonian for quantum annealing:

$$H_u = \sum_{z \in \{0,1\}^n \backslash u} |z\rangle\langle z|$$

Spectrum

1 —— all other strings

0 —— search string

Example: Quantum Adiabatic Search

W. Dam, et. al. arxiv:quant-ph/0206003 (2008).

initial Hamiltonian

$$H_0 = \sum_{z \in \{0,1\}^n \setminus \{0\}^n} |z\rangle\langle z|$$

final Hamiltonian

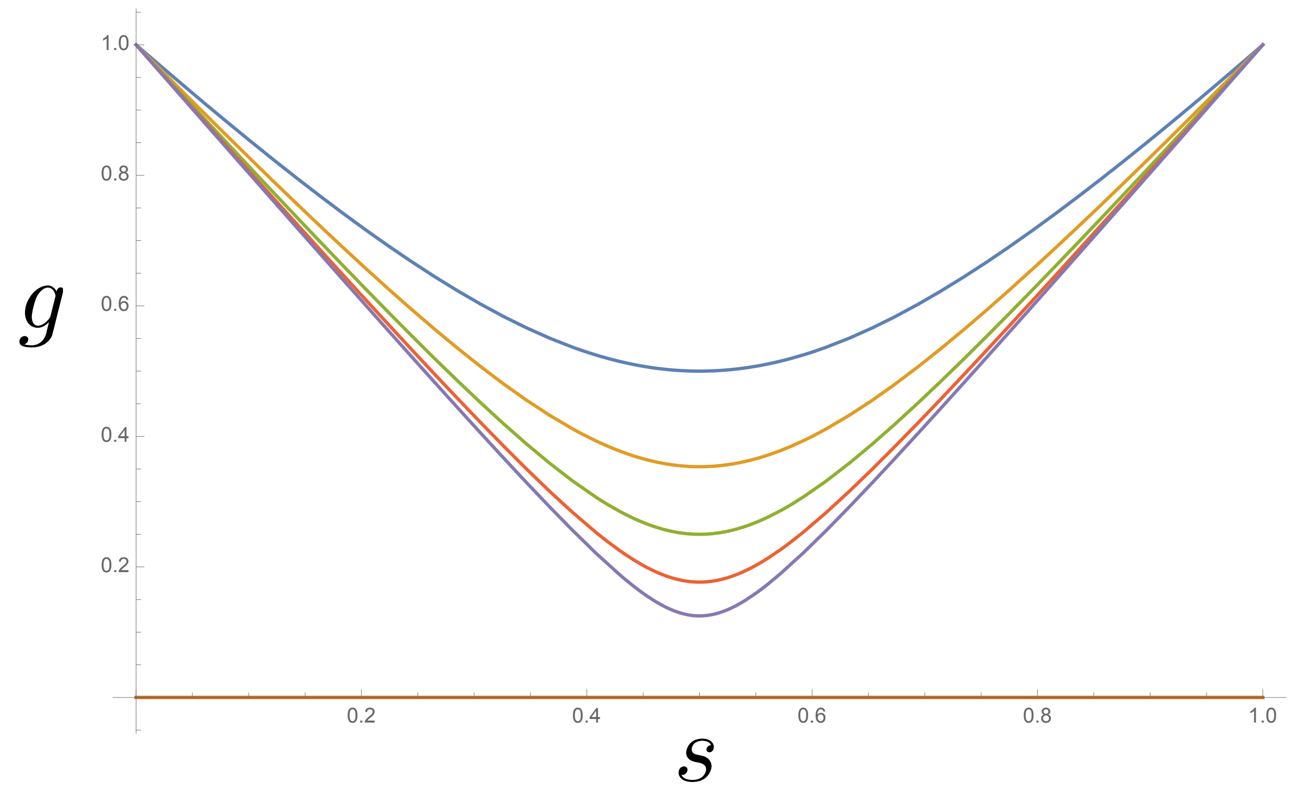
$$H_u = \sum_{z \in \{0,1\}^n \setminus u} |z\rangle\langle z|$$

Adiabatic protocol

$$H(t) = \left(1 - \frac{t}{T}\right) H_0 + \frac{t}{T} H_u$$

Gap as a function of $s = \dfrac{t}{T}$

$$g(s) = \sqrt{\frac{2^n + 4(2^n - 1)(s^2 - s)}{2^n}}$$



Example: Quantum Adiabatic Search

W. Dam, et. al. arxiv:quant-ph/0206003 (2008).

Minimum at $s^* = \frac{t}{T} = \frac{1}{2}$ $\quad g(s^*) = \sqrt{\frac{2^n + 4(2^n-1)(-\frac{1}{4})}{2^n}} = \frac{1}{\sqrt{2^n}}$

Naive :

$$T \approx \mathcal{O}(g(s^*)^{-2}) \sim \mathcal{O}(2^n) = \mathcal{O}(N)$$

Inverse gap schedule

$$T = \int_{s=0}^{1} \frac{ds}{g(s)^2} = \int_{s=0}^{1} \frac{2^n}{2^n + (2^n - 1)(s^2 - s)} ds$$

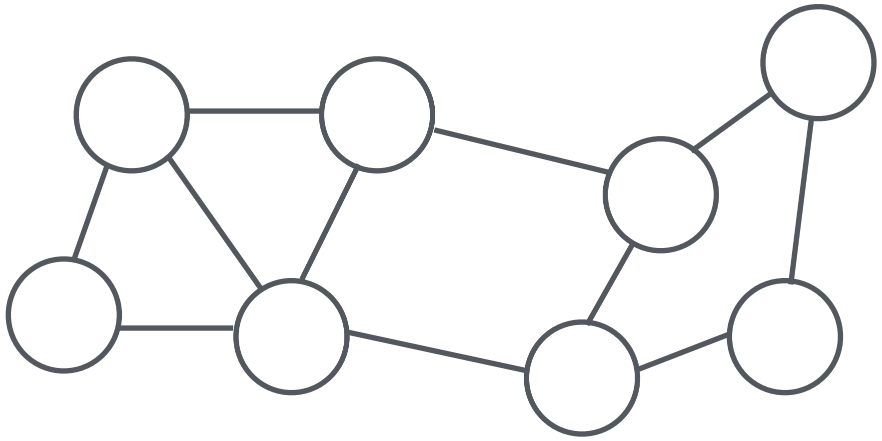
$$= \frac{2^n \ \arctan(\sqrt{2^n - 1})}{\sqrt{2^n - 1}} \sim \mathcal{O}(\sqrt{2^n}) = \mathcal{O}(\sqrt{N})$$

Quadratic Grover Speedup

further reading: D. Wild et.al. Phys. Rev. Lett. **117**, 150501 (2016).

Example: Quantum Optimization

Graph Partitioning



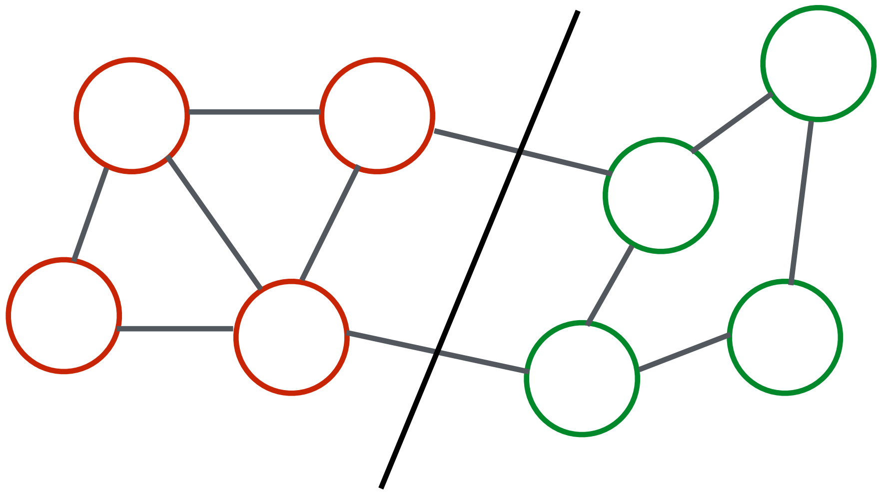
$$G = (V, E)$$

We ask: What is a partition of the set V into two subsets of equal size $N/2$ such that the number of edges connecting the two subsets is minimized?

A. Lucas, Frontiers in Physics ***2****, 00005 (2014).*

Example: Quantum Optimization

Graph Partitioning

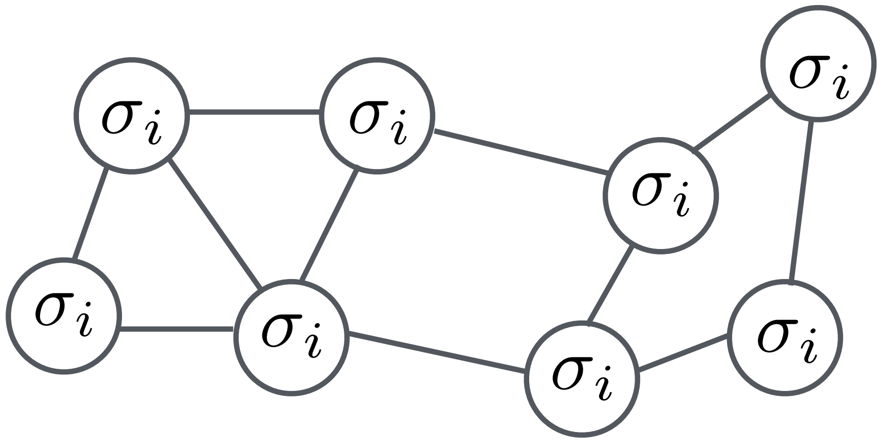


$$G = (V, E)$$

We ask: What is a partition of the set V into two subsets of equal size $N/2$ such that the number of edges connecting the two subsets is minimized?

Example: Quantum Optimization

Graph Partitioning



$$G = (V, E)$$

We ask: What is a partition of the set V into two subsets of equal size $N/2$ such that the number of edges connecting the two subsets is minimized?

Strategy: Cost-function = Energy

$$\sigma_i = +1$$

$$\sigma_i = -1$$

$$H_p = \boxed{C_A \left(\sum_{n=1}^{N} \sigma_i \right)^2} + \boxed{C_B \sum_{\{i,j\} \in E} \frac{1 - \sigma_i \sigma_j}{2}}$$

Two subsets have the same size.

Connection between two sets is minimal.

$$\mathcal{H}(t) = A(t) \sum_i^N b_i \sigma_x^{(i)} + B(t) \left[\sum_{i=1}^{N} h_i \sigma_z^{(i)} + \sum_{i=1}^{N} \sum_{j=1}^{i} J_{ij} \sigma_z^{(i)} \sigma_z^{(j)} \right]$$

Quantum Optimization

NP-complete problems = Infinite range spin glass

$$\mathcal{H}(t) = A(t)\sum_{i}^{N} b_i \sigma_x^{(i)} + B(t)\left[\sum_{i=1}^{N} h_i \sigma_z^{(i)} + \sum_{i=1}^{N}\sum_{j=1}^{i} J_{ij}\sigma_z^{(i)}\sigma_z^{(j)}\right]$$

List of NP-complete problems:

A. Lucas, *Frontiers in Physics* ***2****, 00005 (2014).*

- Traveling Salesman
- Knapsack
- 3SAT
- Number Partitioning
- Graph Coloring
- Minimal Spanning Tree
- Graph Partition
- … and many more

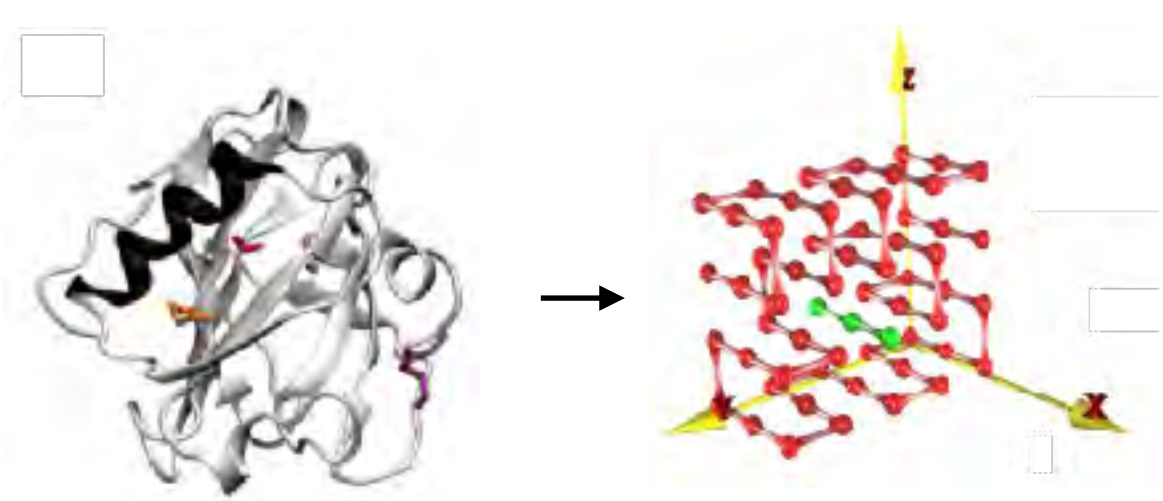
Quantum Optimization

NP-complete problems = Infinite range spin glass

$$\mathcal{H}(t) = A(t)\sum_{i}^{N} b_i \sigma_x^{(i)} + B(t)\left[\sum_{i=1}^{N} h_i \sigma_z^{(i)} + \sum_{i=1}^{N}\sum_{j=1}^{i} J_{ij}\sigma_z^{(i)}\sigma_z^{(j)}\right]$$

Scientific optimization problems:

Protein folding

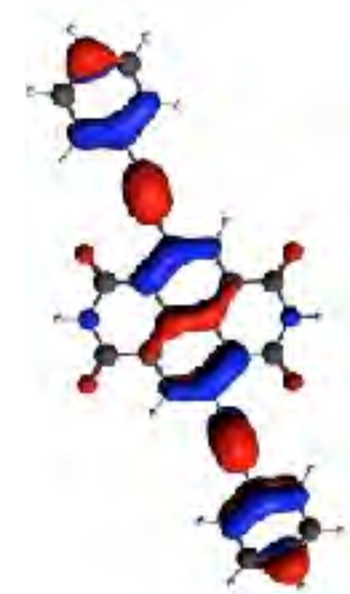


e.g. beta-lactoglobolin (milk protein)
Picture: Peter Bolhuis

I. Coluzza, et.al. Biophys. J. (2007).

Adiabatic Quantum computing: A. Perdomo-Ortiz et. al., Sci. Rep. 2, 571 (2012).

Quantum Chemistry



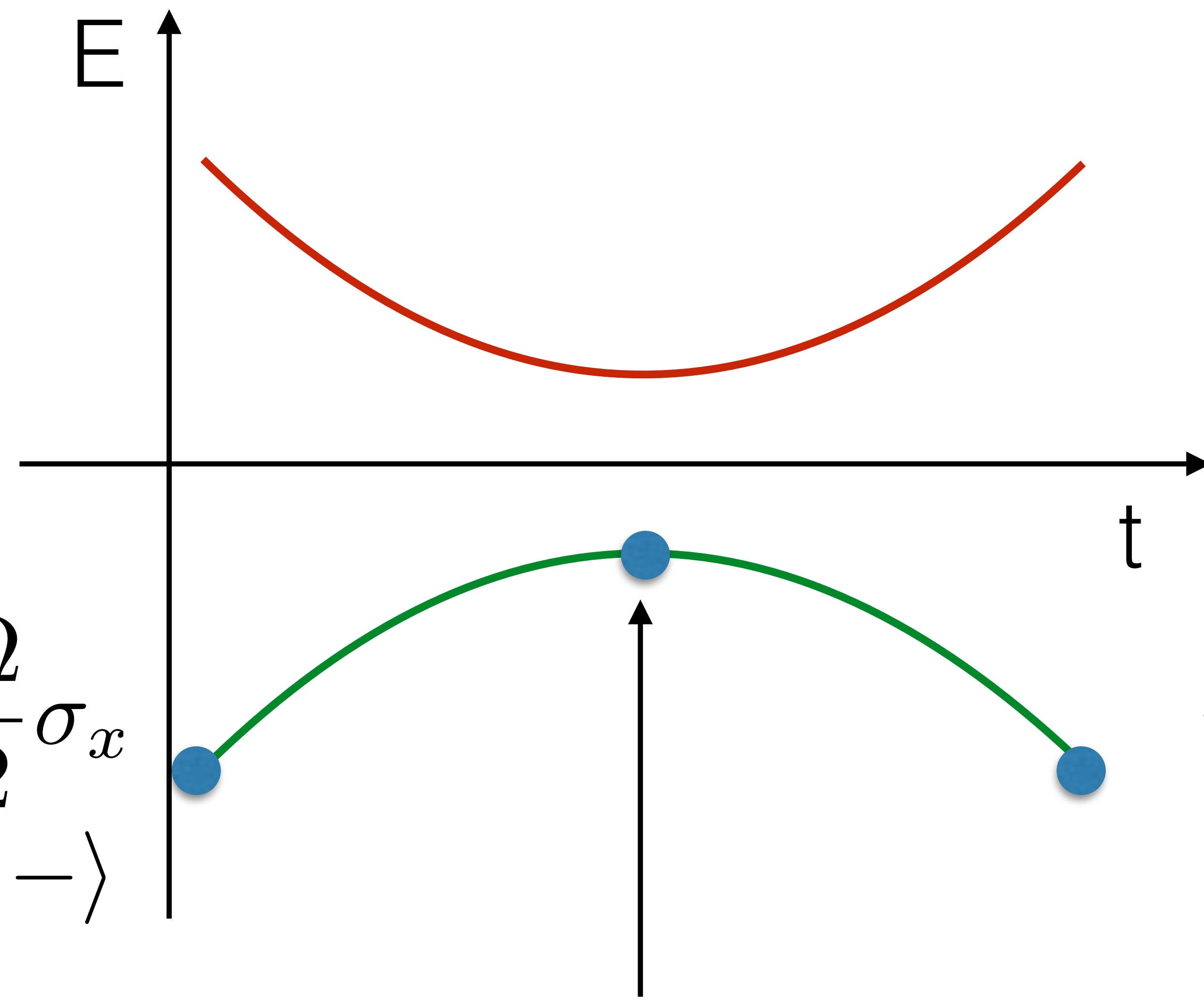
Picture: E. Meijer, University of Amsterdam.

Adiabatic Quantum Computing: R. Babbush et. al., Sci. Rep. 4, 6603 (2014).

4. Entanglement in adiabatic processes

Entanglement in an Adiabatic Protocol

$$H(t) = -\Delta\sigma_z + \frac{\Omega}{2}\sigma_x$$



$$H_0 = \frac{\Omega}{2}\sigma_x$$

$$|\psi_0\rangle = |-\rangle$$

$$H_f = -\Delta\sigma_z$$

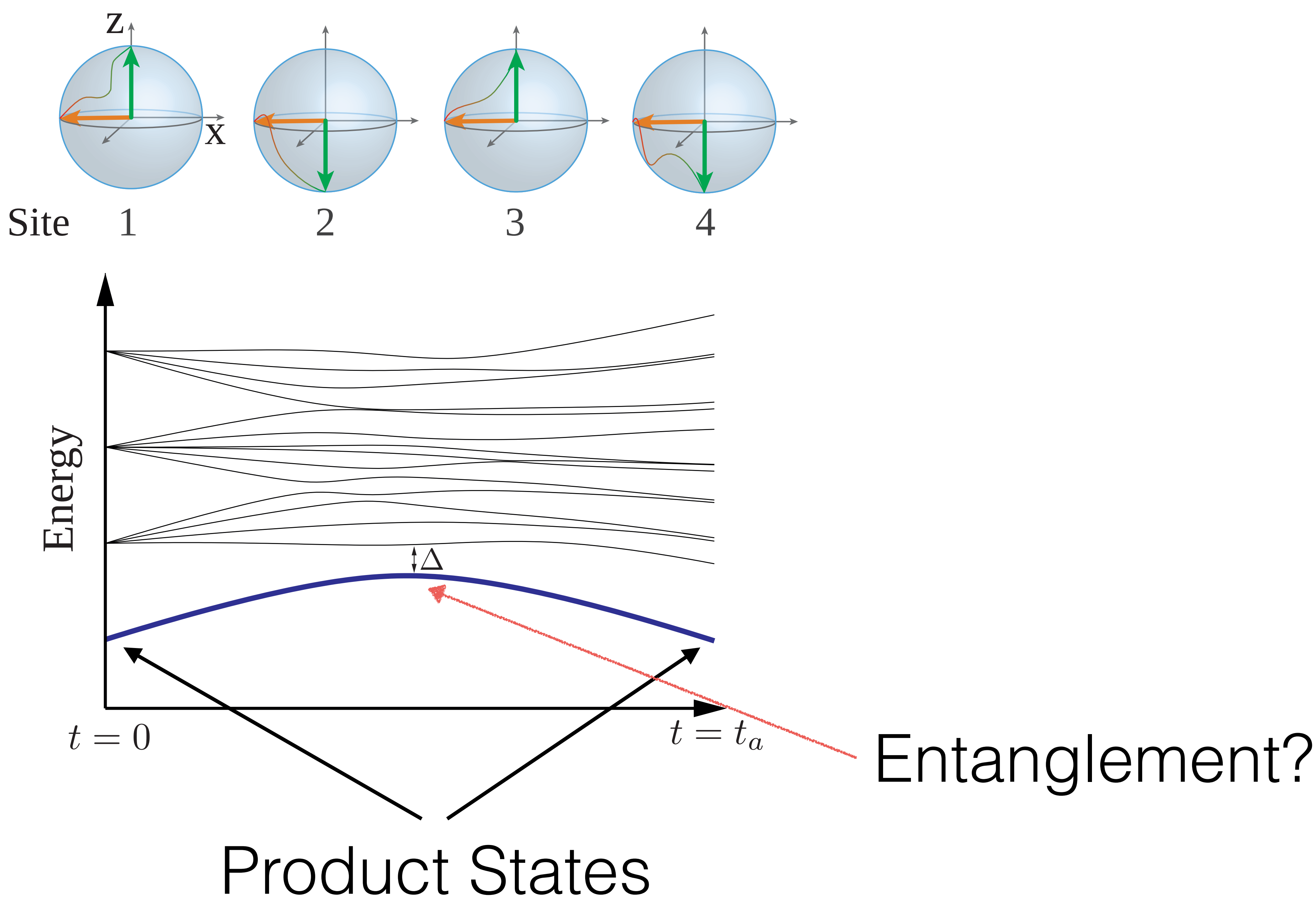
$$|\psi_f\rangle = |\downarrow\rangle$$

maximal entanglement

Entanglement in an Adiabatic Protocol

P. Hauke, L. Bonnes, M. Heyl and W. Lechner, Front. Phys. 3, 21 (2015).

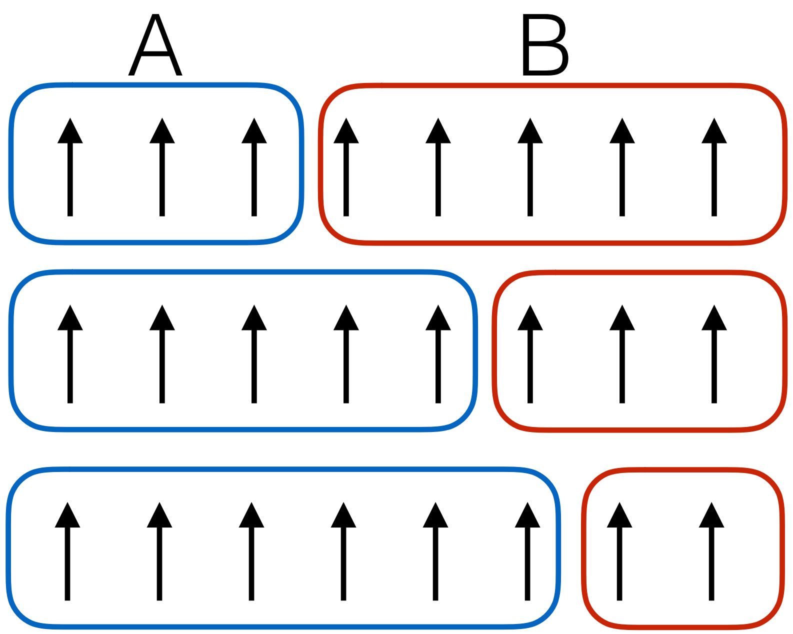
$$\mathcal{H}(t) = A(t)\sum_{i}^{N} b_i\sigma_x^{(i)} + B(t)\left[\sum_{i=1}^{N} h_i\sigma_z^{(i)} + \sum_{i=1}^{N}\sum_{j=1}^{i} J_{ij}\sigma_z^{(i)}\sigma_z^{(j)}\right]$$



Entanglement measures

Van Neumann Entropy

Relative entropy of two subsystems A and B



Entanglement Measure

$$\boxed{S_A = -\mathrm{Tr}\left[\rho_A \log(\rho_A)\right]}$$

$$\rho_A = \mathrm{Tr}_B \rho$$

Take Maximum of all combinations of A and B and maximum at each time $S_2^{\max} = \max_t \left[S_2^{\max}(t)\right]$

Features

+ well-established, simple to calculate
- only closed systems
- only block-pair entanglement

Entanglement measures

Quantum Fisher Information

P. Hyllus et.al., Phys. Rev. A 85, 022321 (2012).

Phase estimation experiment:

$$\rho(\theta) = e^{-i\theta H} \rho e^{i\theta H}$$

Want to measure unknown phase

Estimate θ from m independent POVM (positive operator values measurements) $\{E_\mu\}$

$\theta_{est}(\{\mu_i\}_m)$ with $\{\mu_i\}_m = \{\mu_1, \mu_2, ..., \mu_m\}$

m measurement outcomes μ

Average

$$\langle \theta_{est} \rangle$$

Standard deviation

$$(\Delta\theta_{est})^2 = \langle \theta_{est}^2 \rangle - \langle \theta_{est} \rangle^2$$

Entanglement measures

P. Hyllus et.al., Phys. Rev. A 85, 022321 (2012).

Standard Deviation is limited by the bounds

$$\Delta\theta_{est} \geq \frac{1}{\sqrt{mF_Q}}$$

Quantum Fisher Information

$$\boxed{F_Q[\rho, H] = 2\sum_{l,l'} \frac{(\lambda_l - \lambda_{l'})^2}{\lambda_l + \lambda_{l'}} |\langle l|H|l'\rangle|}$$

for any mixed state

$$\rho = \sum_l \lambda_l |l\rangle\langle l| \qquad \text{with} \quad \sum_l \lambda_l = 1 \qquad \lambda_l > 0$$

Entanglement measures

P. Hyllus et.al., Phys. Rev. A 85, 022321 (2012).

- A pure state is **k-producible** if it can be written as

$$|\psi_{\text{k-prod}}\rangle = \otimes_{l=1}^{M} |\psi_l\rangle \text{ where } |\psi_l\rangle \text{ is a state of } N_l \leq k \text{ particles}$$

$$\text{with } \sum_l^M N_l = N$$

- A pure state is **k-particle entangled** if it is k-producible but not (k-1)-producible.

- A **mixed state** is k-producible if it can be written as

$$\rho_{\text{k-prod}} = \sum_l p_l |\psi_{\text{k-prod}}\rangle\langle\psi_{\text{k-prod}}|$$

Entanglement measures

P. Hyllus et.al., Phys. Rev. A 85, 022321 (2012).

Quantum Fisher Information bounds k-particle entanglement

$$\boxed{F_Q \leq \lfloor \frac{N}{k} \rfloor k^2 + \left(N - \lfloor \frac{N}{k} \rfloor k \right)^2}$$

where $\lfloor \cdot \rfloor$ denotes a floor operation

Features

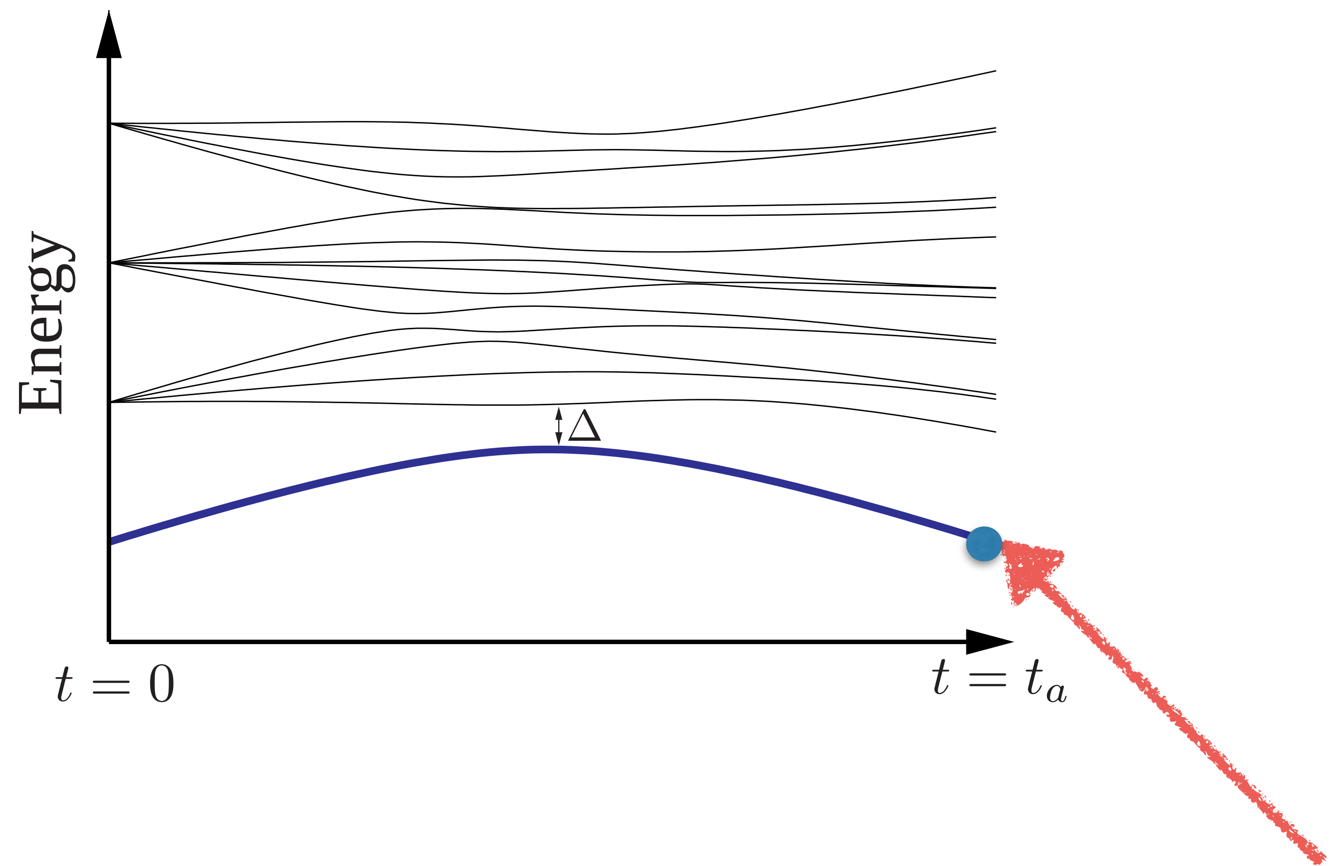
+ measure for multipartite entanglement
+ also density matrices
- more involved to measure

Entanglement and Success Probability

P. Hauke, L. Bonnes, M. Heyl and W. Lechner, Front. Phys. 3, 21 (2015).

Instantaneous probability to be in the ground state

$$P(t) = |\langle \psi(t) | \psi_{\mathrm{gs}} \rangle|^2.$$



Success Probability is the ground state overlap at the final time

$$P_{\mathrm{succ}} = |\langle \psi(T) | \psi_{\mathrm{gs}} \rangle|^2.$$

Entanglement and Success Probability

P. Hauke, L. Bonnes, M. Heyl and W. Lechner, Front. Phys. 3, 21 (2015).
A. Polkovnikov, Annals of Physics 326, 486 (2011).

Ideal solution

$$|\psi(T)\rangle = |s_A^* s_B^*\rangle$$

Finite speed

$$\rho_A = \mathrm{Tr}_B \rho$$

Entropy from population of “wrong” states

$$S_A \leq S_A^d = -\mathrm{tr}[\rho_A^d \log(\rho_A^d)] = -\sum_{\mu=1}^{d_A} p_\mu \log(p_\mu)$$

ρ_A^d only diagonal elements of ρ_A

$d_A = 2^l$ number of spins in A

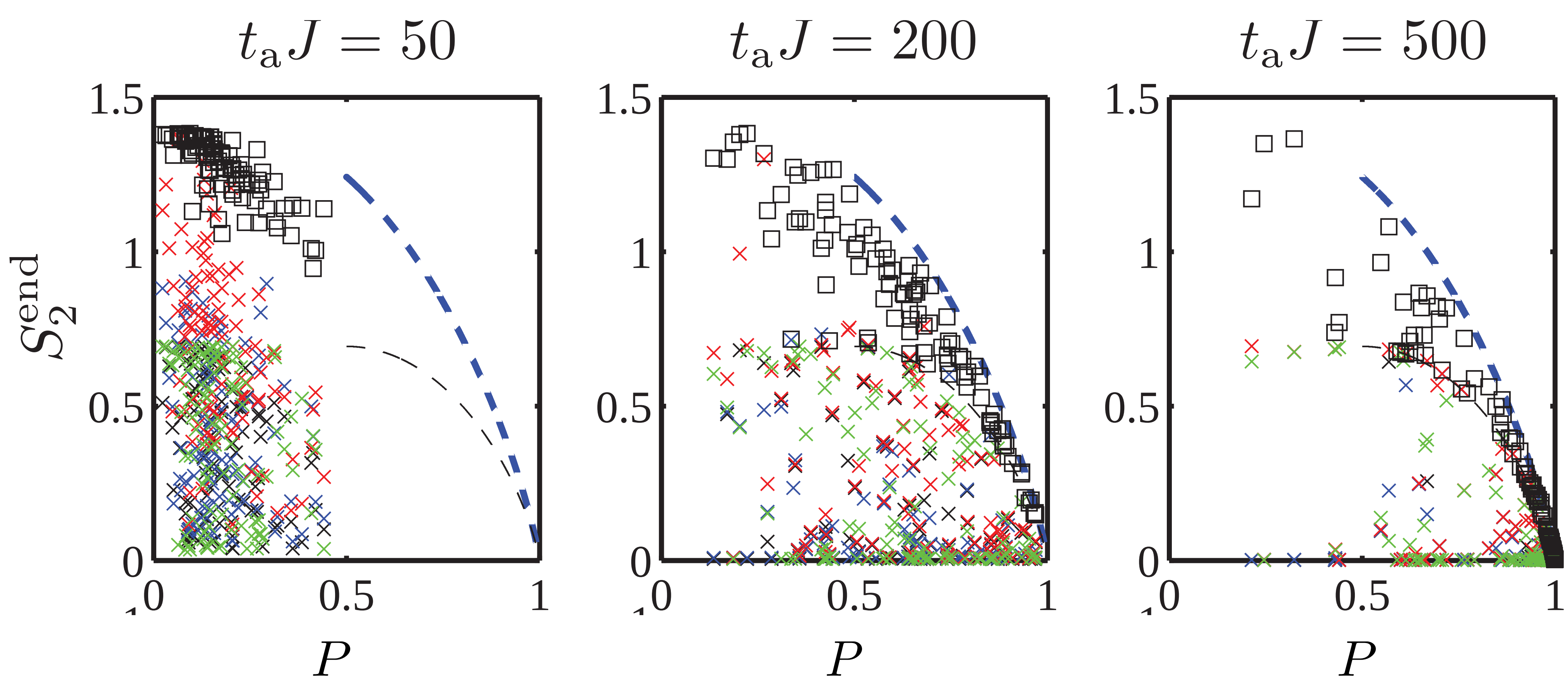
p_μ Eigenvalues of ρ_A^d

Entanglement Entropy as a function of the Success probability

$$S_l^A \leq -P\log(P) - (1-P)\log(1-P) + (1-P)\log(2^l - 1)$$

Entanglement and Success Probability

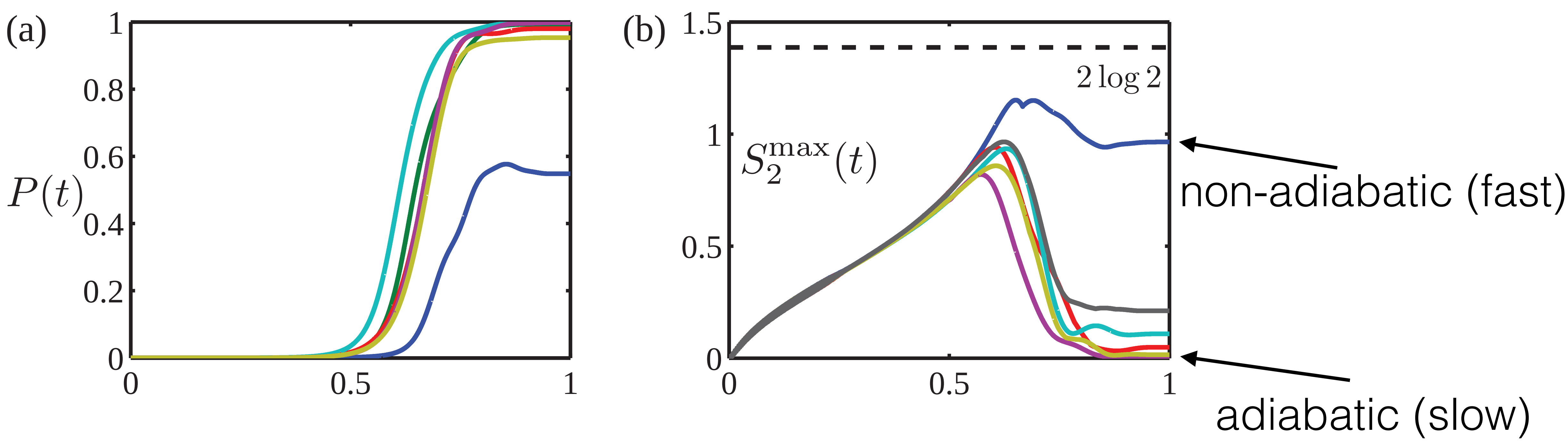
P. Hauke, L. Bonnes, M. Heyl and W. Lechner, Front. Phys. 3, 21 (2015).



Entanglement and Success Probability

P. Hauke, L. Bonnes, M. Heyl and W. Lechner, Front. Phys. 3, 21 (2015).

Individual Run: Connection between irreversibility and entanglement



5.Quantum Annealing with Parity Constraints

WL, P. Hauke, P. Zoller, Science Advances 1, 1500838 (2015).

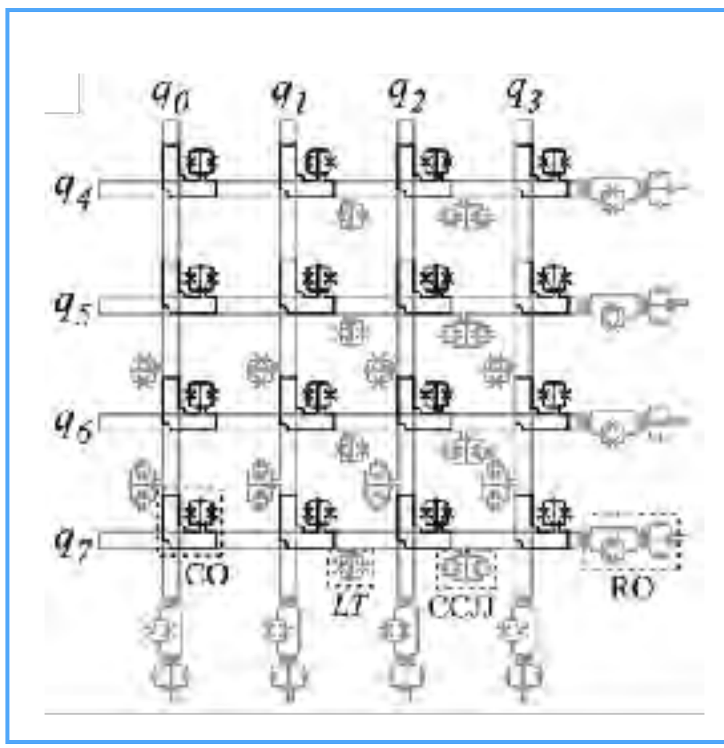
Adiabatic Quantum Annealing

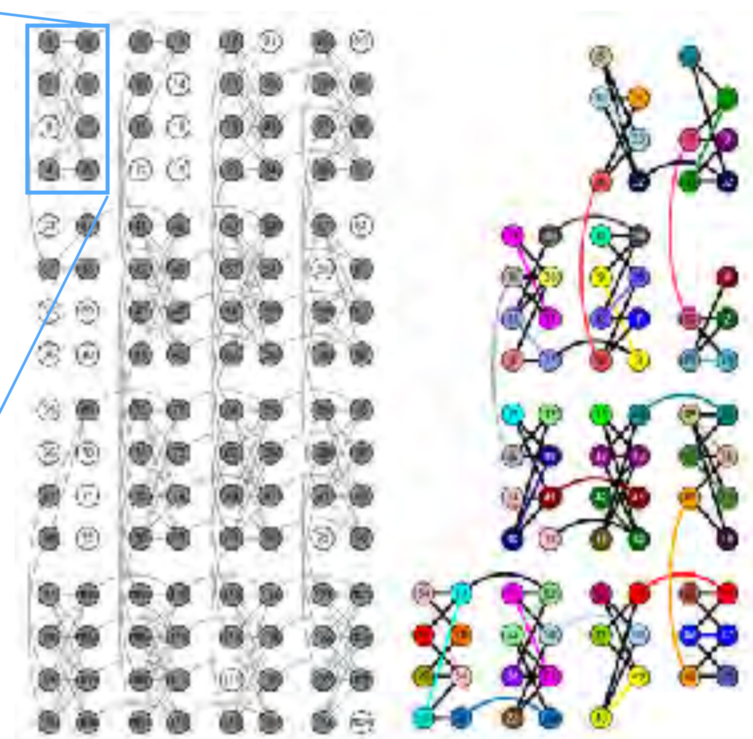
Spin Glass paradigm of AQC

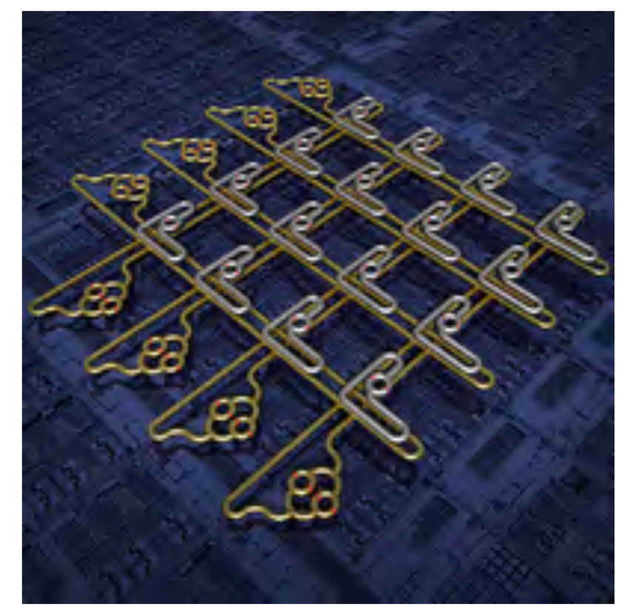
$$\mathcal{H}(t) = A(t)\sum_{i}^{N} b_i \sigma_x^{(i)} + B(t)\left[\sum_{i=1}^{N} h_i \sigma_z^{(i)} + \sum_{i=1}^{N}\sum_{j=1}^{i} J_{ij}\sigma_z^{(i)}\sigma_z^{(j)}\right]$$

Superconducting Qubits

Chimera-graphs







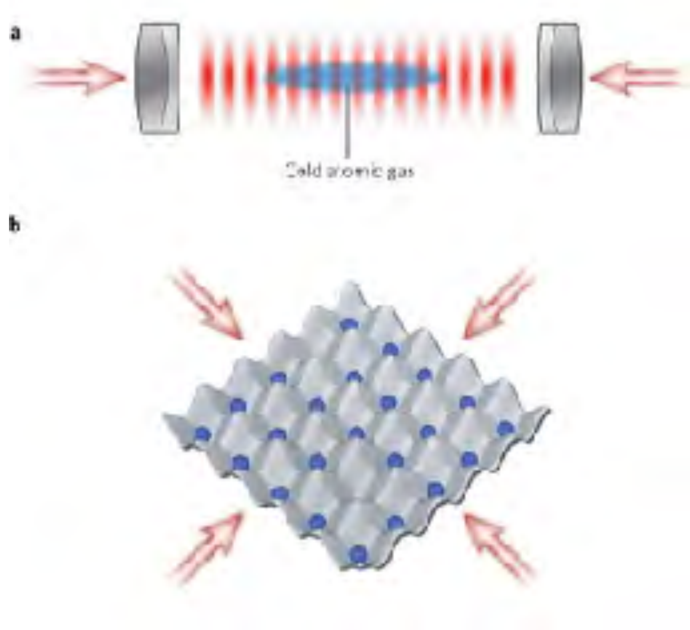
(D-Wave Systems)

M. W. Johnson et. al. Supercond. Sci. Technol. **23** 065004 (2010).

Z. Bian,et. al. Phys. Rev. Lett. ***111****, 130505 (2013).*

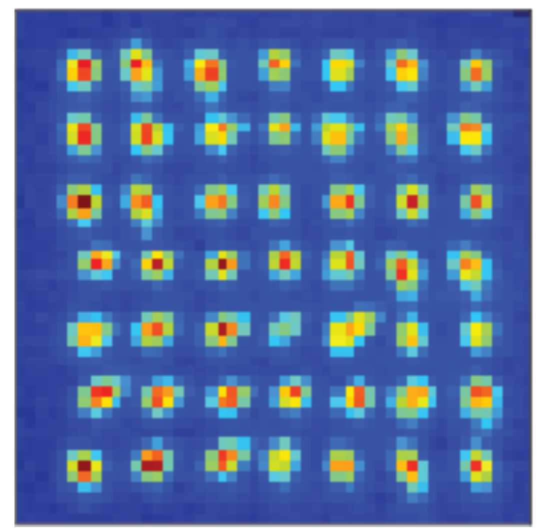
New platforms for Quantum Annealing

Ultracold atoms in optical lattices



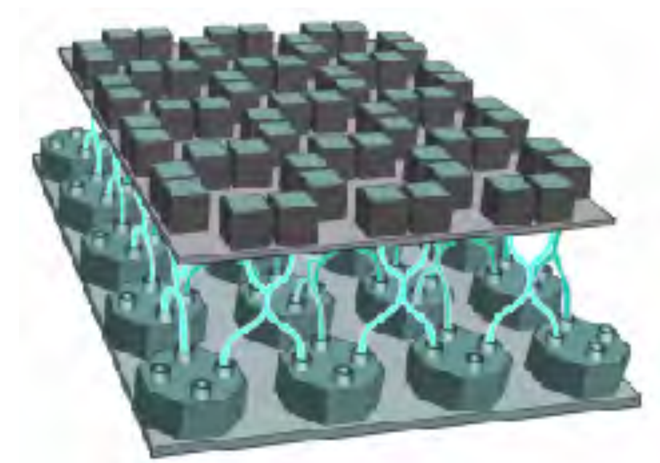
Bloch, Munich

Rydberg atoms



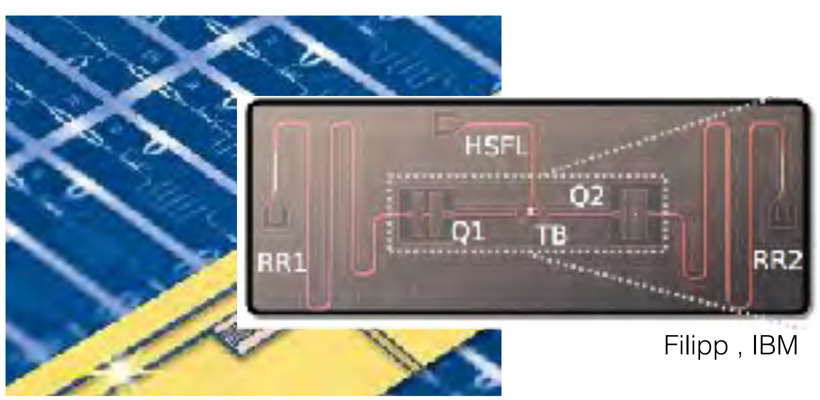
Saffman, Madison

Hybrid Ion-traps



S. Benjamin, Oxford

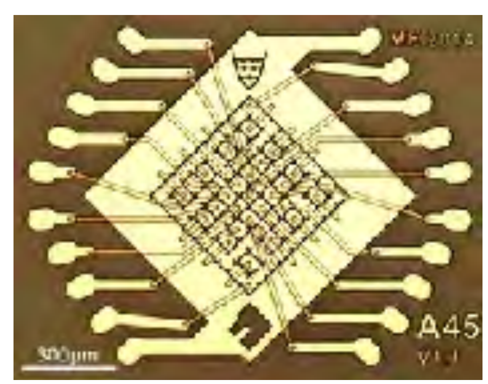
Transmons



Filipp , IBM

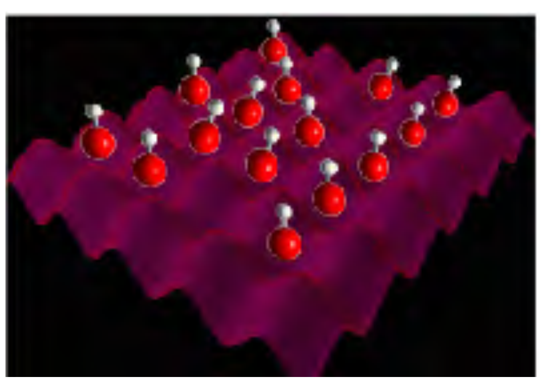
A. Wallraff

Ions in surface traps



Blatt, Innsbruck

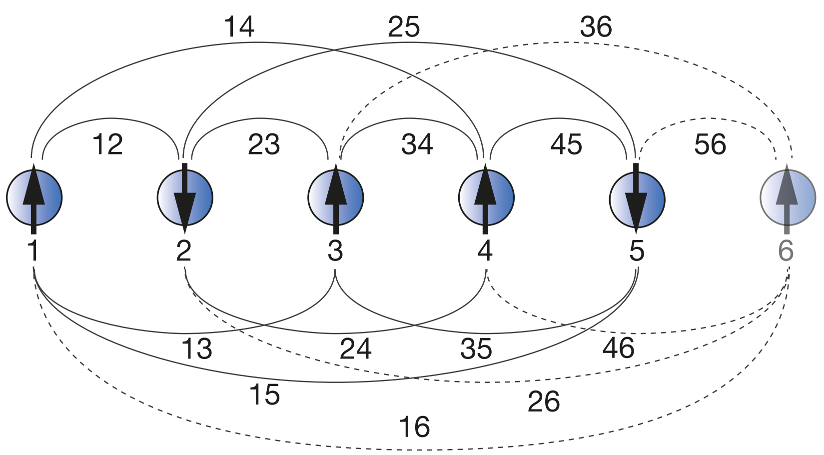
Polar Molecules



J. Ye, Boulder

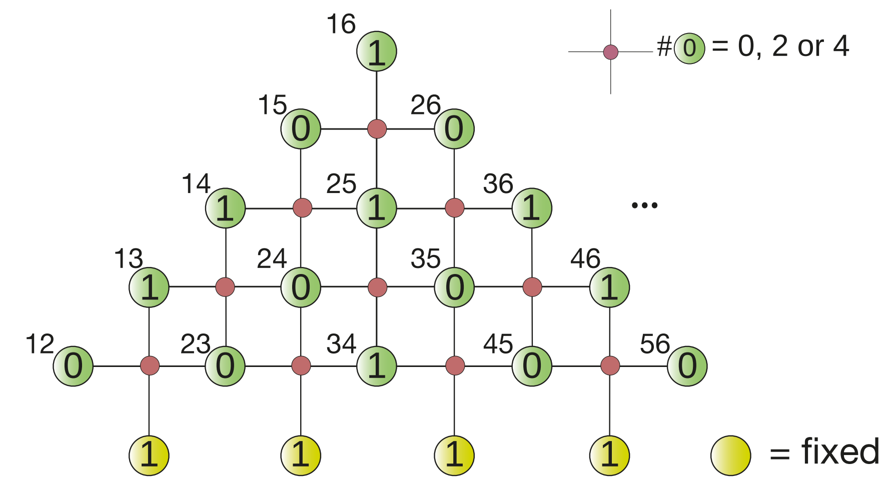
Parity Based Quantum Annealing

Spin glass paradigm



$$\mathcal{H}(t) = A(t)\sum_{i}^{N} b_i \sigma_x^{(i)} + B(t)\left[\sum_{i=1}^{N} h_i \sigma_z^{(i)} + \sum_{i=1}^{N}\sum_{j=1}^{i} J_{ij}\sigma_z^{(i)}\sigma_z^{(j)}\right]$$

Parity paradigm



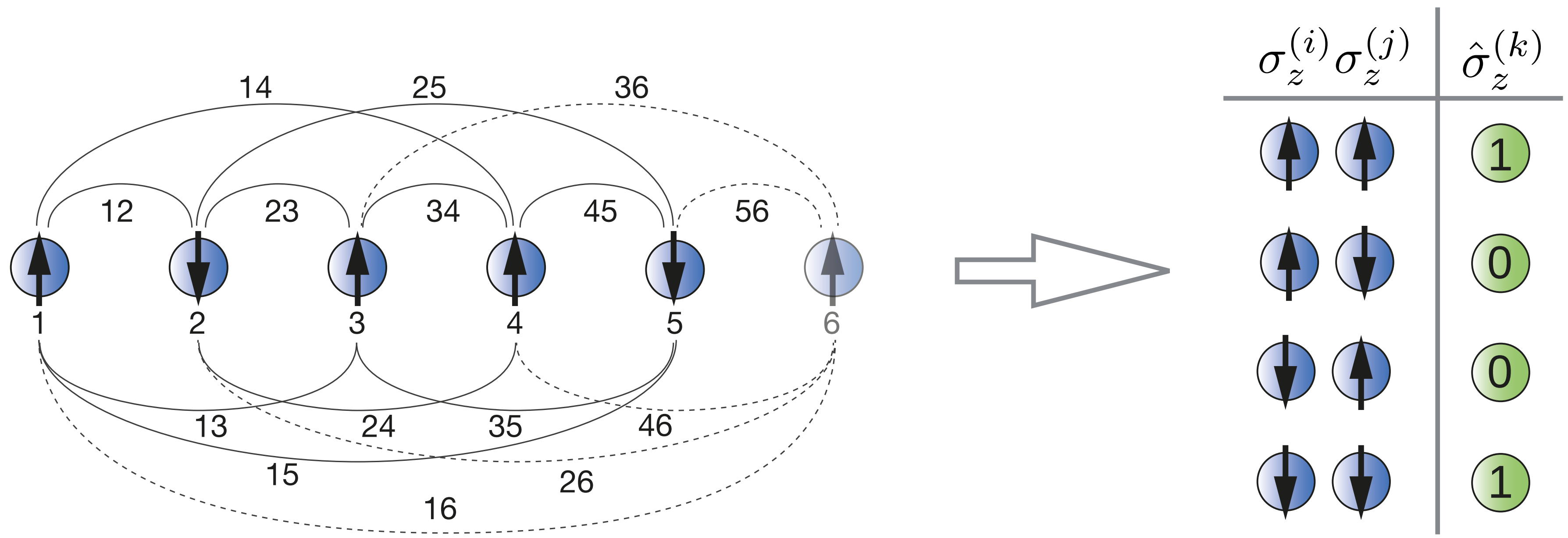
$$\mathcal{H}(t) = A(t)\sum_{i=1}^{K} b_i \sigma_x^{(i)} + B(t)\sum_{i=1}^{K} J_i \sigma_z^{(i)} + C(t)\sum_{l=1}^{K-N} C_l$$

WL, P. Hauke, P. Zoller, Science Advances 1, 1500838 (2015).

- encode **all-to-all interaction matrix** in local fields
- architecture comprises **only local interaction** on 2D geometry
- can be realized in current **qubit platform**
  (e.g. flux qubits, transmon qubits, ultracold atoms, hybrids systems,…)
- **intrinsic error tolerance** from redundant encoding

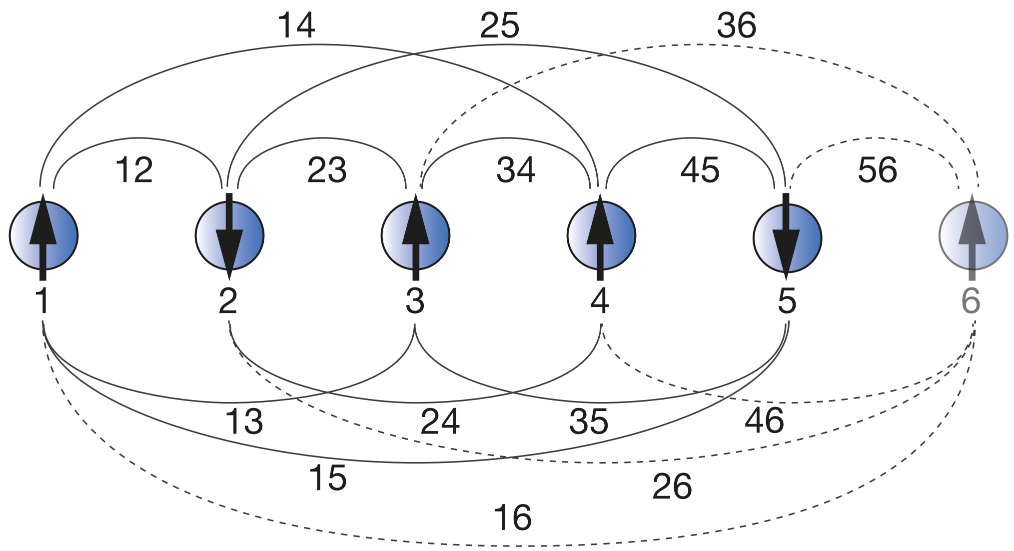
All-to-all programmable architecture

Transformation to parity variables.



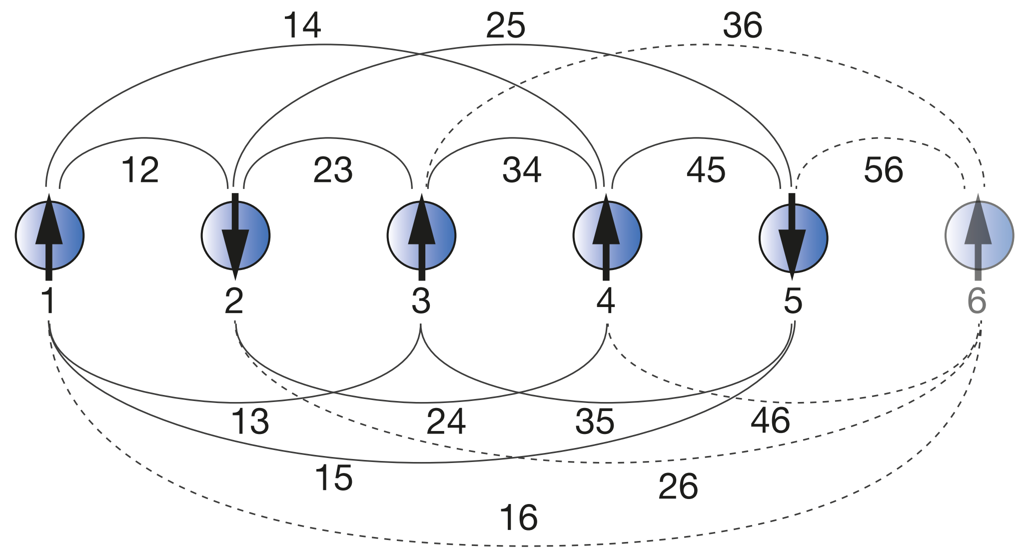
Physical qubits $\hat{\sigma}_z^{(k)}$ represent parity of **logical qubits.**

All-to-all programmable architecture



$$\mathcal{H} = \sum_{i=1}^{N} \sum_{j=1}^{i} J_{ij} \sigma_z^{(i)} \sigma_z^{(j)} = \sum_{i=1}^{K} J_i \hat{\sigma}_z^{(i)}$$

All-to-all programmable architecture

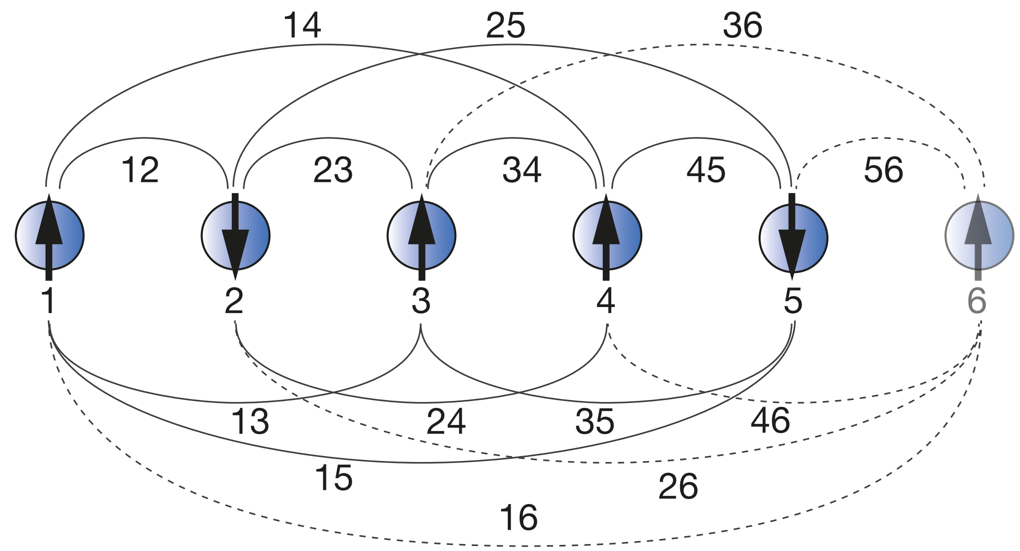


$$\mathcal{H} = \sum_{i=1}^{N} \sum_{j=1}^{i} J_{ij} \sigma_z^{(i)} \sigma_z^{(j)} = \sum_{i=1}^{K} J_i \hat{\sigma}_z^{(i)}$$

N degrees of freedom

$K = \dfrac{N(N-1)}{2}$ degrees of freedom

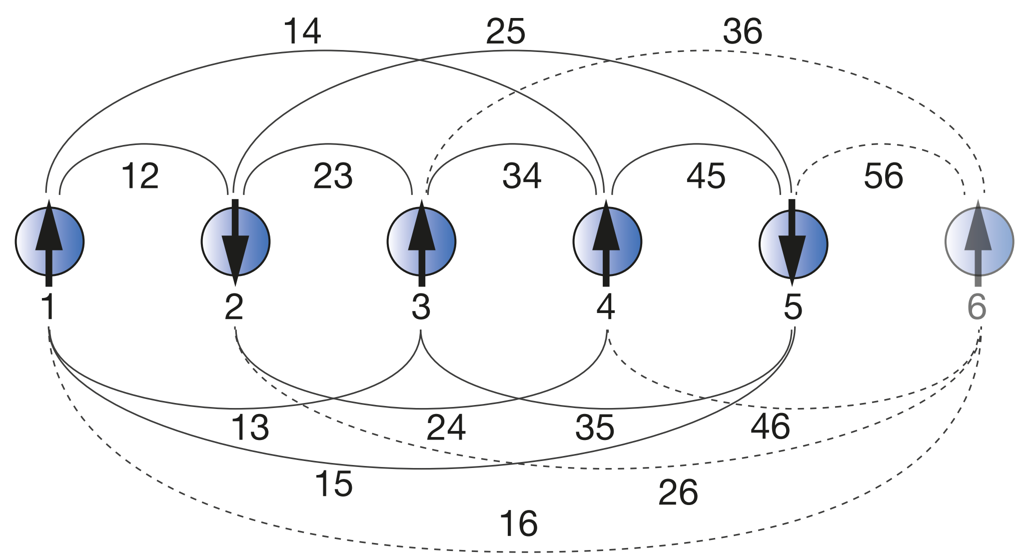
All-to-all programmable architecture



$$\mathcal{H} = \sum_{i=1}^{N} \sum_{j=1}^{i} J_{ij} \sigma_z^{(i)} \sigma_z^{(j)} = \sum_{i=1}^{K} J_i \hat{\sigma}_z^{(i)} + \sum_{l=1}^{K-N} C_l$$

K-N constraints

All-to-all programmable architecture



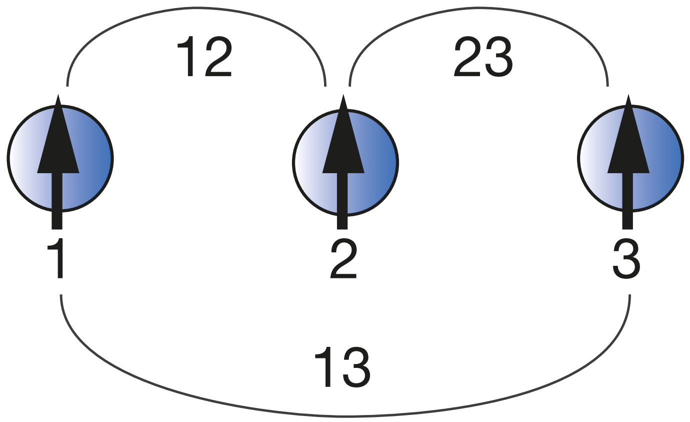
$$\mathcal{H} = \sum_{i=1}^{N} \sum_{j=1}^{i} J_{ij} \sigma_z^{(i)} \sigma_z^{(j)} = \sum_{i=1}^{K} J_i \hat{\sigma}_z^{(i)} + \sum_{l=1}^{K-N} C_l$$

K-N constraints

What are these constraints?

Constraints

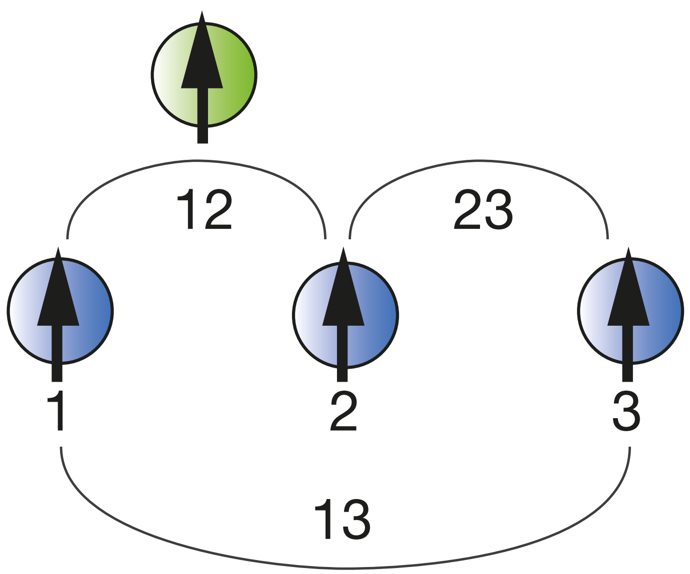
Conditions on closed loops



$\sigma_z^{(1)}$	$\sigma_z^{(2)}$	$\sigma_z^{(3)}$	$\hat{\sigma}_z^{(12)}$	$\hat{\sigma}_z^{(13)}$	$\hat{\sigma}_z^{(23)}$
1	1	1			

Constraints

Conditions on closed loops

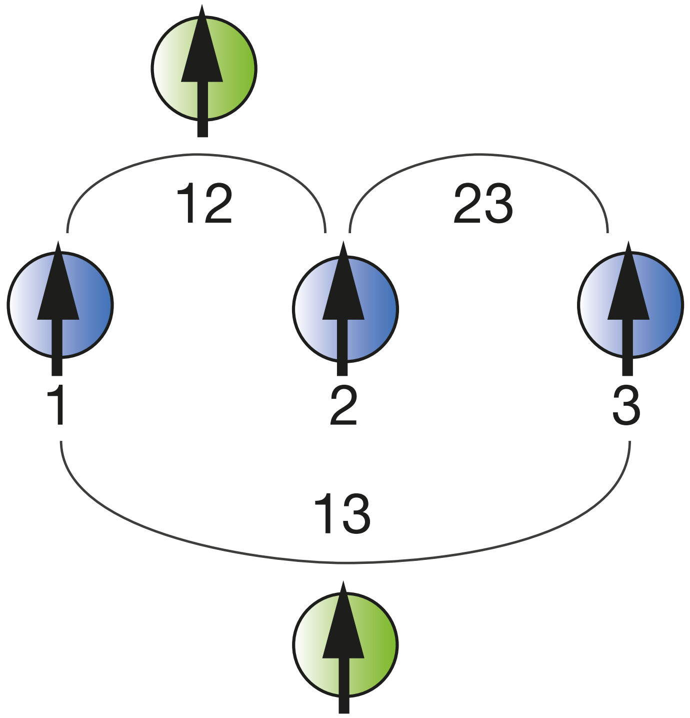


$\sigma_z^{(1)}$	$\sigma_z^{(2)}$	$\sigma_z^{(3)}$	$\hat{\sigma}_z^{(12)}$	$\hat{\sigma}_z^{(13)}$	$\hat{\sigma}_z^{(23)}$
1	1	1	1		

Constraints

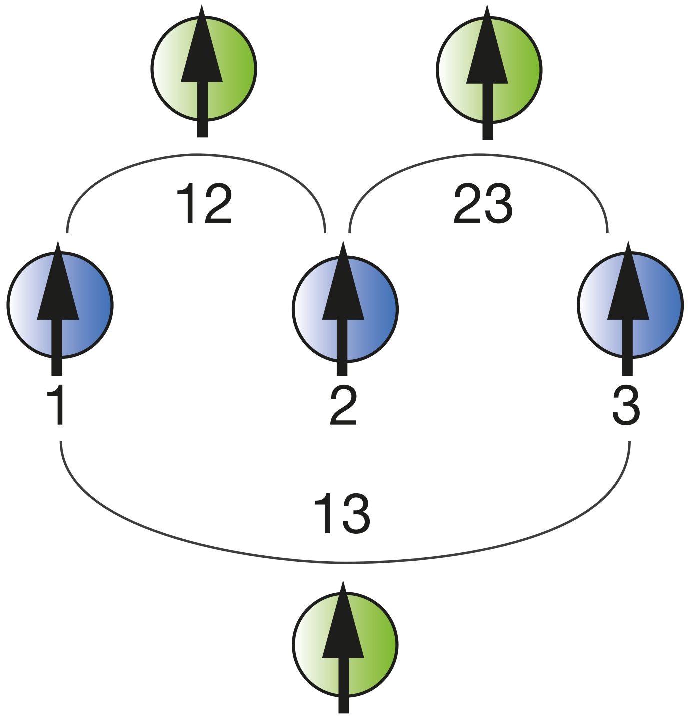
Conditions on closed loops

$\sigma_z^{(1)}$	$\sigma_z^{(2)}$	$\sigma_z^{(3)}$	$\hat{\sigma}_z^{(12)}$	$\hat{\sigma}_z^{(13)}$	$\hat{\sigma}_z^{(23)}$
1	1	1	1	1	



Constraints

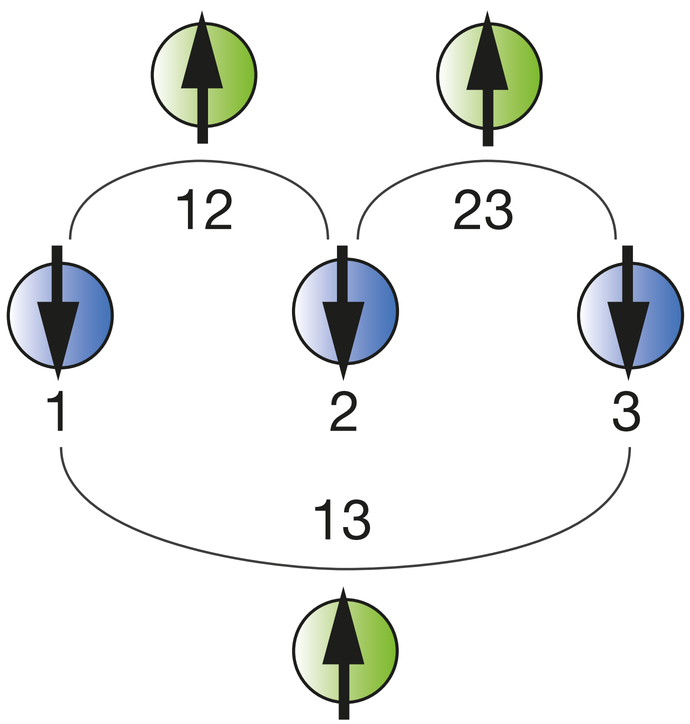
Conditions on closed loops



$\sigma_z^{(1)}$	$\sigma_z^{(2)}$	$\sigma_z^{(3)}$	$\hat{\sigma}_z^{(12)}$	$\hat{\sigma}_z^{(13)}$	$\hat{\sigma}_z^{(23)}$
1	1	1	1	1	1

Constraints

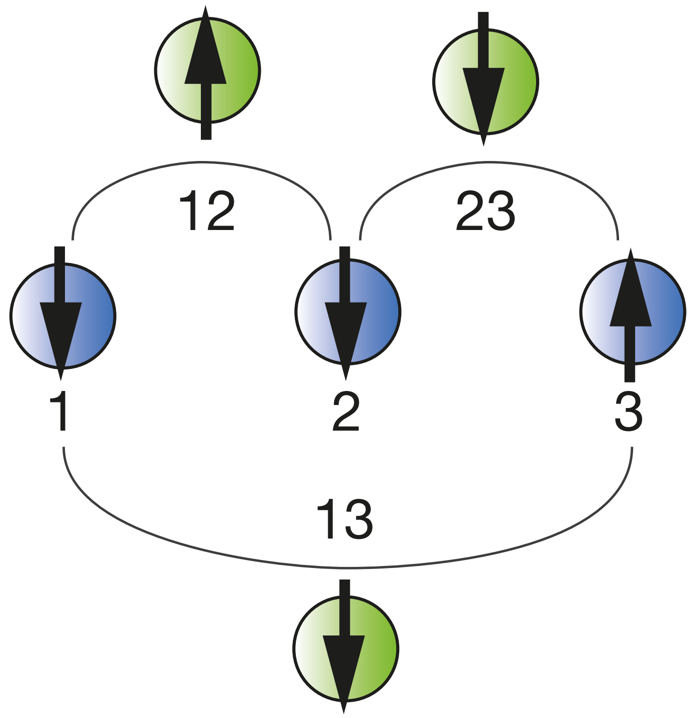
Conditions on closed loops



$\sigma_z^{(1)}$	$\sigma_z^{(2)}$	$\sigma_z^{(3)}$	$\hat{\sigma}_z^{(12)}$	$\hat{\sigma}_z^{(13)}$	$\hat{\sigma}_z^{(23)}$
1	1	1	1	1	1
0	0	0	1	1	1

Constraints

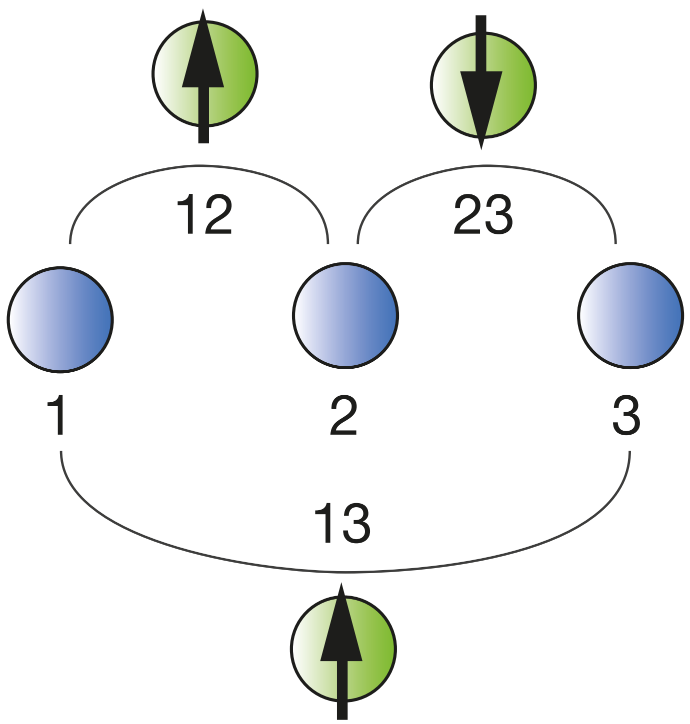
Conditions on closed loops



$\sigma_z^{(1)}$	$\sigma_z^{(2)}$	$\sigma_z^{(3)}$	$\hat{\sigma}_z^{(12)}$	$\hat{\sigma}_z^{(13)}$	$\hat{\sigma}_z^{(23)}$
1	1	1	1	1	1
0	0	0	1	1	1
0	0	1	1	0	0

Constraints

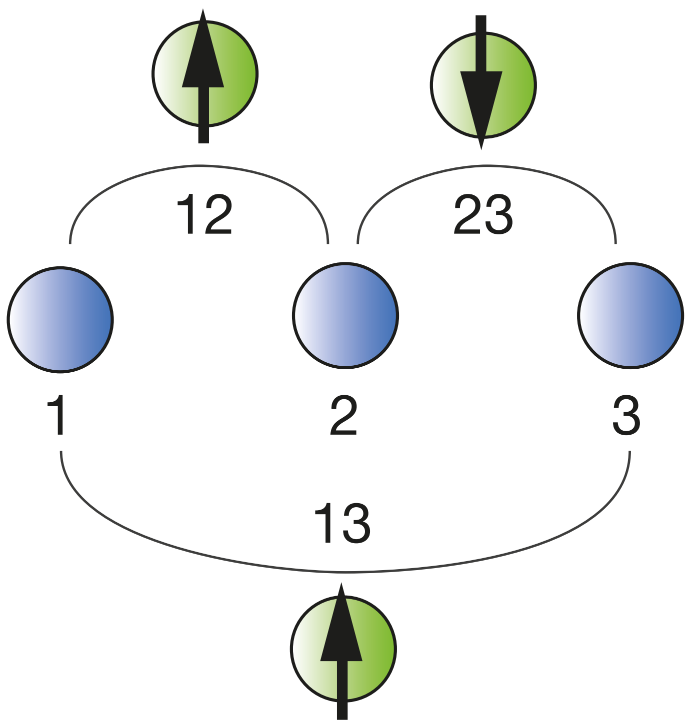
Conditions on closed loops



$\sigma_z^{(1)}$	$\sigma_z^{(2)}$	$\sigma_z^{(3)}$	$\hat{\sigma}_z^{(12)}$	$\hat{\sigma}_z^{(13)}$	$\hat{\sigma}_z^{(23)}$
1	1	1	1	1	1
0	0	0	1	1	1
0	0	1	1	0	0
	$\ldots$			$\ldots$	
			1	1	0

Constraints

Conditions on closed loops

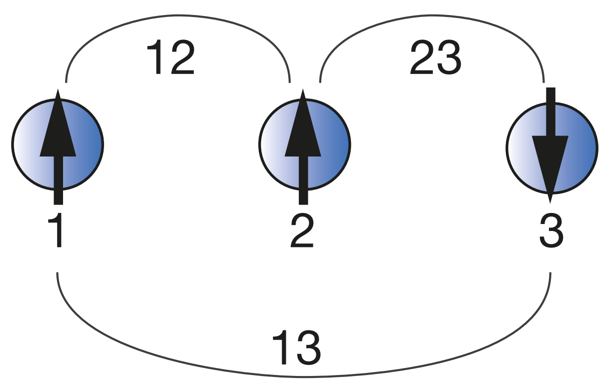


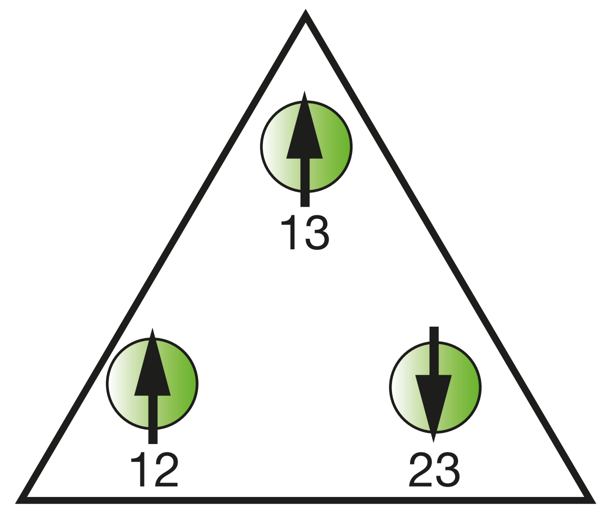
$\sigma_z^{(1)}$	$\sigma_z^{(2)}$	$\sigma_z^{(3)}$	$\hat{\sigma}_z^{(12)}$	$\hat{\sigma}_z^{(13)}$	$\hat{\sigma}_z^{(23)}$
1	1	1	1	1	1
0	0	0	1	1	1
0	0	1	1	0	0
	…			…	
			1	1	0

In **each closed loop**, the number of **spin-down** has to be an **even** number or 0.

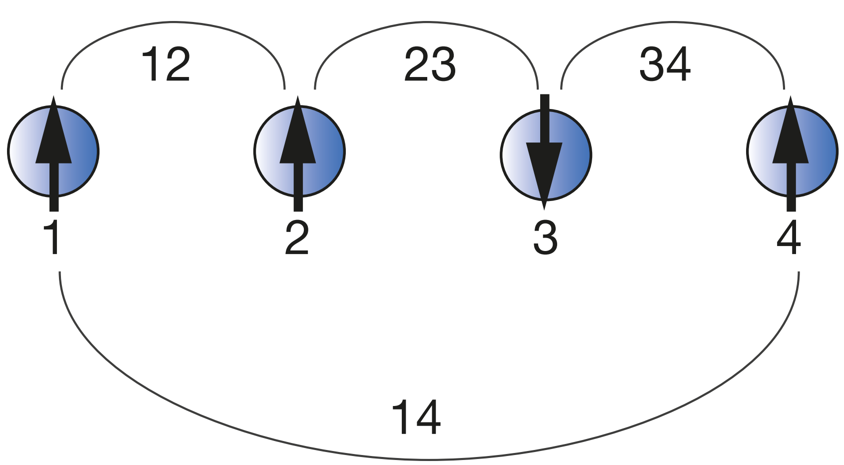
Constraints

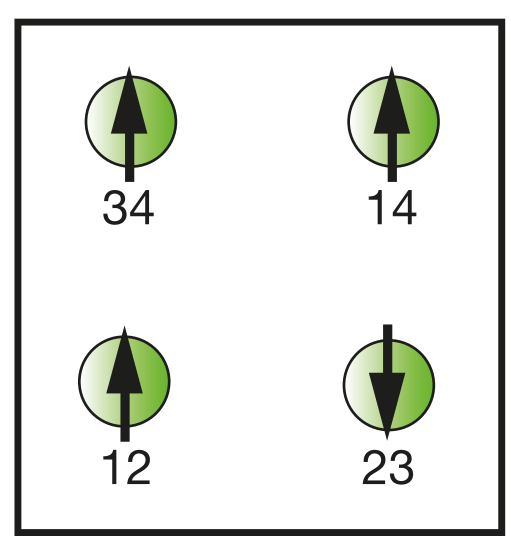
Logical Qubits	Physical Qubits





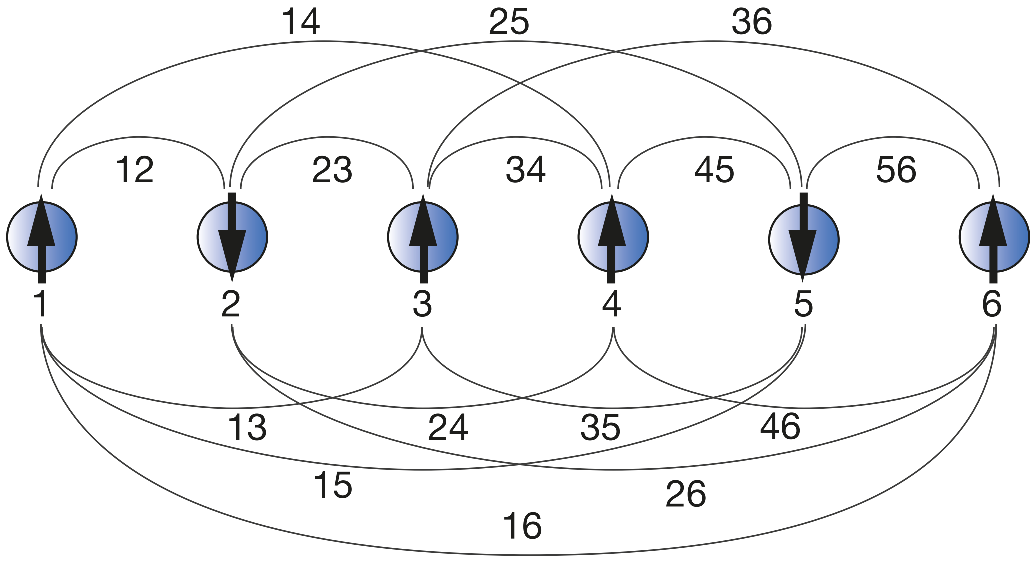
$$C_l = -C\hat{\sigma}_z^{(12)}\hat{\sigma}_z^{(23)}\hat{\sigma}_z^{(13)}$$



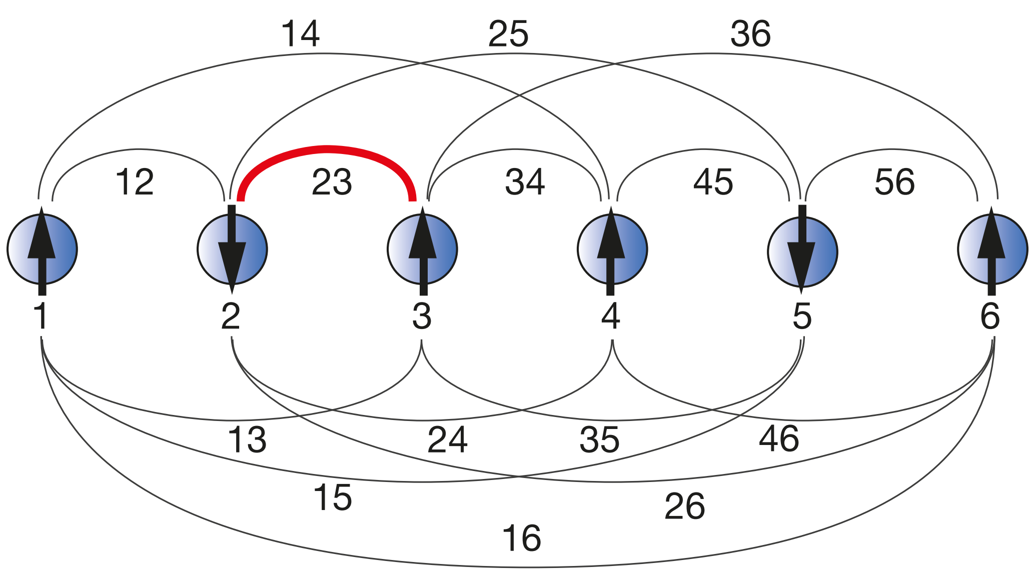


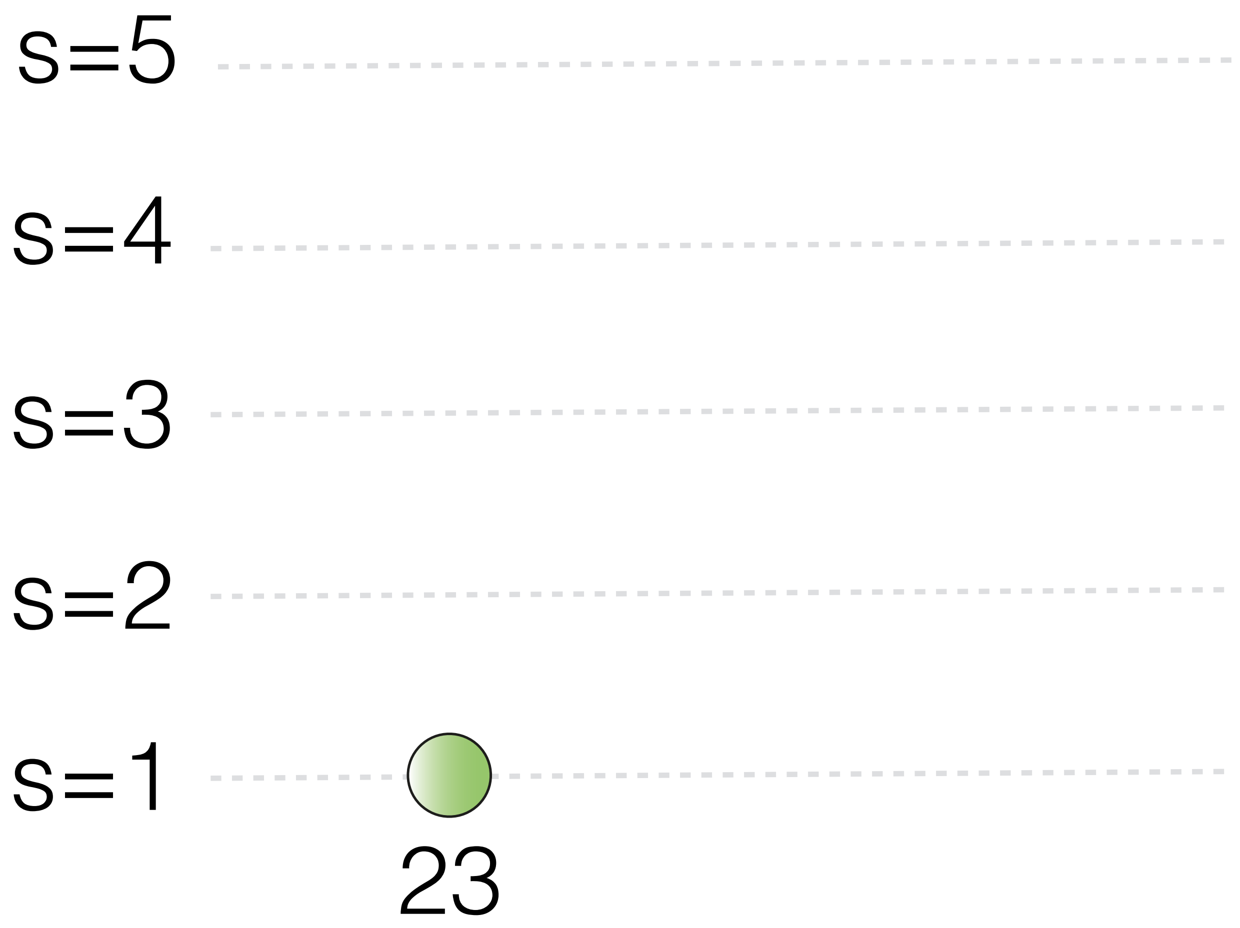
$$C_l = -C\hat{\sigma}_z^{(12)}\hat{\sigma}_z^{(23)}\hat{\sigma}_z^{(34)}\hat{\sigma}_z^{(14)}$$

Constraints

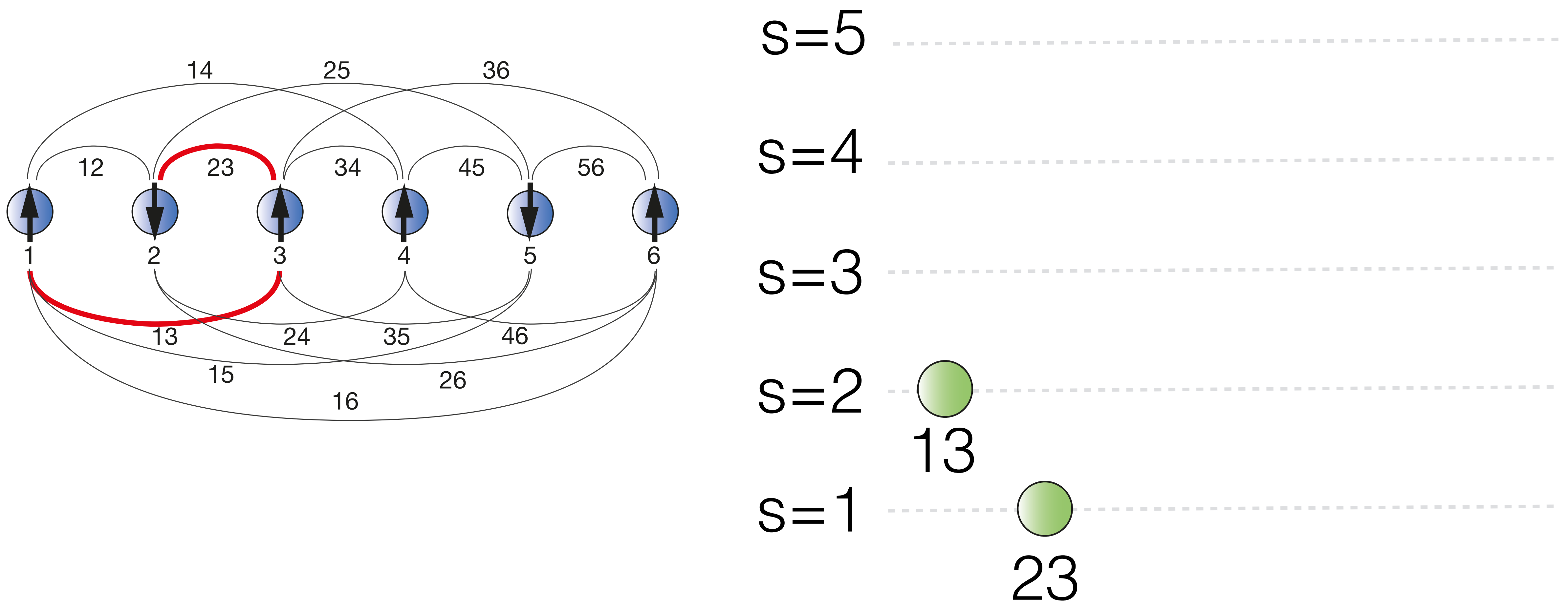


Constraints

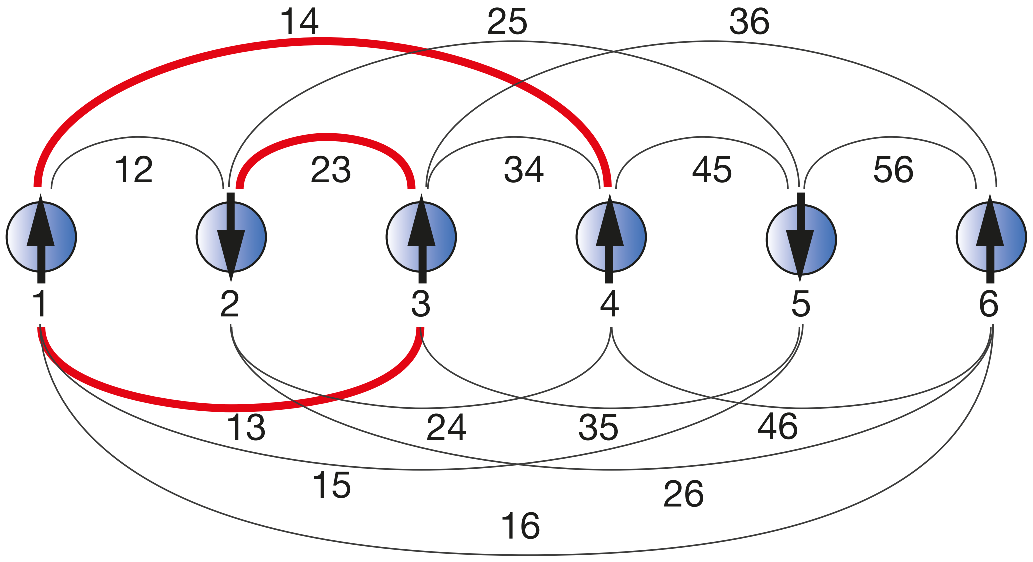


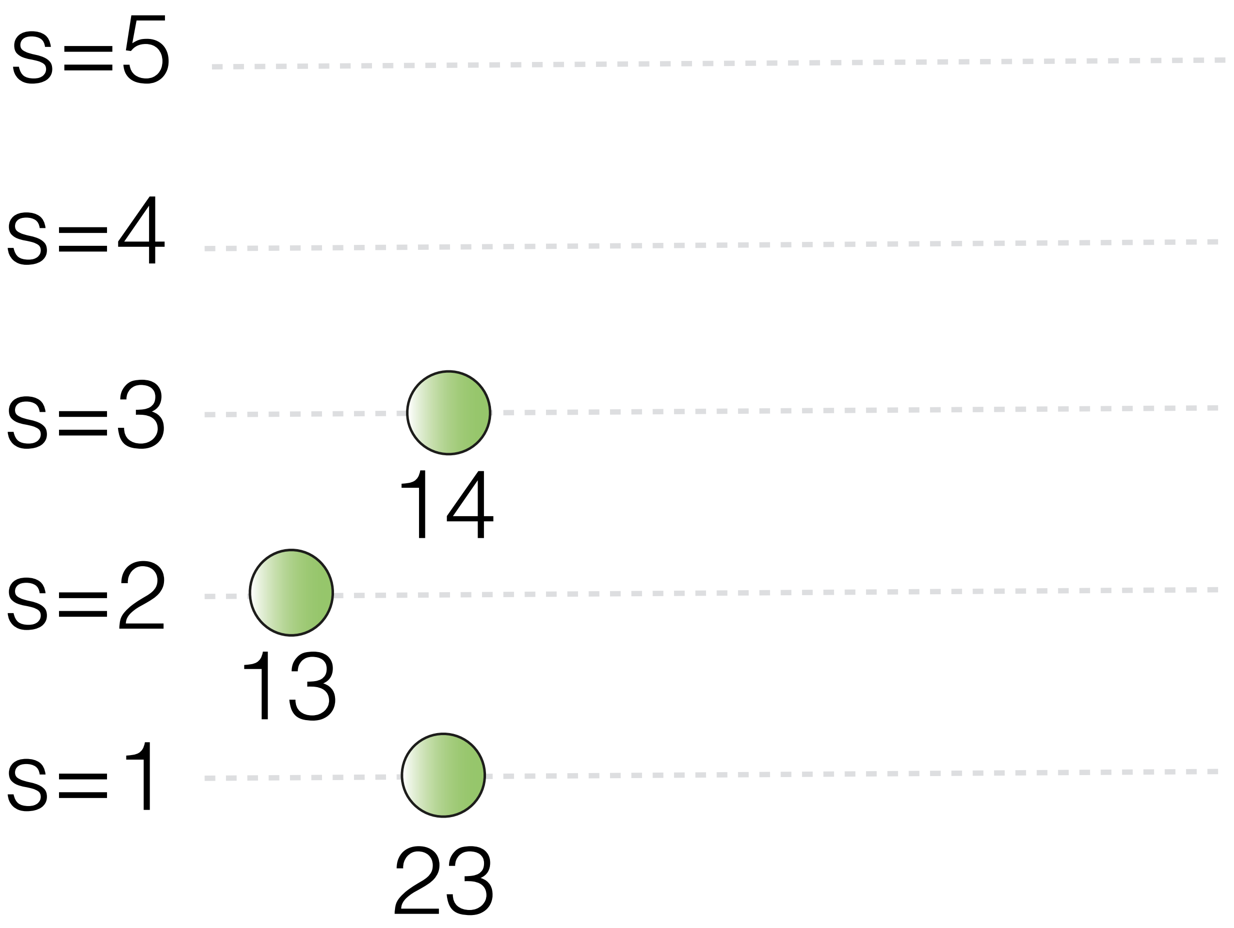


Constraints

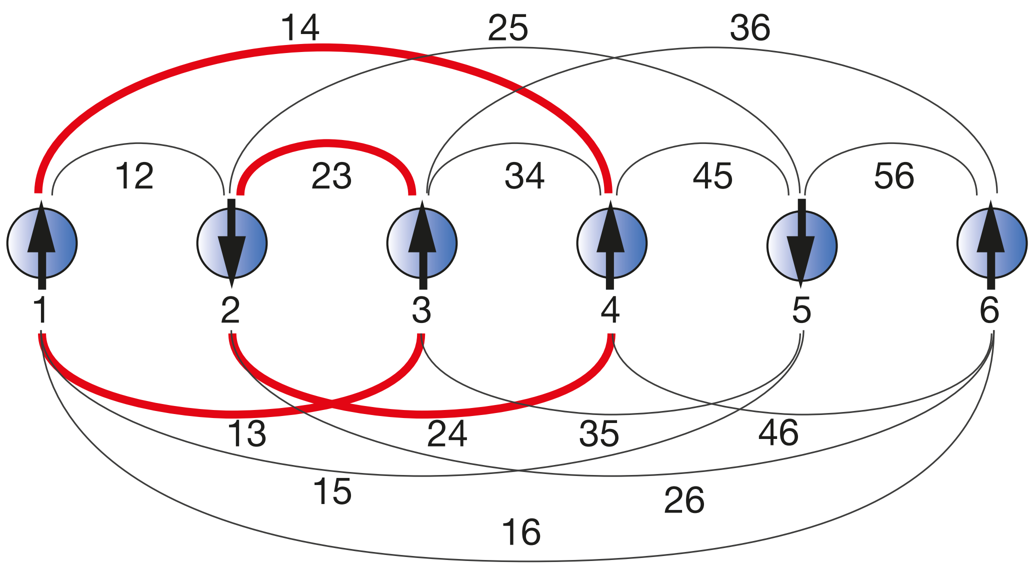


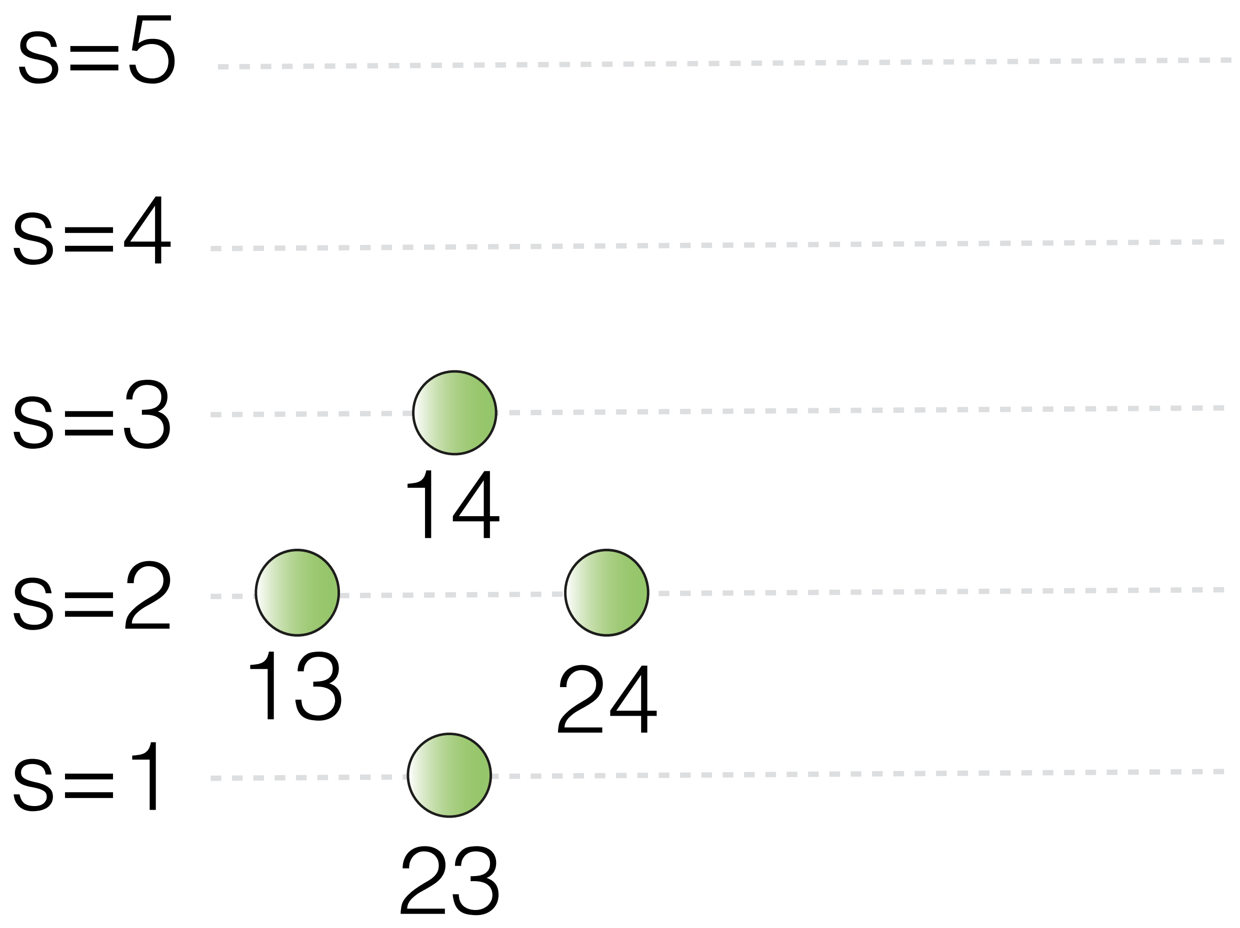
Constraints



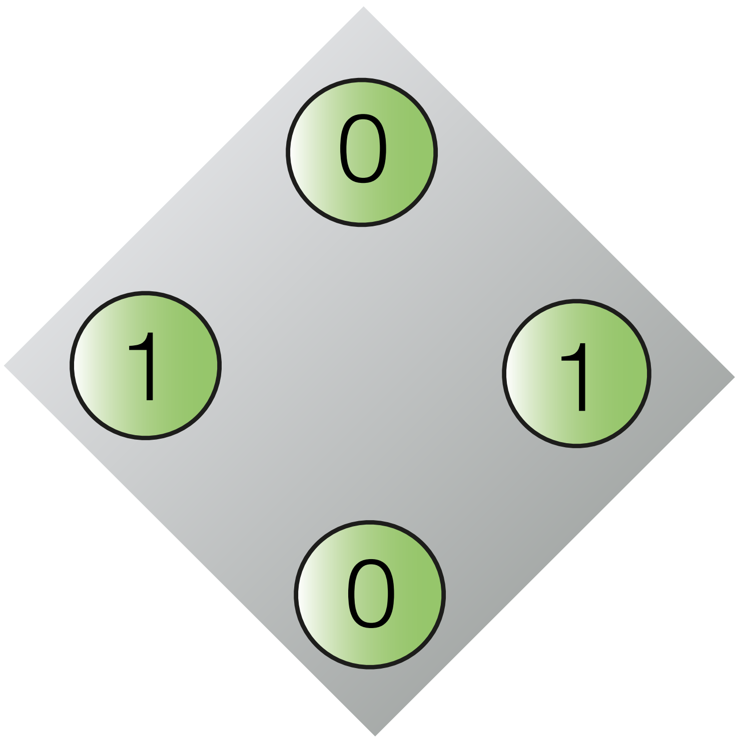


Constraints

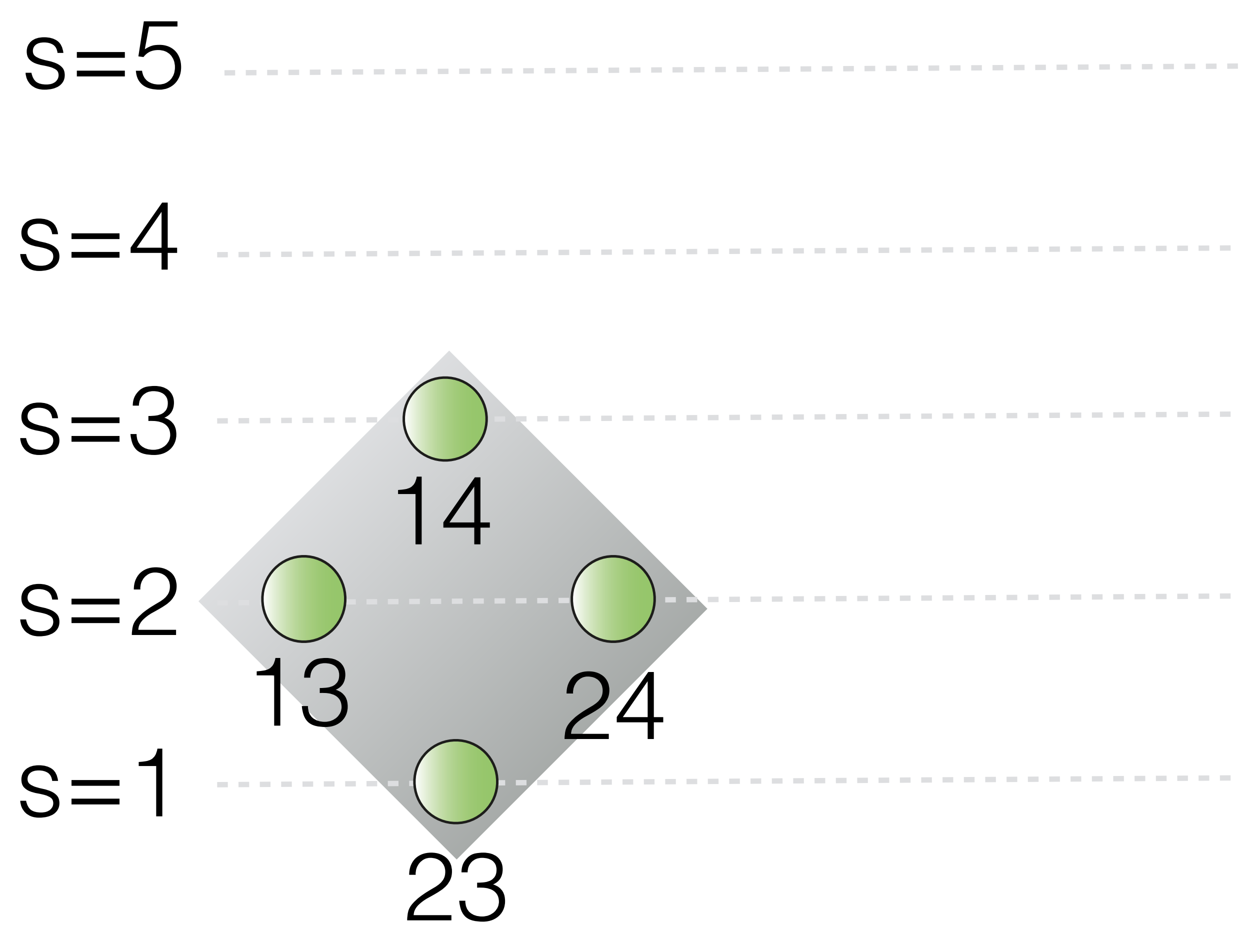




Constraints

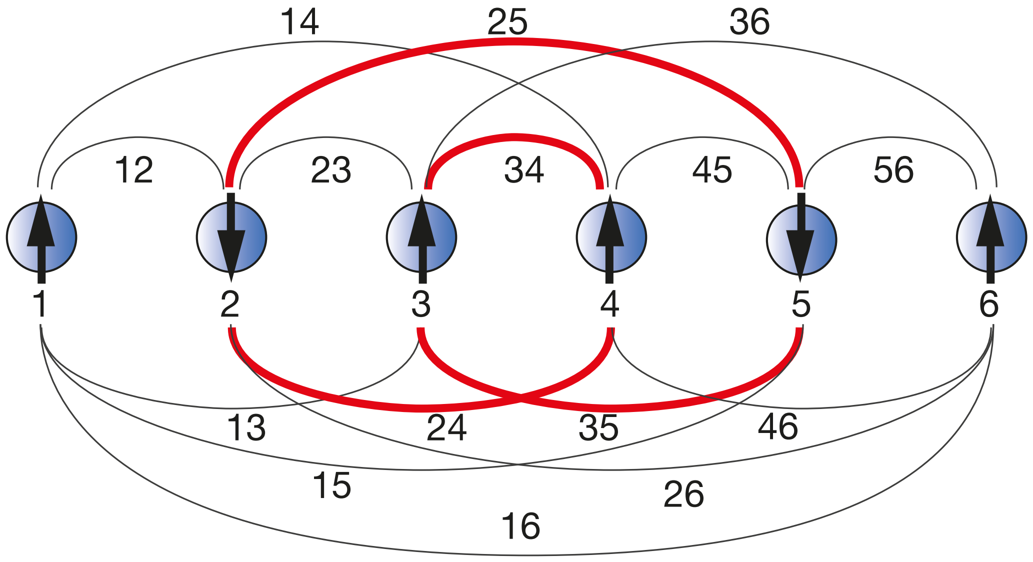


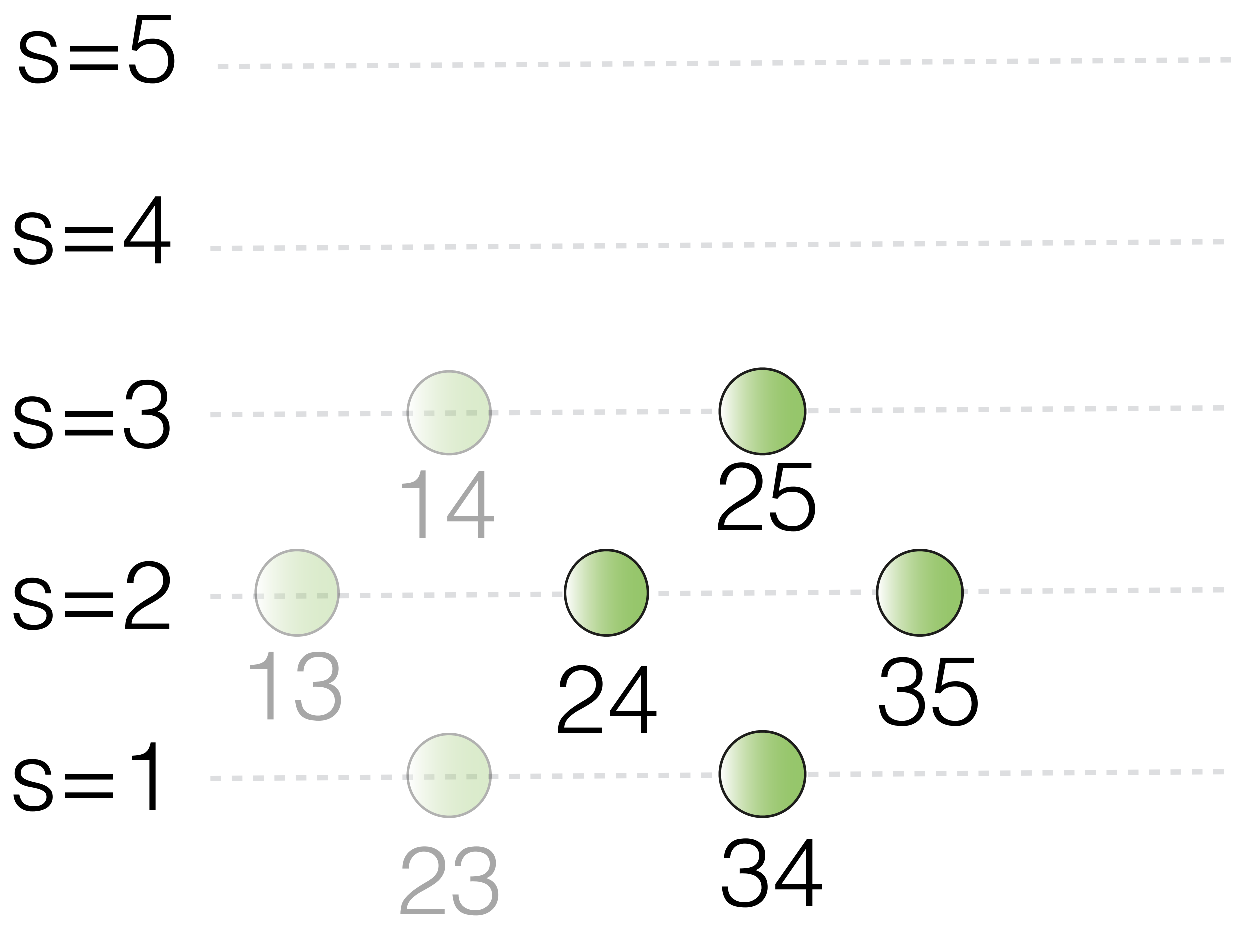
Number of spin-down
is even



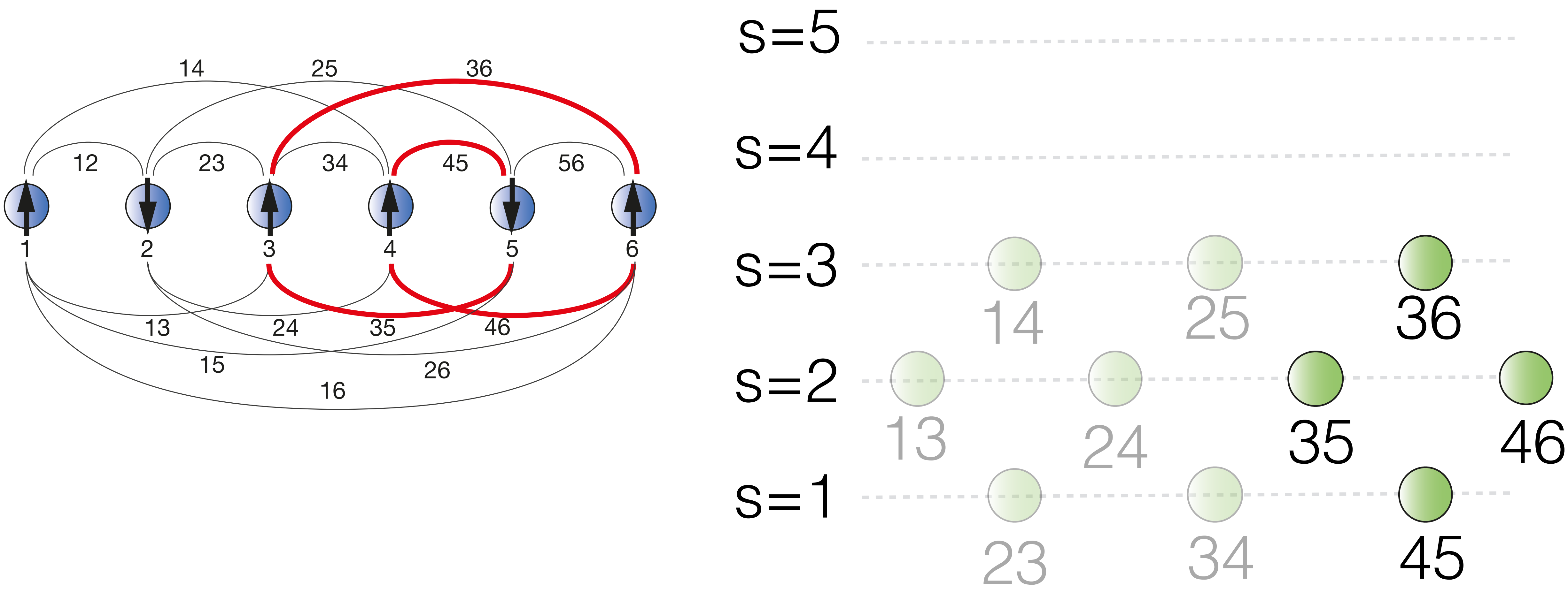
$$C_l = -C\left(\sum_{m=n,e,s,w}\tilde{\sigma}_z^{(l,m)} + S_z^l\right)^2 = -C\tilde{\sigma}_z^{(l,n)}\tilde{\sigma}_z^{(l,e)}\tilde{\sigma}_z^{(l,s)}\tilde{\sigma}_z^{(l,w)}$$

Constraints

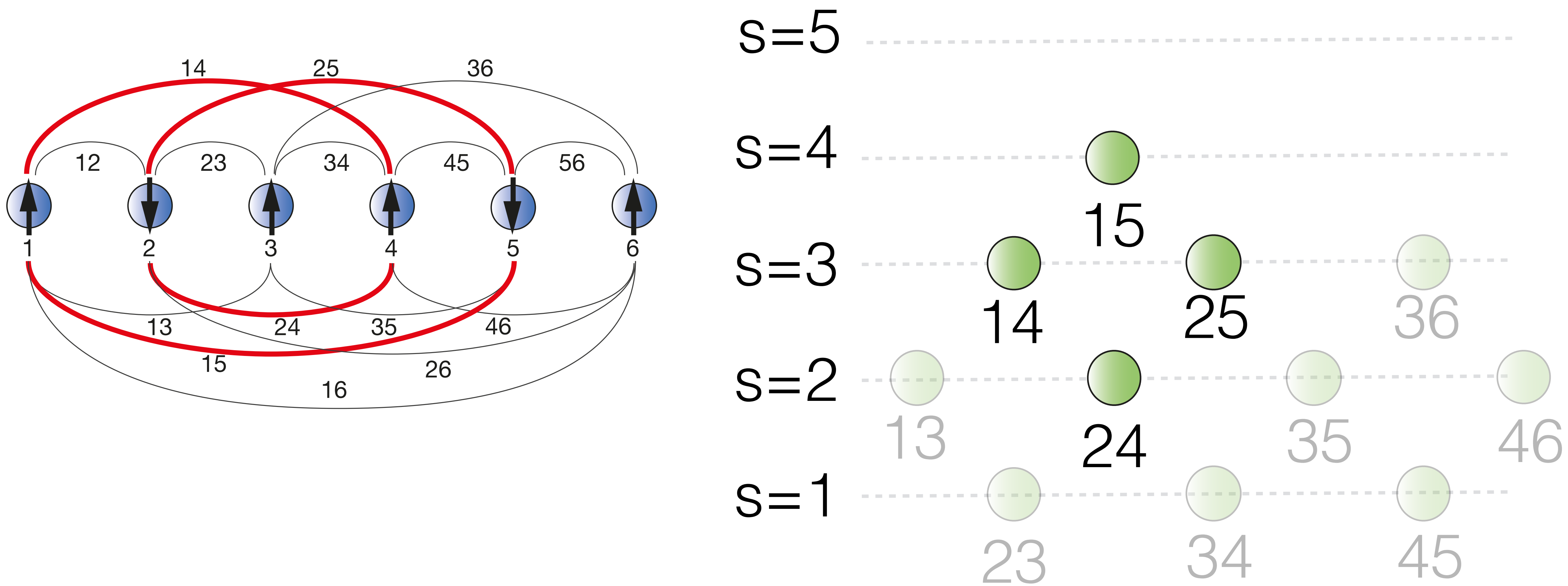




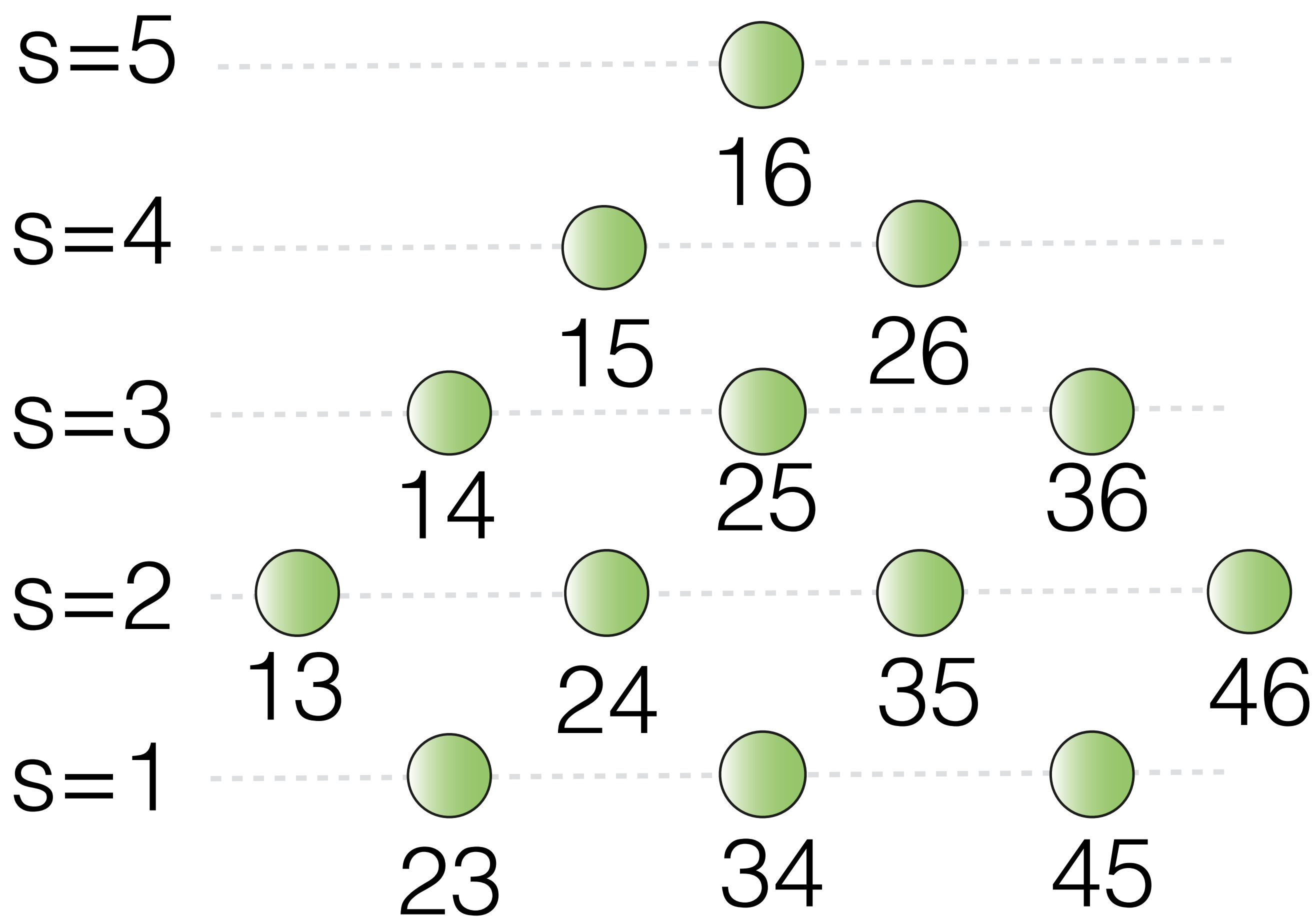
Constraints



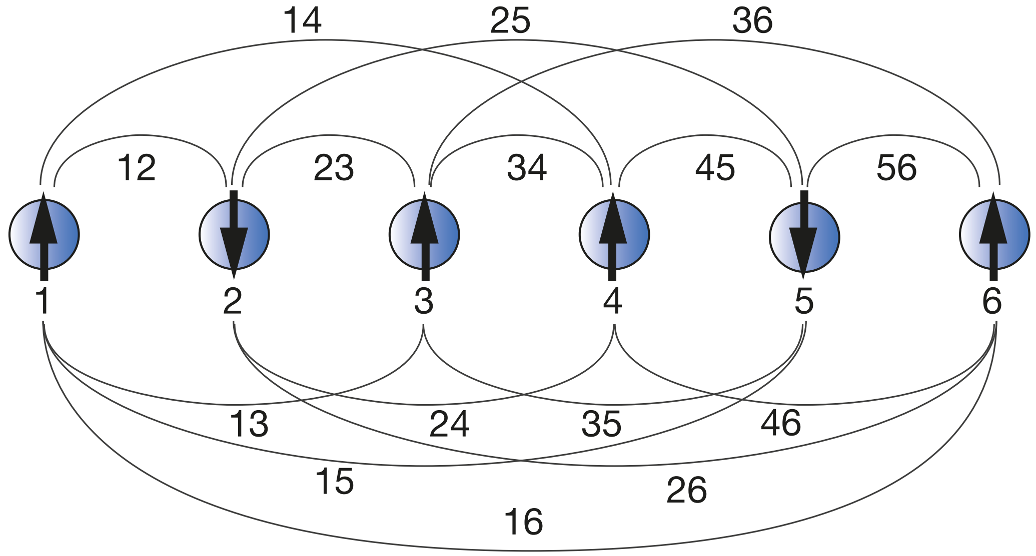
Constraints

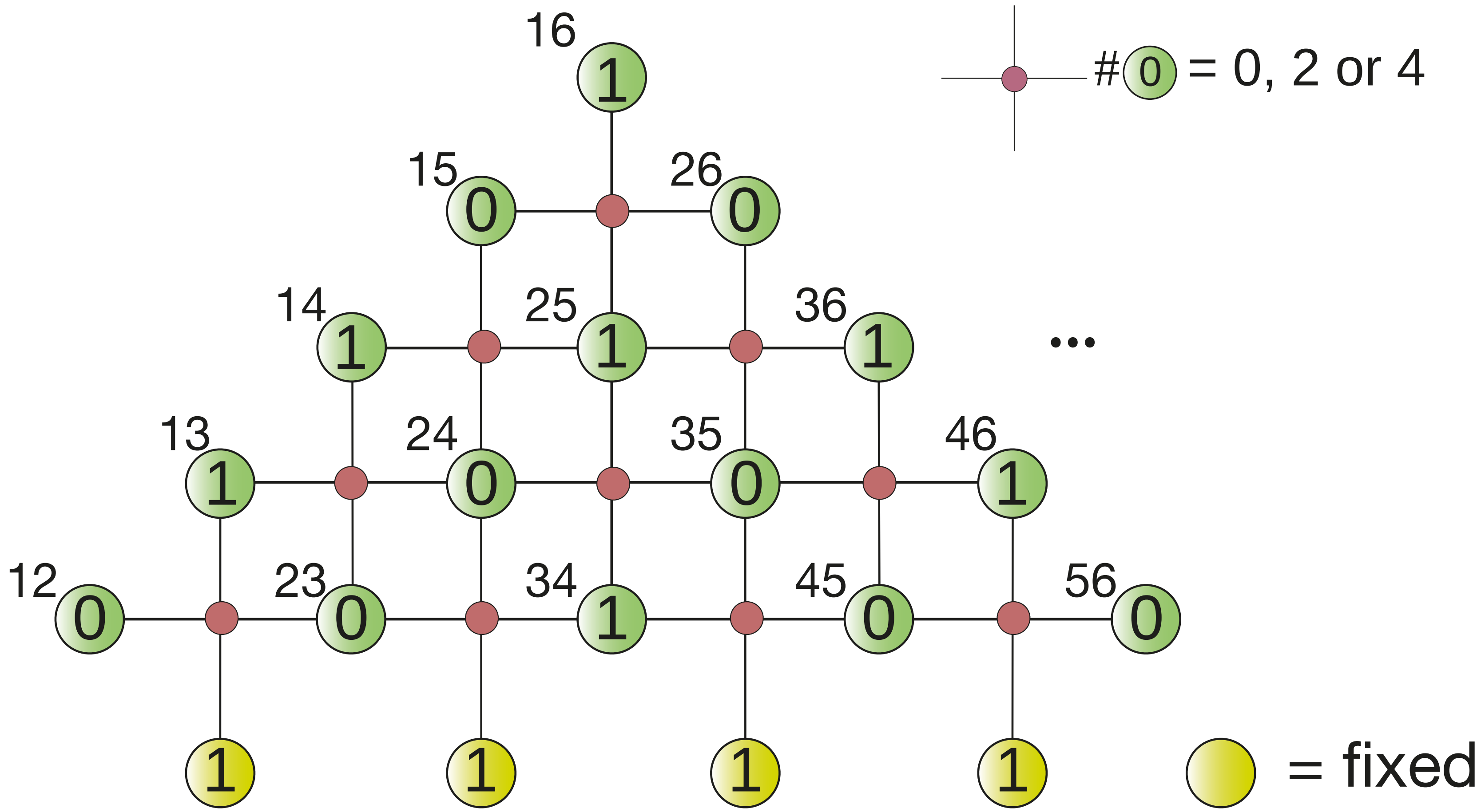


Constraints



Constraints

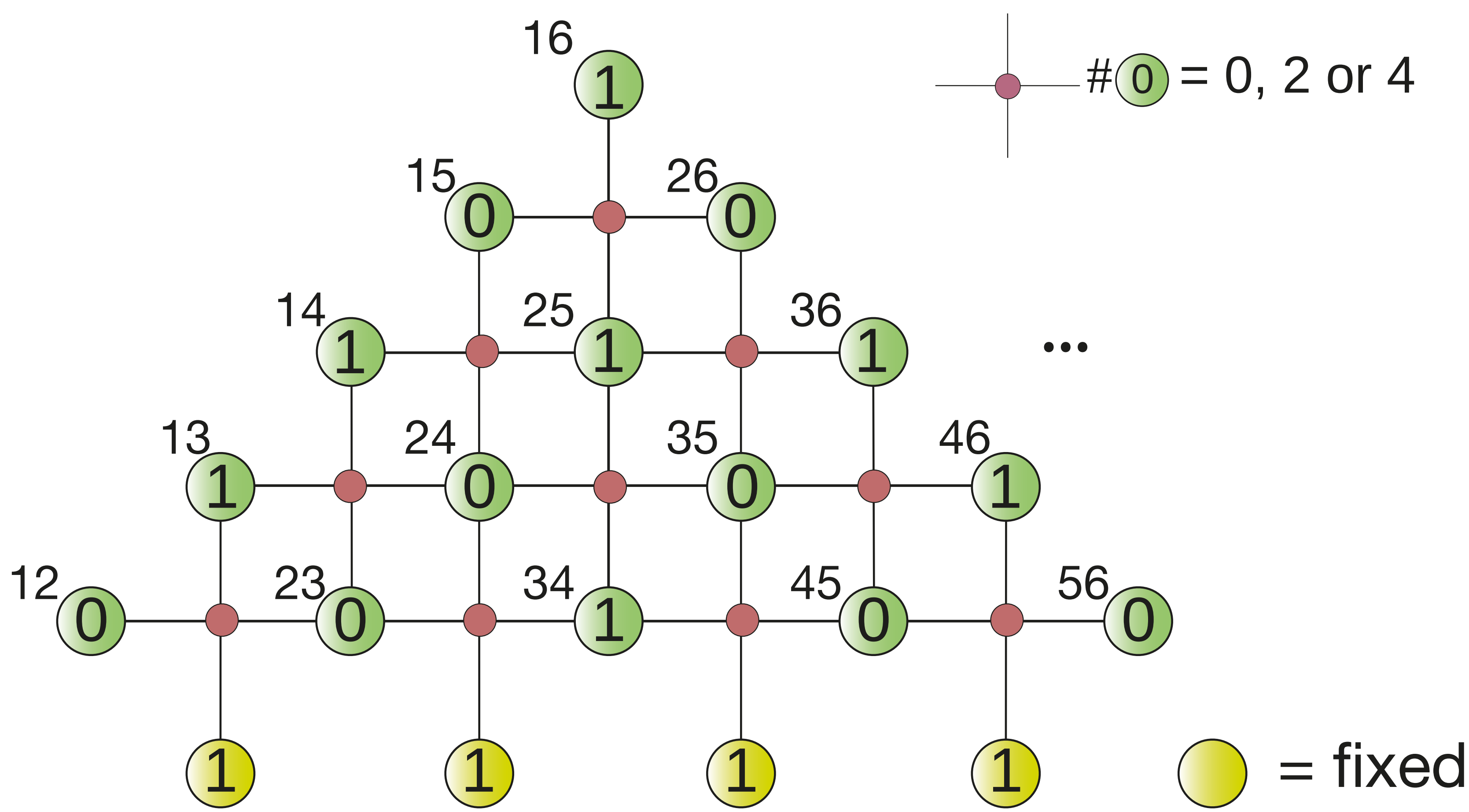




Parity AQC

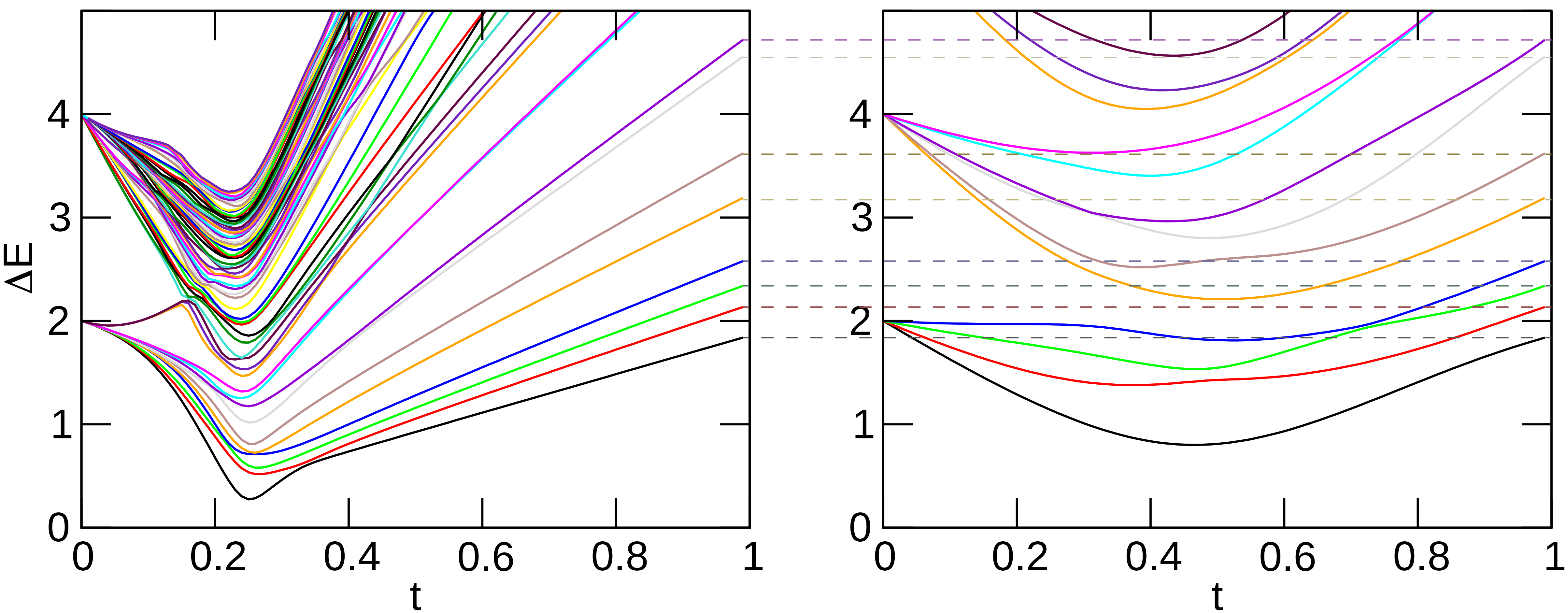
$$\mathcal{H}(t) = A(t)\sum_{i=1}^{K} b_i \sigma_x^{(i)} + B(t)\sum_{i=1}^{K} J_i \sigma_z^{(i)} + C(t)\sum_{l=1}^{K-N} C_l$$

$$K = \frac{N(N-1)}{2} \qquad C_l = -C\tilde{\sigma}_z^{(l,n)}\tilde{\sigma}_z^{(l,e)}\tilde{\sigma}_z^{(l,s)}\tilde{\sigma}_z^{(l,w)}$$



Spectrum

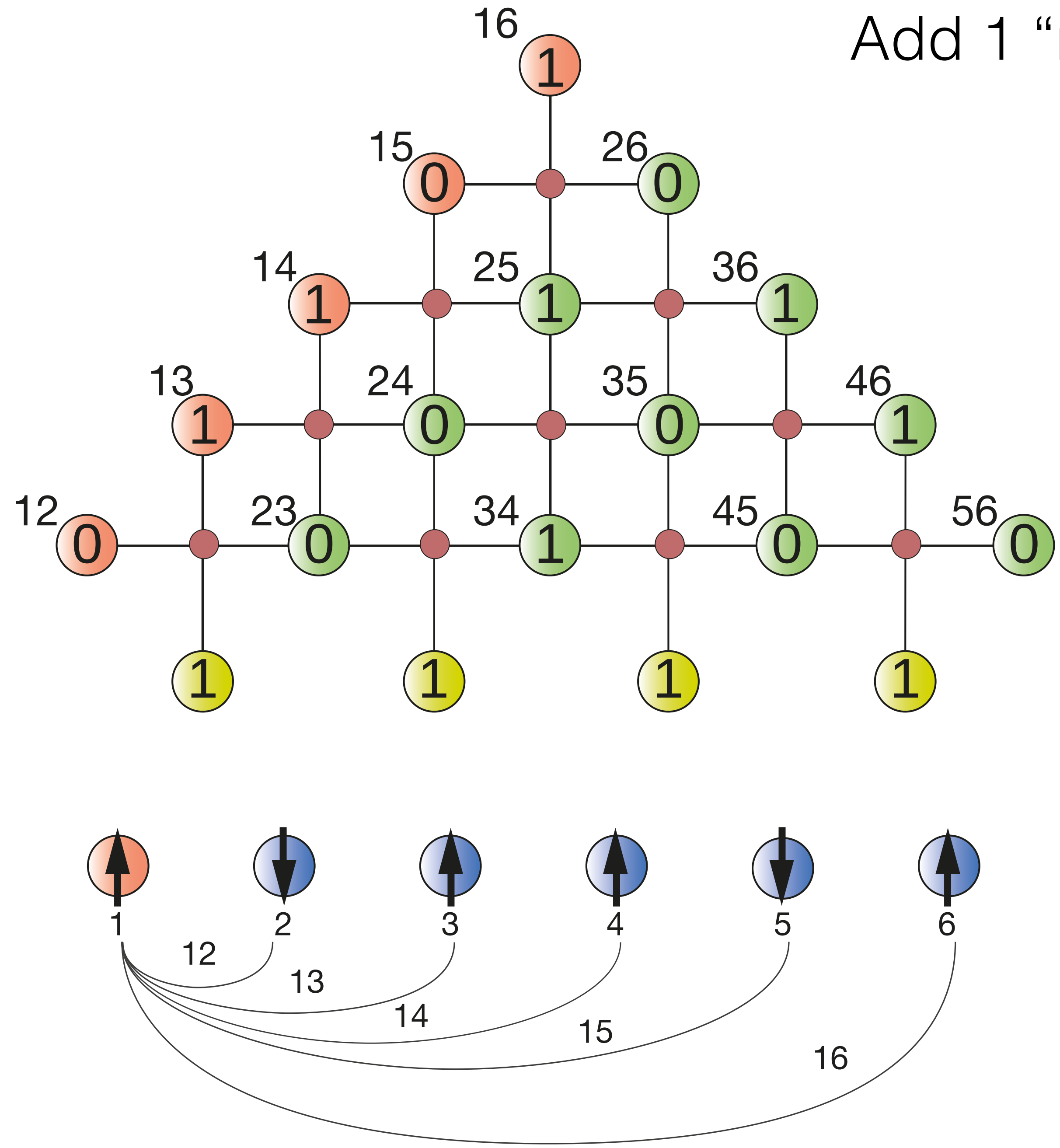
$$H(t) = A(t)\sum_{k=1}^{K} h_k \tilde{\sigma}_x^{(k)} + B(t)\left[\sum_{k=1}^{K} J_k \tilde{\sigma}_z^{(k)} + \sum_{l=1}^{K-N} C_l\right]$$



Magnetic Field term

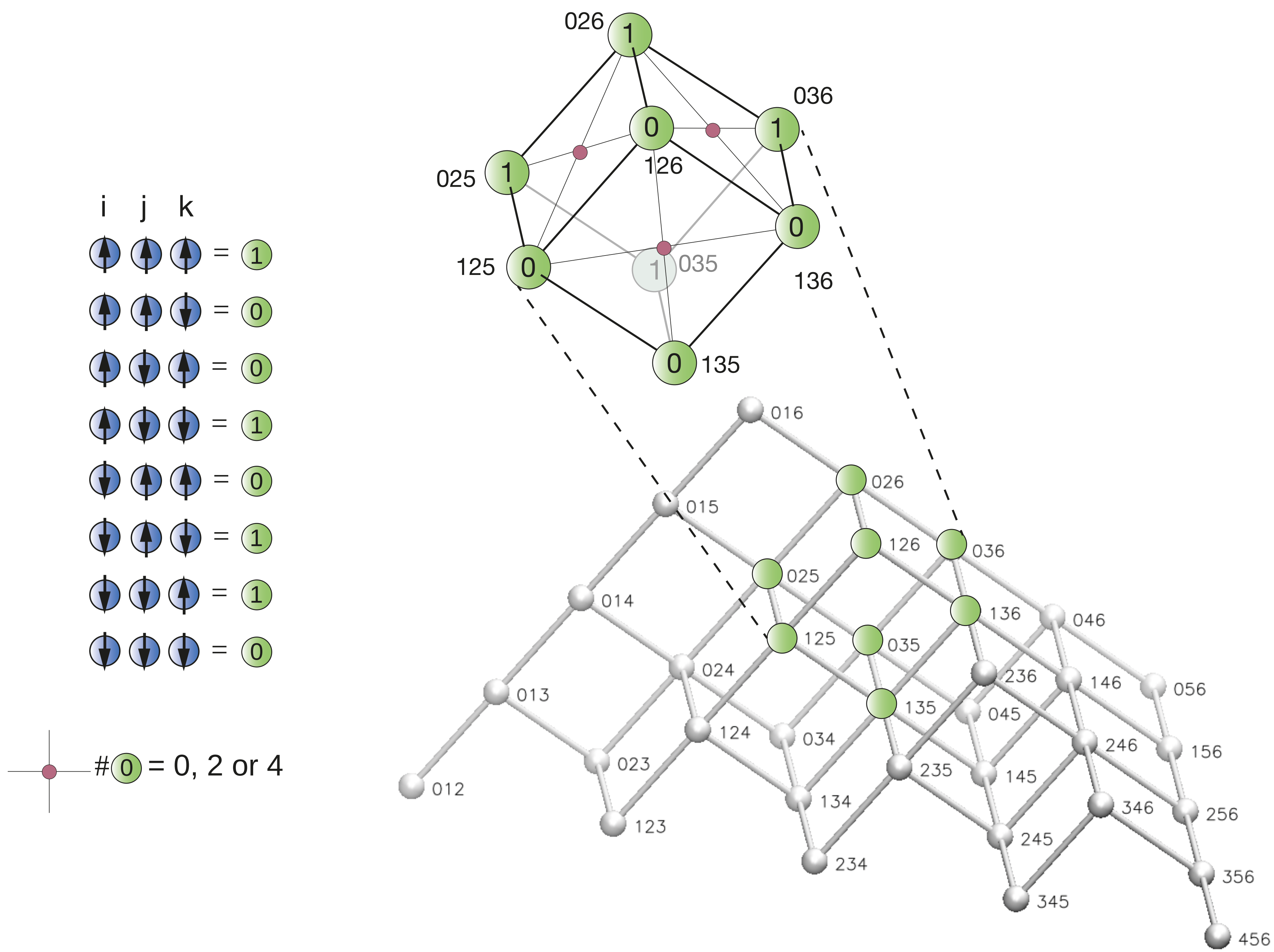
$$H = \sum_{i=1}^{N} h_i \sigma_z^{(i)} + \sum_{i=1}^{N} \sum_{j=1}^{i} J_{ij} \sigma_z^{(i)} \sigma_z^{(j)}$$

Add 1 “row” of qubits.

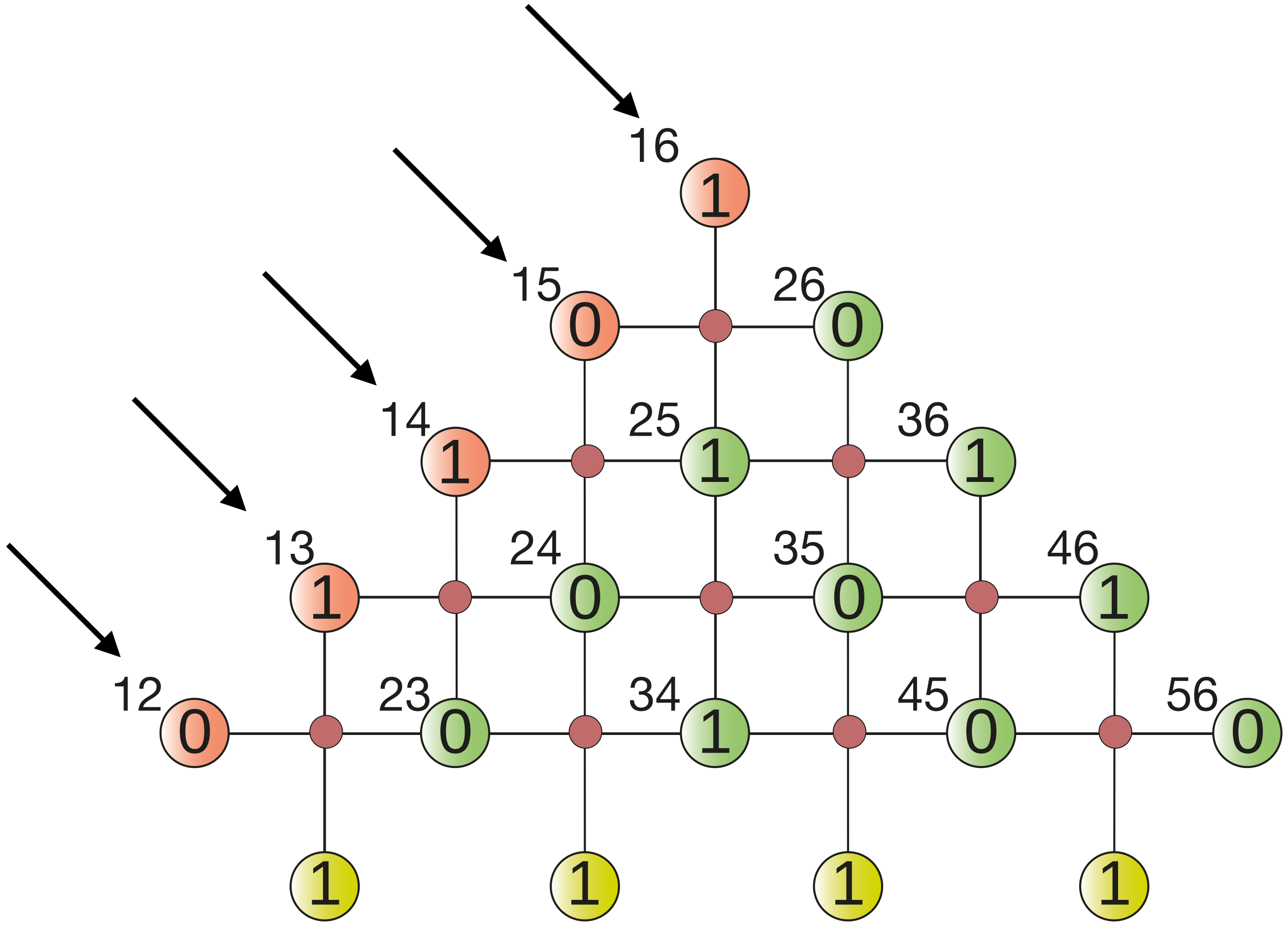


Three-Body interactions

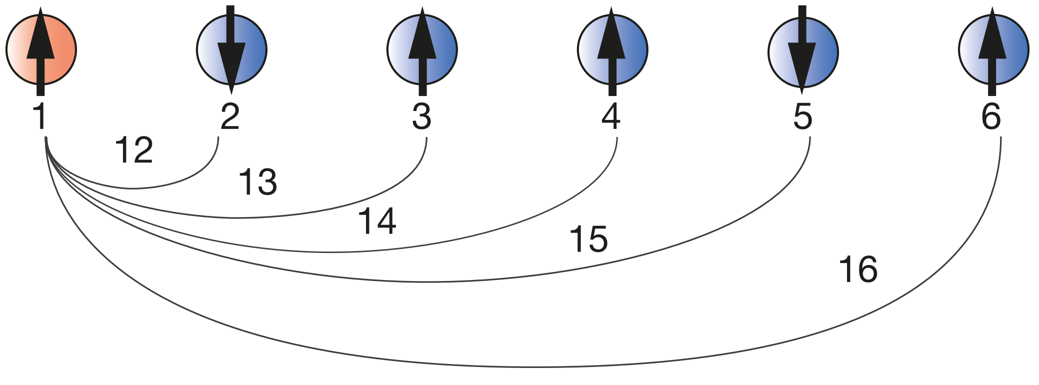
$$H = \sum_{i=1}^{N} h_i \sigma_z^{(i)} + \sum_{i=1}^{N} \sum_{j=1}^{i} J_{ij} \sigma_z^{(i)} \sigma_z^{(j)} + \sum_{i}^{N} \sum_{j}^{i} \sum_{k}^{j} J_{ijk} \sigma_z^{(i)} \sigma_z^{(j)} \sigma_z^{(k)}$$



Readout

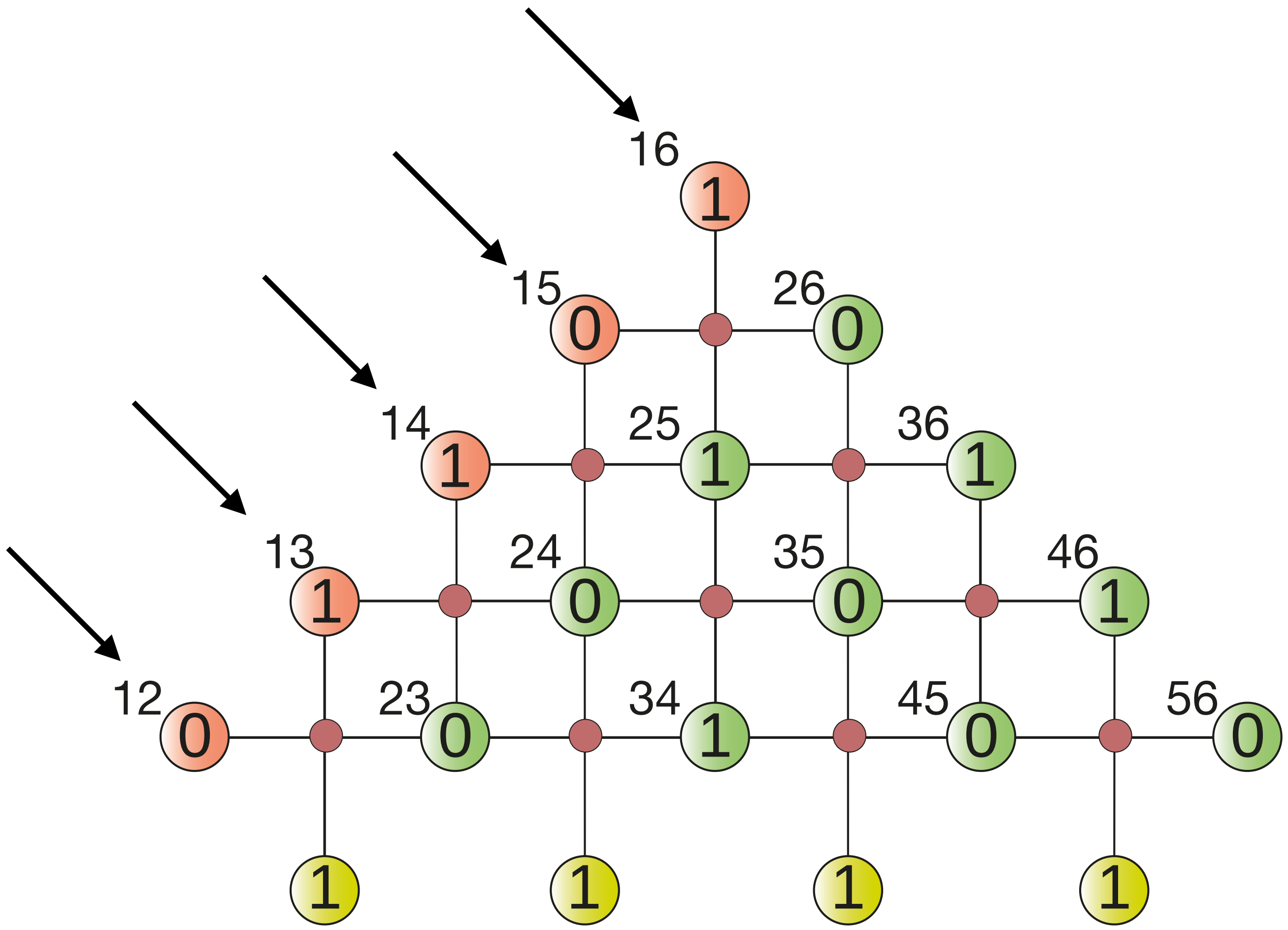


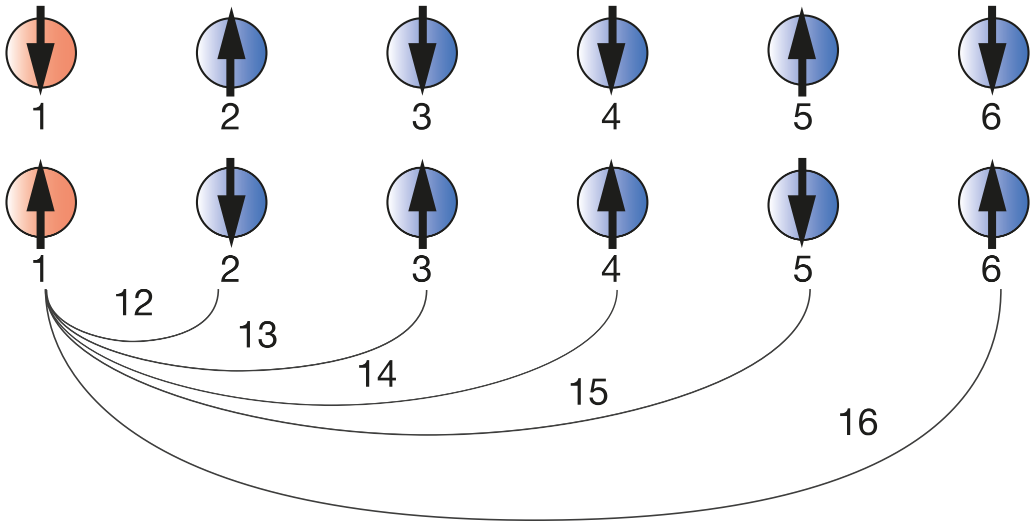
16
1
15
0
26
0
14
1
25
1
36
1
13
1
24
0
35
0
46
1
12
0
23
0
34
1
45
0
56
0
1
1
1
1



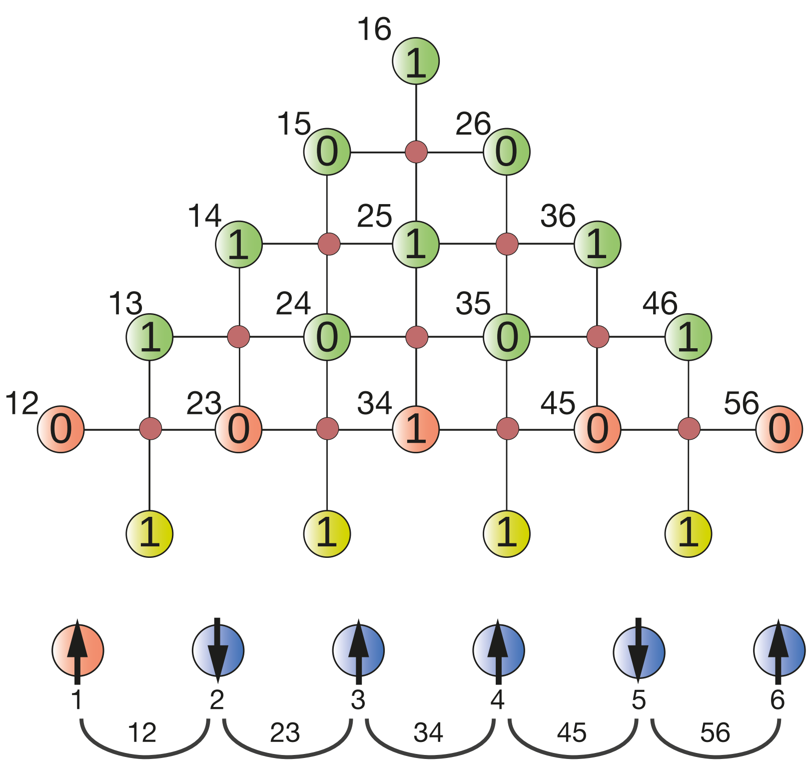
1
2
3
4
5
6
12
13
14
15
16

Readout

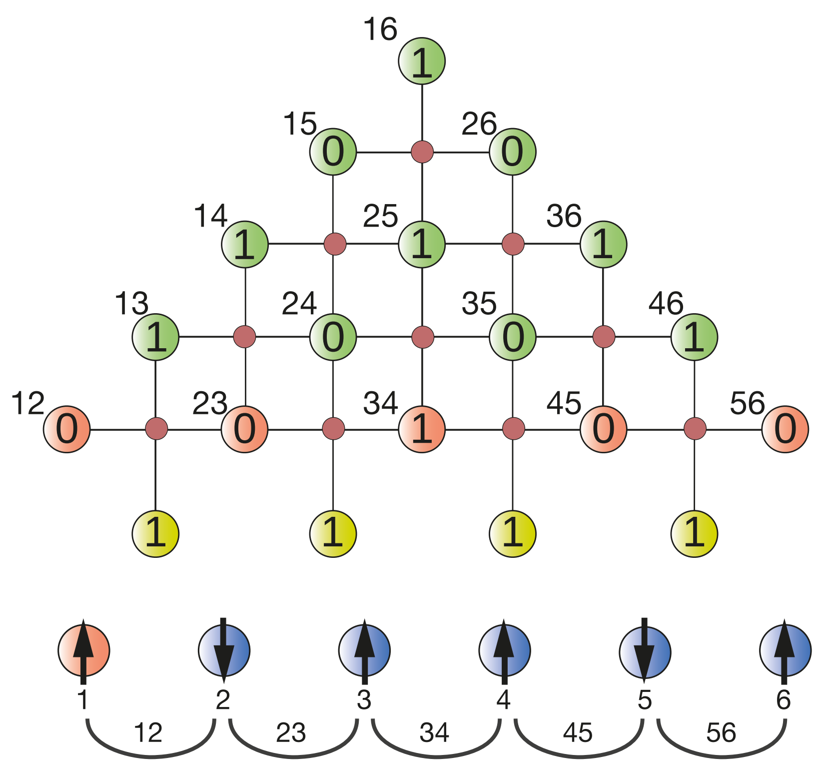

16
1
15
0
26
0
14
1
25
1
36
1
13
1
24
0
35
0
46
1
12
0
23
0
34
1
45
0
56
0
1
1
1
1

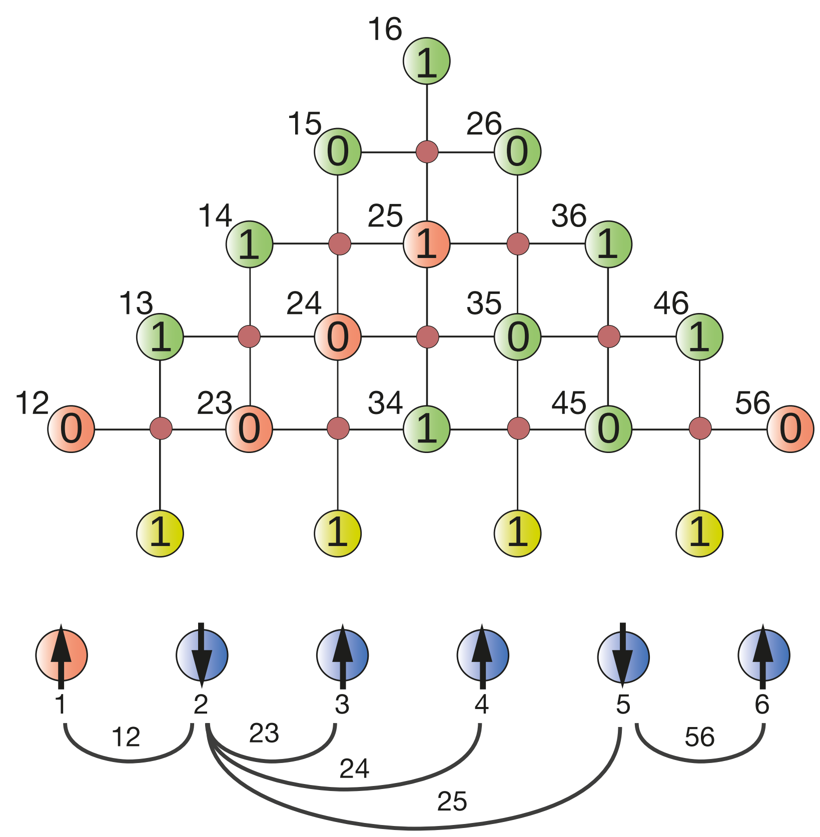

1
2
3
4
5
6
1
2
3
4
5
6
12
13
14
15
16

Determining and Non-determining Readouts

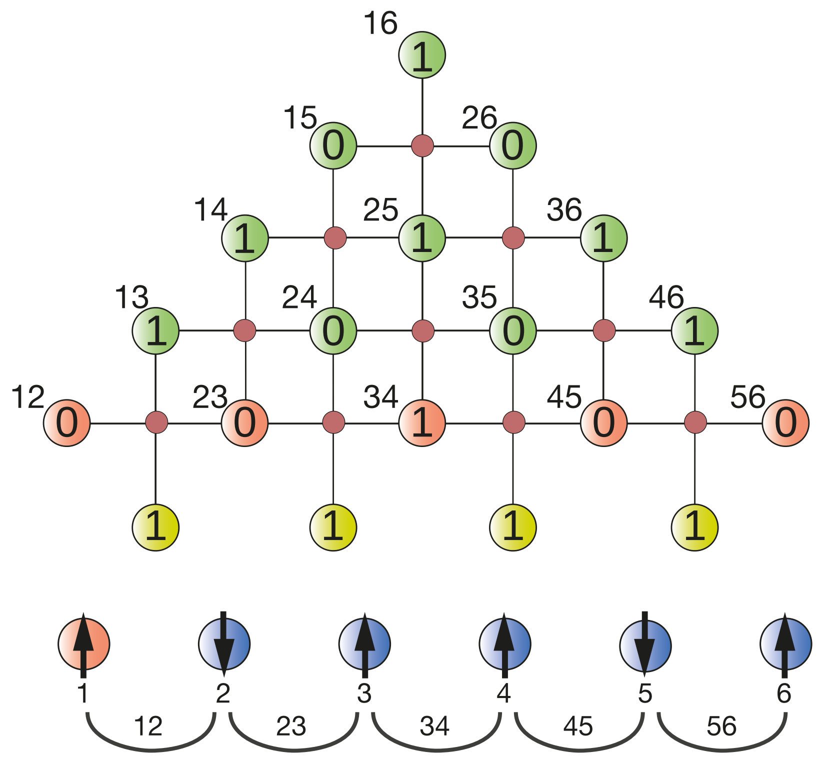


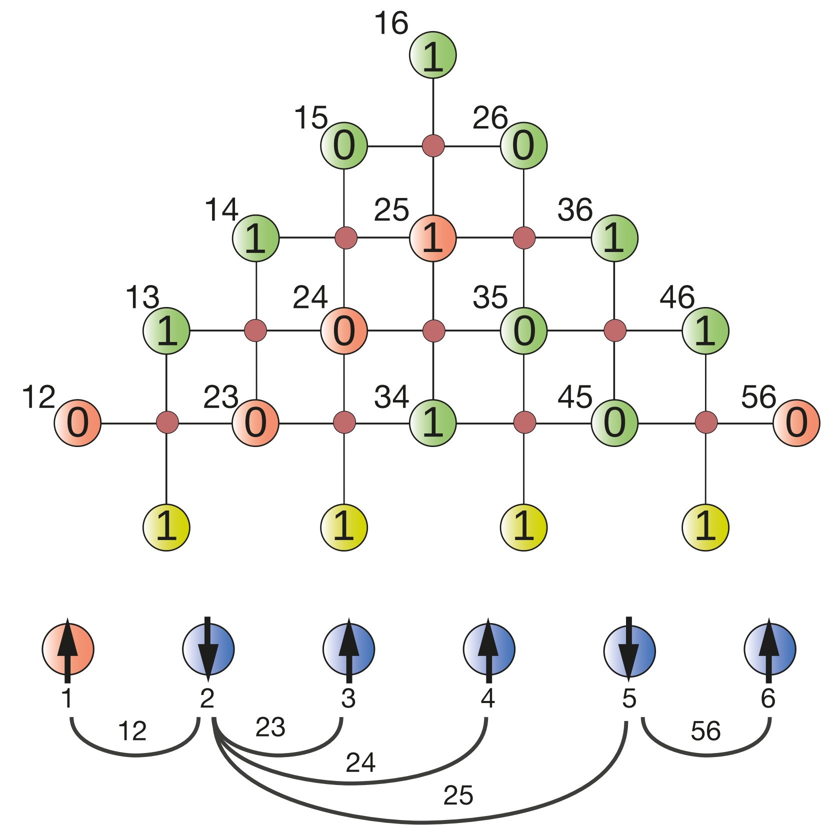
Determining and Non-determining Readouts

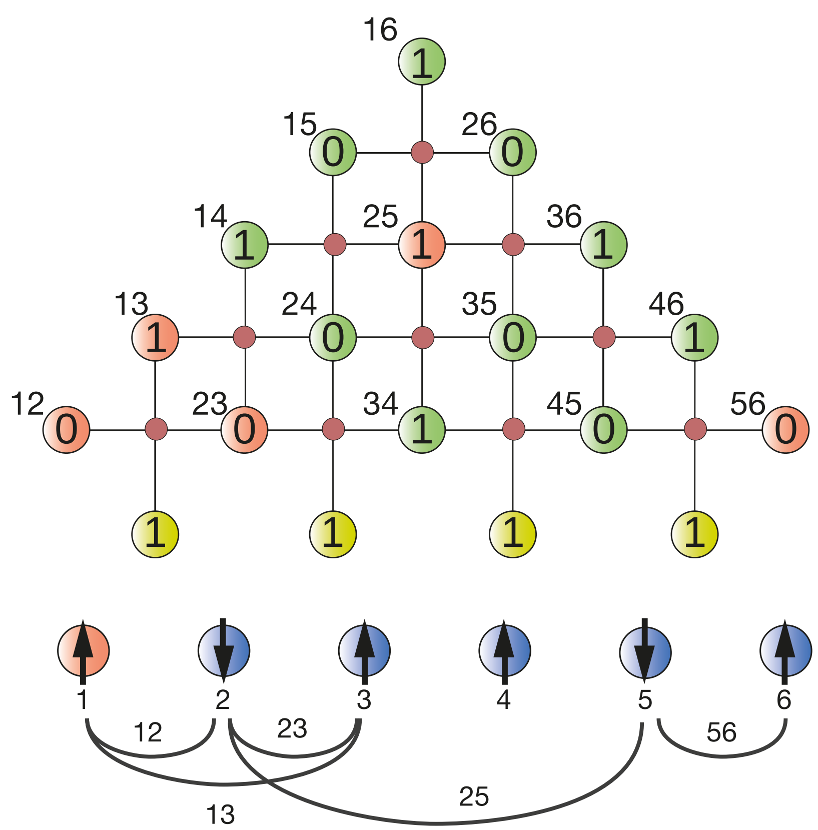




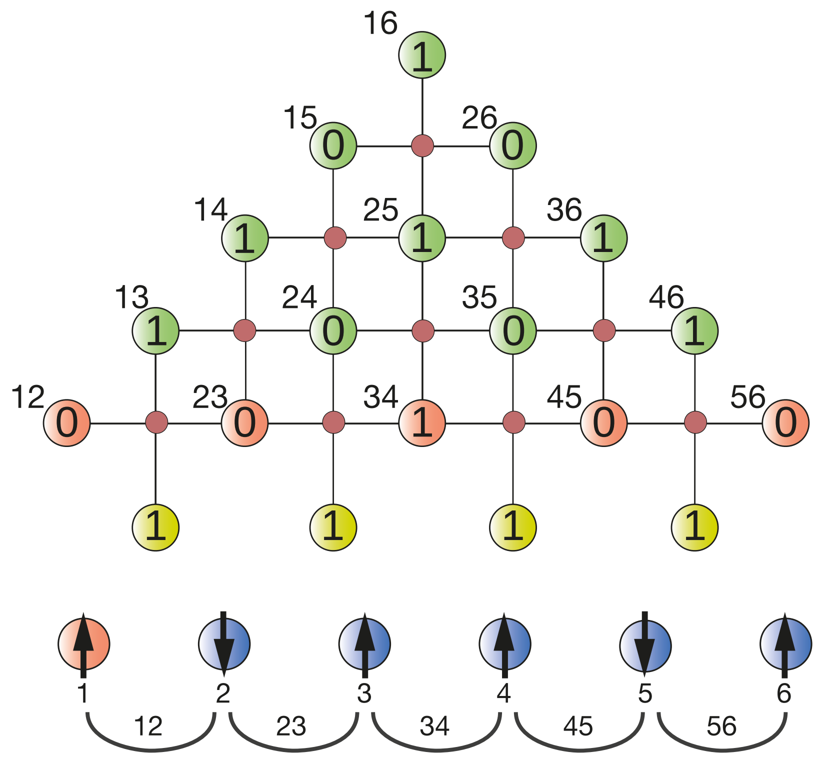
Determining and Non-determining Readouts

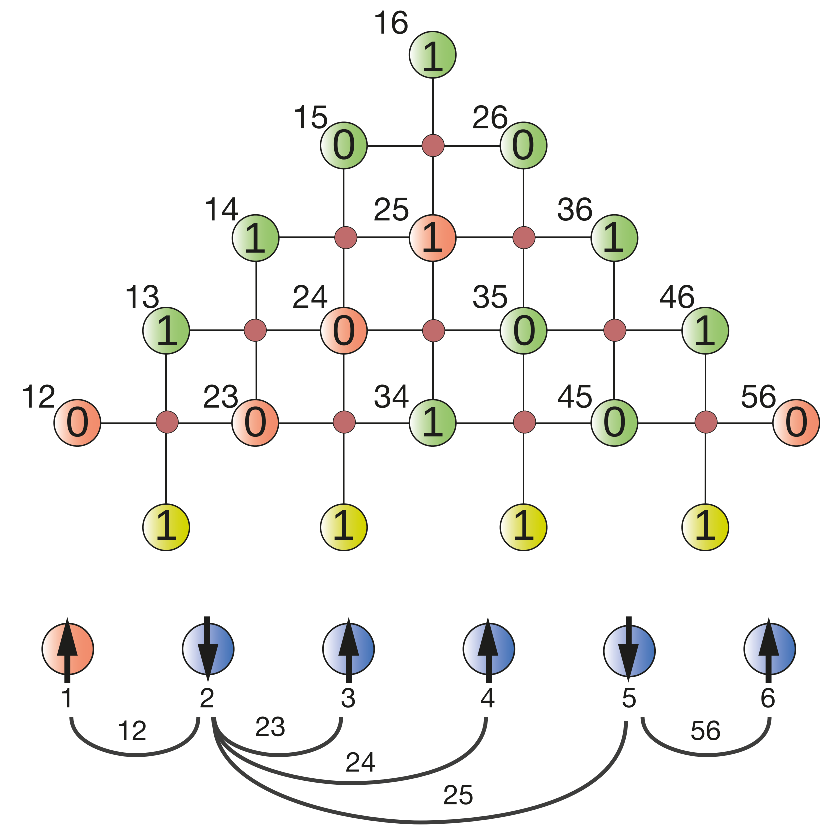


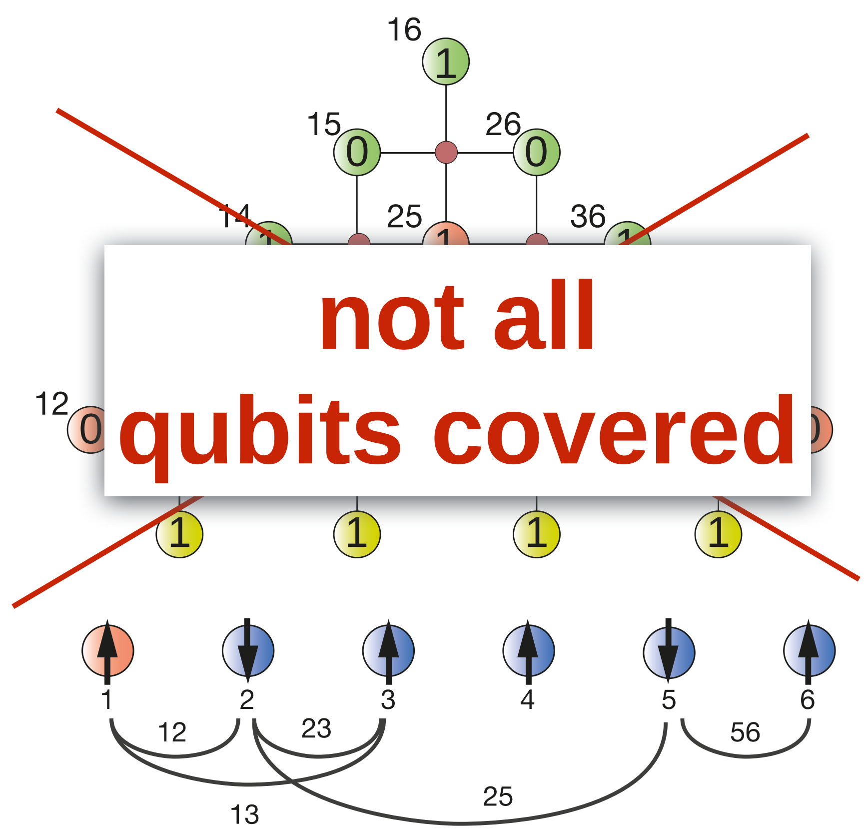




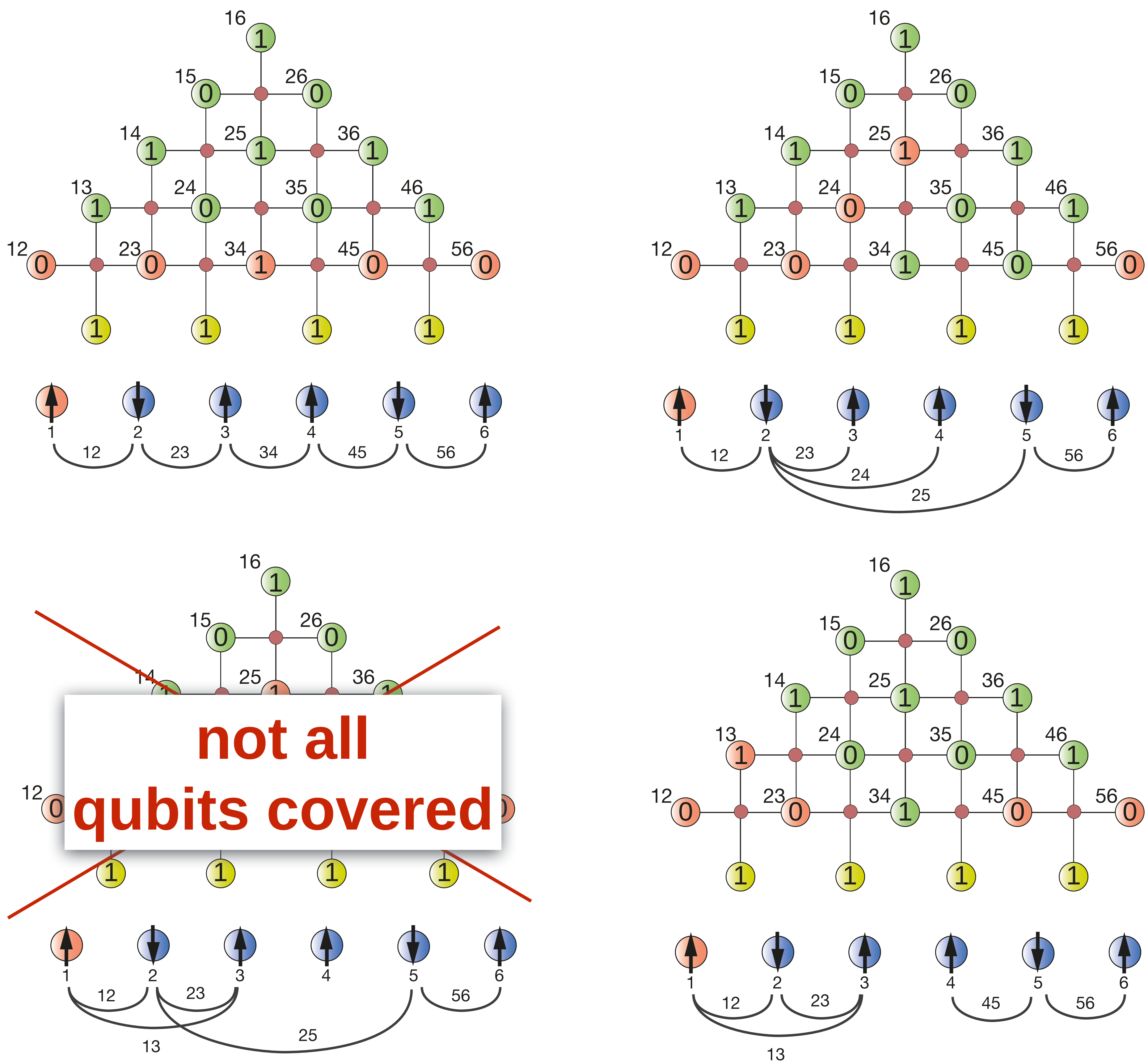
Determining and Non-determining Readouts



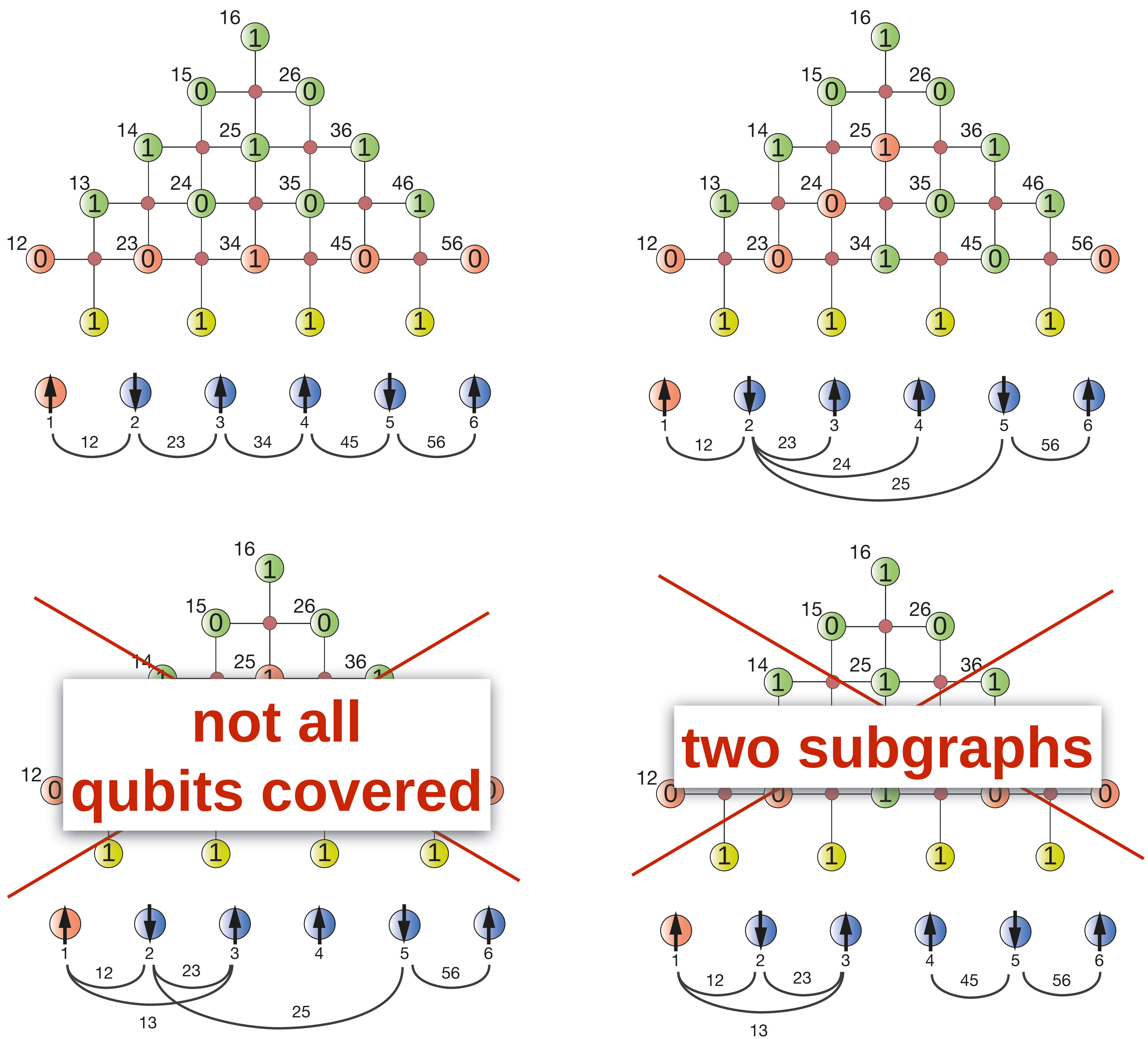




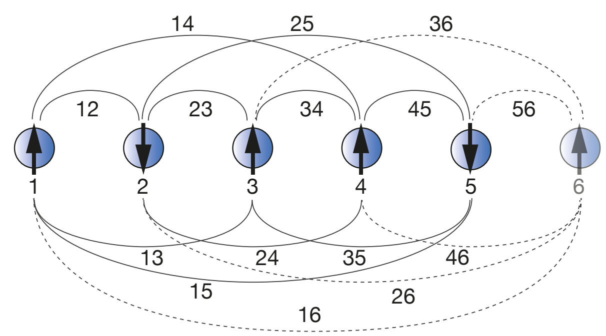
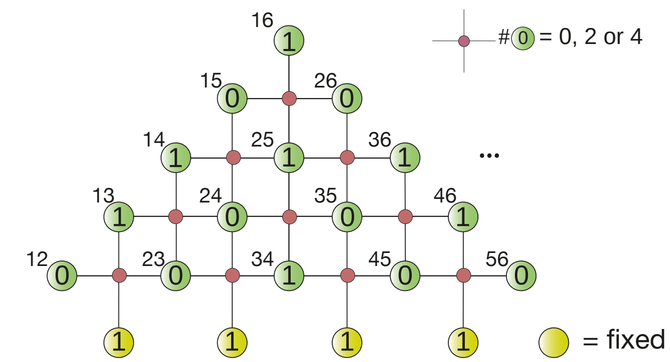
Determining and Non-determining Readouts



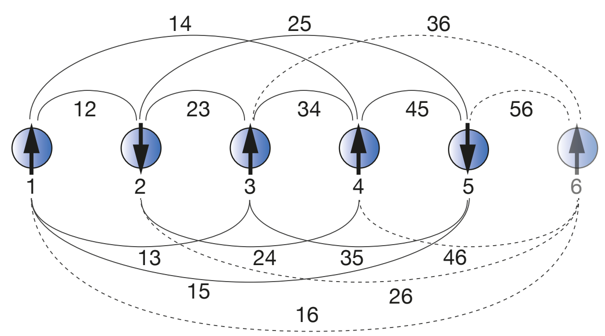
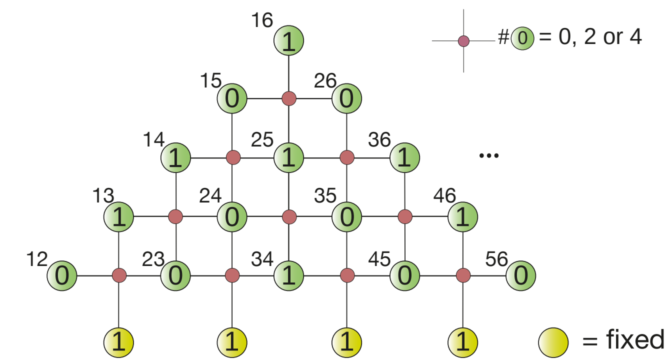
Determining and Non-determining Readouts



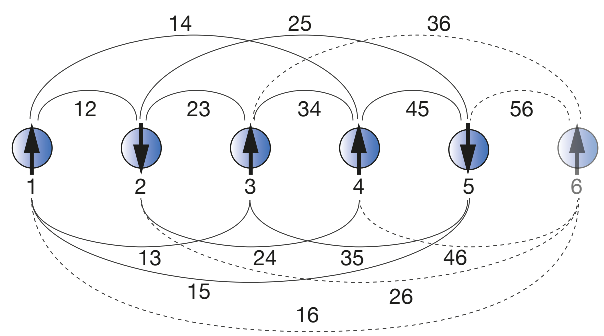
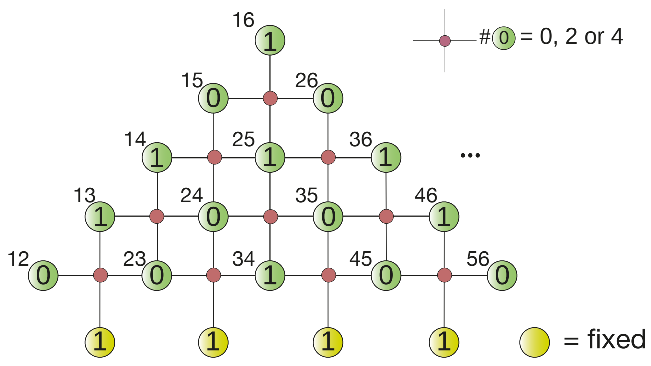
Error correction

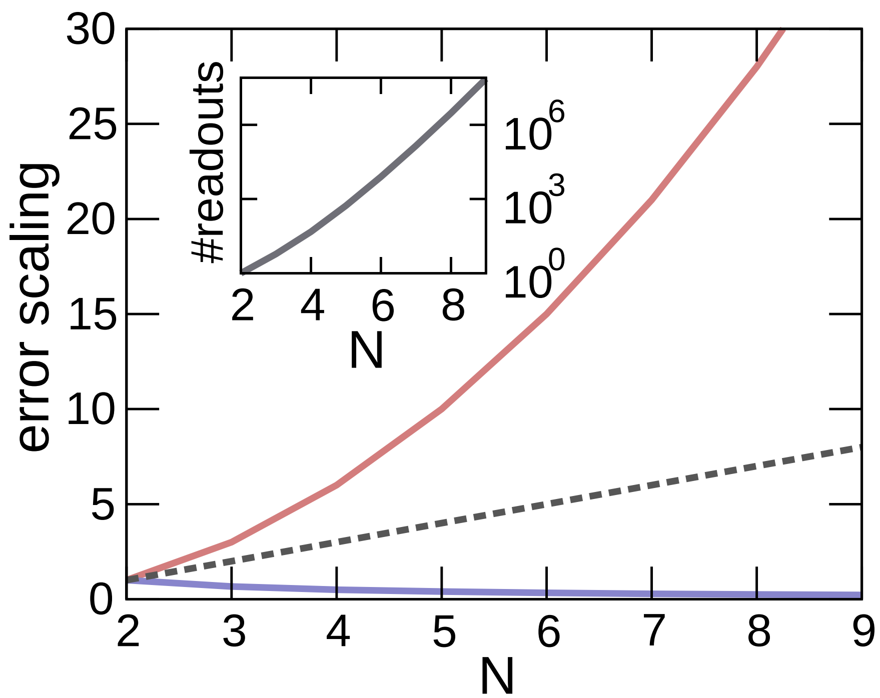
	Spin glass annealing	Gauge field annealing
		
Number of qubits	N	$\frac{N(N-1)}{2}$
Individual Qubit error rate	Γ	Γ
Success Probability	$P_s = 1 = \sum_i^N \Gamma \tau I_i = 1 - \Gamma \tau I_s N$	$P_s = 1 - \Gamma \tau I_g \frac{N(N-1)}{2}$

Error correction

	Spin glass annealing	Gauge field annealing
		
Success Probability	$P_s = 1 - \Gamma\tau N I_s$ $I_s = 1$	$P_s = 1 - \Gamma\tau \frac{N(N-1)}{2} I_g$

Error correction

	Spin glass annealing	Gauge field annealing
		
Success Probability	$P_s = 1 - \Gamma\tau N I_s$ $I_s = 1$	$P_s = 1 - \Gamma\tau \frac{N(N-1)}{2} I_g$



$\Gamma\tau \frac{N(N-1)}{2}$

I_g

$\Gamma\tau \frac{N(N-1)}{2} I_g$

$$\boxed{P_s = 1 - \Gamma\tau(N-1)}$$

Error correction

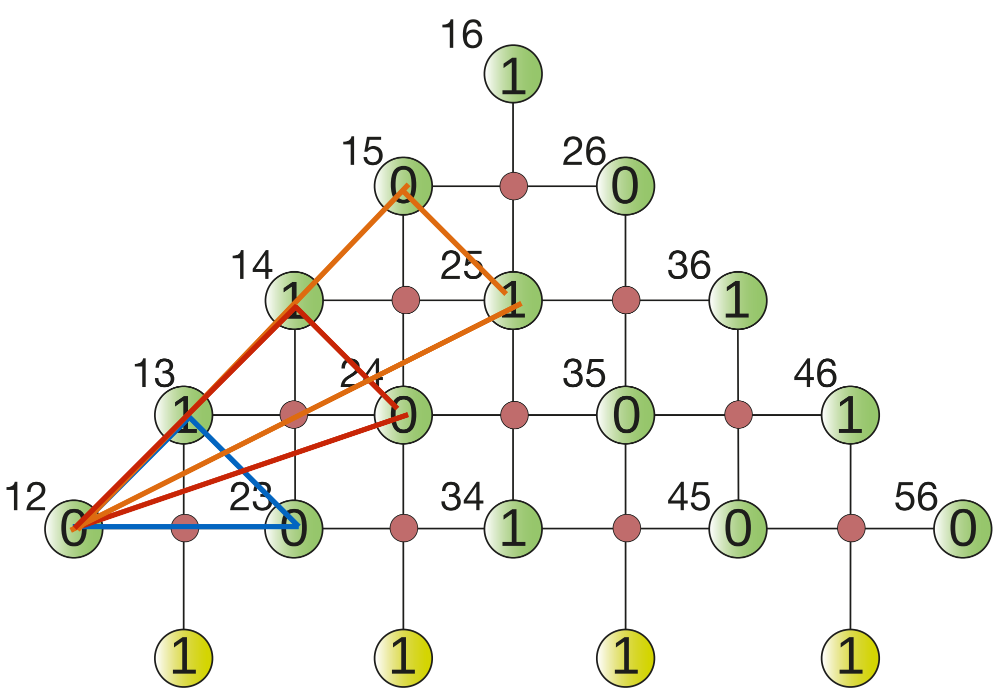
Fernando Pastawski, John Preskill, arXiv/1511.00004 (2015).

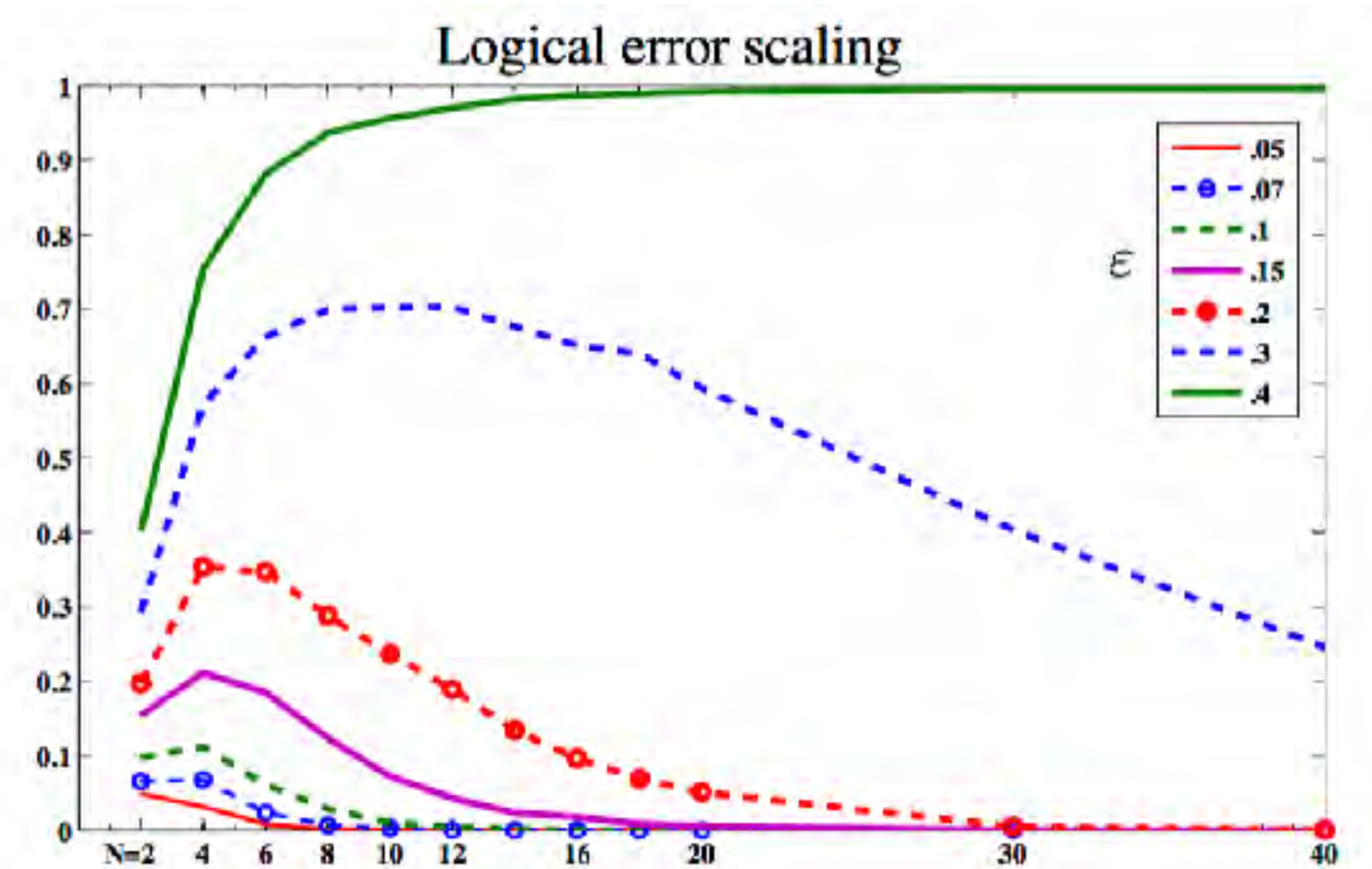
Belief propagation algorithm

Parity is preserved for any closed loop: $0 = (12) \oplus (23) \oplus (13) = (12) \oplus (24) \oplus (14) = \ldots = (12) \oplus (2N) \oplus (1N)$

Estimate value of (12) from

$$g_{12} = (23) \oplus (13) = (24) \oplus (14) = \ldots \qquad \text{loops of 3}$$
$$g_{12} = (23) \oplus (34) \oplus (14) = (23) \oplus (35) \oplus (15) = \ldots = \qquad \text{loops of 4}$$
$$g_{12} = (23) \oplus (34) \oplus (45) \oplus (15) = \ldots \qquad \text{loops of 5}$$
$$\ldots$$





6.QRAM

Many-Body Spin Superposition

List of Bit-Strings

Quantum Superposition

1011
1100
1111
...

$\longrightarrow$

Oracle
?

$\longrightarrow$

$$|\psi\rangle = a_1|1011\rangle + \\ + a_2|1100\rangle + \\ + a_3|1111\rangle$$

QRAM - V. Giovannetti, S. Lloyd, and L. Maccone, PRL 100, 160501 (2008).

Many-Body Spin Superposition

List of Bit-Strings

1011
1100
1111
…

Oracle

Quantum Superposition

$$|\psi\rangle = a_1|1011\rangle + \\ + a_2|1100\rangle + \\ + a_3|1111\rangle$$

This Talk!

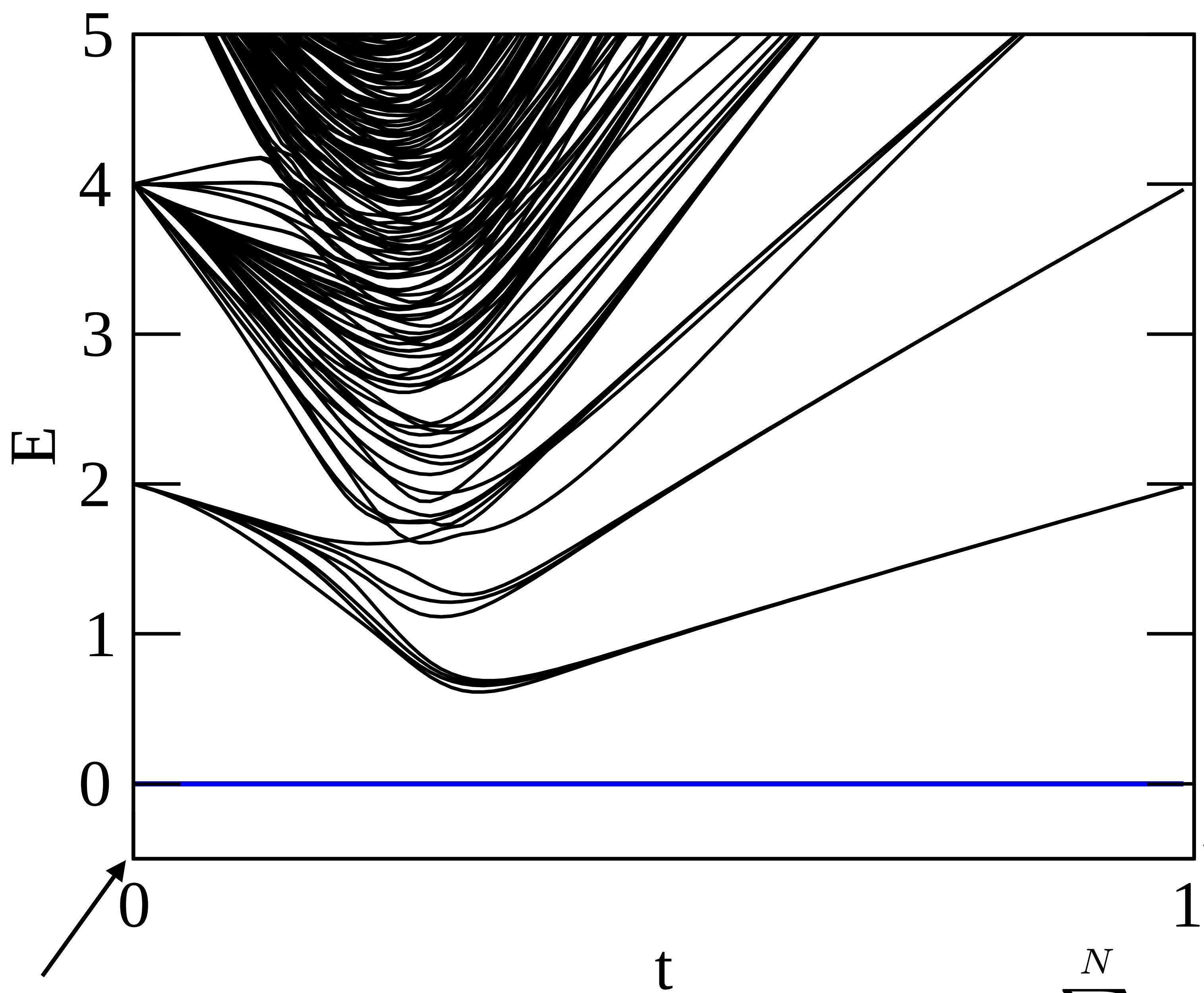
arXiv:1708.02533 (2017).

QRAM - V. Giovannetti, S. Lloyd, and L. Maccone, PRL 100, 160501 (2008).

Adiabatic Theorem

$$\mathcal{H}(t) = A(t)\sum_i^N b_i\sigma_x^{(i)} + B(t)\left[\sum_{i=1}^N h_i\sigma_z^{(i)} + \sum_{i=1}^N\sum_{j=1}^i J_{ij}\sigma_z^{(i)}\sigma_z^{(j)}\right]$$

H. Nishimori (1998).
E. Farhi (2001).

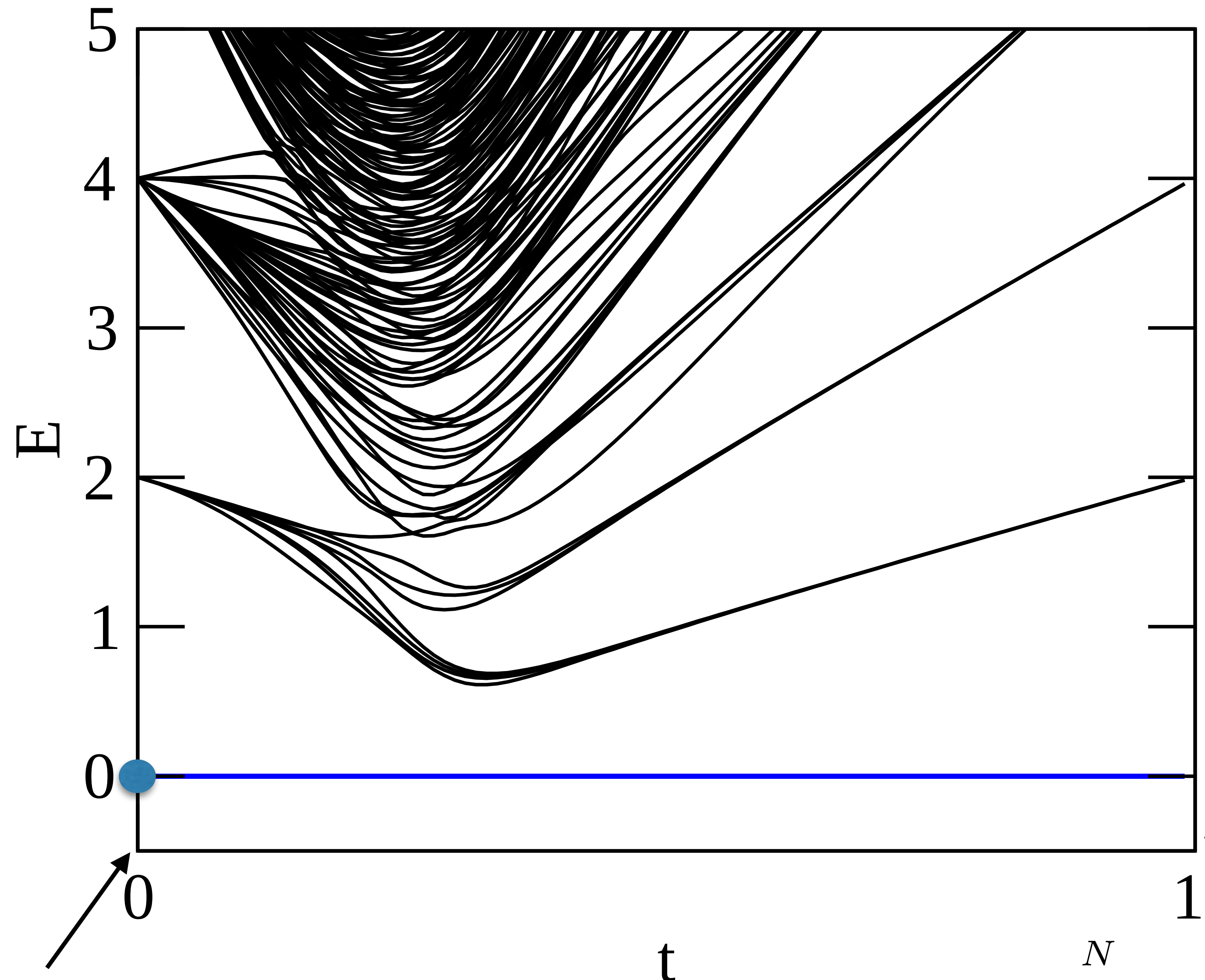


$$\sum_i^N b_i\sigma_x^{(i)}$$

$$\sum_{i=1}^N h_i\sigma_z^{(i)} + \sum_{i=1}^N\sum_{j=1}^i J_{ij}\sigma_z^{(i)}\sigma_z^{(j)}$$

Adiabatic Theorem

$$\mathcal{H}(t) = A(t)\sum_{i}^{N} b_i \sigma_x^{(i)} + B(t)\left[\sum_{i=1}^{N} h_i \sigma_z^{(i)} + \sum_{i=1}^{N}\sum_{j=1}^{i} J_{ij}\sigma_z^{(i)}\sigma_z^{(j)}\right]$$

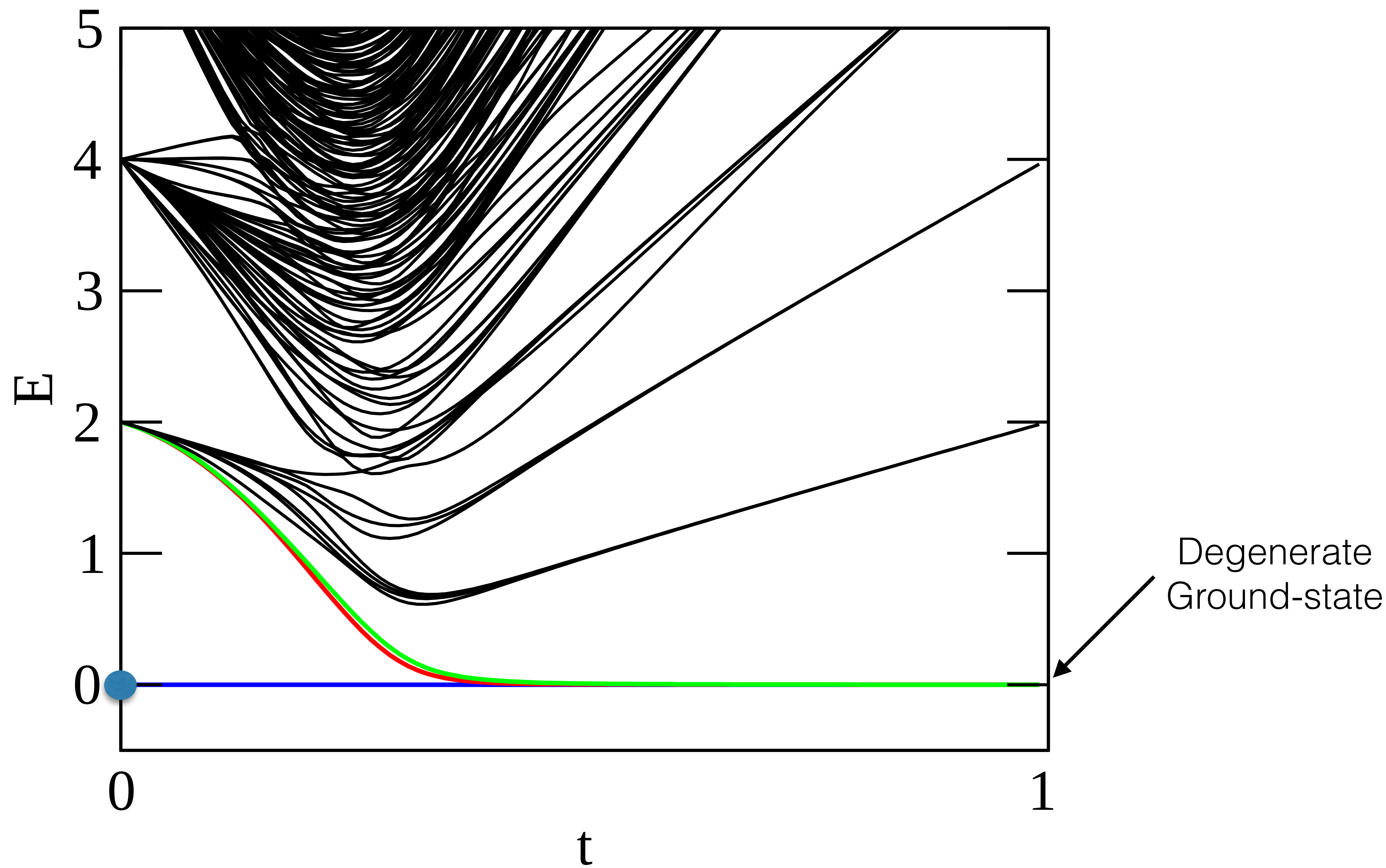


$$\sum_{i}^{N} b_i \sigma_x^{(i)}$$

$$\sum_{i=1}^{N} h_i \sigma_z^{(i)} + \sum_{i=1}^{N}\sum_{j=1}^{i} J_{ij}\sigma_z^{(i)}\sigma_z^{(j)}$$

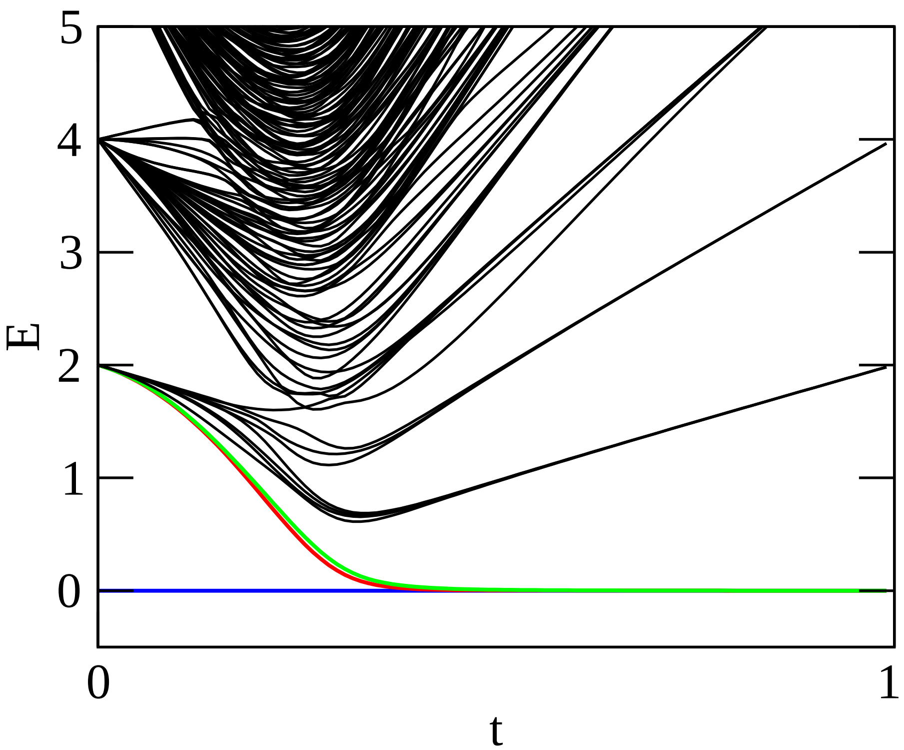
Adiabatic Theorem

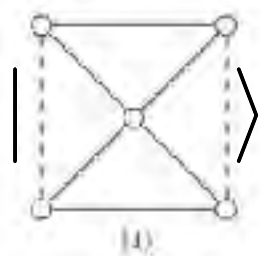
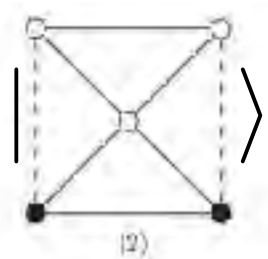
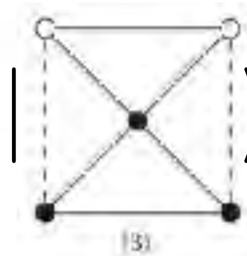
$$\mathcal{H}(t) = A(t)\sum_{i}^{N} b_i\sigma_x^{(i)} + B(t)\left[\sum_{i=1}^{N} h_i\sigma_z^{(i)} + \sum_{i=1}^{N}\sum_{j=1}^{i} J_{ij}\sigma_z^{(i)}\sigma_z^{(j)}\right]$$



Fair Sampling

$$\mathcal{H}(t) = A(t)\sum_{i}^{N} b_i \sigma_x^{(i)} + B(t)\left[\sum_{i=1}^{N} h_i \sigma_z^{(i)} + \sum_{i=1}^{N}\sum_{j=1}^{i} J_{ij}\sigma_z^{(i)}\sigma_z^{(j)}\right]$$



$|\psi\rangle = a_1|$  $\rangle + a_2|$  $\rangle + a_3|$  $\rangle$

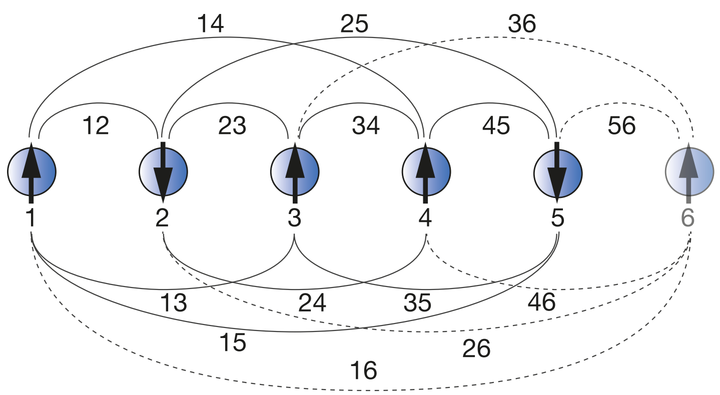
Exponential bias in amplitudes

H. Katzgraber et.al. Phys. Rev. Lett. **118**, 070502 (2017).

Can we make these amplitudes tunable?

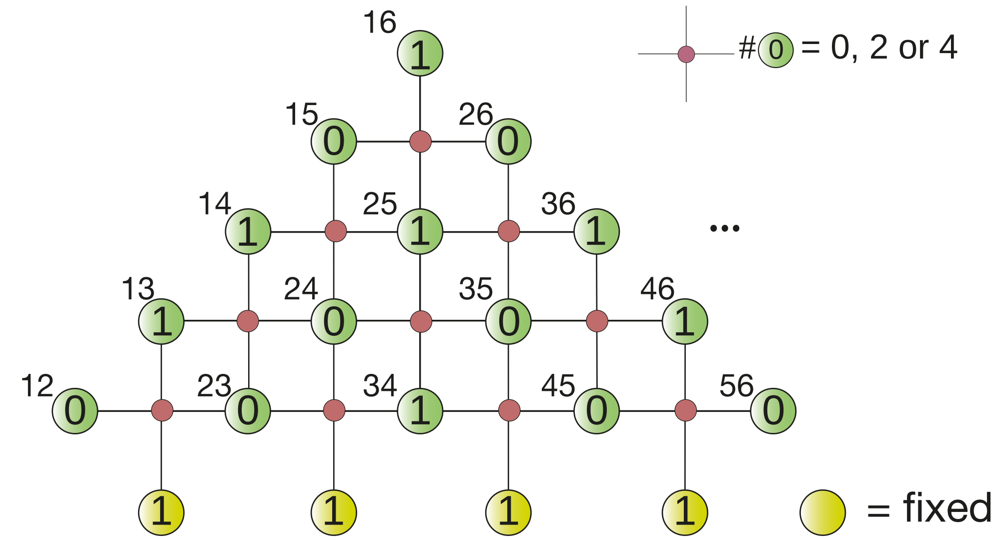
Parity Constraints

Spin glass paradigm



$$\mathcal{H}(t) = A(t)\sum_{i}^{N} b_i \sigma_x^{(i)} + B(t)\left[\sum_{i=1}^{N} h_i \sigma_z^{(i)} + \sum_{i=1}^{N}\sum_{j=1}^{i} J_{ij}\sigma_z^{(i)}\sigma_z^{(j)}\right]$$

Parity constraints



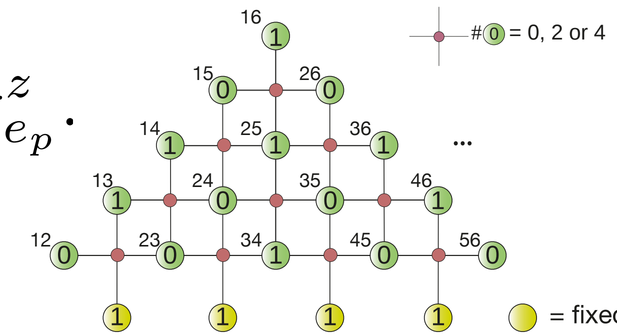
$$\mathcal{H}(t) = A(t)\sum_{i=1}^{K} b_i \sigma_x^{(i)} + B(t)\sum_{i=1}^{K} J_i \sigma_z^{(i)} + C(t)\sum_{l=1}^{K-N} C_l$$

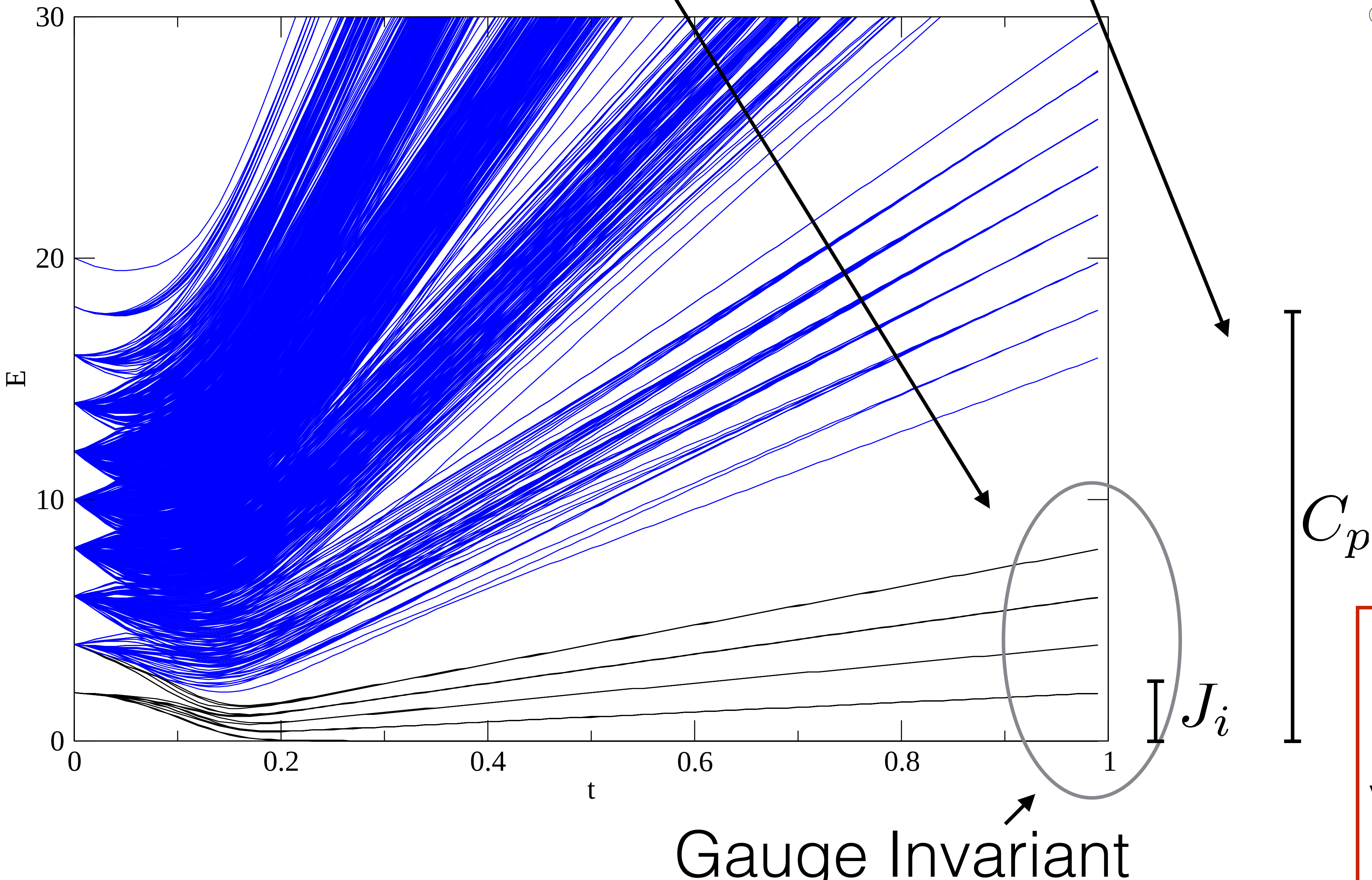
- comprises **problem-independent**, **local interaction -> Gauge invariance**

WL, P. Hauke, P. Zoller, Science Advances 1, 1500838 (2015).

Spectrum

$$H(t) = A(t)\sum_{i=1}^{K}\sigma_i^x + B(t)\sum_{i=1}^{K}J_i\sigma_i^z + C(t)\sum_{p=1}^{K-N+1} C_p\sigma_{n_p}^z\sigma_{w_p}^z\sigma_{s_p}^z\sigma_{e_p}^z.$$

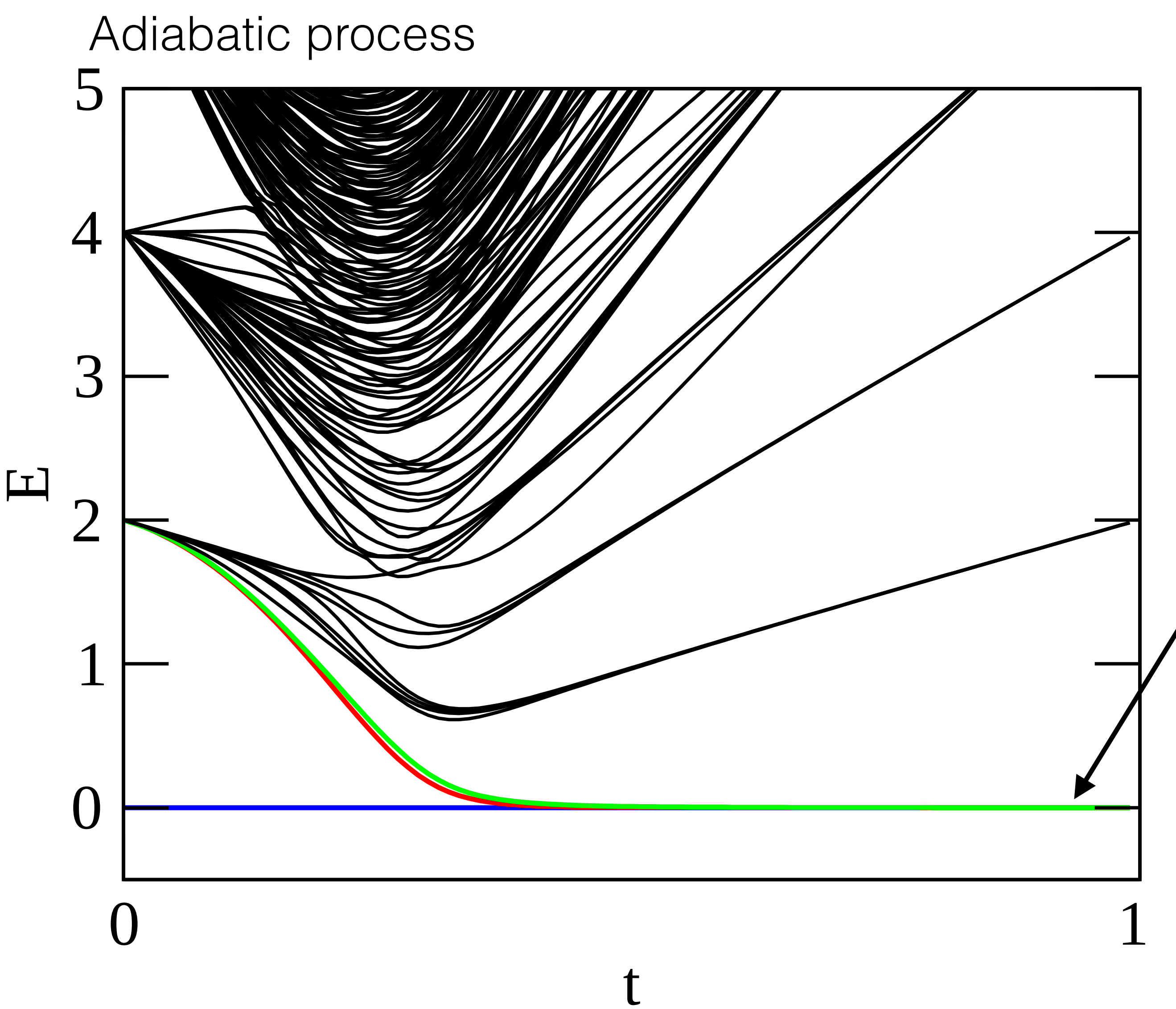




Invariant under variation of individual C_p

Spectrum

$$H(t) = A(t)\sum_{i=1}^{K}\sigma_i^x + B(t)\sum_{i=1}^{K}J_i\sigma_i^z + C(t)\sum_{p=1}^{K-N+1}C_p\sigma_{n_p}^z\sigma_{w_p}^z\sigma_{s_p}^z\sigma_{e_p}^z.$$



Adiabatic manifold

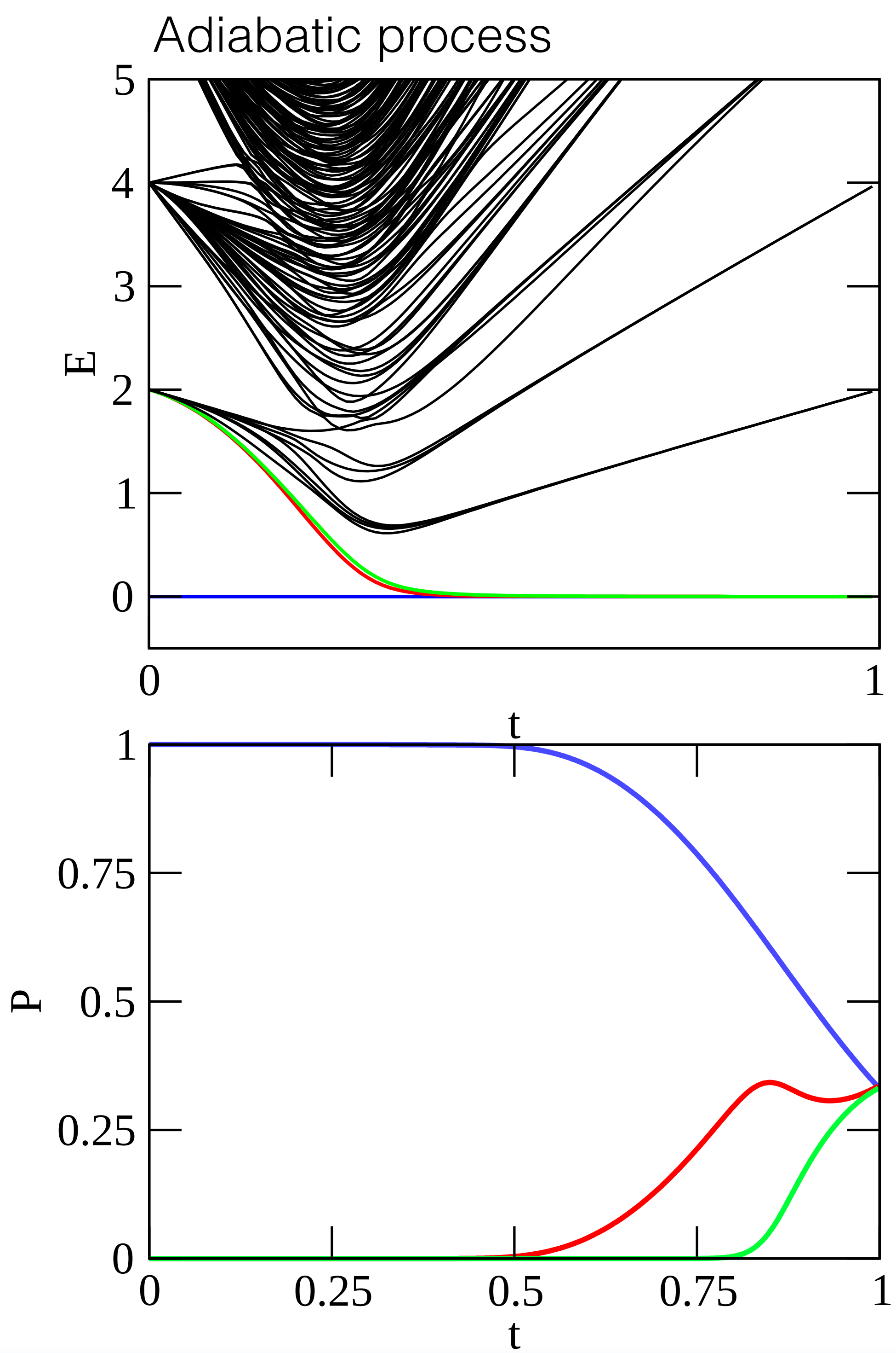
↓

Population of final state depends on acquired phases.

Strategy: make use of gauge degrees of freedom to tune the quantum phase of the paths.

Effective Theory

$$H(t) = A(t)\sum_{i=1}^{K}\sigma_i^x + B(t)\sum_{i=1}^{K}J_i\sigma_i^z + C(t)\sum_{p=1}^{K-N+1}C_p\sigma_{n_p}^z\sigma_{w_p}^z\sigma_{s_p}^z\sigma_{e_p}^z.$$



Adiabatic manifold:

- Projection

$$\mathcal{P} = \sum_{n=1}^{M}|z_n\rangle\langle z_n|$$

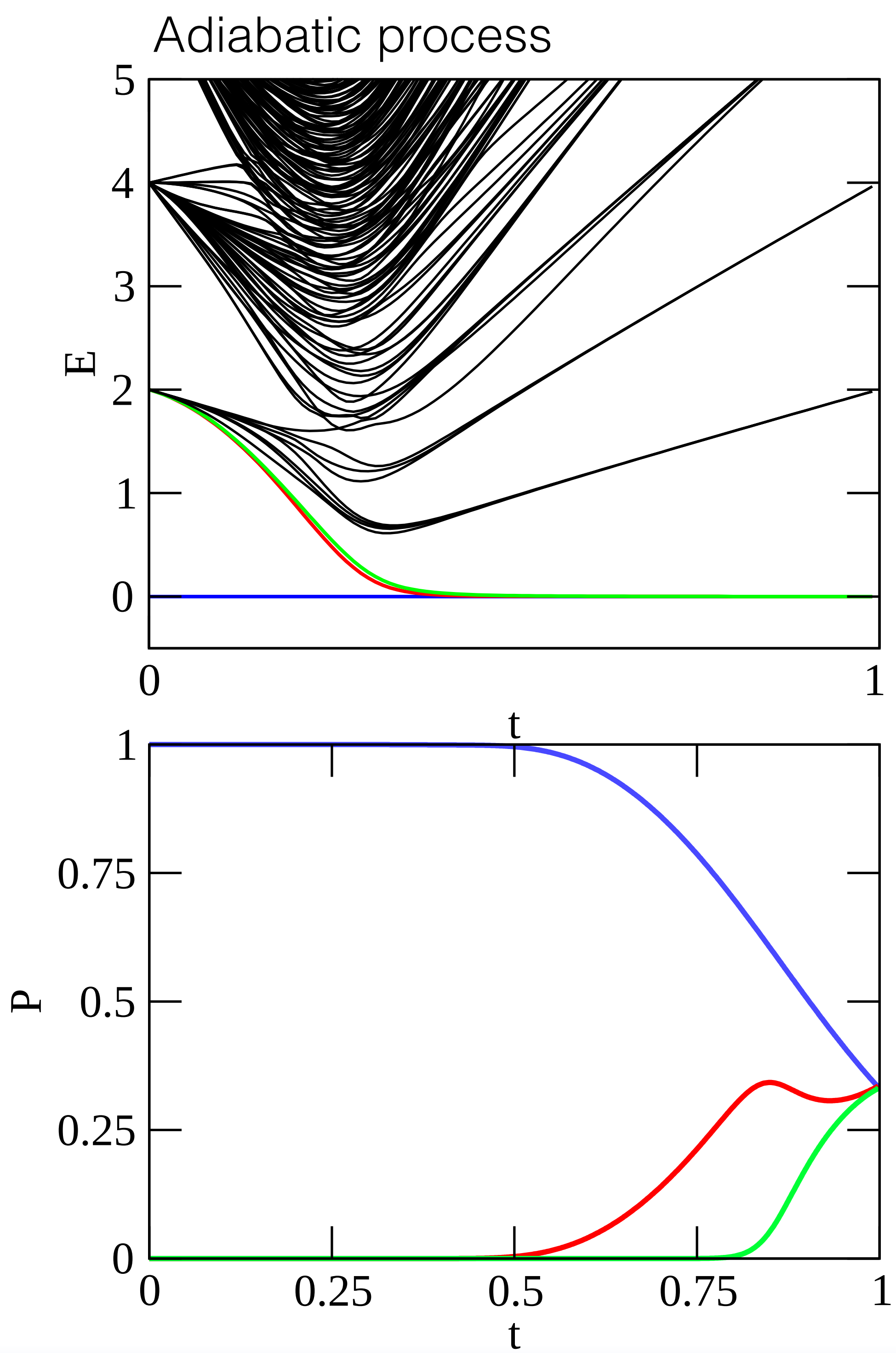
$$Q = 1 - \mathcal{P}$$

- Schrieffer-Wolff

- Perturbation Theory

Effective Theory

$$H(t) = A(t)\sum_{i=1}^{K}\sigma_i^x + B(t)\sum_{i=1}^{K} J_i\sigma_i^z + C(t)\sum_{p=1}^{K-N+1} C_p\sigma_{n_p}^z\sigma_{w_p}^z\sigma_{s_p}^z\sigma_{e_p}^z.$$

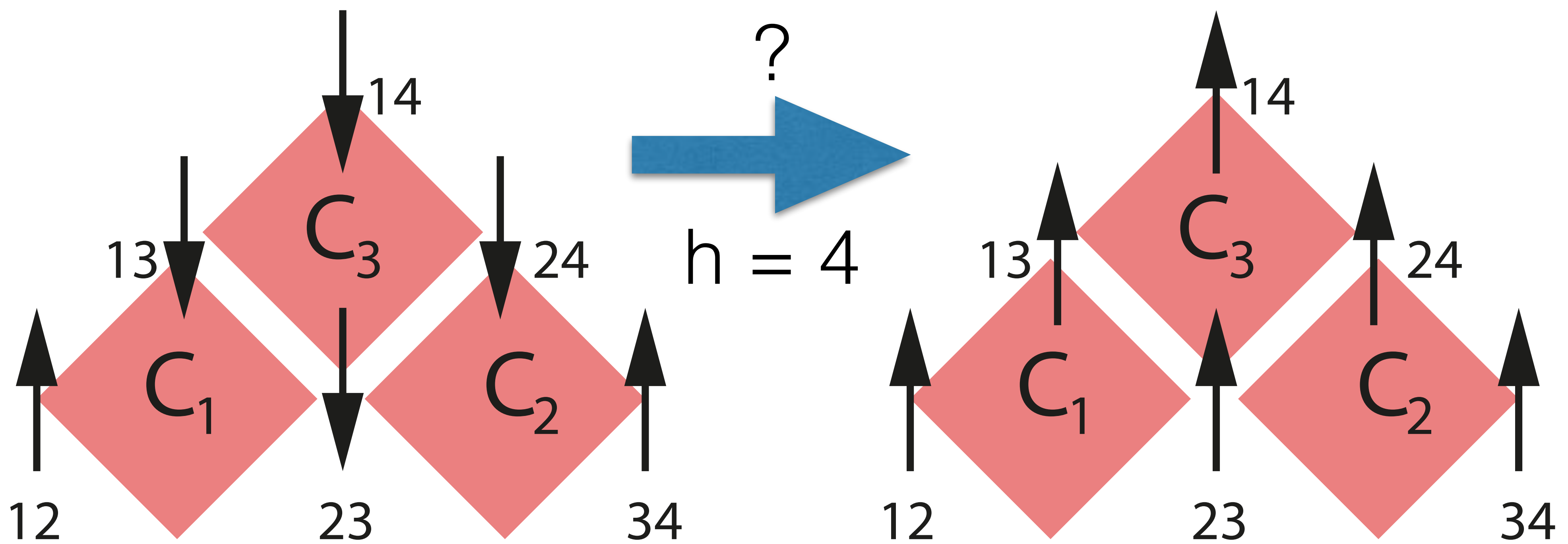


$$H_{\text{eff}}(t) = \begin{pmatrix} e_1 t^{-2} & g_{12}t^{-h_{12}} & g_{13}t^{-h_{13}} \\ g_{21}t^{-h_{21}} & e_2 t^{-2} & g_{23}t^{-h_{23}} \\ g_{31}t^{-h_{31}} & g_{32}t^{-h_{32}} & e_3 t^{-2} \end{pmatrix}$$

Matrix Elements

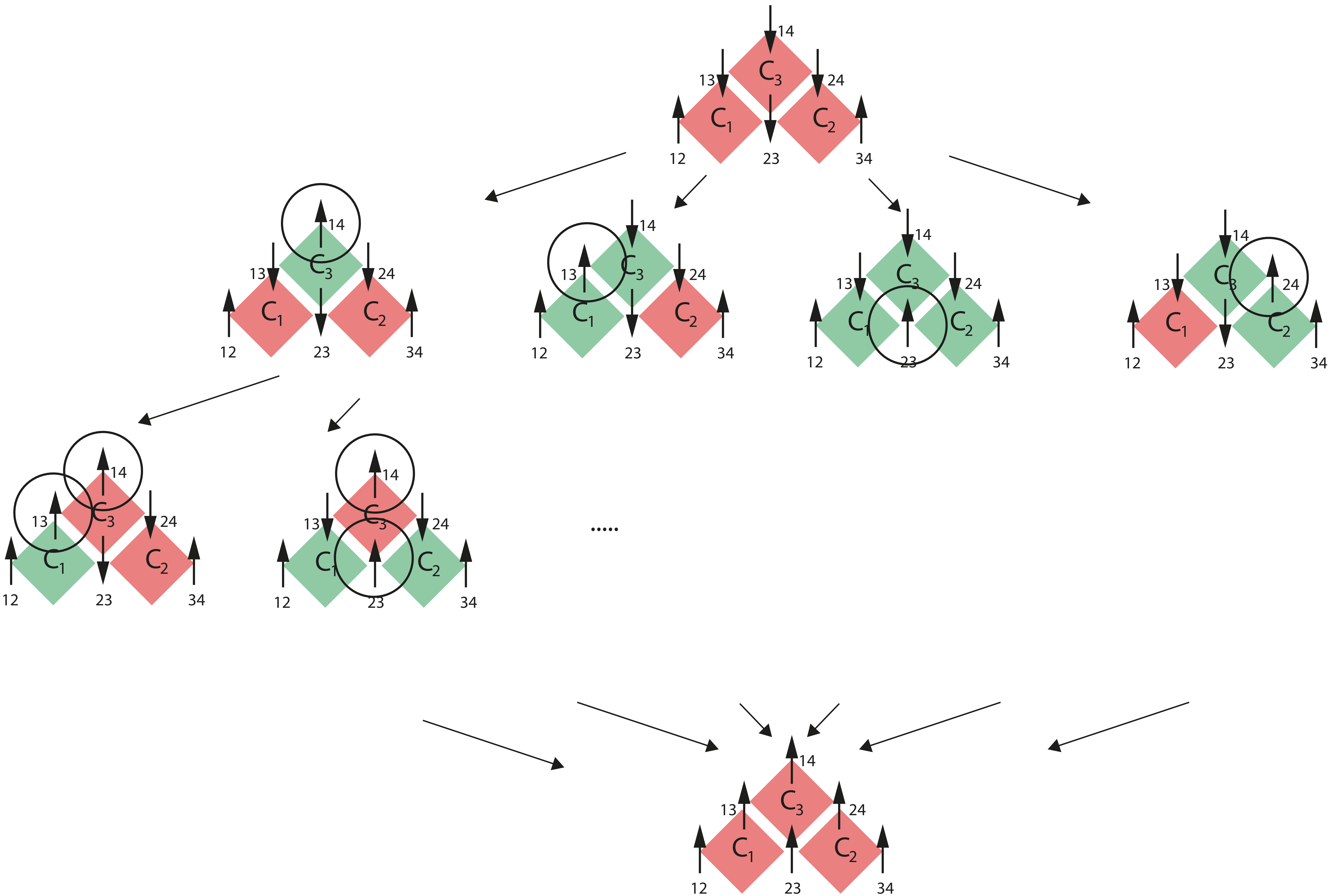
Spin-Model 1100 1111

Lattice Gauge



Count Feynman Paths!

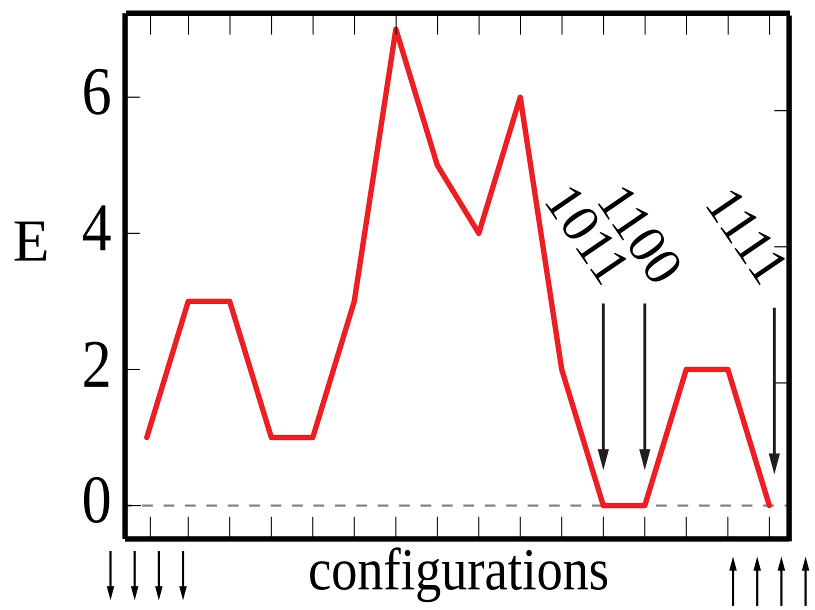
Feynman Paths



Matrix Elements

$$
\begin{aligned}
g_{12} = \frac{1}{2^3} \Bigg(& \frac{-\frac{\frac{1}{C_2+C_3}+\frac{1}{1+C_1+C_3}}{1+C_1+C_2} + \frac{\frac{1}{1-C_1-C_2-C_3}-\frac{1}{1+C_1+C_3}}{C_2} - \frac{\frac{1}{1-C_1-C_2-C_3}-\frac{1}{C_2+C_3}}{1-C_1}}{C_3} \\
& - \frac{\frac{-\frac{1}{C_2+C_3}-\frac{1}{C_3}}{C_2} - \frac{\frac{1}{1-C_1-C_2-C_3}-\frac{1}{C_2+C_3}}{1-C_1} - \frac{\frac{1}{1-C_1-C_2-C_3}-\frac{1}{C_3}}{1-C_1-C_2}}{1-C_1-C_3} \\
& + \frac{-\frac{\frac{1}{1+C_1+C_3}+\frac{1}{C_3}}{1+C_1} + \frac{\frac{1}{1-C_1-C_2-C_3}-\frac{1}{1+C_1+C_3}}{C_2} - \frac{\frac{1}{1-C_1-C_2-C_3}-\frac{1}{C_3}}{1-C_1-C_2}}{C_2+C_3} \\
& + \frac{-\frac{\frac{1}{1+C_1+C_3}+\frac{1}{C_3}}{1+C_1} - \frac{\frac{1}{C_2+C_3}+\frac{1}{C_3}}{C_2} - \frac{\frac{1}{C_2+C_3}+\frac{1}{1+C_1+C_3}}{1+C_1+C_2}}{1+C_1+C_2+C_3} \Bigg)
\end{aligned}
$$

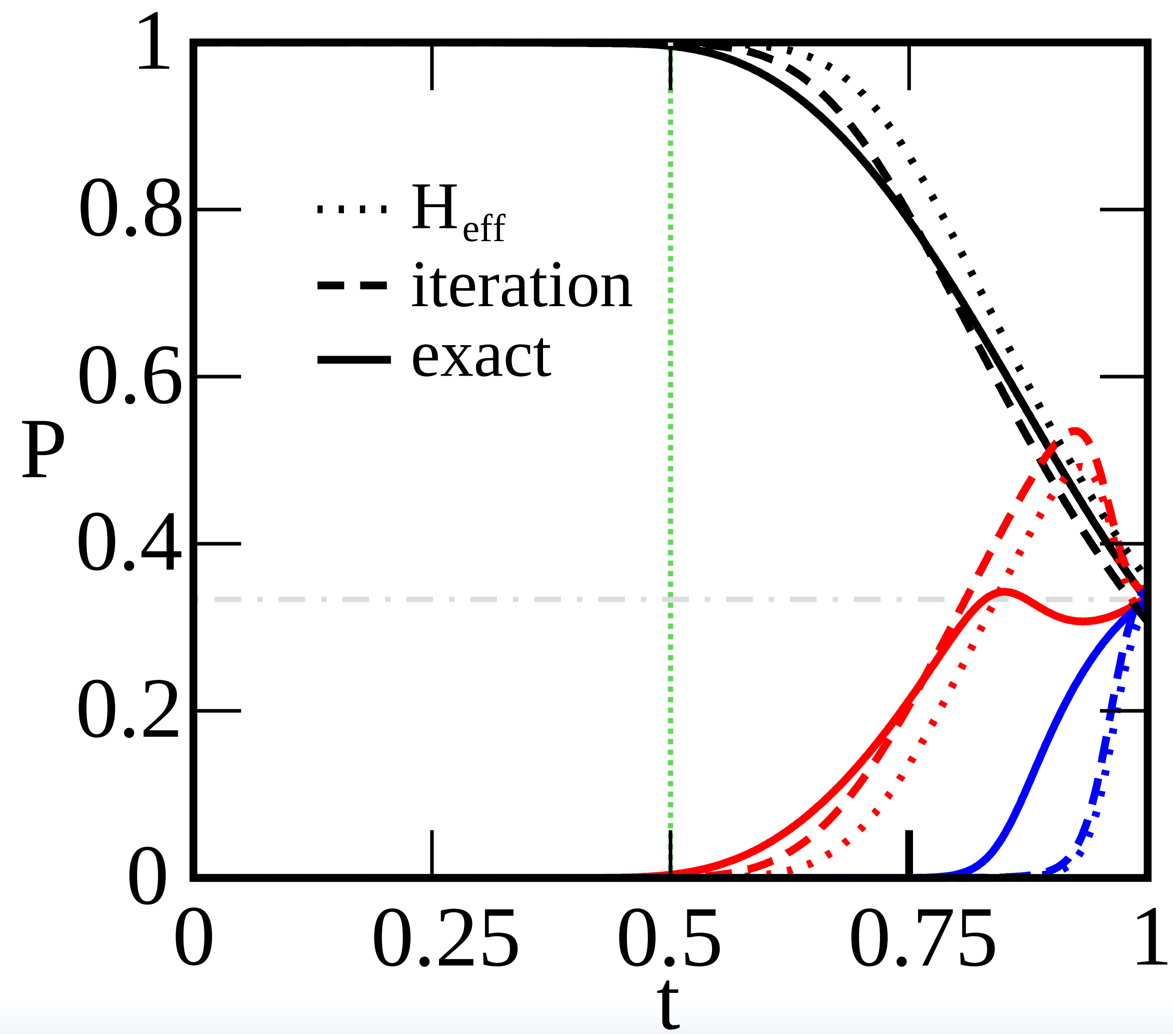
Example



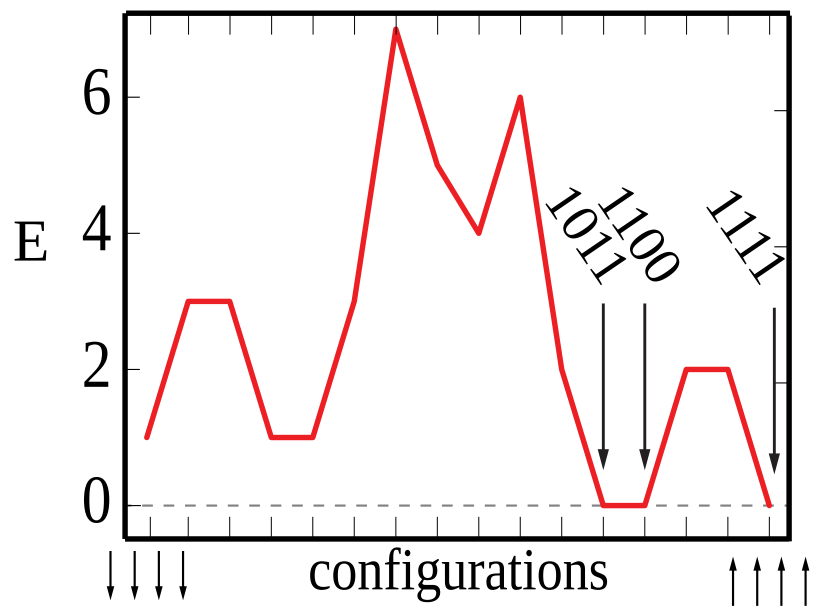
Fair Sampling

$$|\psi\rangle = \sqrt{\frac{1}{3}}|1011\rangle + \sqrt{\frac{1}{3}}|1100\rangle + \sqrt{\frac{1}{3}}|1111\rangle$$

Result: $C_1 = 9.4 \quad C_2 = 0.4 \quad C_3 = 9.2$



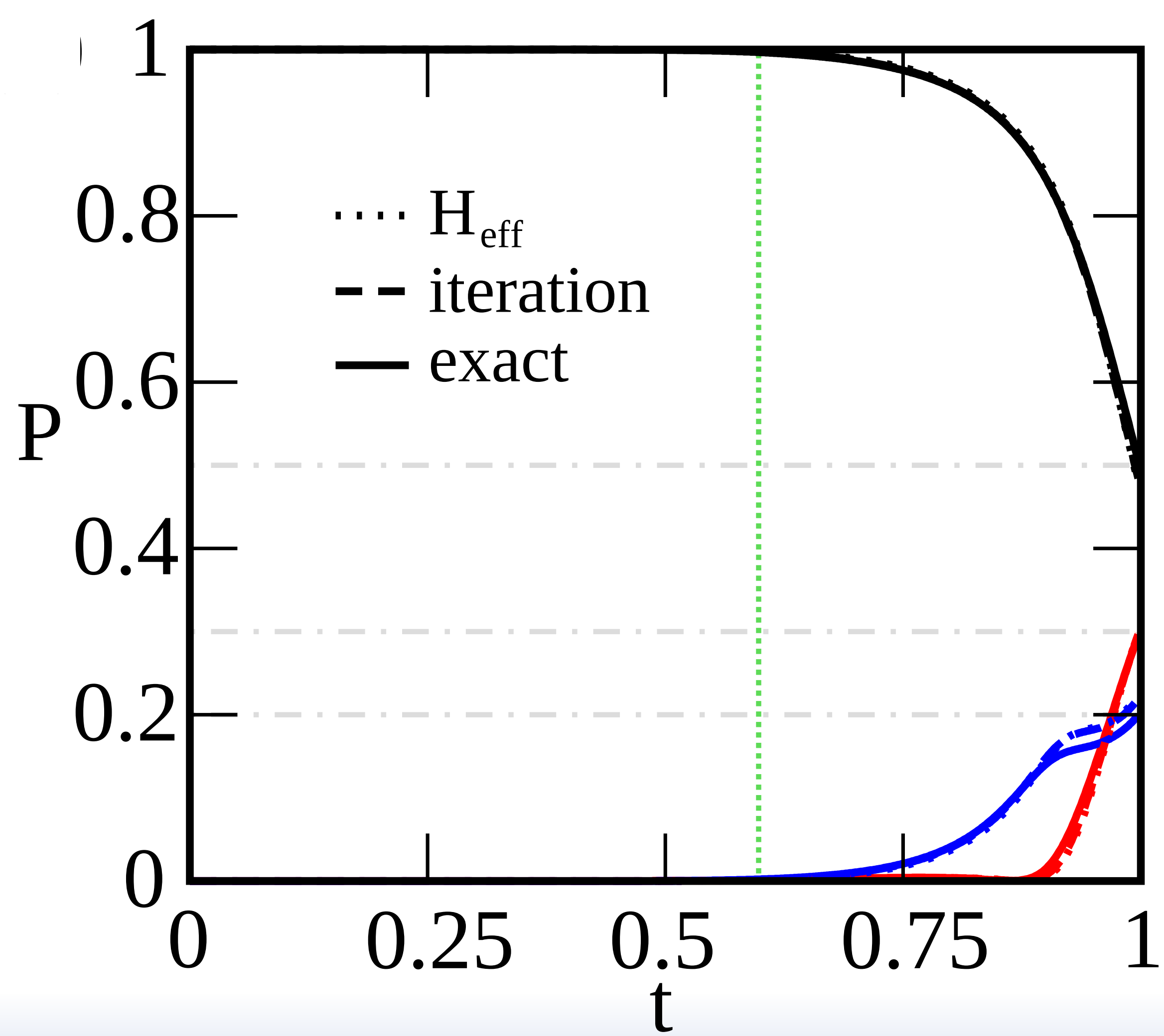
Example



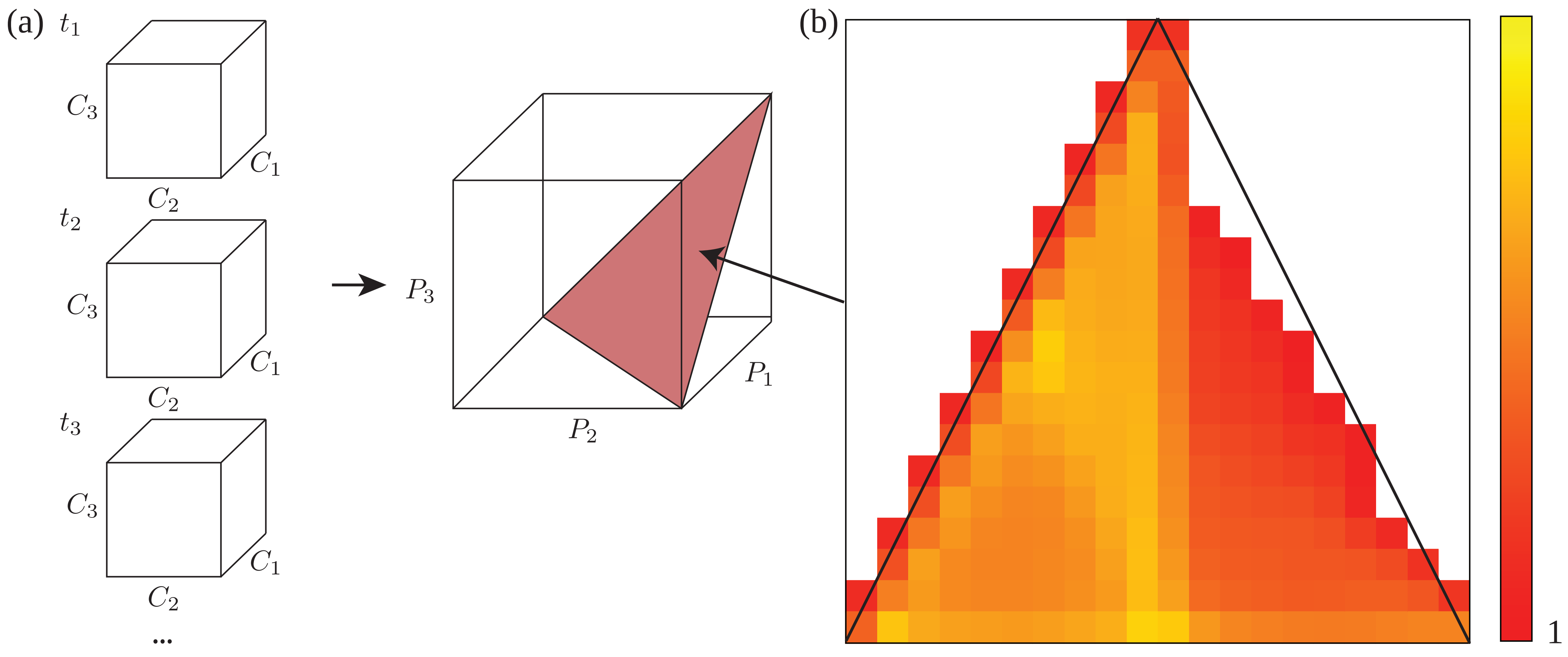
Programable Weights

$$|\psi\rangle = \sqrt{0.2}|1011\rangle + \sqrt{0.3}|1100\rangle + \sqrt{0.5}|1111\rangle$$

Result: $C_1 = 5.6 \quad C_2 = 3.0 \quad C_3 = 6.2$



Ergodicity

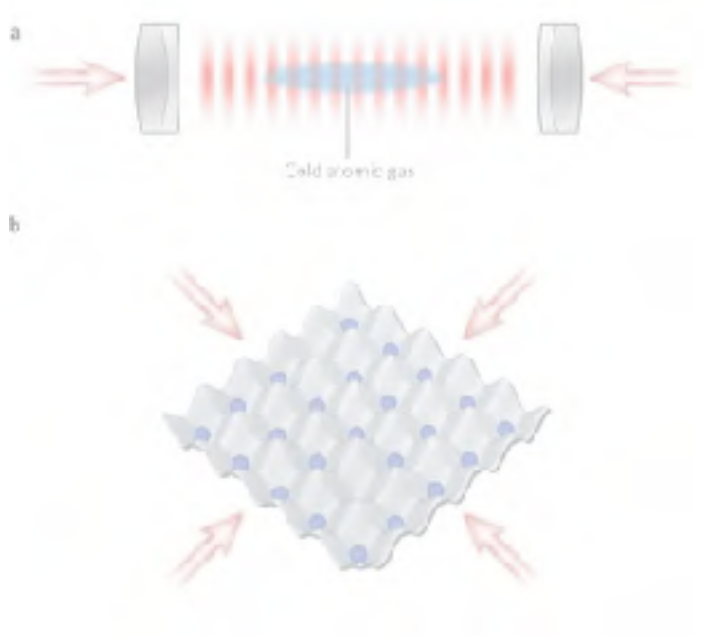


7.Implementations

New platforms for Quantum Annealing

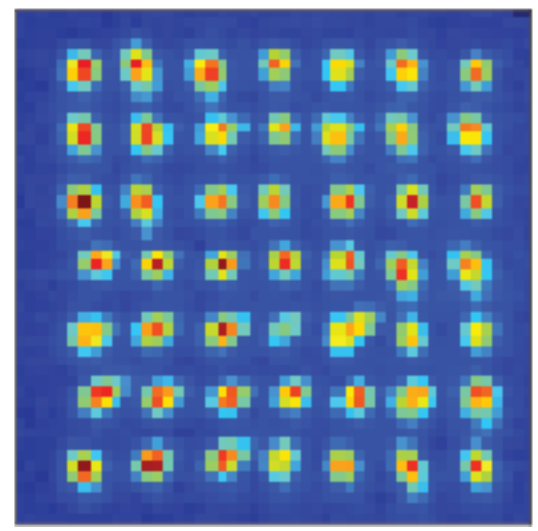
A. Glätzle, R. van Bijnen, P. Zoller and WL, arXiv:1611.02594 (2016).

Ultracold atoms in optical lattices



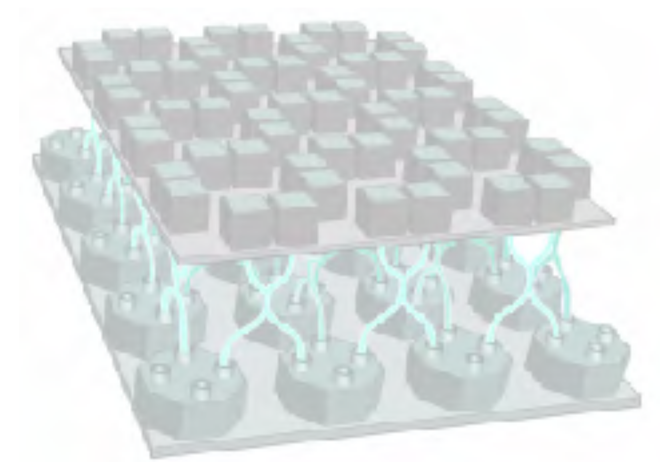
Bloch, Munich

Rydberg atoms



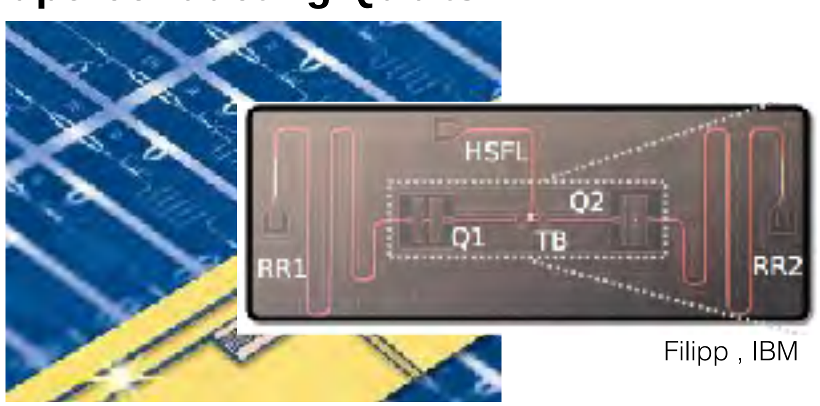
Saffman, Madison

Hybrid Ion-traps



S. Benjamin, Oxford

Superconducting Qubits



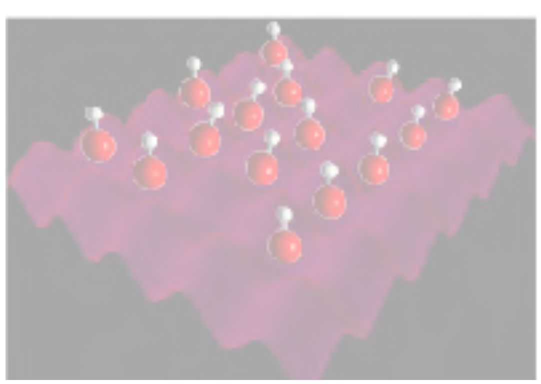
Filipp , IBM

A. Wallraff

Ions in surface traps

Blatt, Innsbruck

Polar Molecules

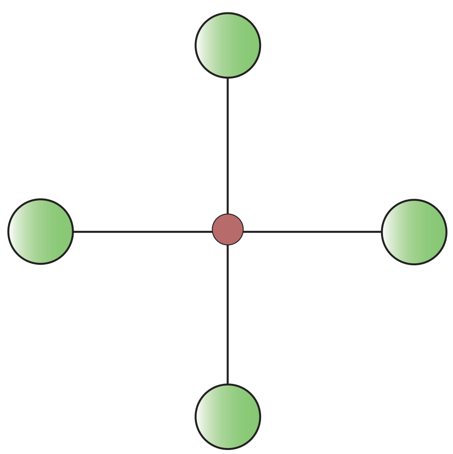


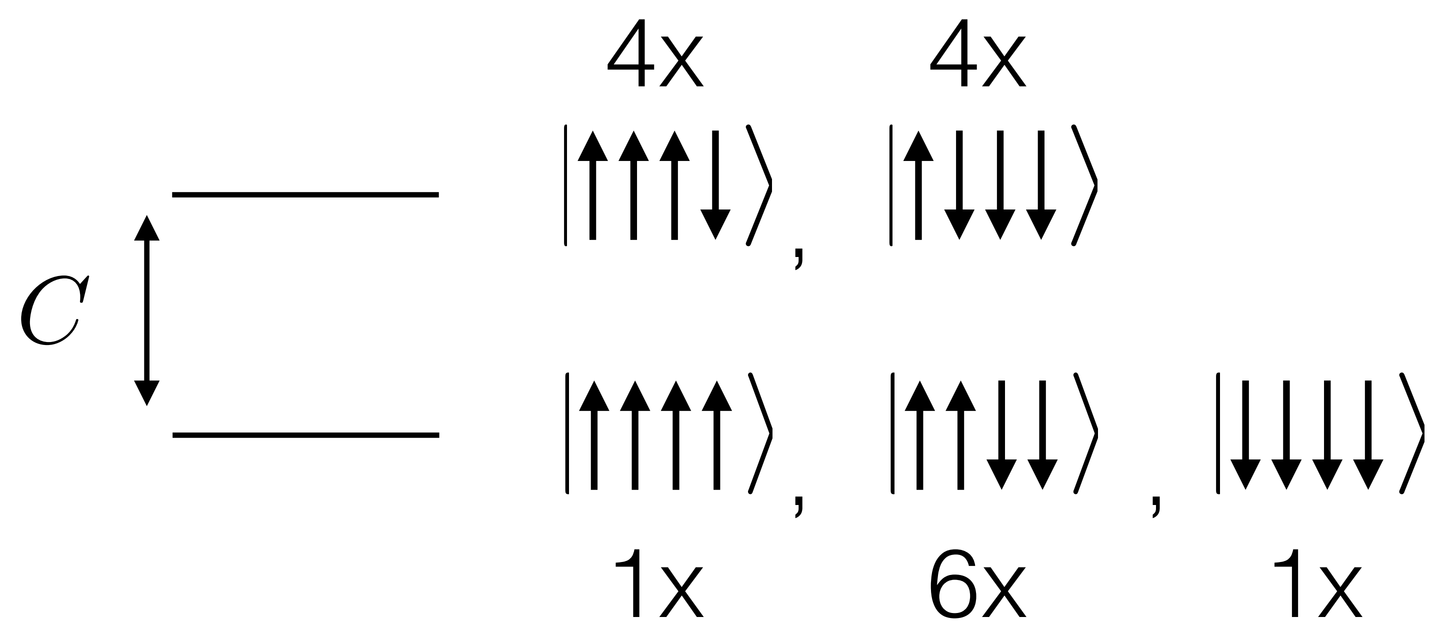
J. Ye, Boulder

Martin Leib, P. Zoller, and WL, arXiv:1604.02359(2016).

Strategies for constraint design with Rydberg atoms

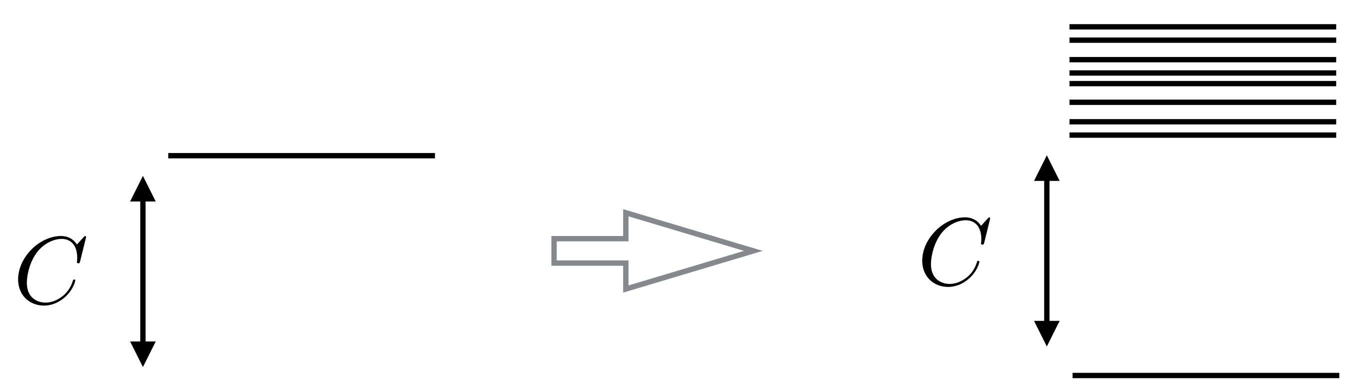
4-body constraints





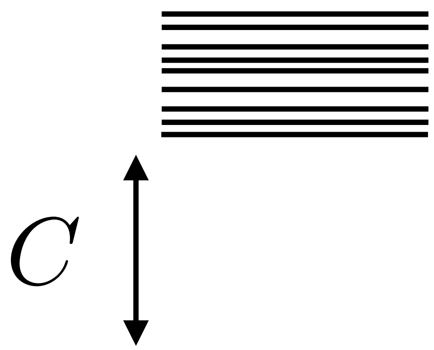
$$H_c = -C\sigma_z^1\sigma_z^2\sigma_z^3\sigma_z^4$$

Unphysical states do not matter



Strategies for constraint design with Rydberg atoms

Andrea Rocchetto, Simon C. Benjamin and Ying Li, Science Advances 2, e1601246 (2016).



even parity

$$|\uparrow\uparrow\uparrow\uparrow\rangle, |\uparrow\uparrow\downarrow\downarrow\rangle, |\downarrow\downarrow\downarrow\downarrow\rangle$$

$$\sum_{i}^{4} \sigma_z^{(i)}$$

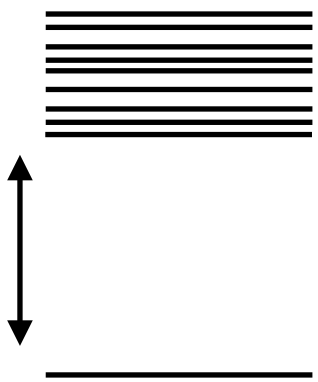
$$\uparrow$$

-4,0,4

Strategies for constraint design with Rydberg atoms

Andrea Rocchetto, Simon C. Benjamin and Ying Li, Science Advances 2, e1601246 (2016).

C



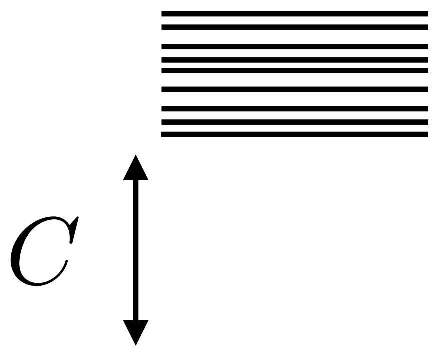
even parity

$|\uparrow\uparrow\uparrow\uparrow\rangle, |\uparrow\uparrow\downarrow\downarrow\rangle, |\downarrow\downarrow\downarrow\downarrow\rangle$

$$S = \left(\underbrace{4S_z}_{\text{qutrit}} + \underbrace{\sum_i^4 \sigma_z^{(i)}}_{-4,0,4} \right)^2$$

Strategies for constraint design with Rydberg atoms

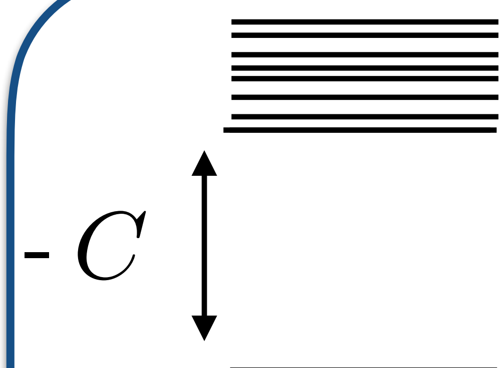
Andrea Rocchetto, Simon C. Benjamin and Ying Li, Science Advances 2, e1601246 (2016).



even parity

$|\uparrow\uparrow\uparrow\uparrow\rangle, |\uparrow\uparrow\downarrow\downarrow\rangle, |\downarrow\downarrow\downarrow\downarrow\rangle$

$$S = \left(\underbrace{4S_z}_{\text{qutrit}} + \underbrace{\sum_i^4 \sigma_z^{(i)}}_{-4,0,4} \right)^2$$

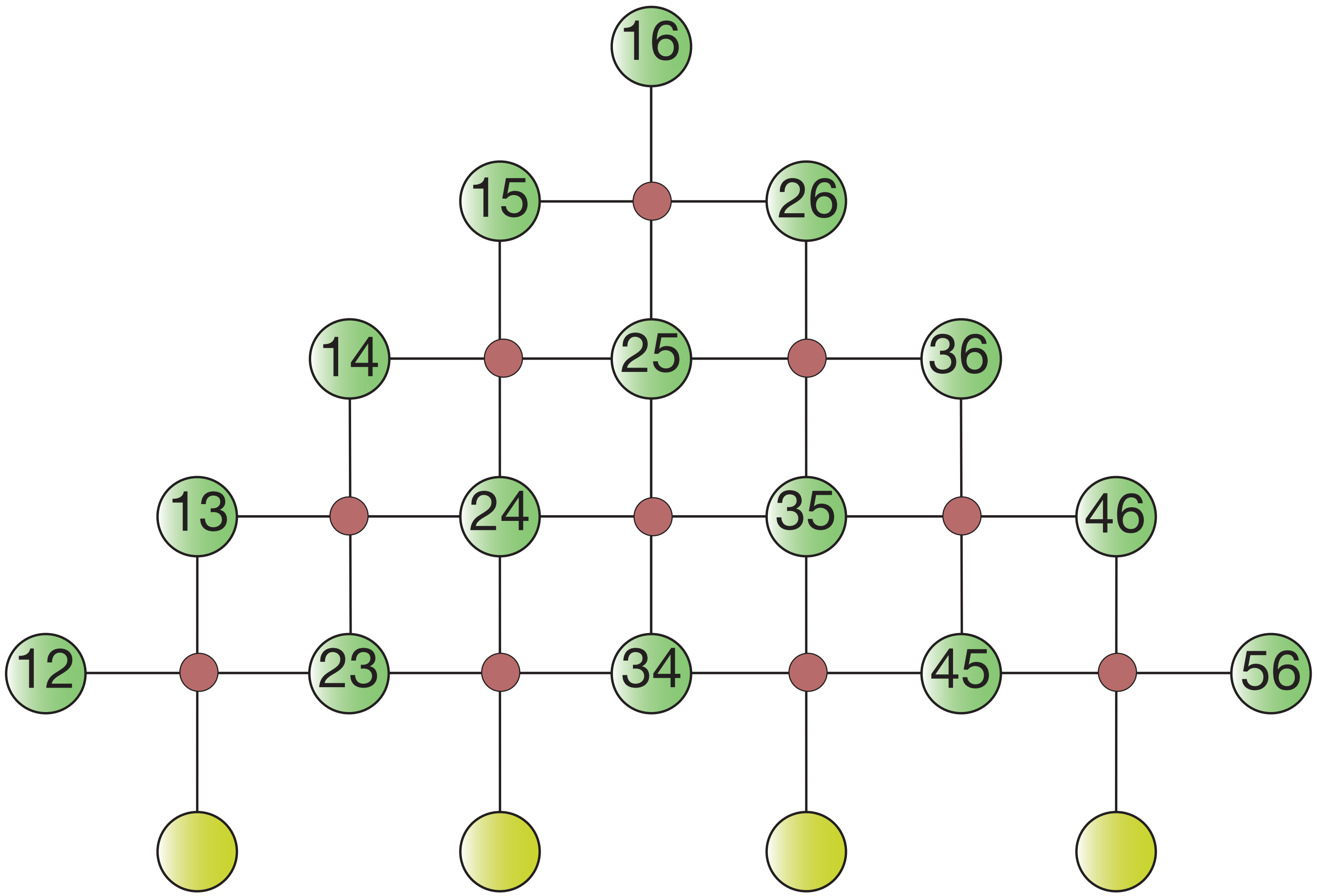


odd parity

$|\uparrow\uparrow\uparrow\downarrow\rangle, \quad |\uparrow\downarrow\downarrow\downarrow\rangle$

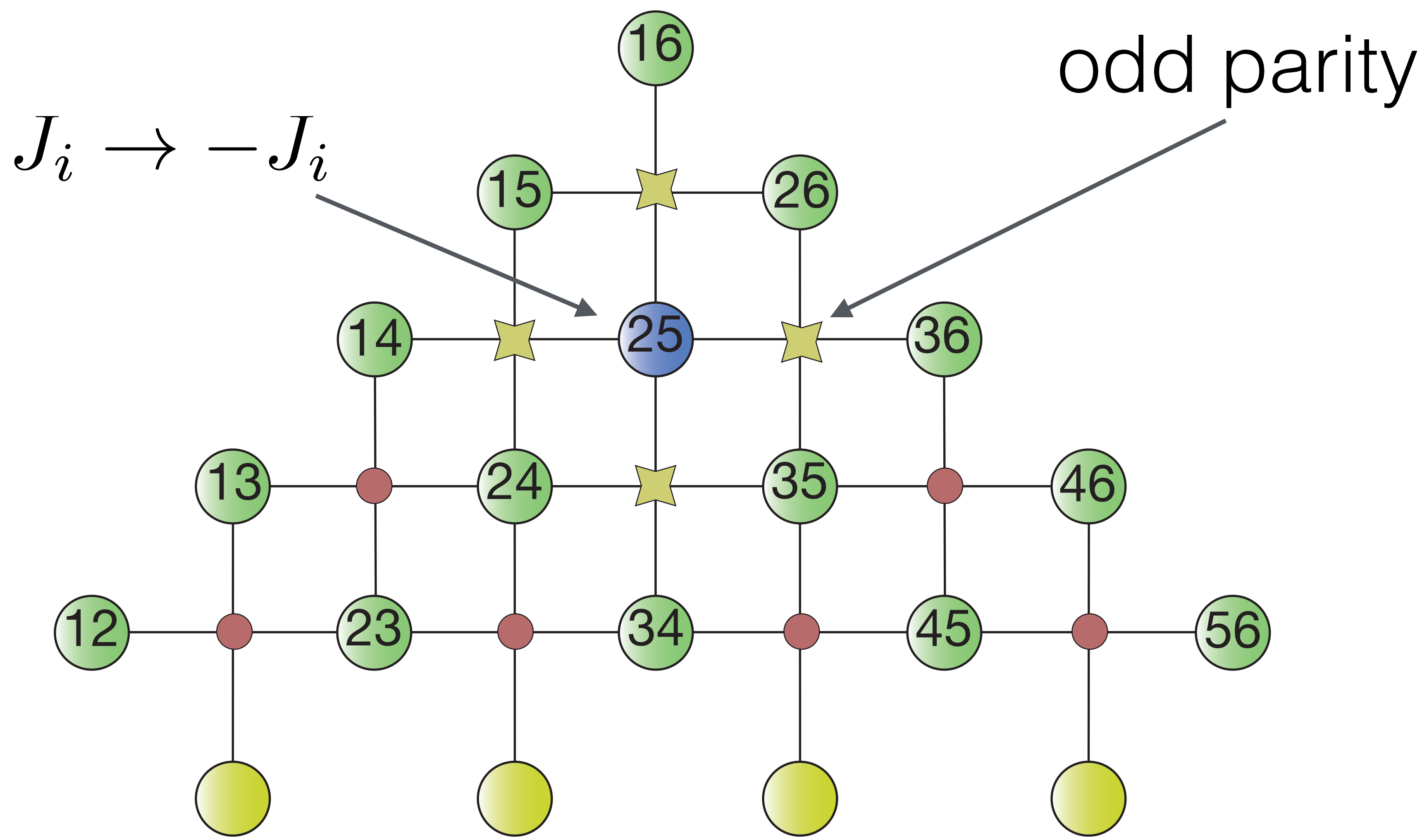
$$S = \left(\underbrace{2\sigma_z^{(a)}}_{\text{qubit}} + \underbrace{\sum_i^4 \sigma_z^{(i)}}_{-2,2} \right)^2$$

Odd parity representation



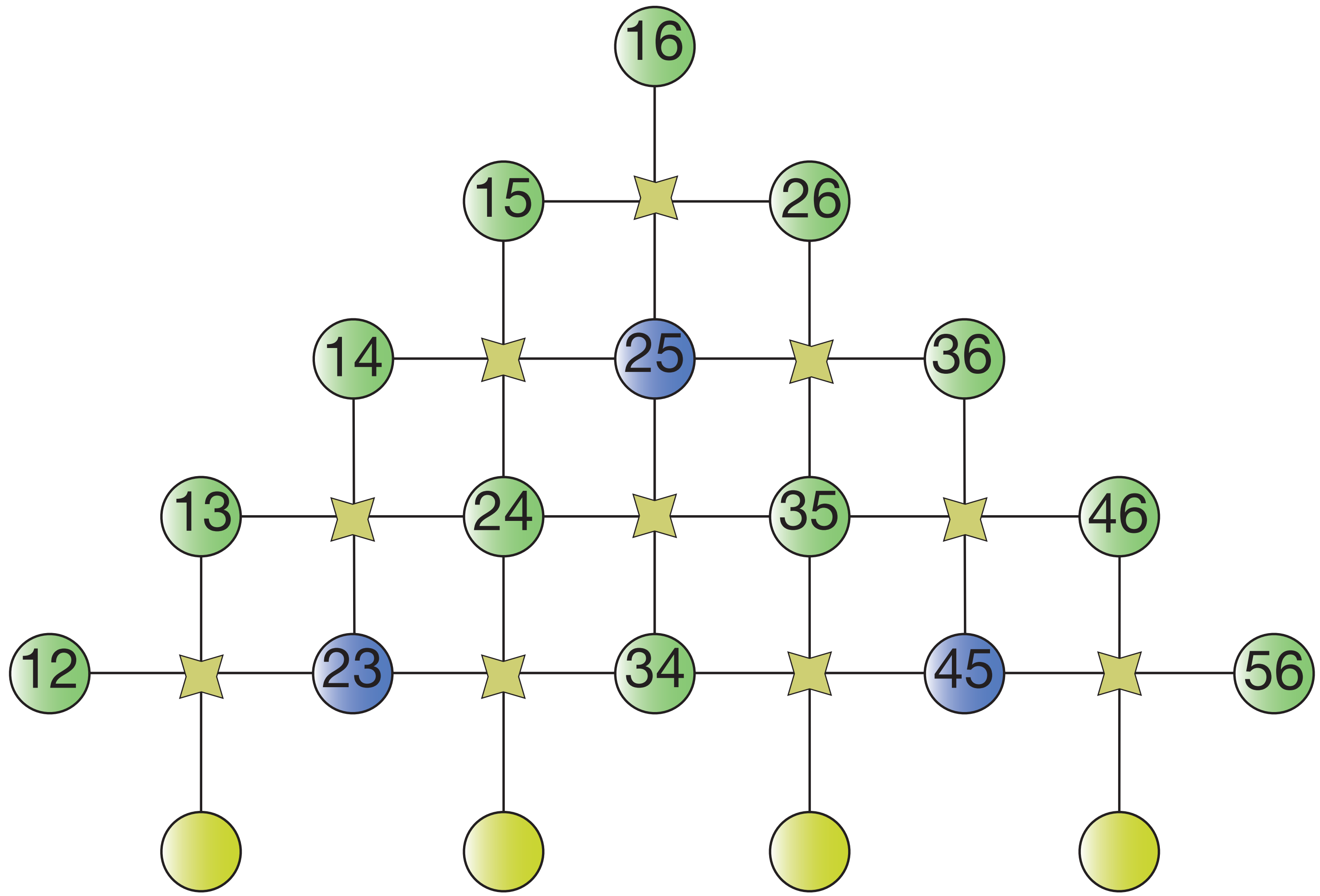
Odd parity representation

Andrea Rocchetto, Simon C. Benjamin and Ying Li, Science Advances 2, e1601246 (2016).



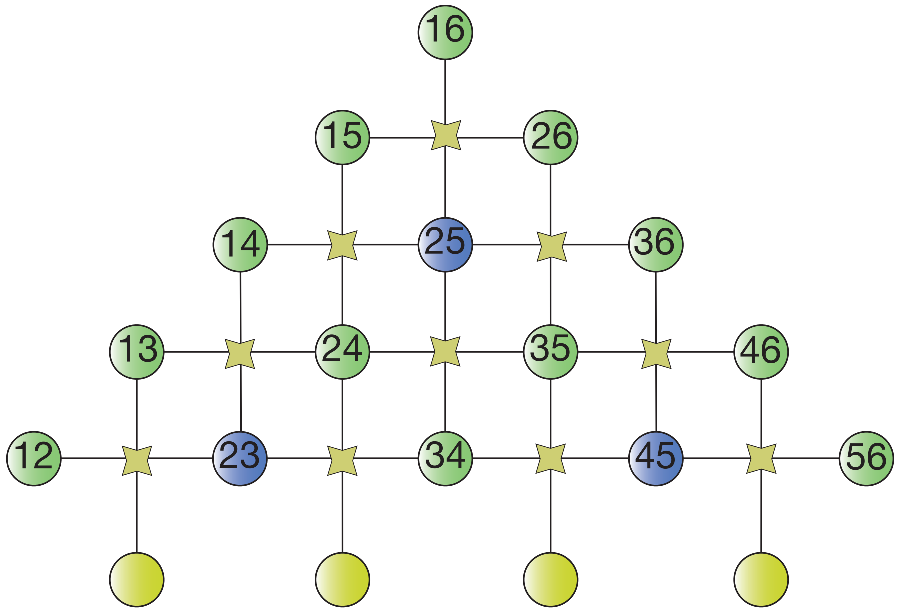
Odd parity representation

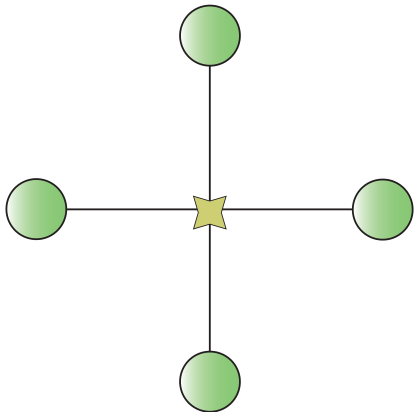
Andrea Rocchetto, Simon C. Benjamin and Ying Li, Science Advances 2, e1601246 (2016).



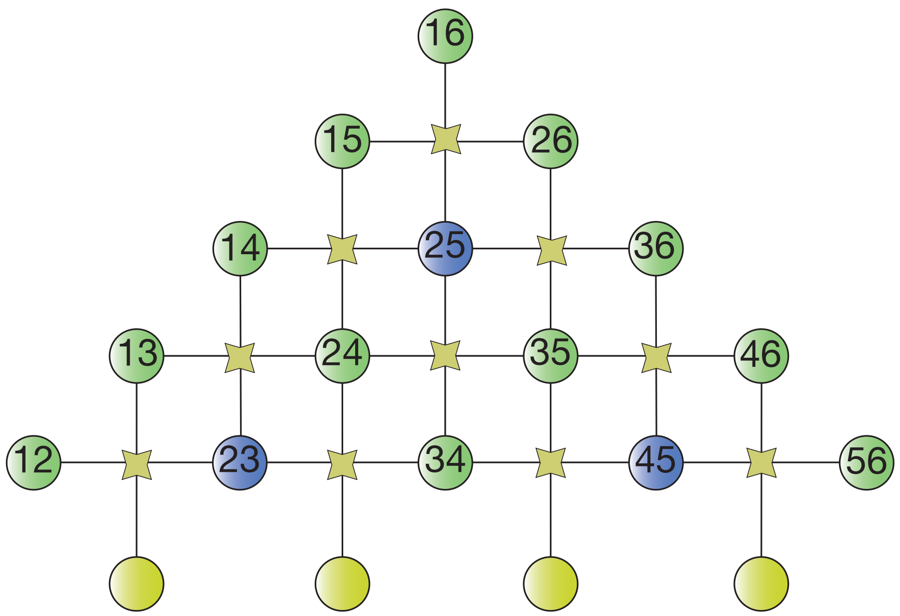
Odd parity representation

$$S = \left(2\sigma_z^{(a)} + \sum_{i}^{4} \sigma_z^{(i)} \right)^2$$



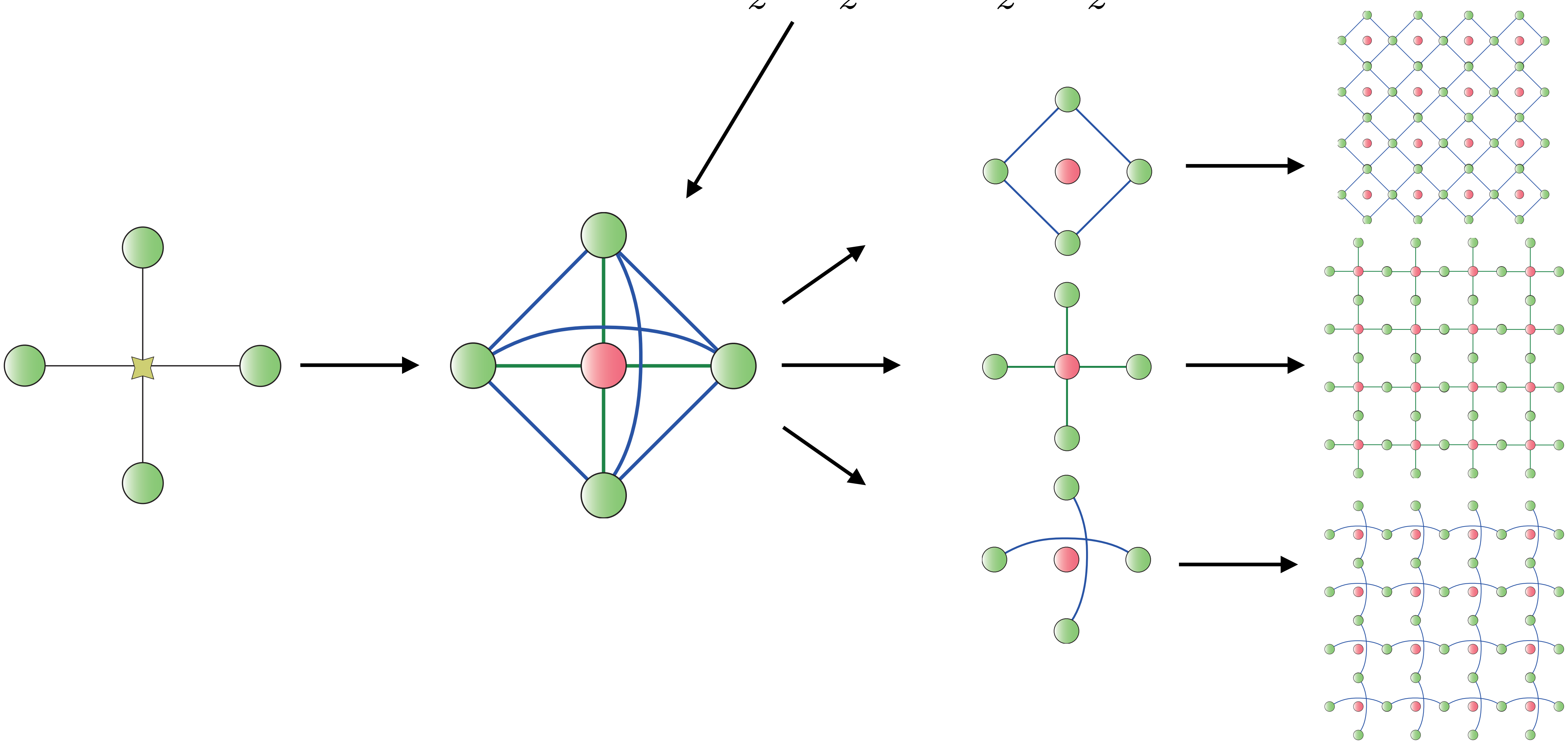


Odd parity representation

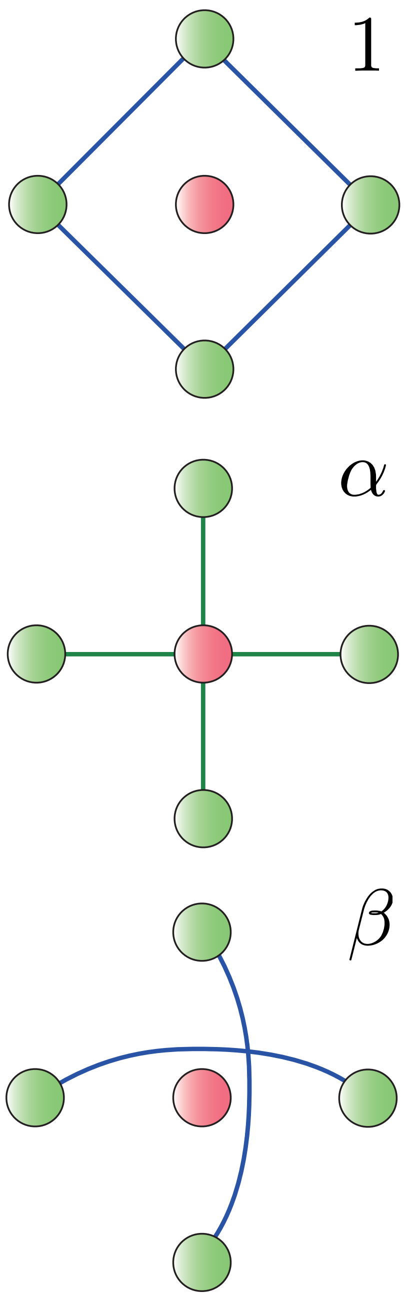


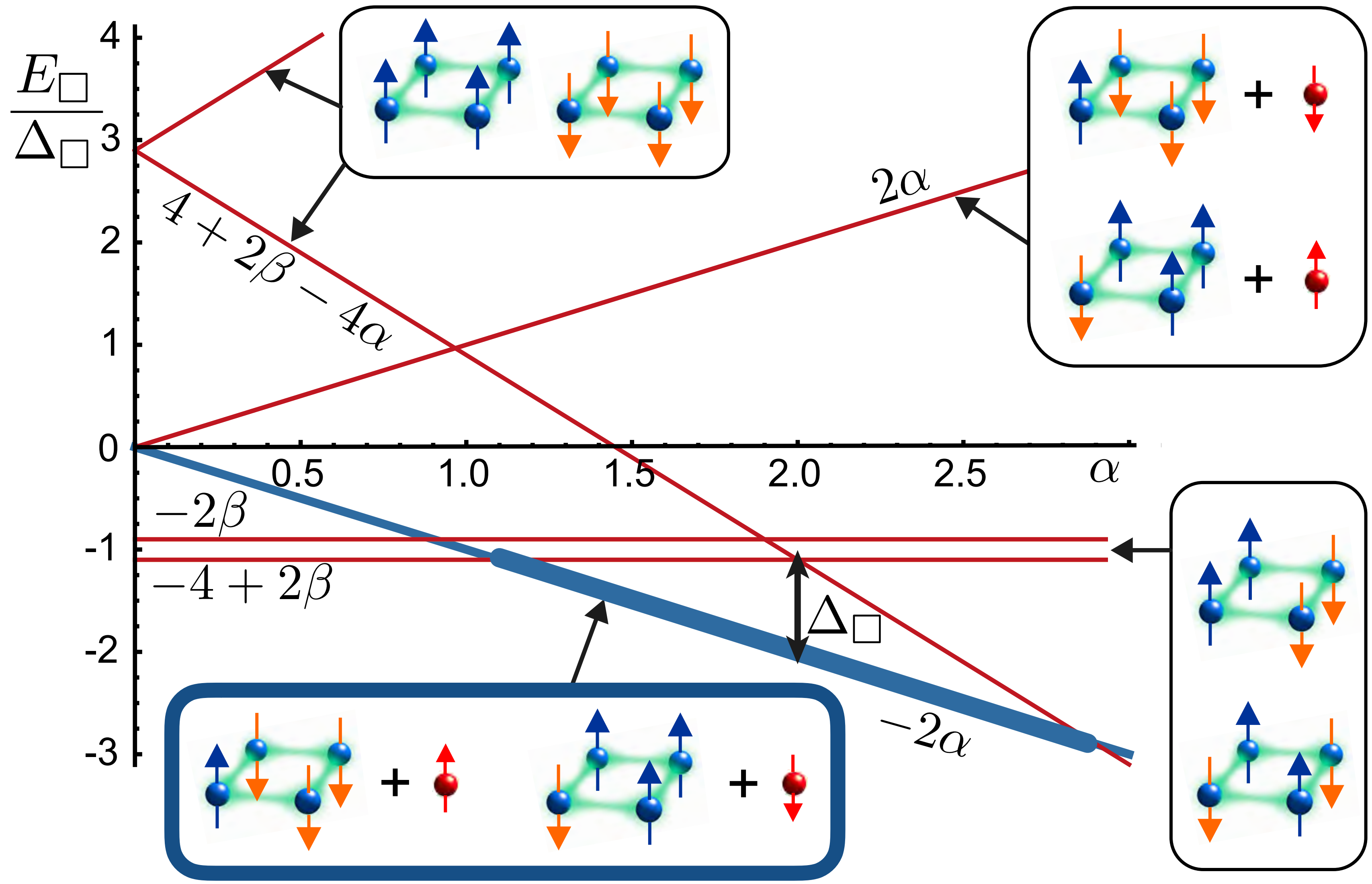
$$S = \left(2\sigma_z^{(a)} + \sum_i^4 \sigma_z^{(i)}\right)^2$$

$$2\sigma_z^{(a)}\sigma_z^{(1)} + 2\sigma_z^{(a)}\sigma_z^{(2)} + 2\sigma_z^{(a)}\sigma_z^{(3)} + 2\sigma_z^{(a)}\sigma_z^{(4)} +$$
$$\sigma_z^{(1)}\sigma_z^{(2)} + \sigma_z^{(1)}\sigma_z^{(3)} + \sigma_z^{(1)}\sigma_z^{(4)} + \sigma_z^{(2)}\sigma_z^{(3)} +$$
$$\sigma_z^{(2)}\sigma_z^{(4)} + \sigma_z^{(3)}\sigma_z^{(4)}$$



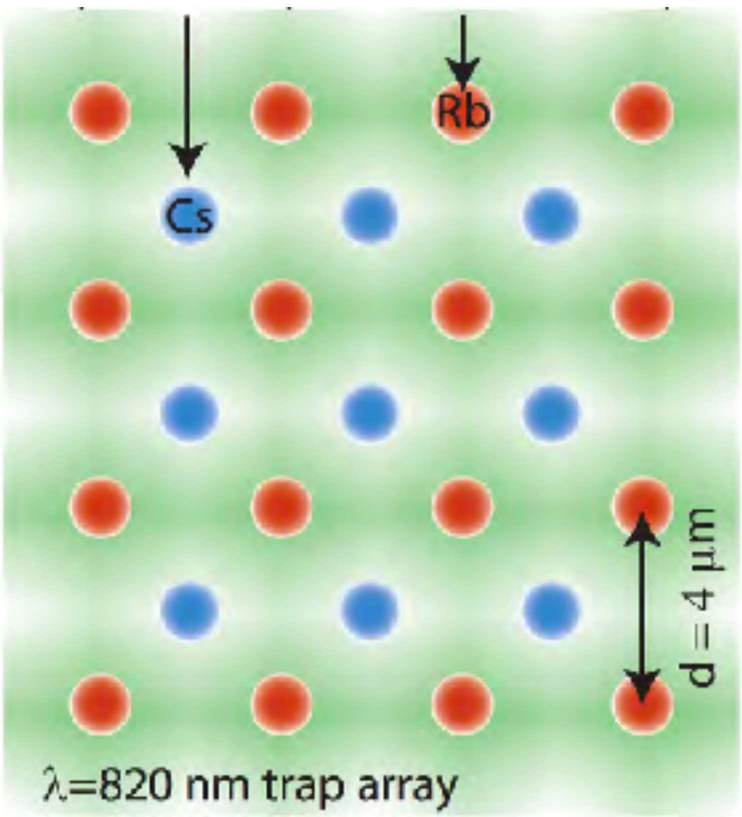
Error Robustness



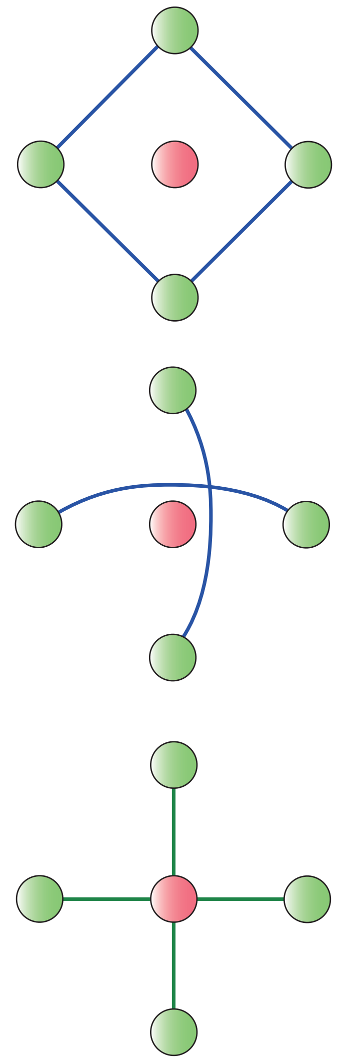


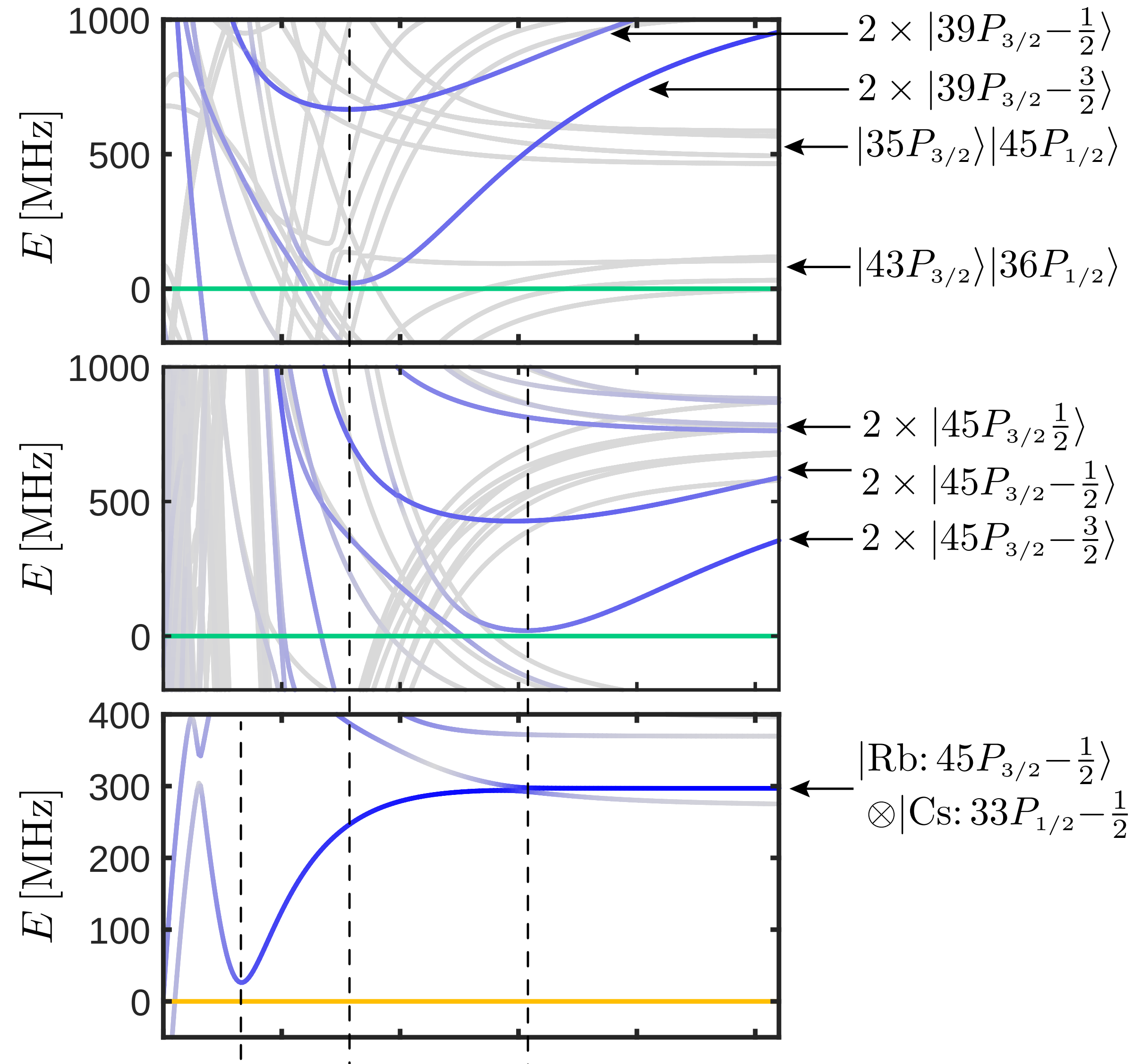
Neutral Atoms in Optical Lattices

A. Glätzle, R. van Bijnen, P. Zoller and WL, arXiv:1611.02594 (2016).



I. I. Beterov and M. Saffman,
Phys. Rev. A 92, 042710 (2015).



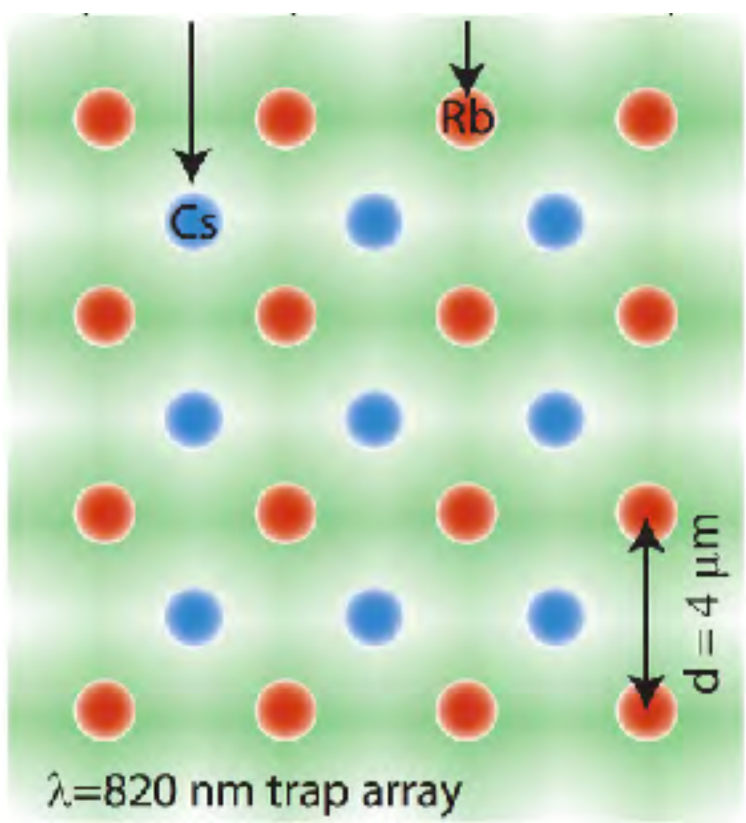


$$\hat{H} = \hat{H}_A^{(1)} \otimes \hat{I} + \hat{I} \otimes \hat{H}_A^{(2)} + \hat{V}_{dd}$$

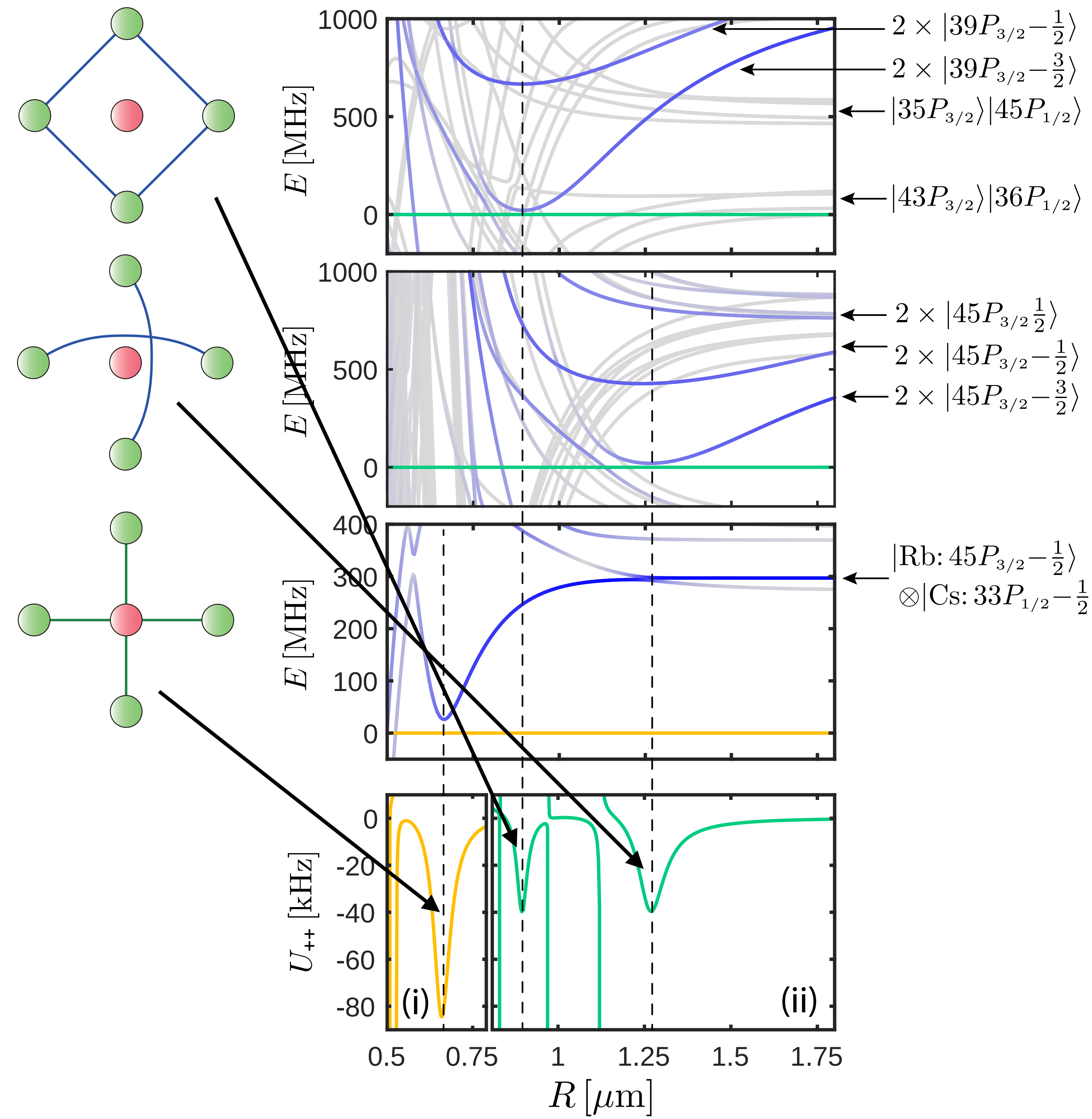
R. M. W. van Bijnen and T. Pohl
Phys. Rev. Lett. **114**, 243002 (2015)

Neutral Atoms in Optical Lattices

A. Glätzle, R. van Bijnen, P. Zoller and WL, arXiv:1611.02594 (2016).

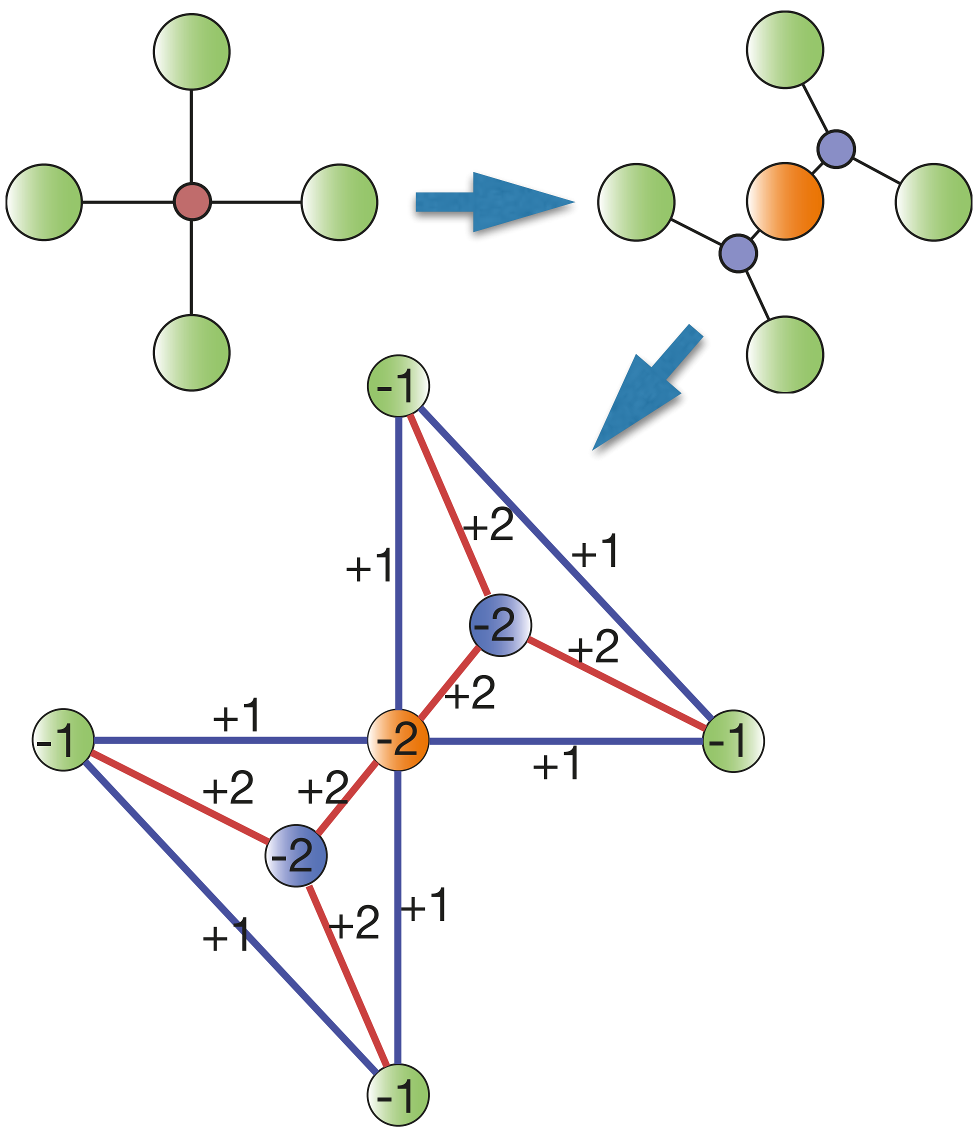


I. I. Beterov and M. Saffman, Phys. Rev. A 92, 042710 (2015).

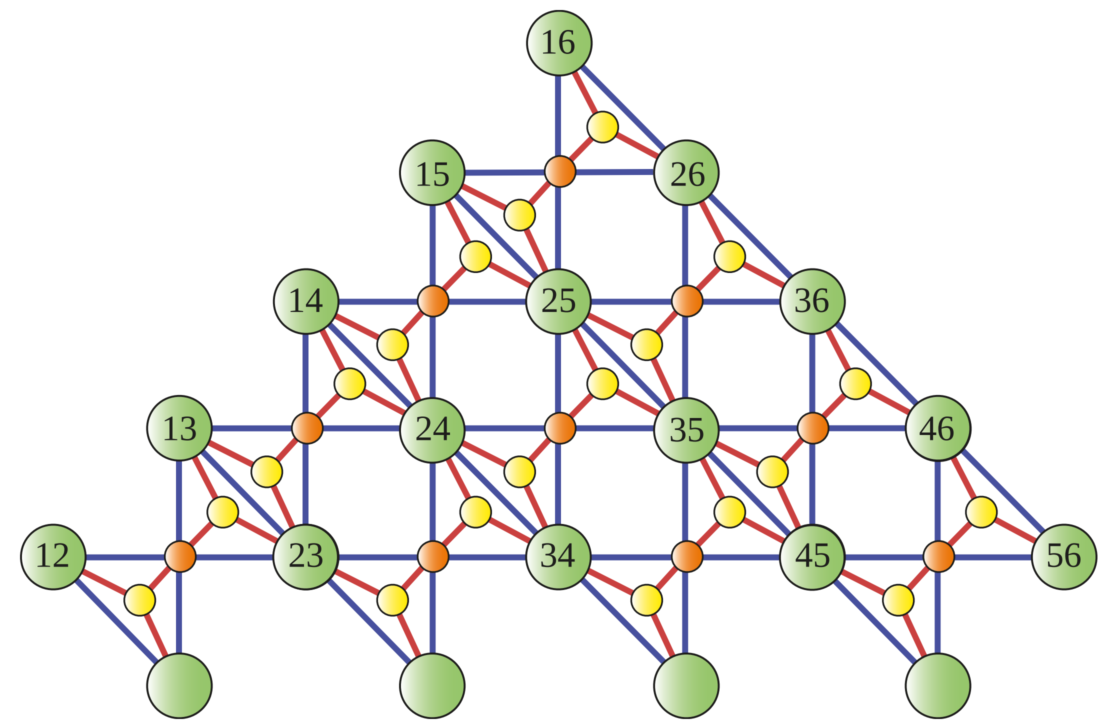


Transmon Implementations

Martin Leib, P. Zoller, WL, arXiv:1604.02359(2016).



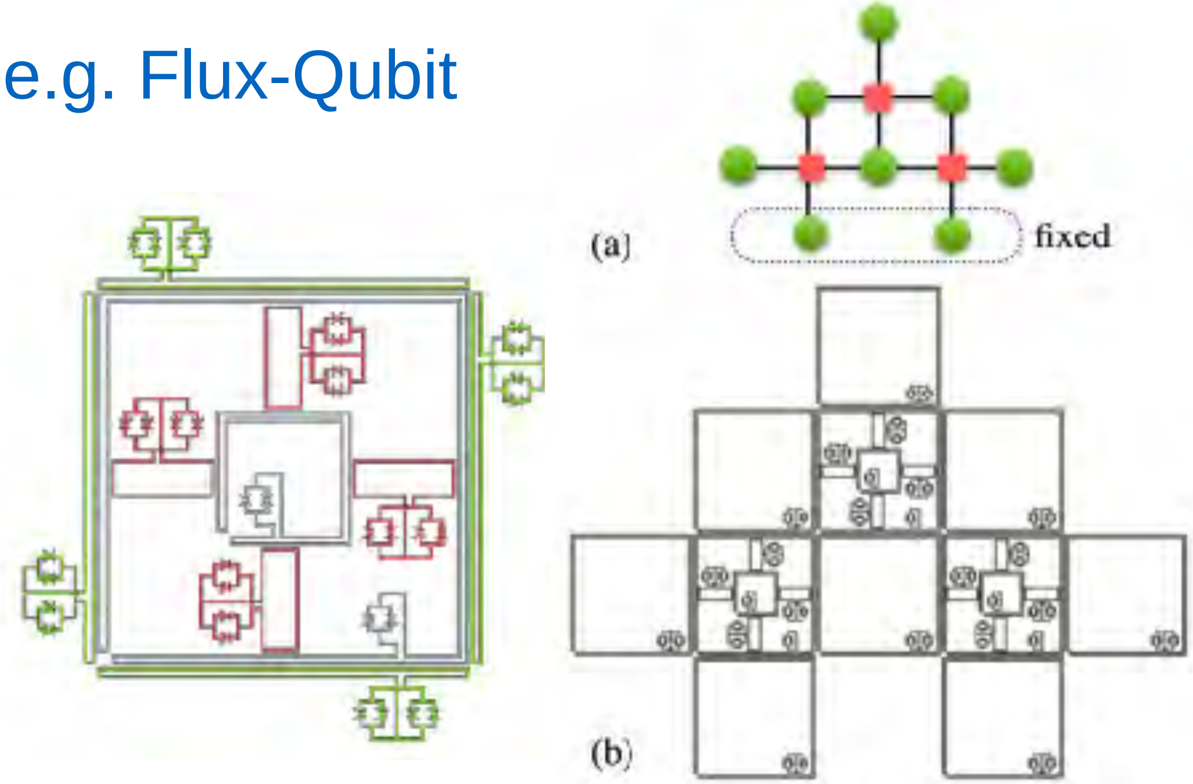
$$\sigma_z^{(1)}\sigma_z^{(2)}\sigma_z^{(3)} \to \sigma_z^{(1)}\sigma_z^{(2)} + \sigma_z^{(2)}\sigma_z^{(3)} + \sigma_z^{(3)}\sigma_z^{(1)} + \sum_{i=1}^{3}[2\sigma_z^{(i)}\sigma_z^{(a)} - \sigma_z^{(i)}] - 2\sigma_z^{(a)}$$



- rotating frame
- only **pair interactions**
- **no crossings**
- all **local fields** have the **same sign**
- all **interactions** have the **same sign**
- programming [0,2]

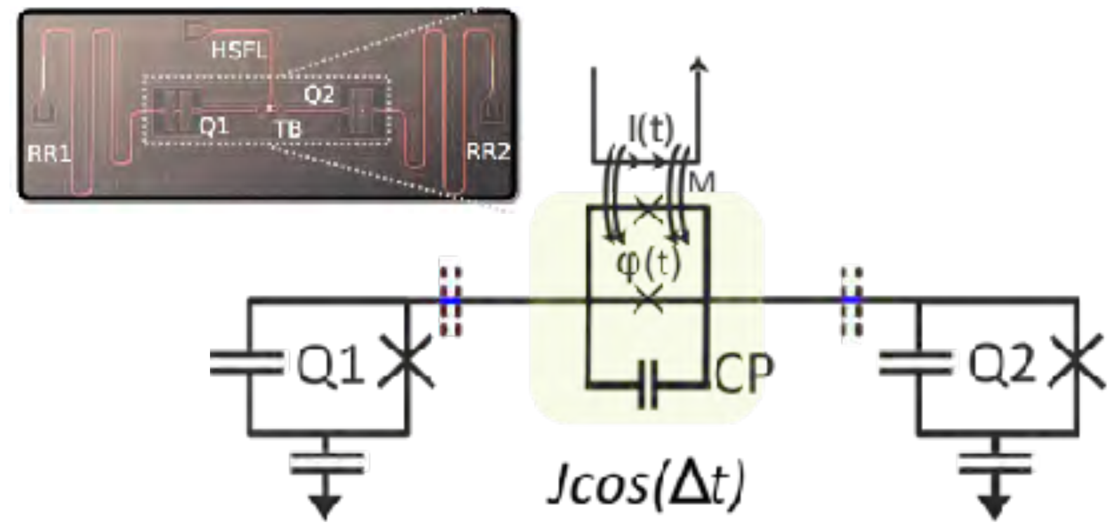
Quantum Circuit for LHZ

e.g. Flux-Qubit



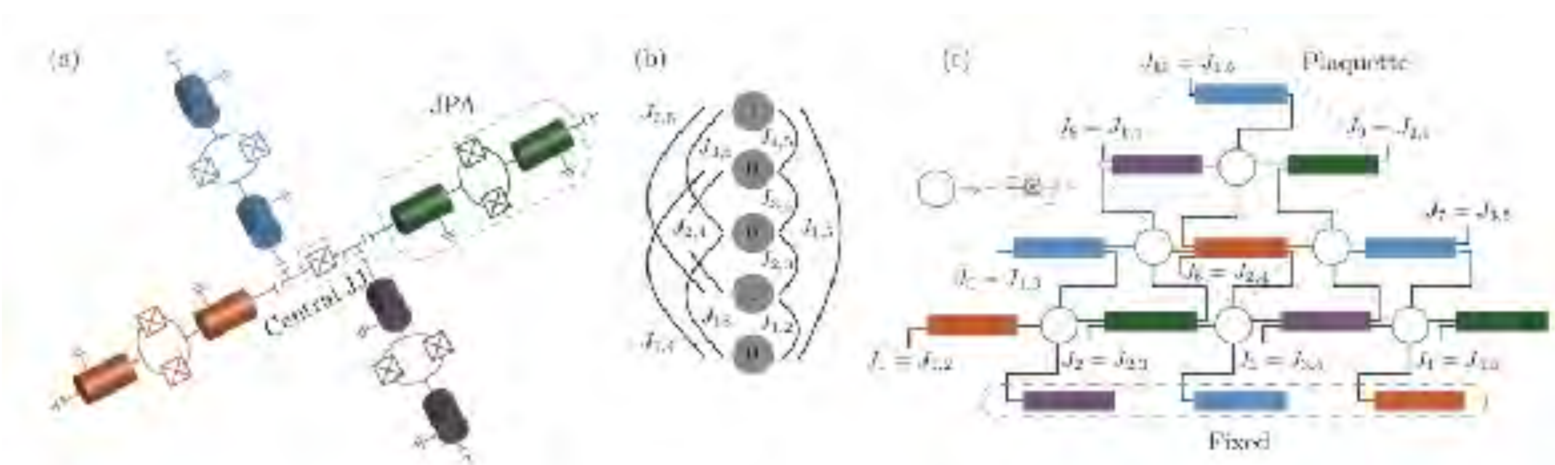
Nicholas Chancellor, Stefan Zohren, Paul A. Warburton arXiv:1603:09521 (2016).

e.g. Driven fixed frequency Transmons



D. C. McKay, S. Filipp, A. Mezzacapo, F. Solgun, J. Chow, and J. M. Gambetta. arXiv:1604.03076 (2016).

e.g. Driven Kerr-nonlinearities



Shruti Puri, Christian Kraglund Andersen, Arne L. Grimsmo, Alexandre Blais, arXiv:1609.07117 (2016).