

*arXiv:1505.07432*

# Classical Verification of Quantum Proofs



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IQC, UWaterloo

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QCIS, UTS

# Classical and quantum proof verification

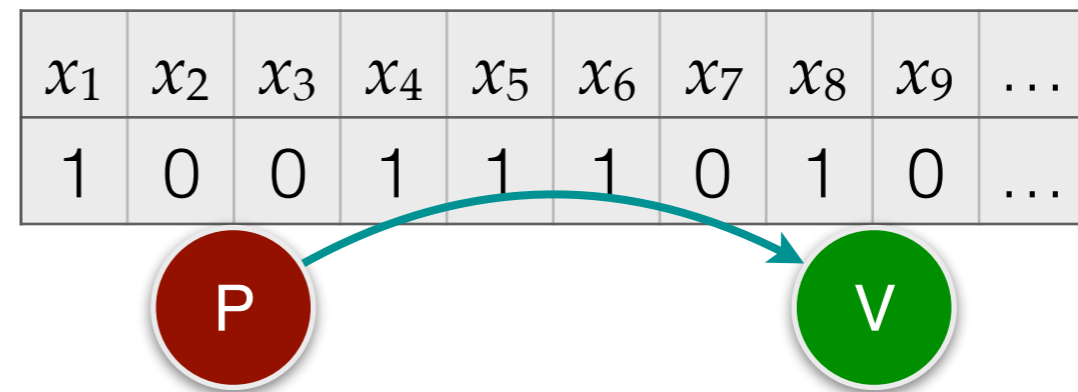
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- Proof verification is a central concept in computer science
  - **NP, IP, MIP, PCP, ...**

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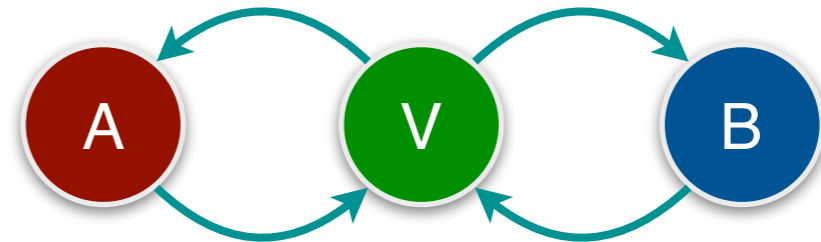
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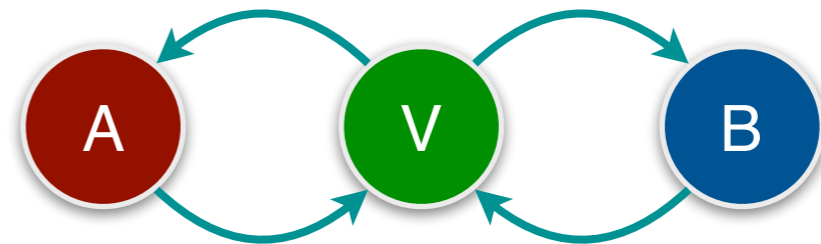
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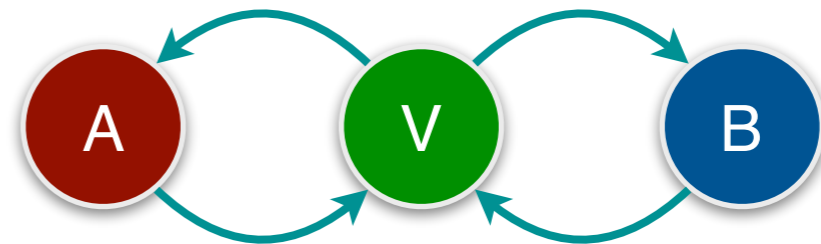


- Cook-Levin theorem: 3-SAT is **NP**-complete
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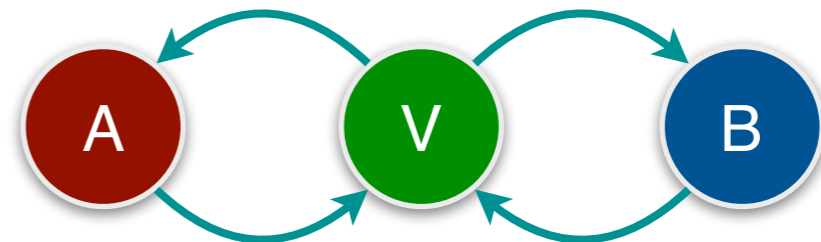
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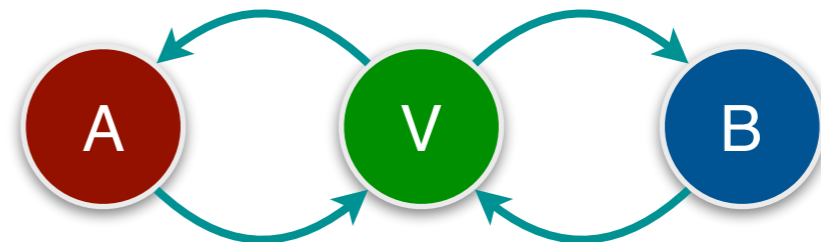


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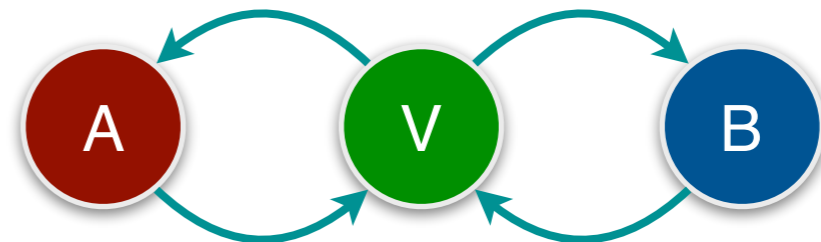


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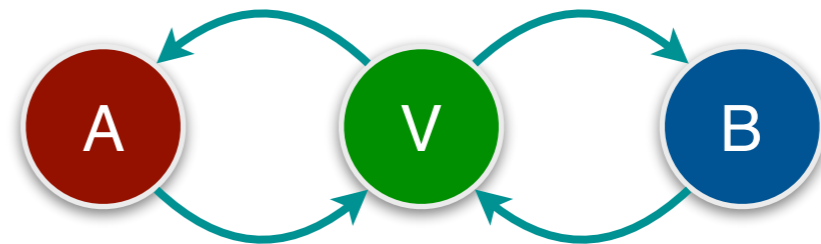


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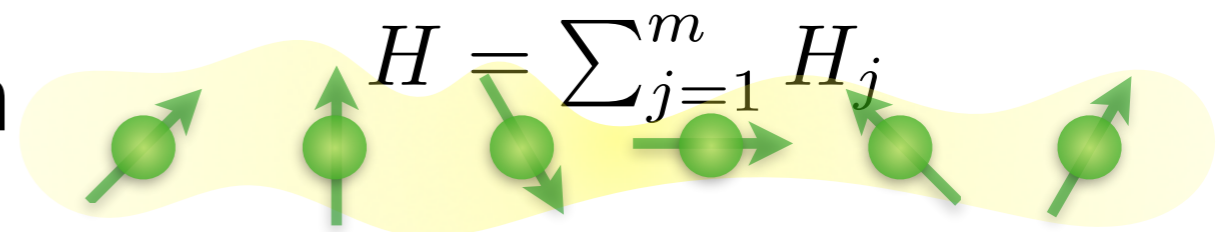
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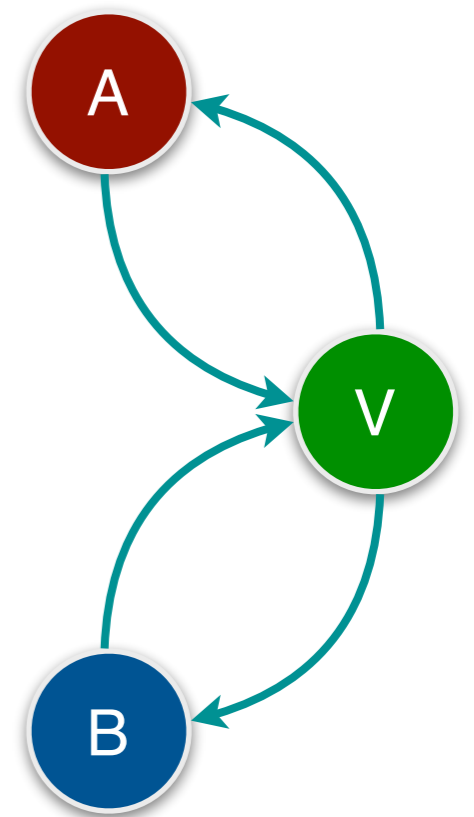
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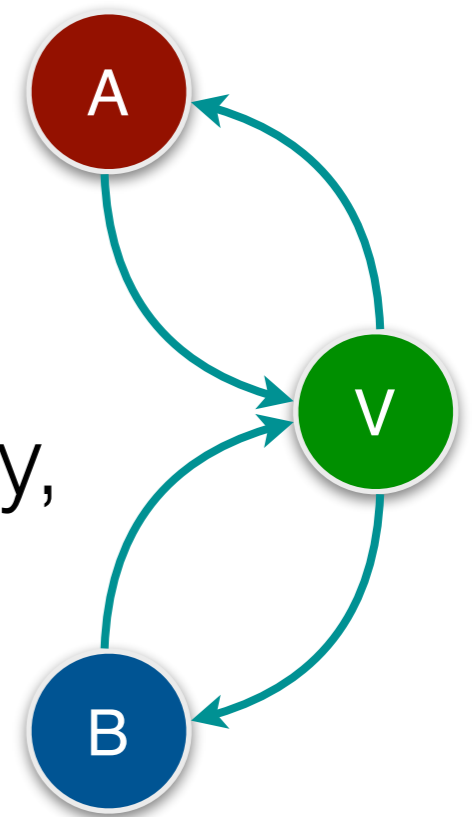
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- Query a variable in the clause randomly, check consistency (**oracularization**)



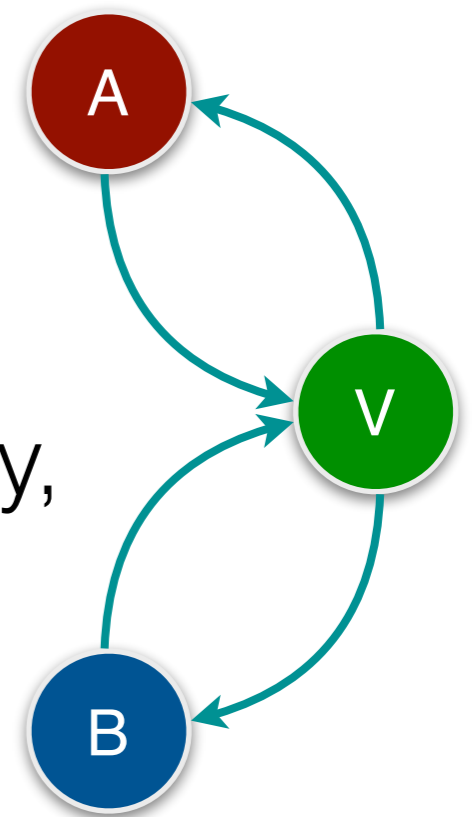
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- **NP**-hardness of multi-player games

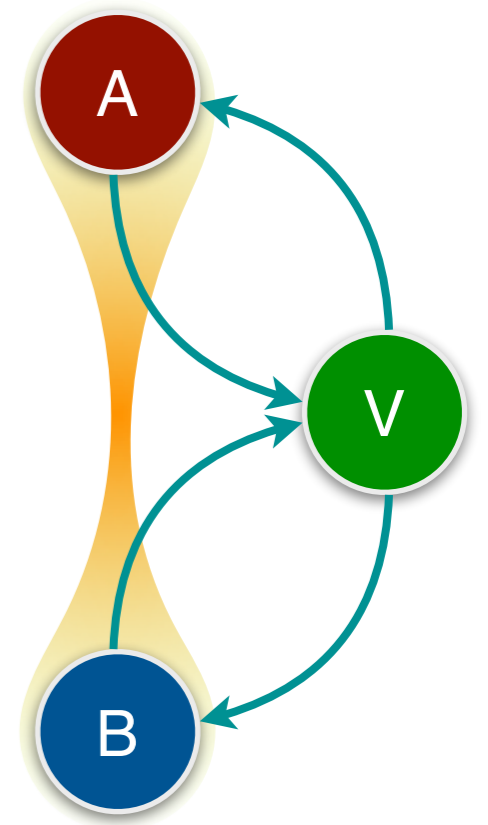


The power of multiple  
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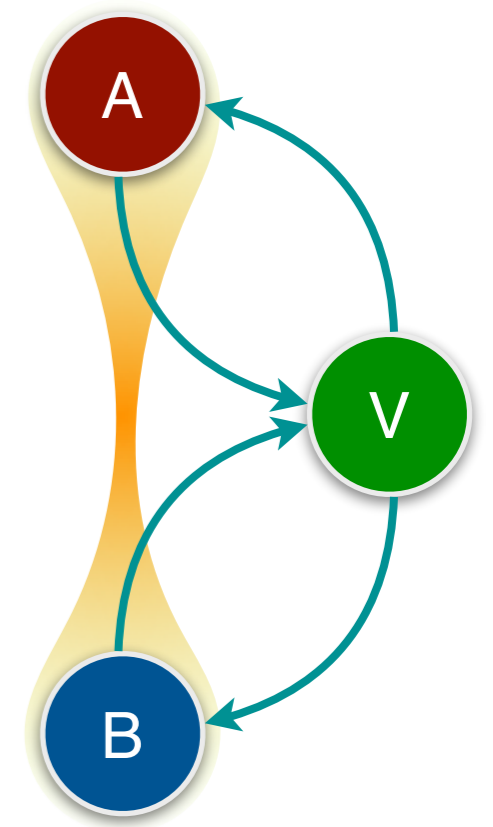
- Bell inequalities
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[Cleve et al. 04]



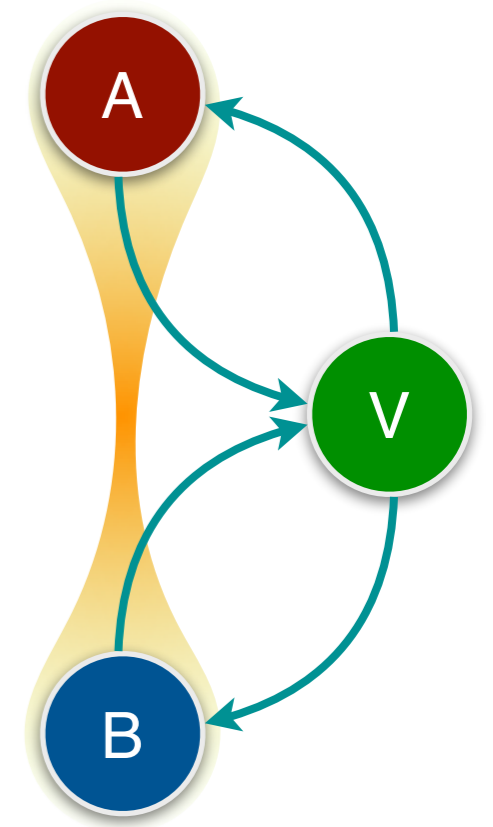
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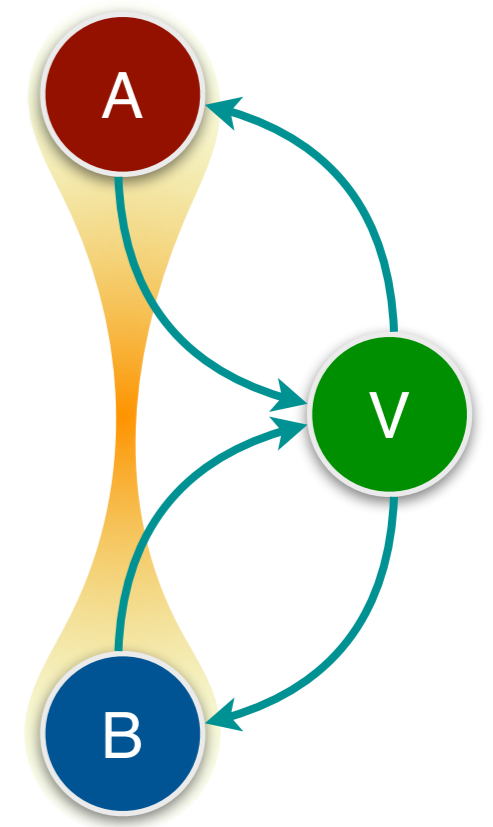
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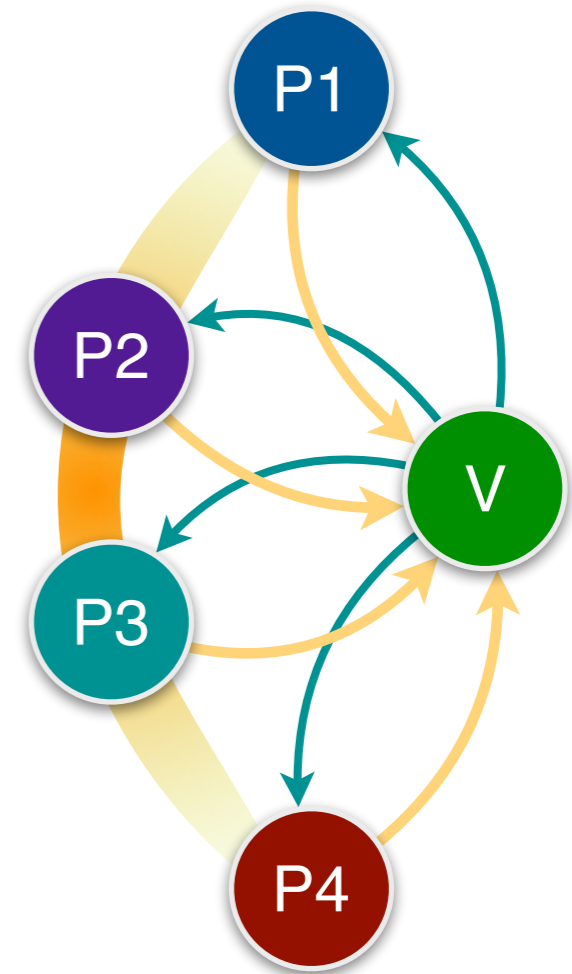


# Multi-player games for QMA



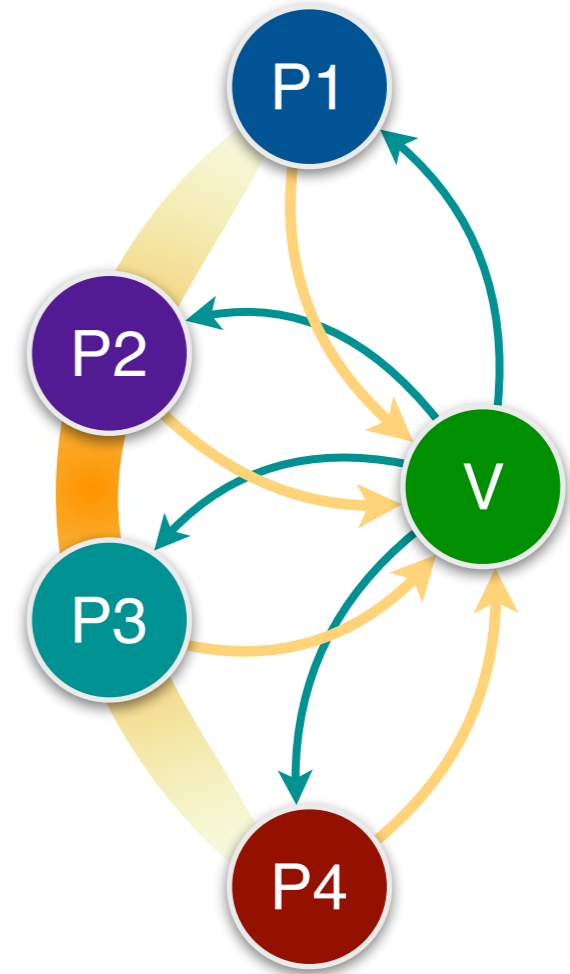
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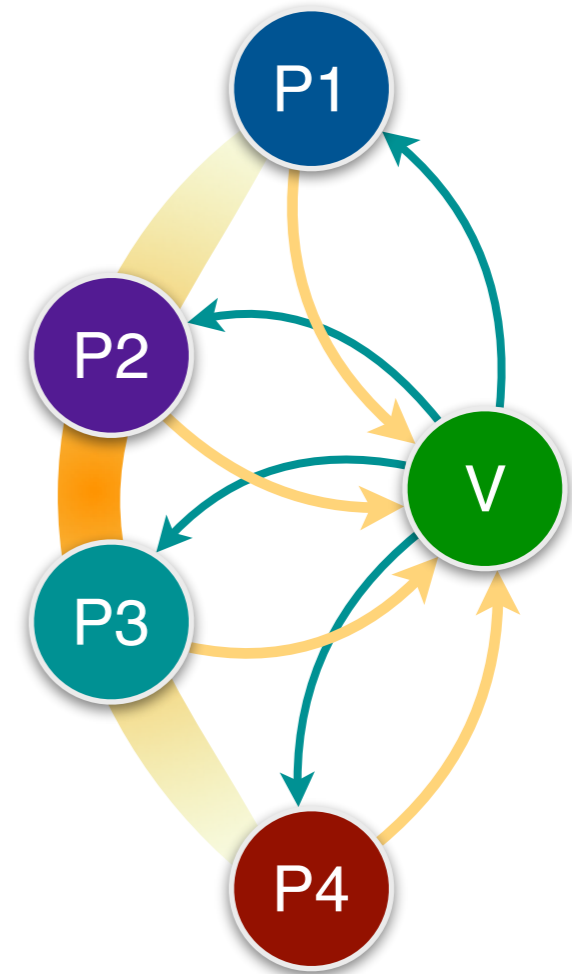
- Fitzsimons-Vidick protocol
- Encode the proof using the 4-qubit **quantum error detecting code** and do the following with equal probability:
  - Perform the **encoding** check
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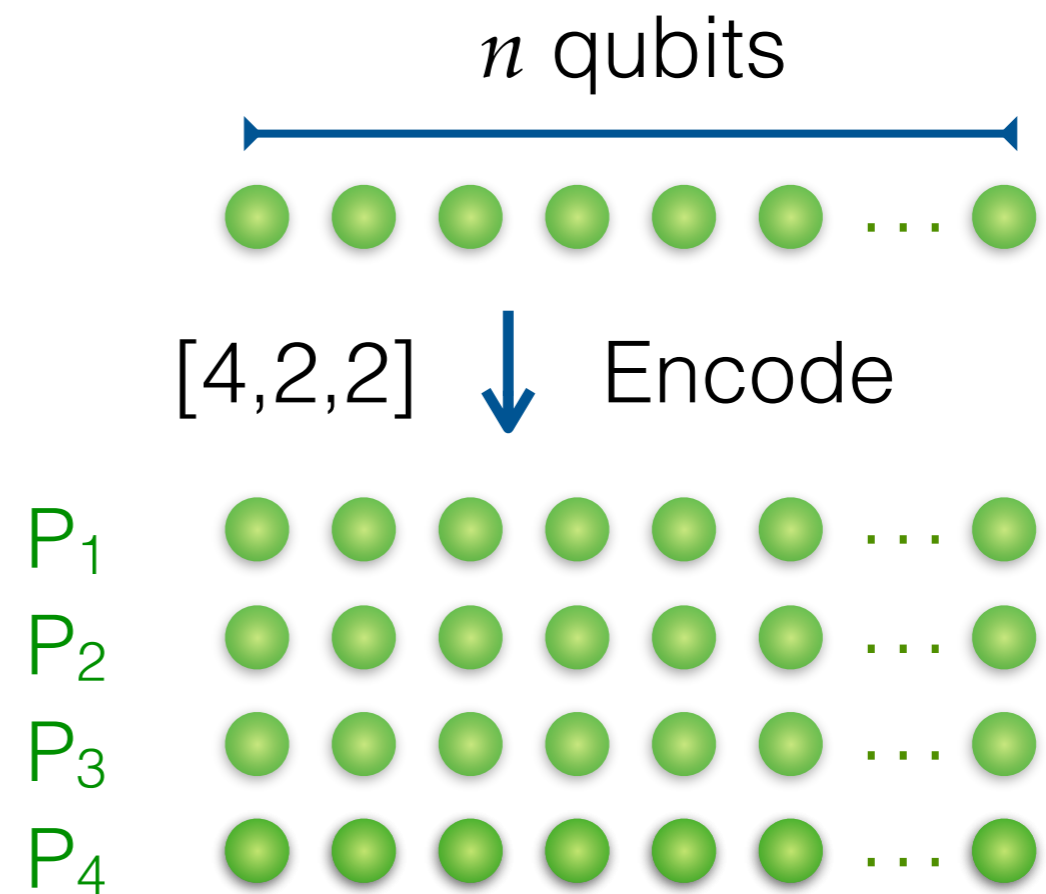
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- Quantum **oracularization**
  - Classical oracularization as an error detecting code

$0 \mapsto 00, 1 \mapsto 11$

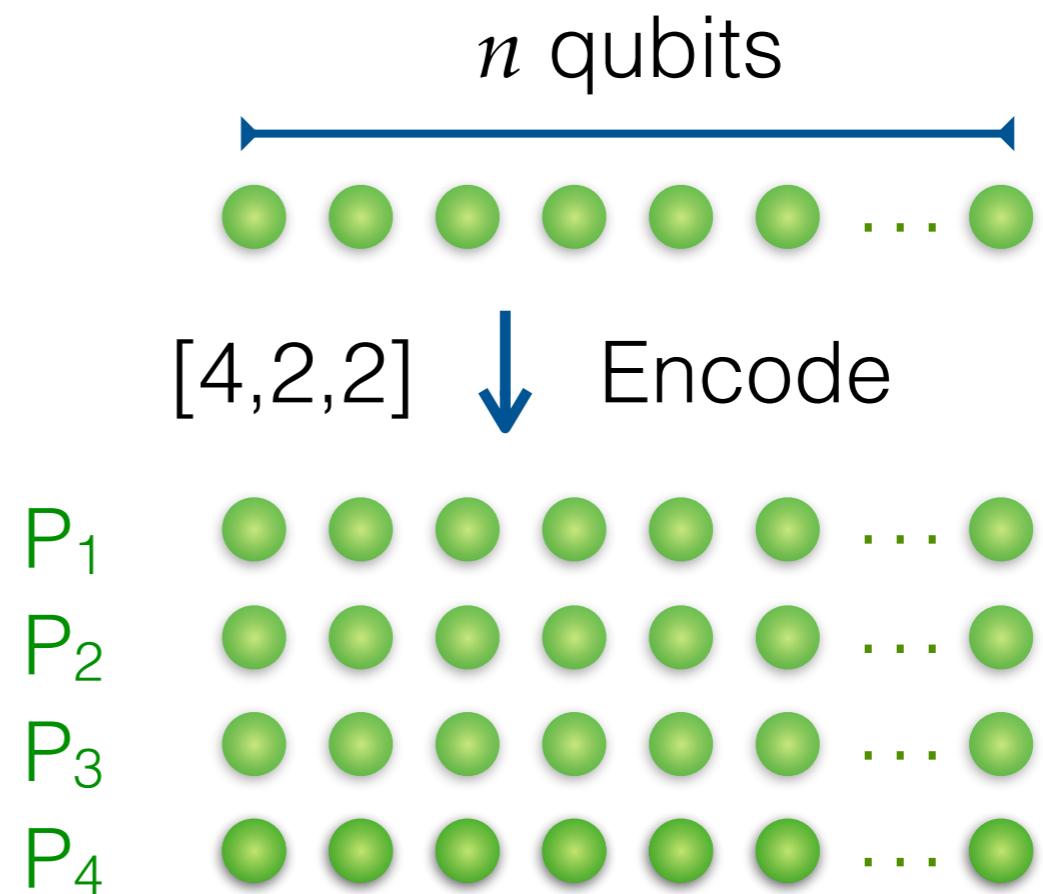


# Fitzsimons-Vidick protocol



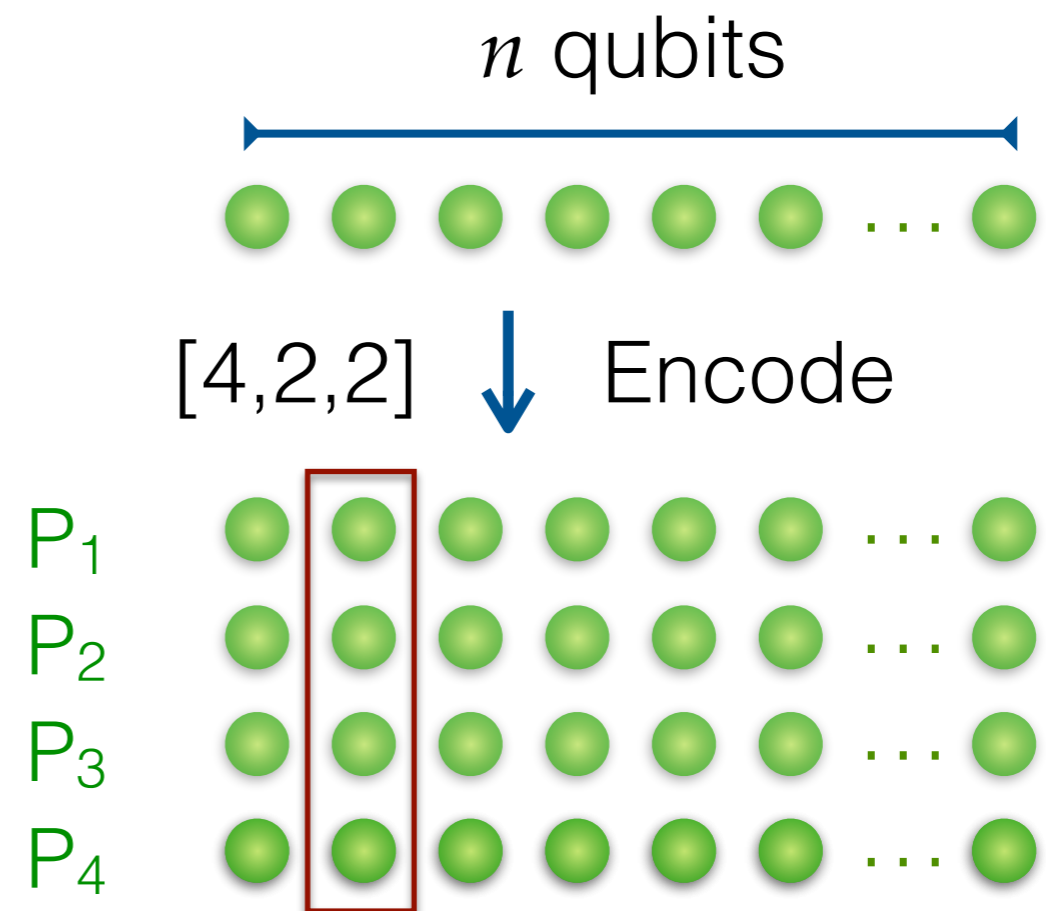
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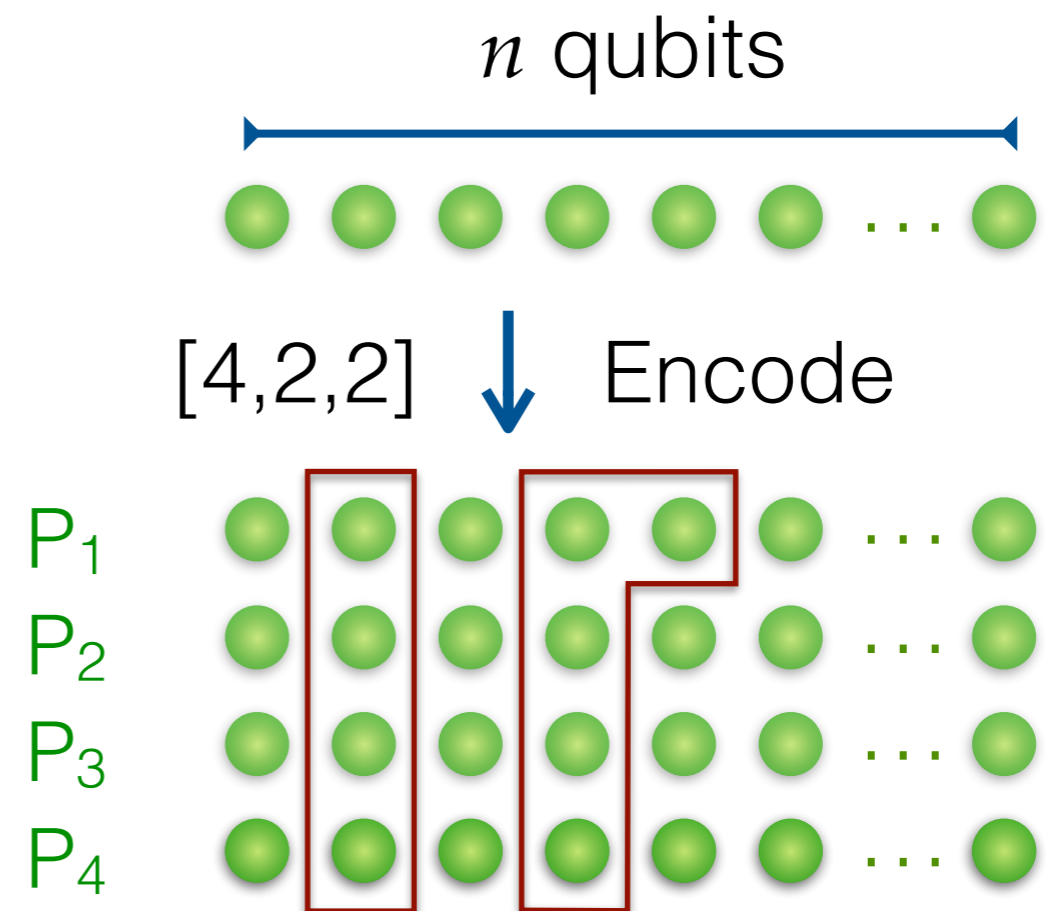
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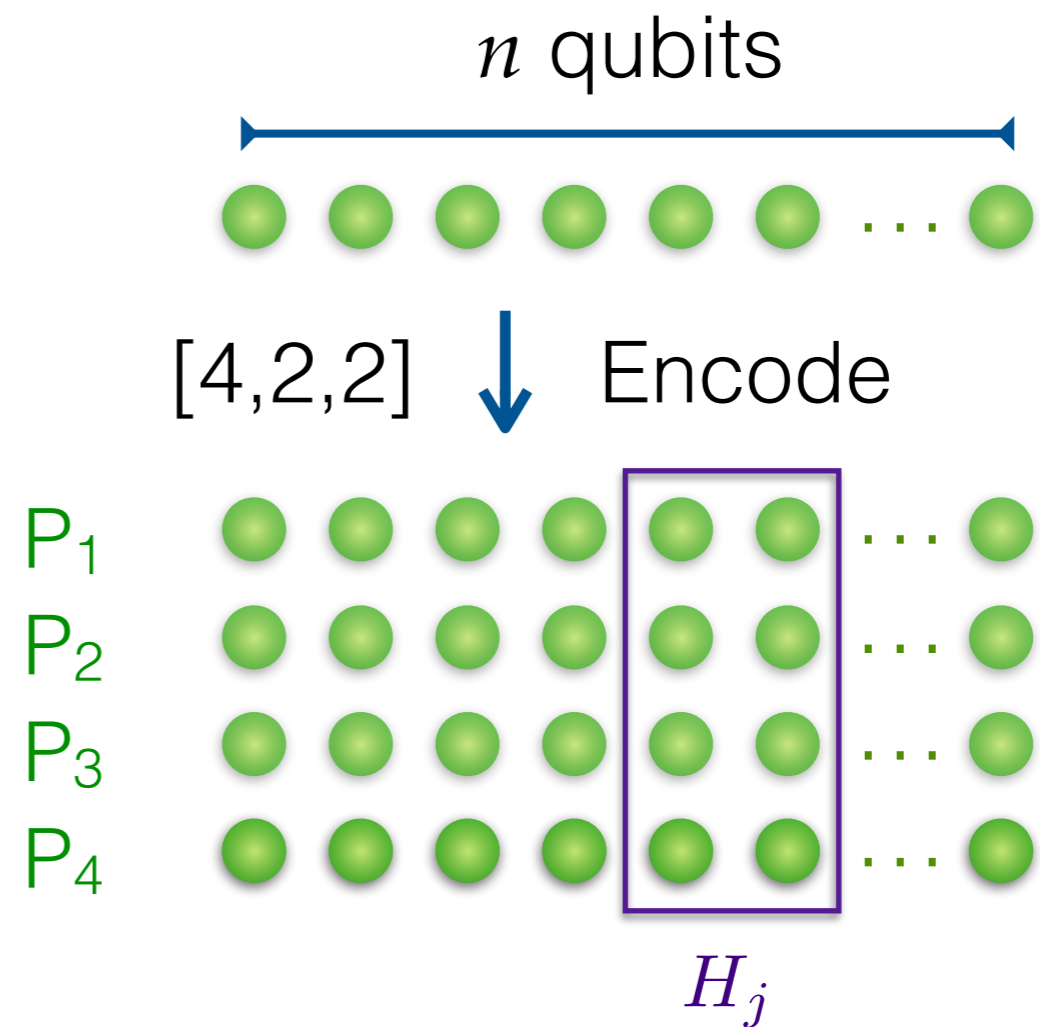
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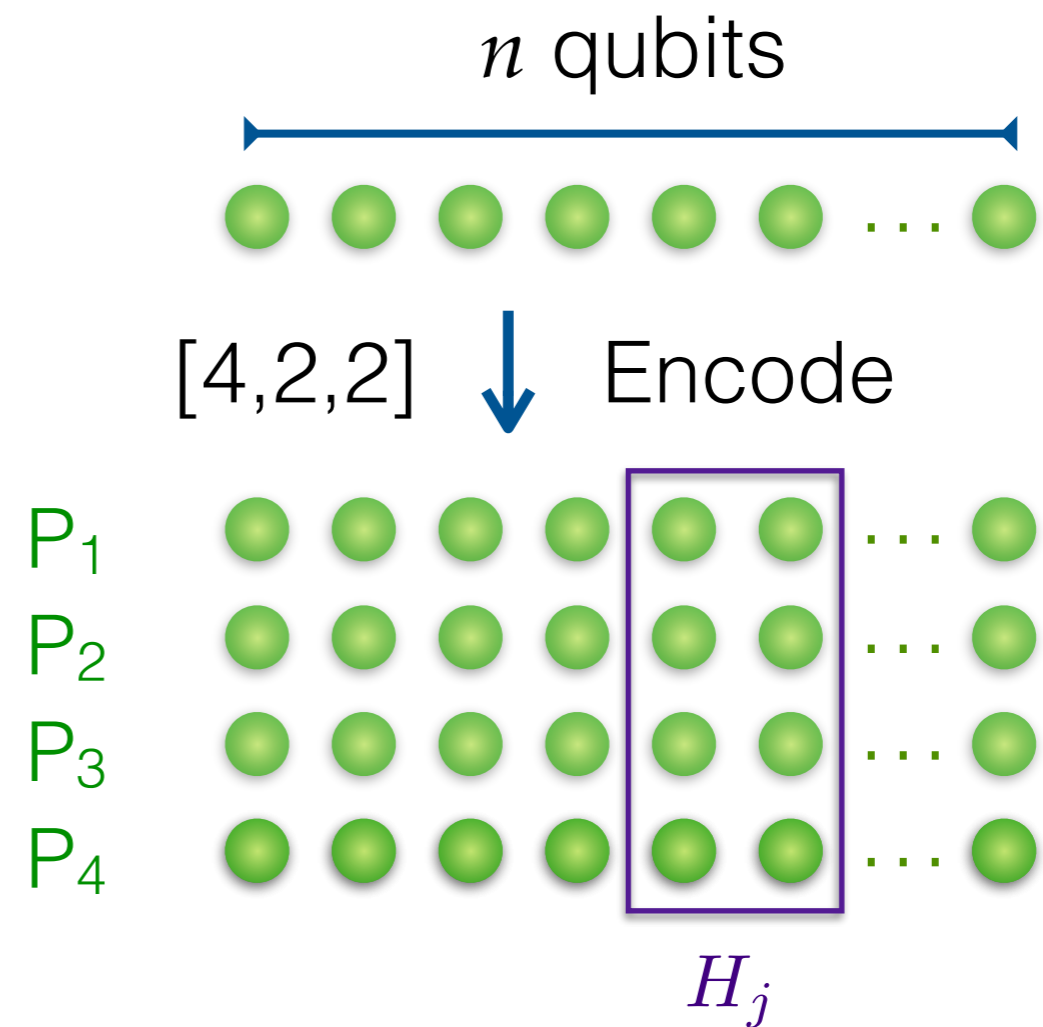
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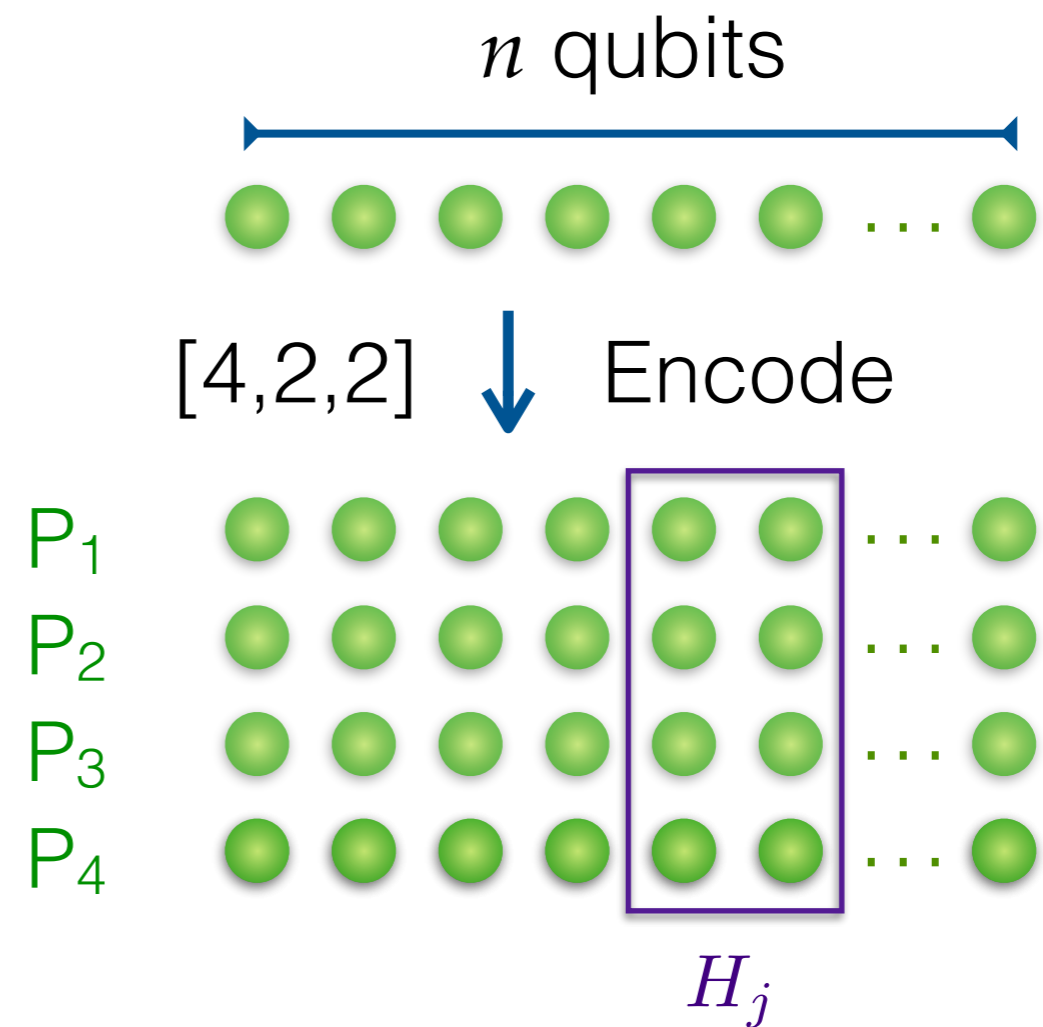
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- Questions:  $O(\log n)$  bits
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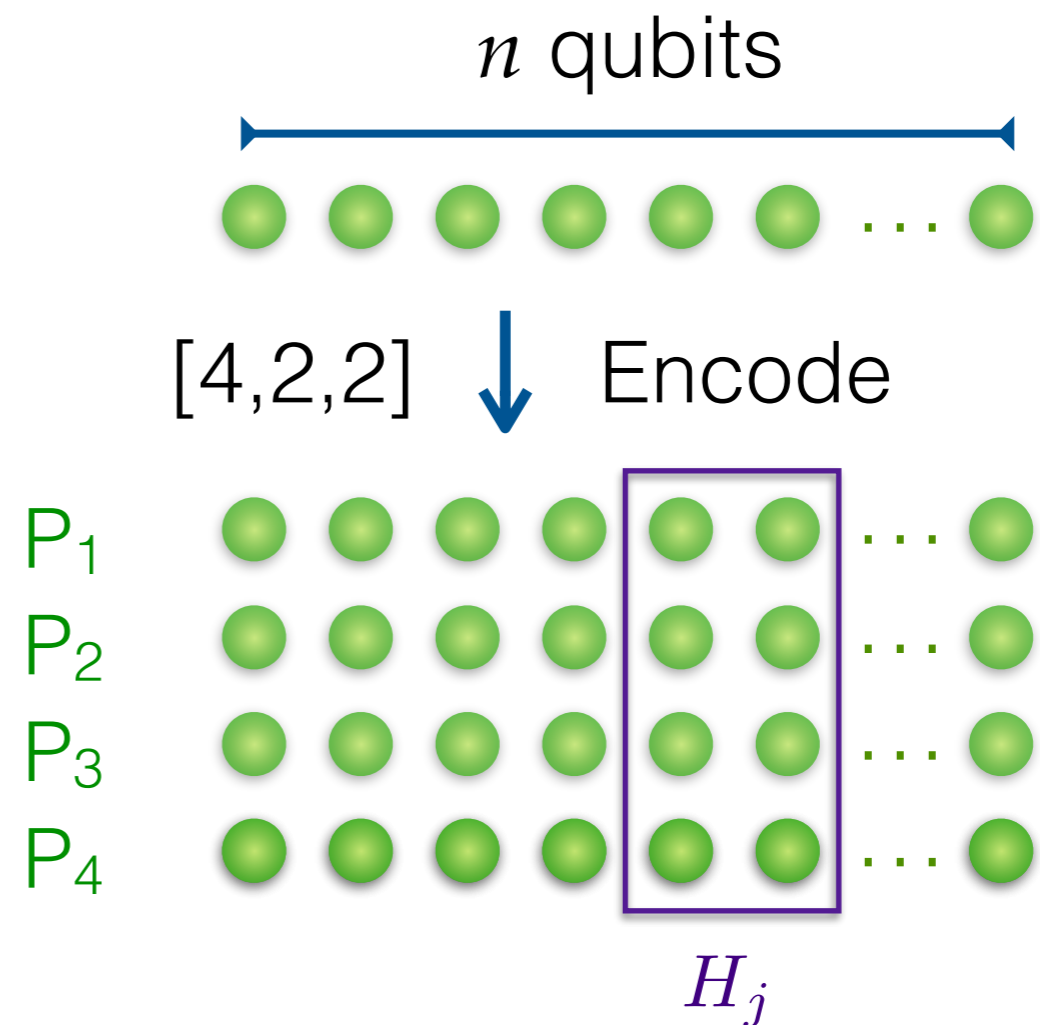
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- De-quantization of both the answer messages and verifier

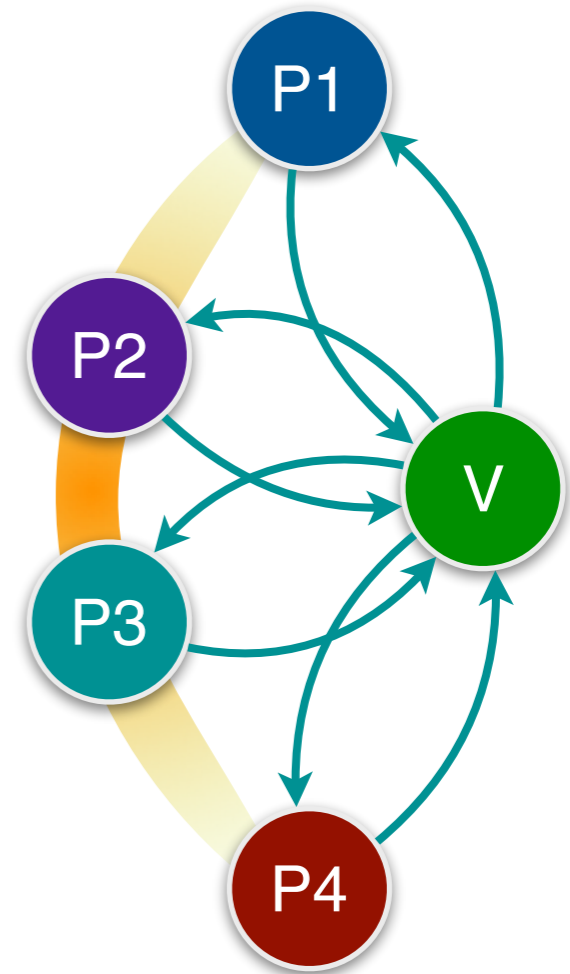


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- A 4-player protocol for the local Hamiltonian problem

Questions: **logarithmic** number of bits,  
Answers: **constant** number of bits

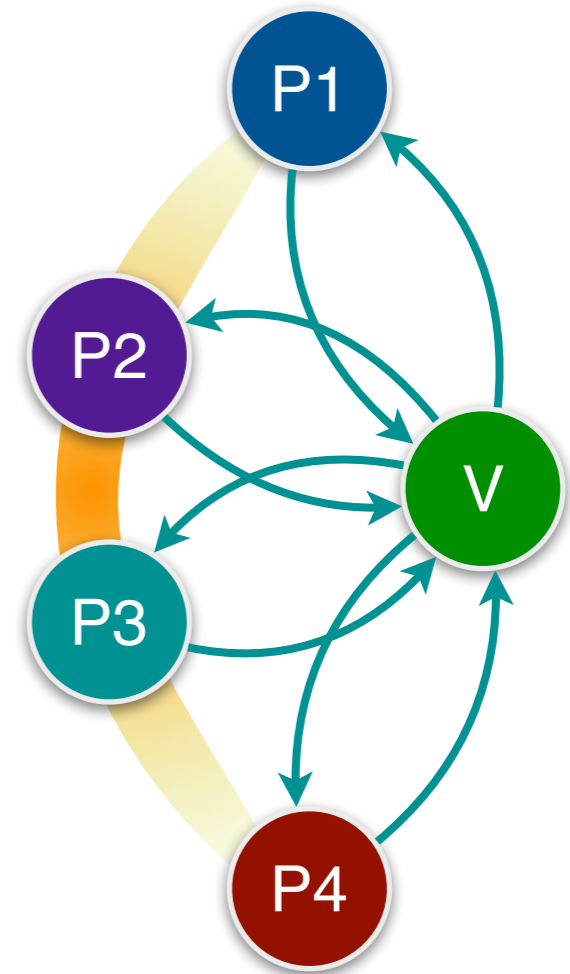


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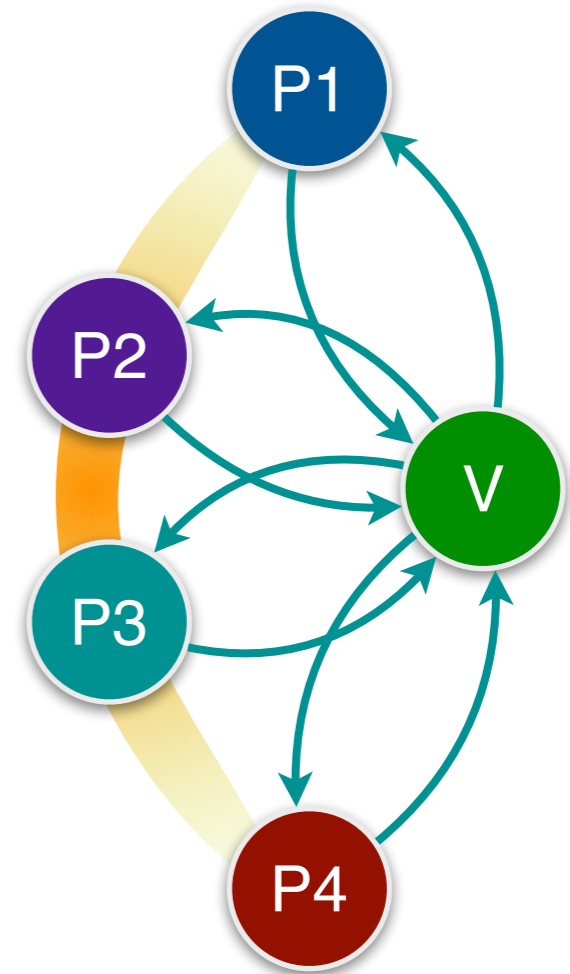


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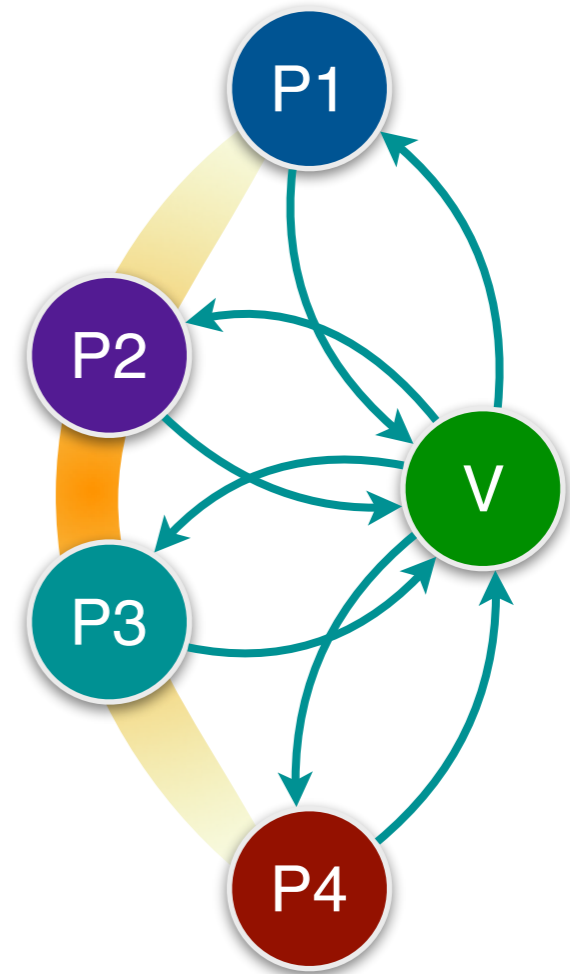


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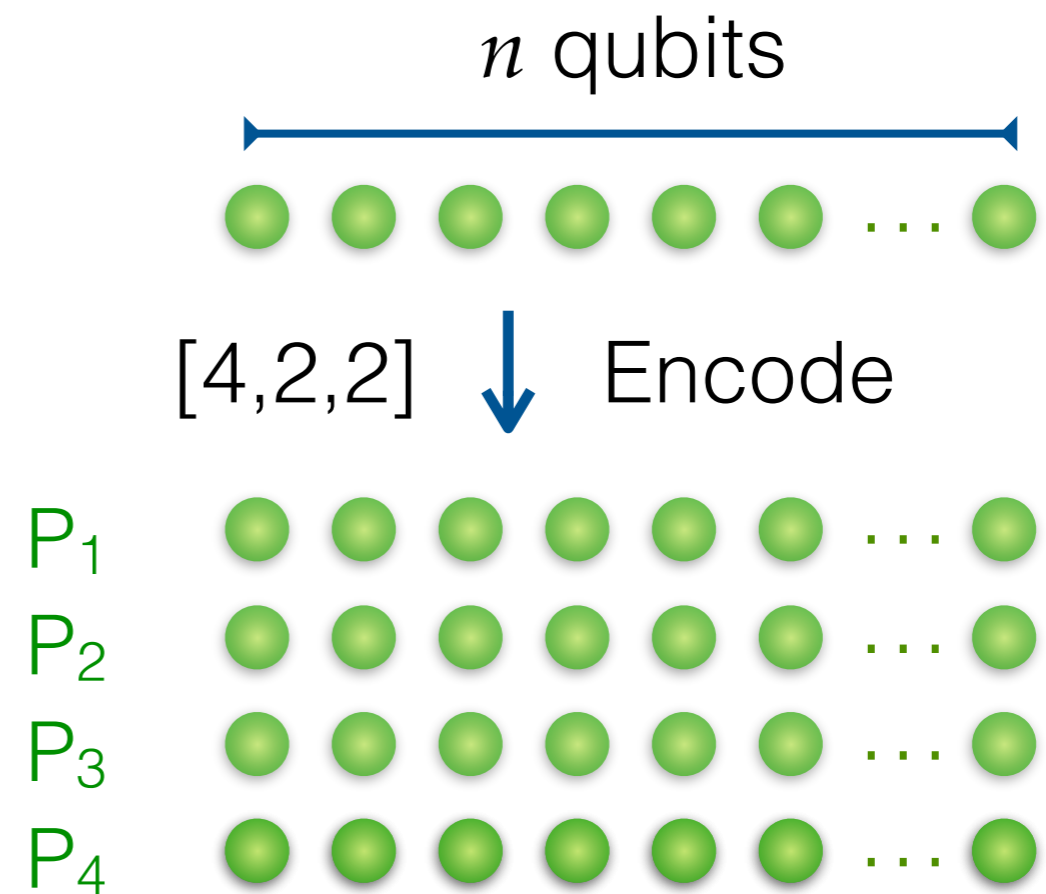
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- For **exponentially** small gapped  $c,s$ ,  $\mathbf{MIP} \subsetneq \mathbf{MIP}^*(4, 1, c, s)$  under assumptions



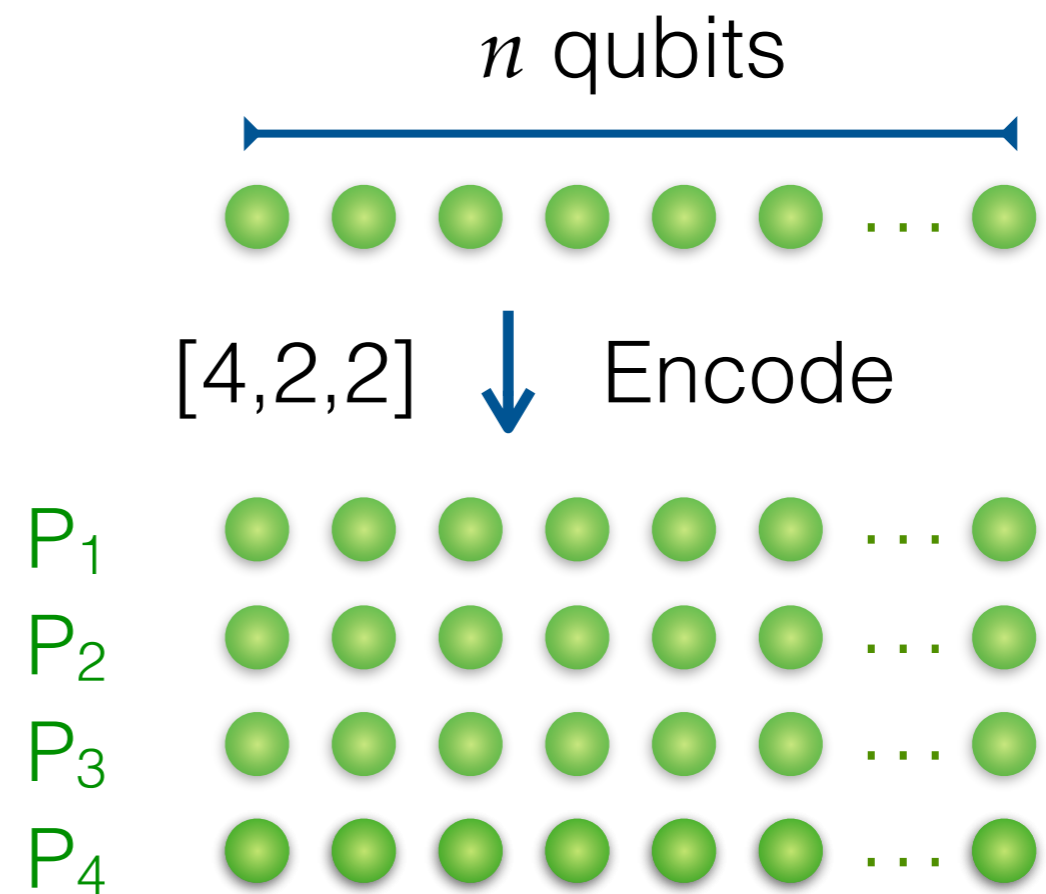


# Overview of the protocol



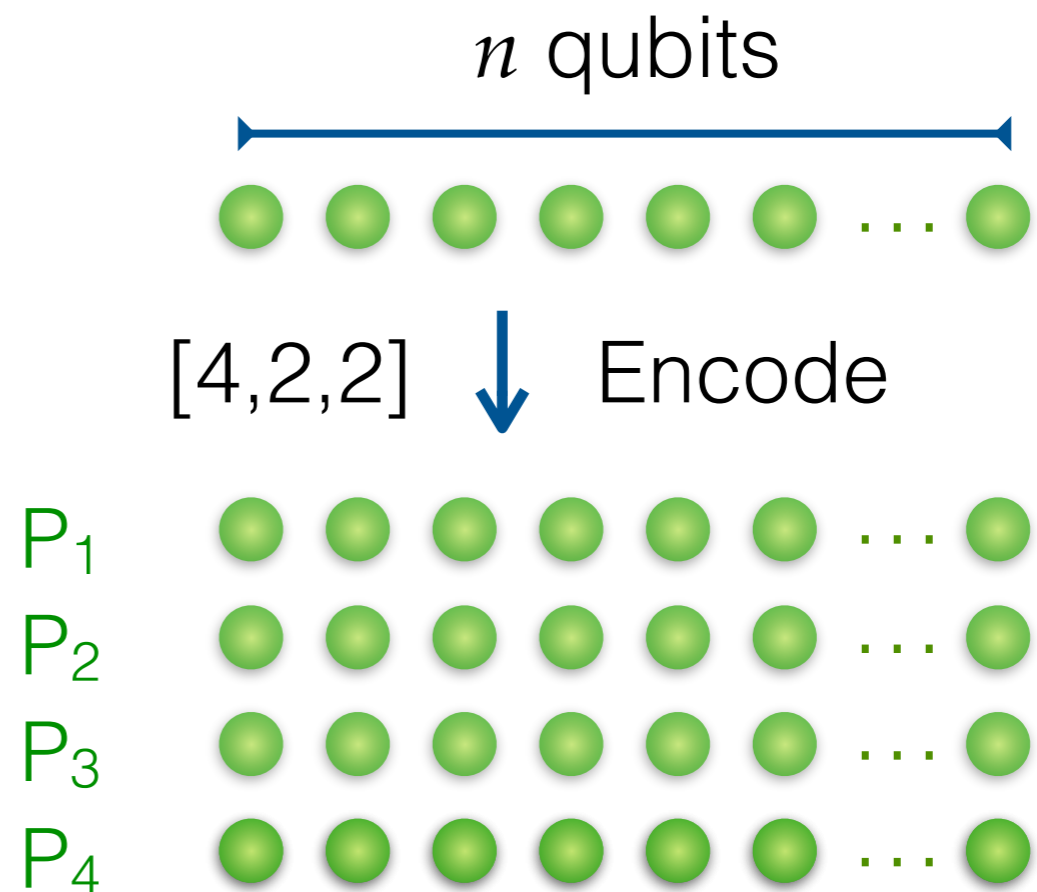
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- Follows Fitzsimons-Vidick protocol very closely



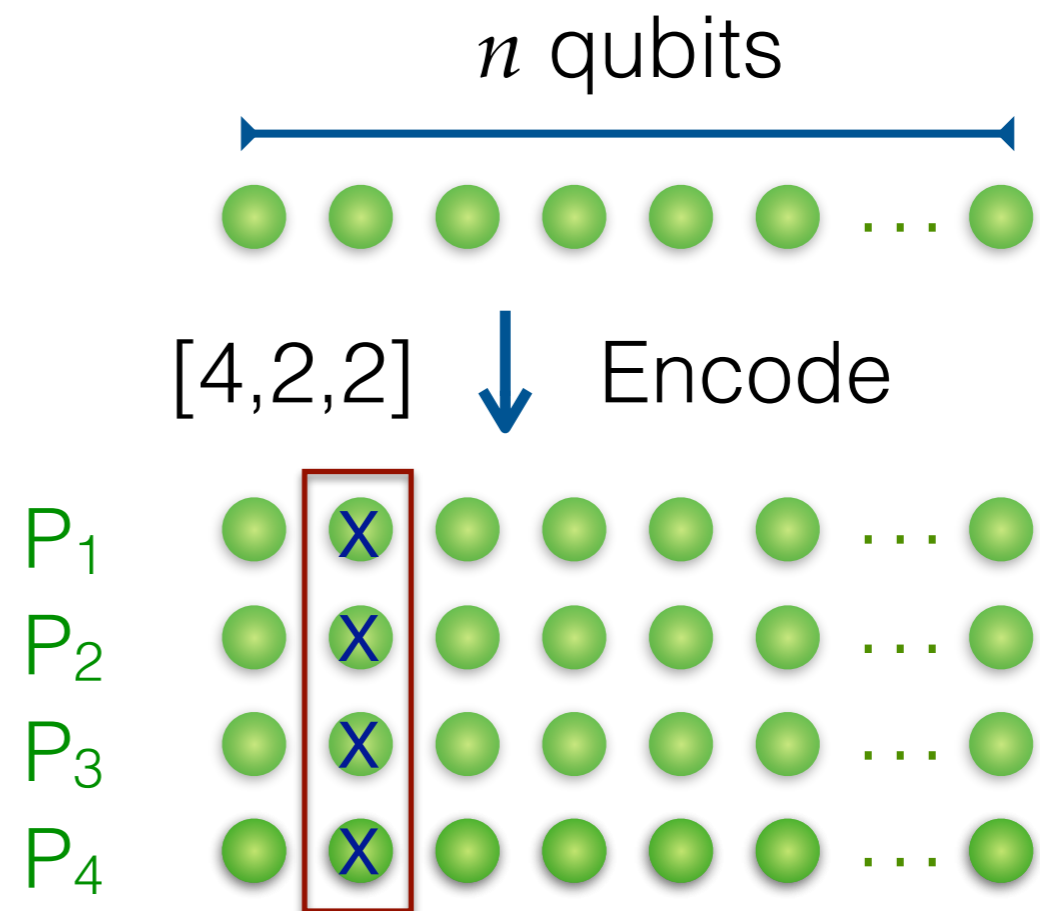
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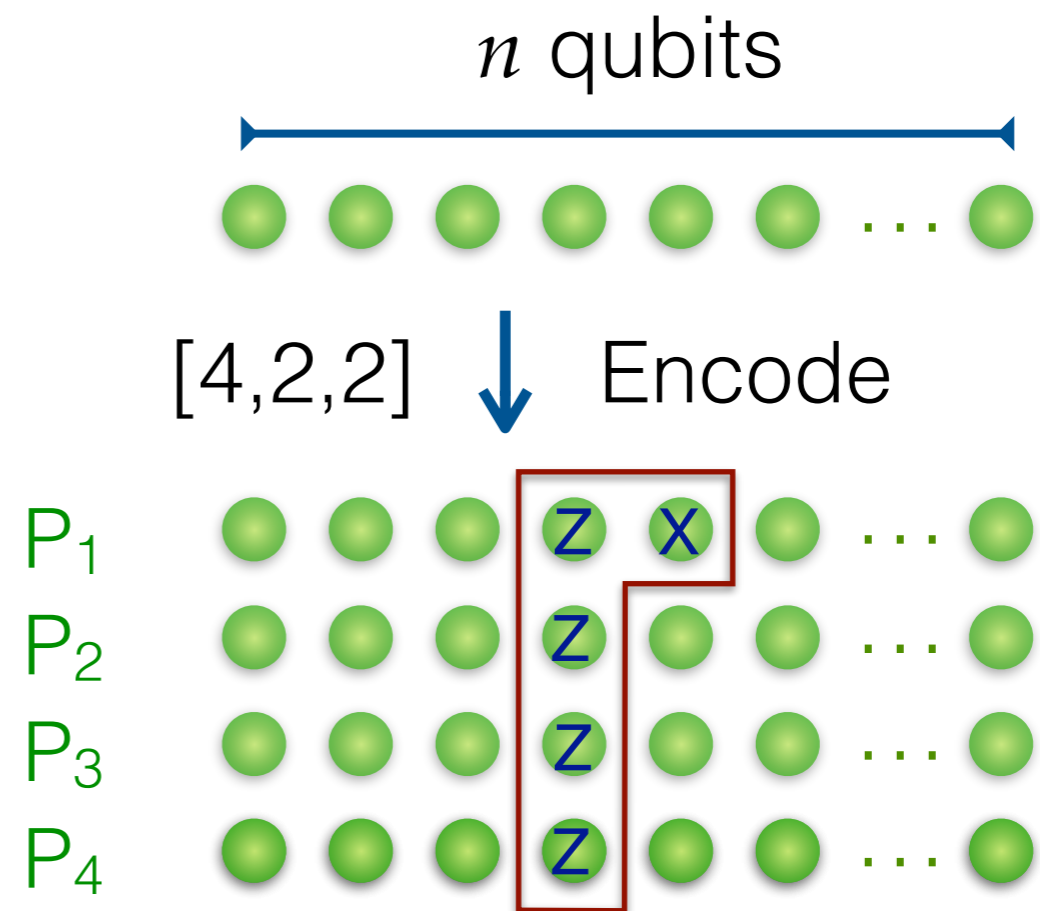
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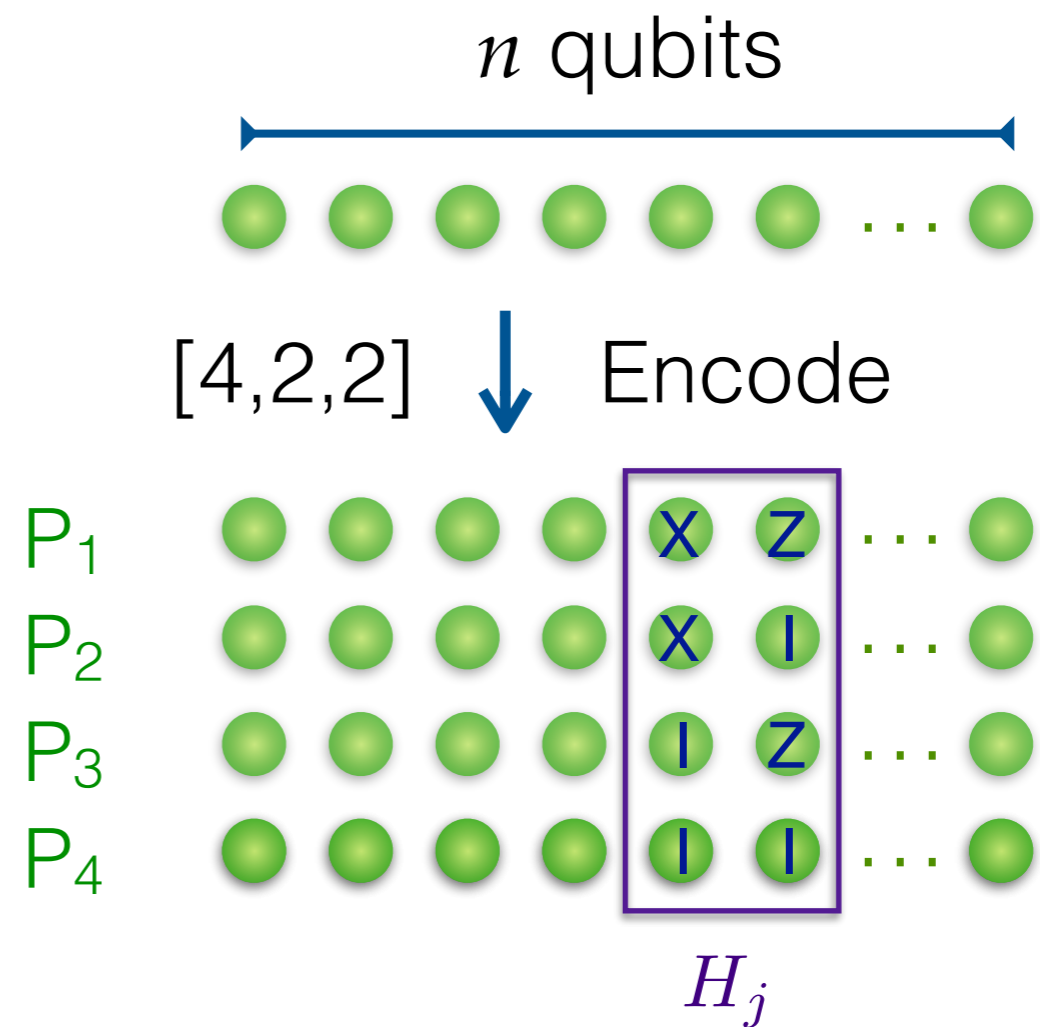
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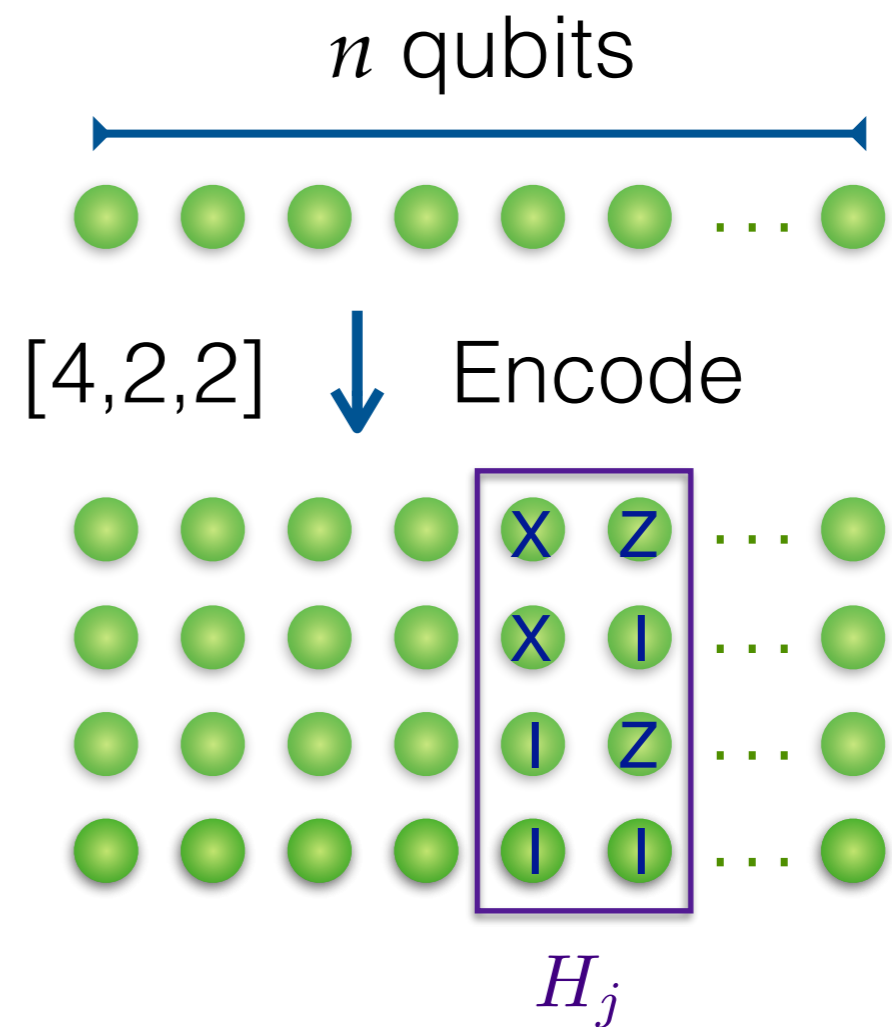
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# Overview of the protocol

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- Sends measurement specifications and asks for the outcome instead of asking for qubits from the provers
- How can we trust the provers?



# Where are the qubits?





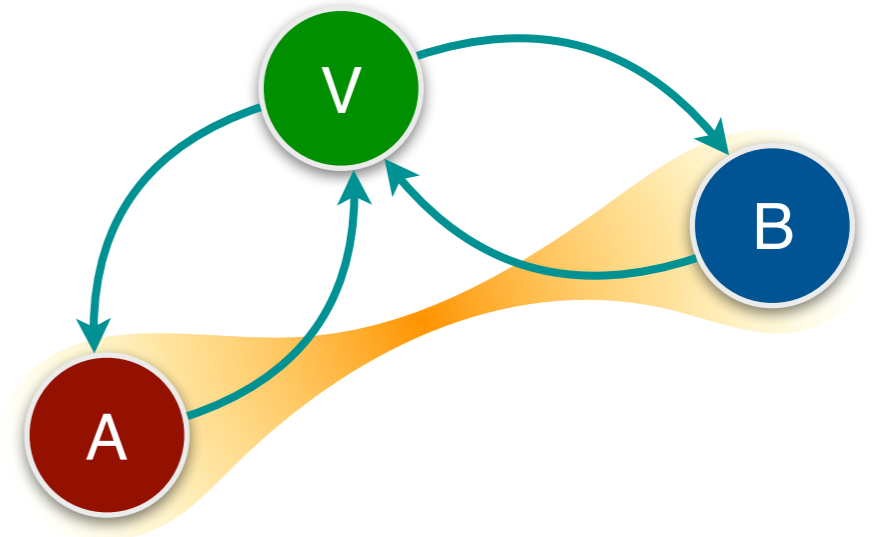
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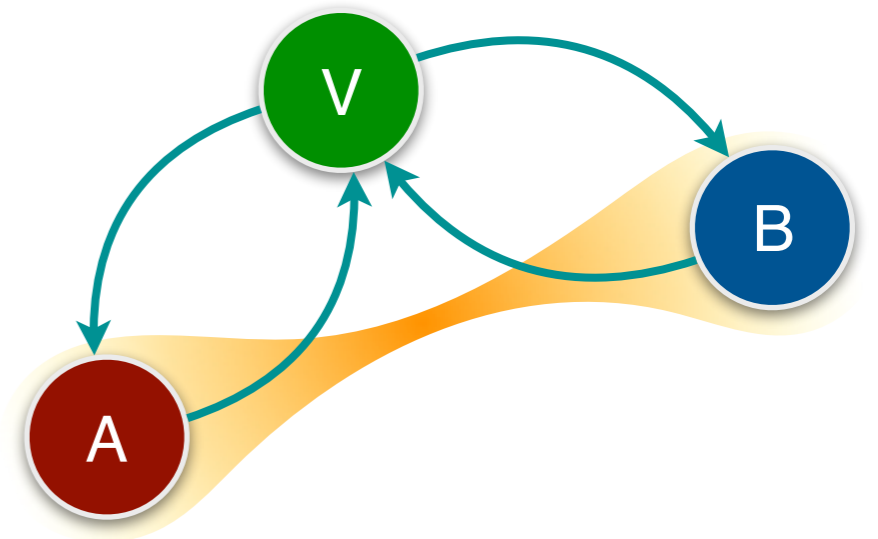
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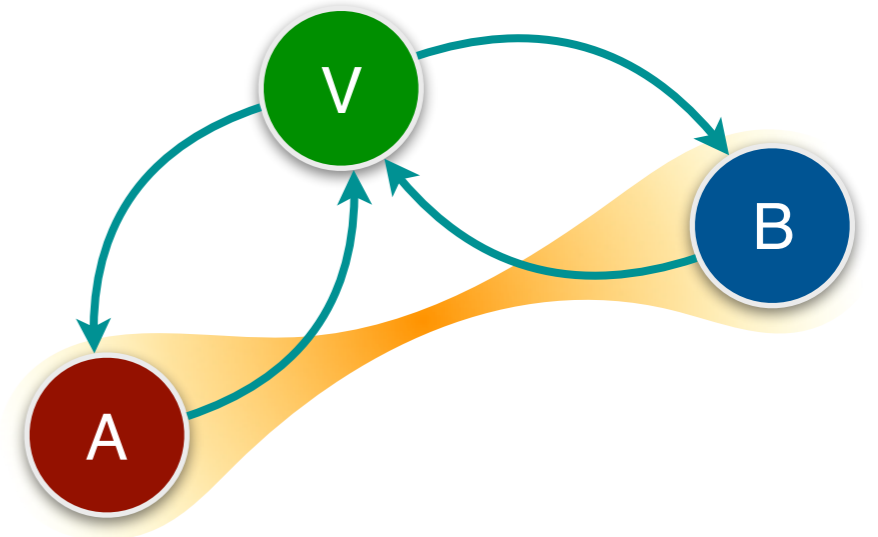
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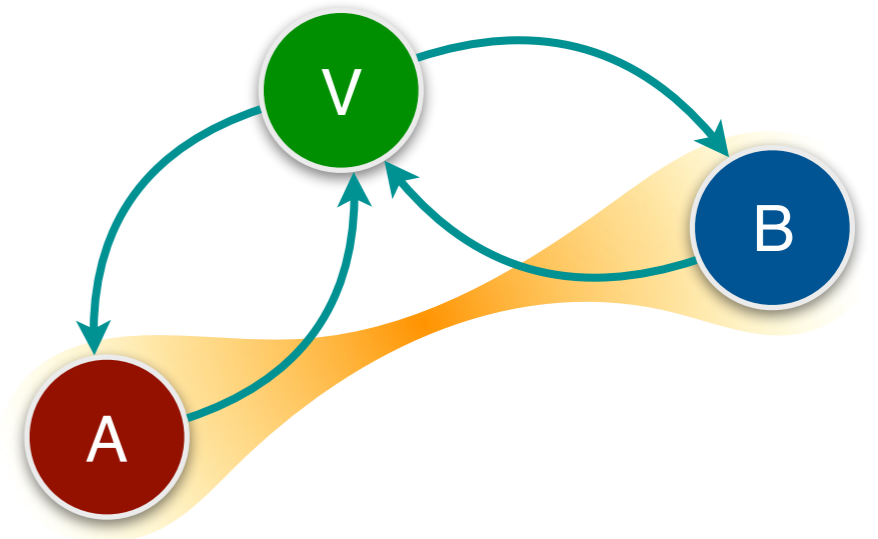
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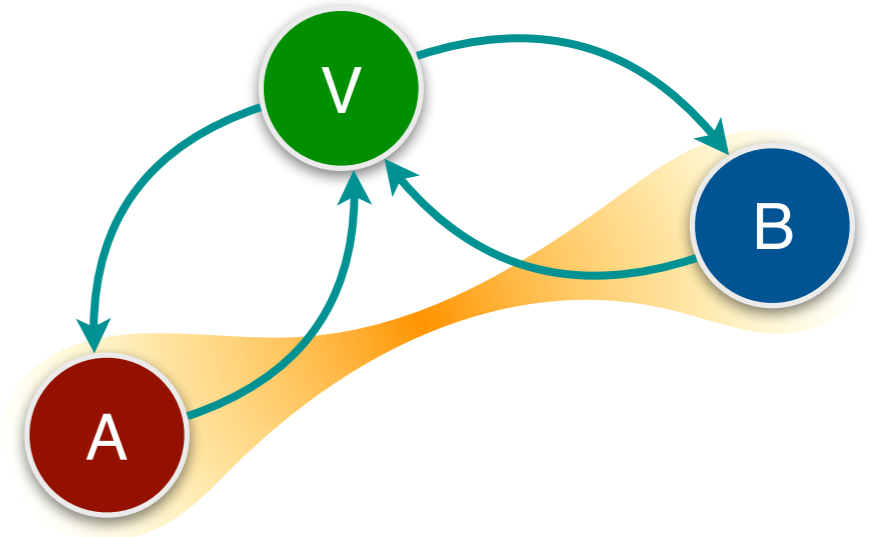
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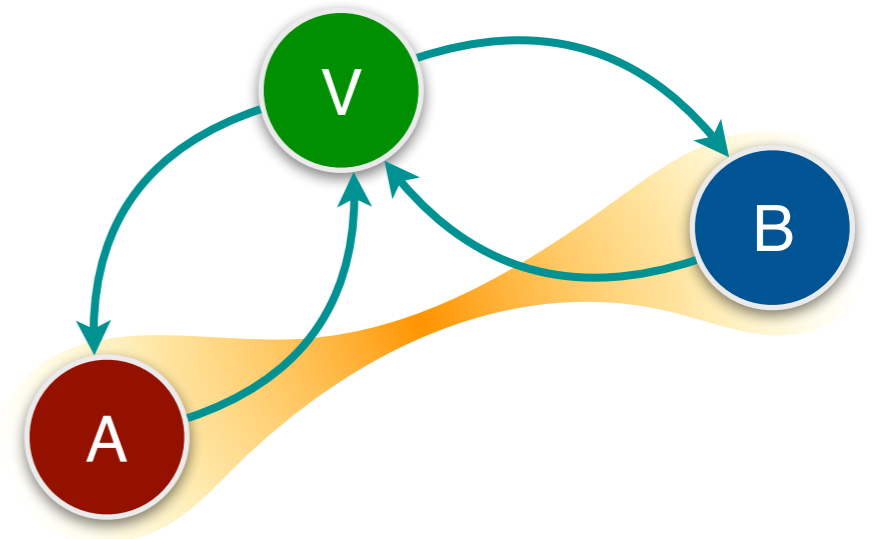
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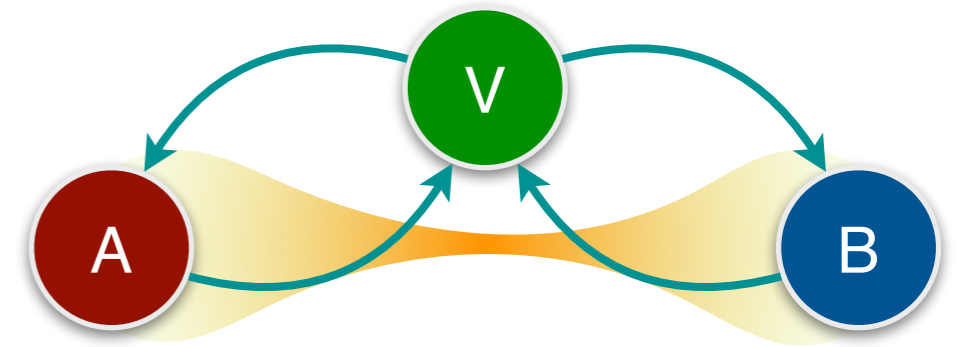


CHSH game in terms of  
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# CHSH game in terms of stabilizers

CHSH:  $a \oplus b \stackrel{?}{=} s \wedge t$

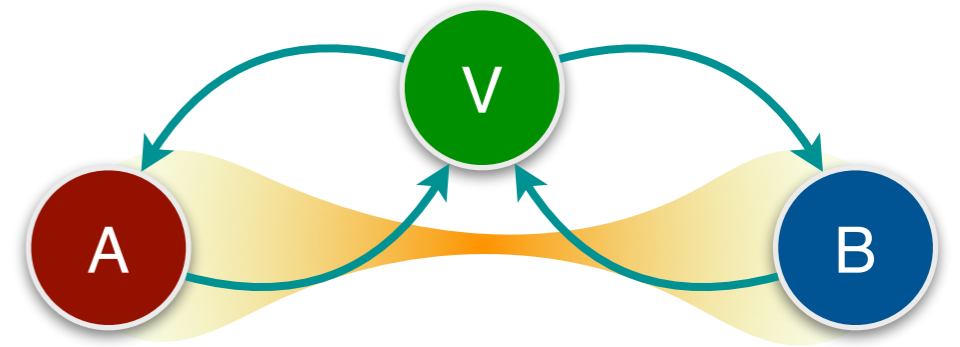


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- The **EPR** state as a stabilizer

X	X
Z	Z

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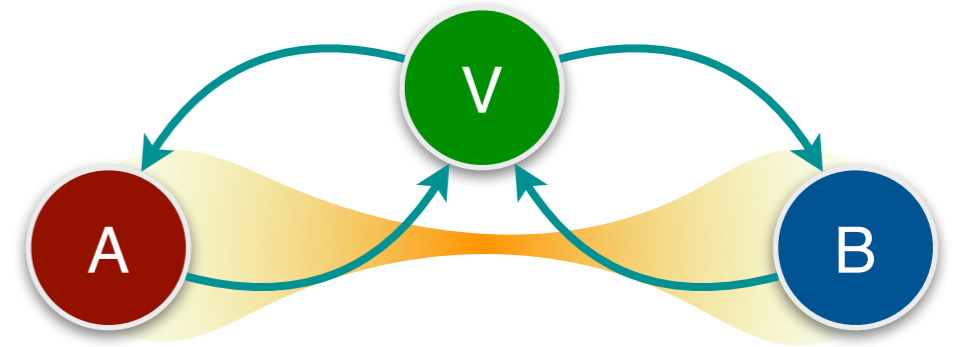
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$$\begin{array}{|c|c|} \hline X & X \\ \hline Z & Z \\ \hline \end{array}$$

$$\langle XX + ZZ \rangle = 2$$

$$\text{CHSH: } a \oplus b \stackrel{?}{=} s \wedge t$$



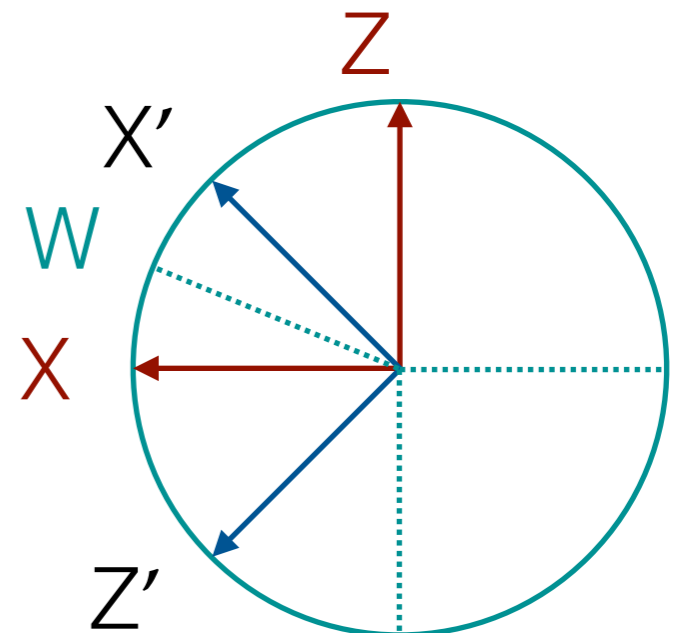
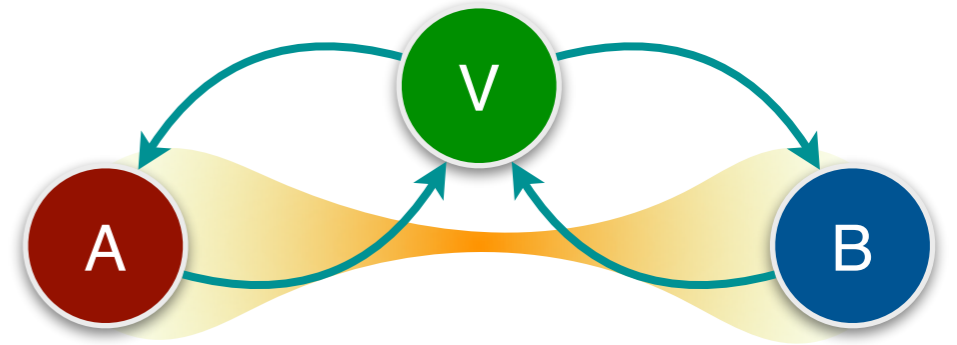
# CHSH game in terms of stabilizers

- The **EPR** state as a stabilizer

X X	$\langle XX + ZZ \rangle = 2$
Z Z	

$$X = \frac{X' + Z'}{\sqrt{2}} \quad Z = \frac{X' - Z'}{\sqrt{2}}$$

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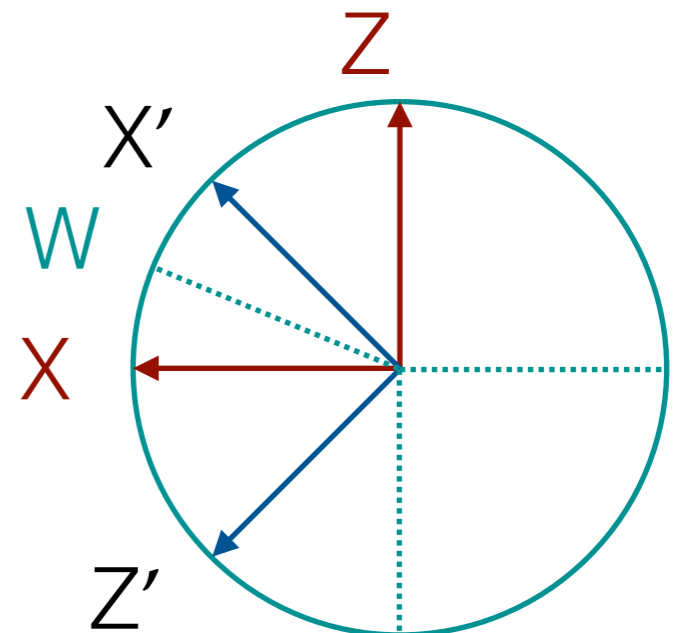
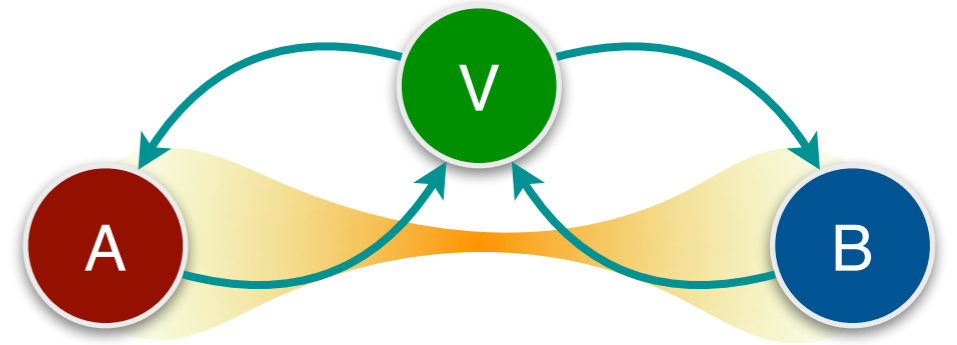
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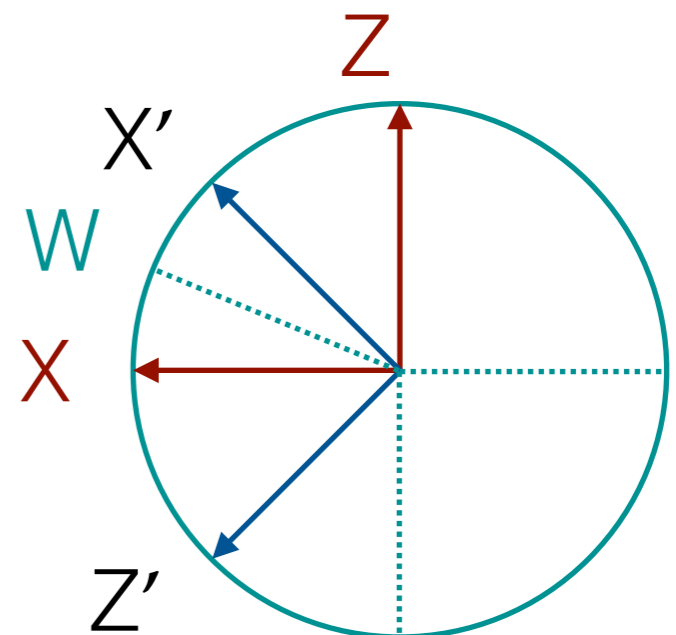
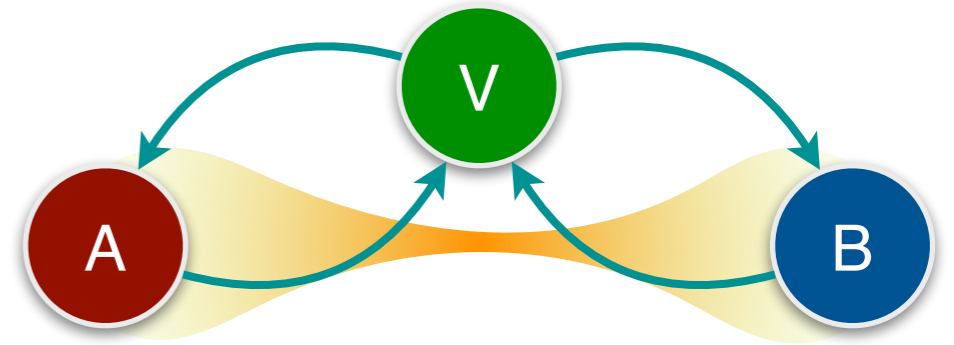
 $\longrightarrow$ 

+	X	X'
+	X	Z'
+	Z	X'
-	Z	Z'

 $\longrightarrow$ 

+	0	0
+	0	1
+	1	0
-	1	1

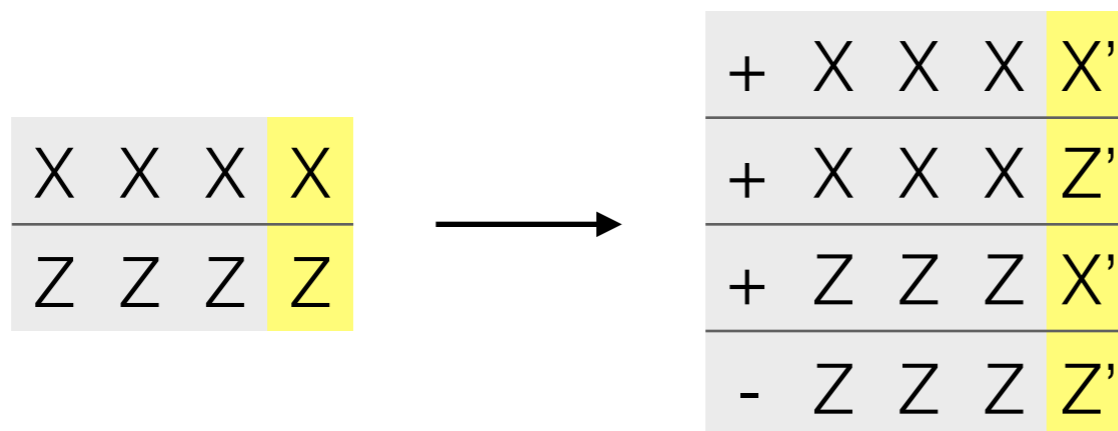
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# Stabilizer games with a special player

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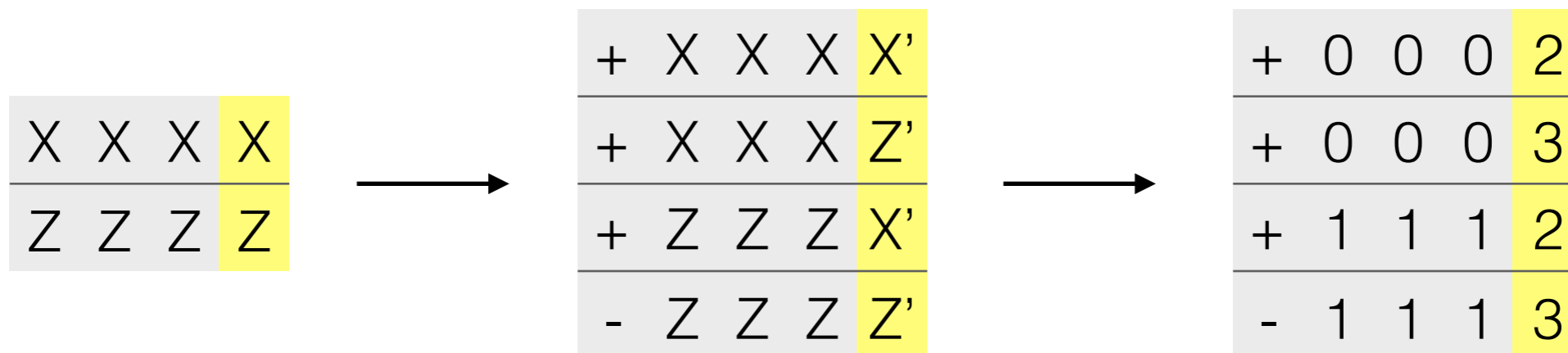
- Apply the 45-degree rotation trick to the stabilizers of the  $[4,2,2]$  code





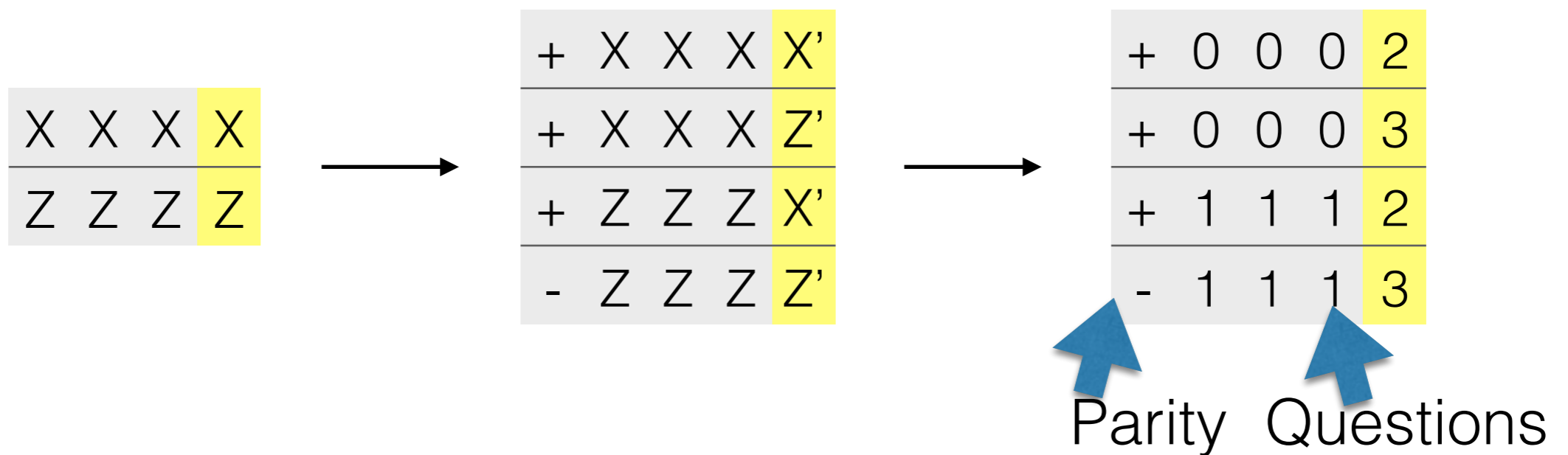
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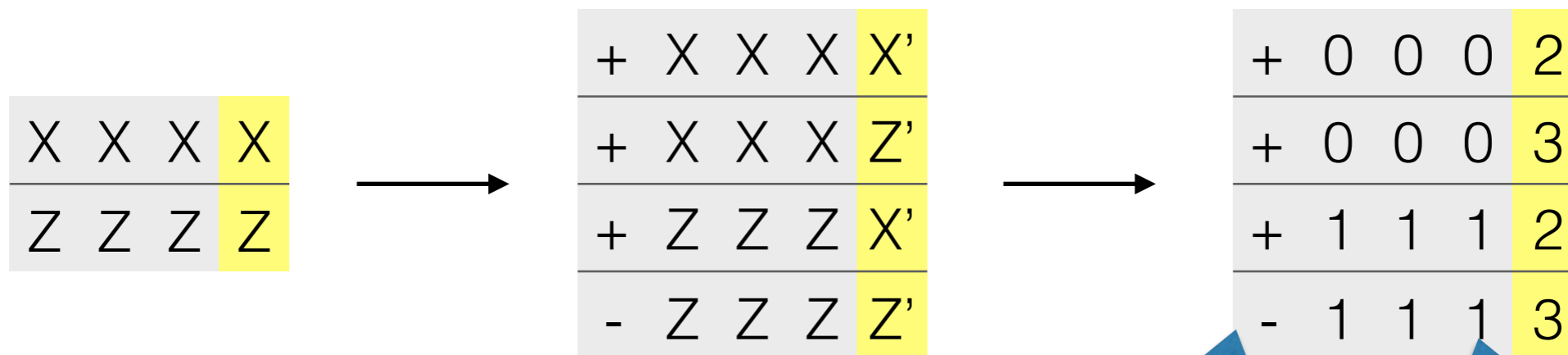
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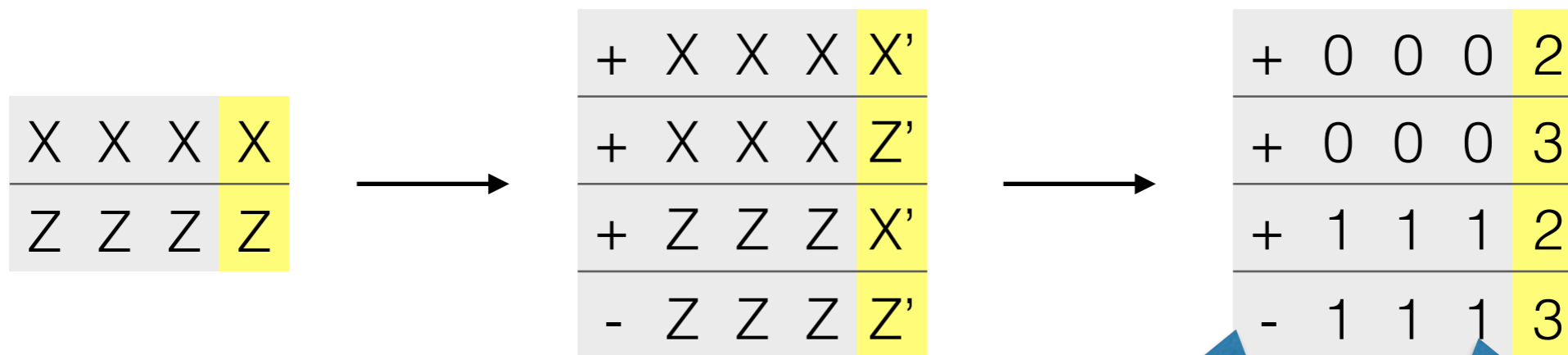


- Special player: the 4-th player

Parity Questions

# Stabilizer games with a special player

- Apply the 45-degree rotation trick to the stabilizers of the  $[4,2,2]$  code



Parity Questions

- Special player: the 4-th player
- No full rigidity, but **partial rigidity**: the special player must measure honestly

# Partial rigidity of the special player stabilizer game

Lemma (Partial Rigidity). For any strategy  $\mathcal{S} = (\rho, \{R_w^{(i)}\})$  of the special player stabilizer game whose value is at least  $\omega_{\text{sps}}^* - \varepsilon$  there exists an isometry  $V : \mathcal{H}_4 \rightarrow \mathbb{C}^2 \otimes \hat{\mathcal{H}}_4$  such that

$$R_3^{(4)} = V^\dagger (Z' \otimes I) V,$$

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Proof of the lemma uses the Jordan's lemma and a proof technique for the CHSH rigidity from [Reichardt, Unger, Vazirani 13]

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- Encoded Werner states are certifiable!

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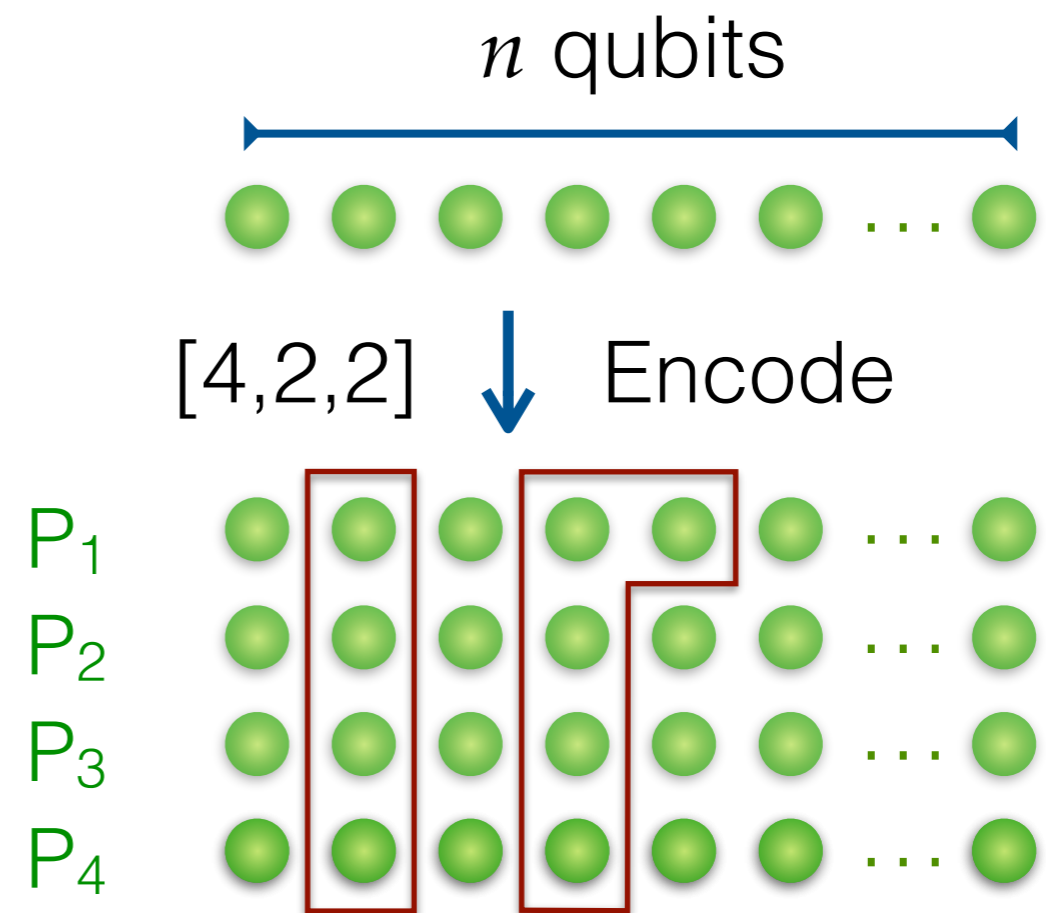
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The proof uses the **consistency** properties of the game

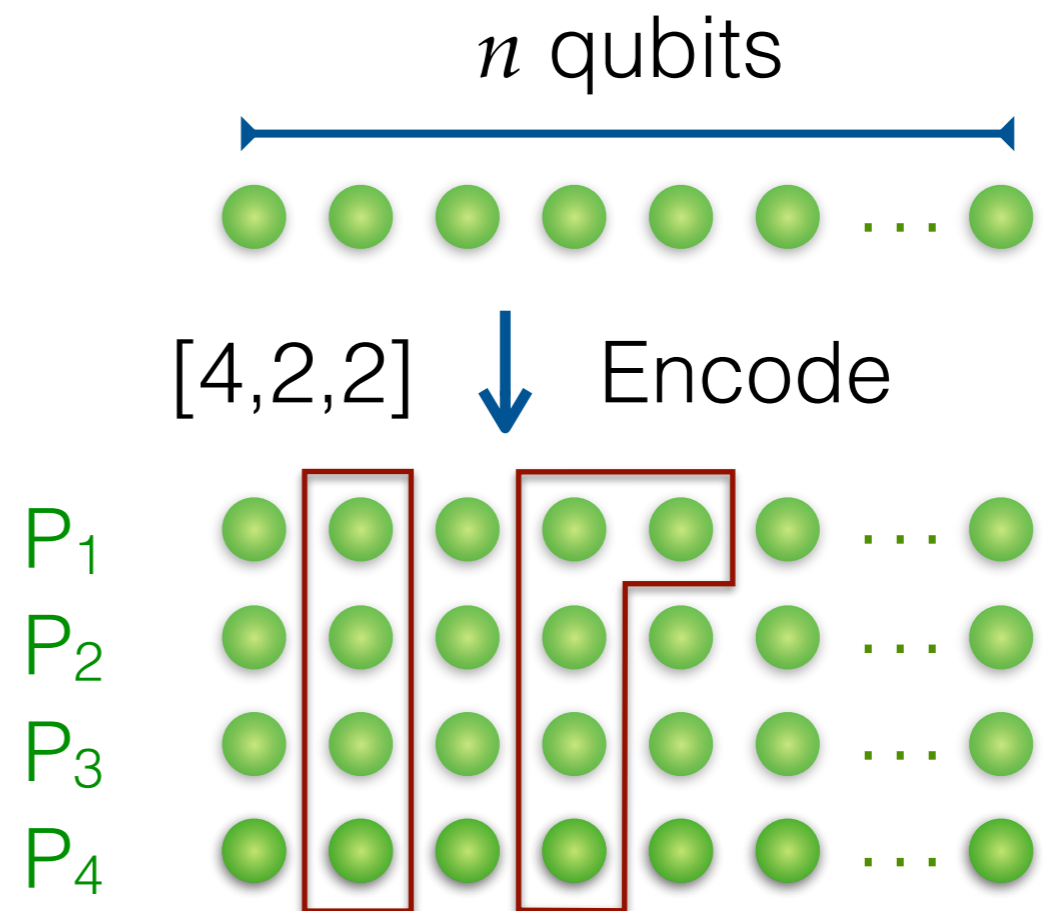


# Multi-qubit stabilizer game



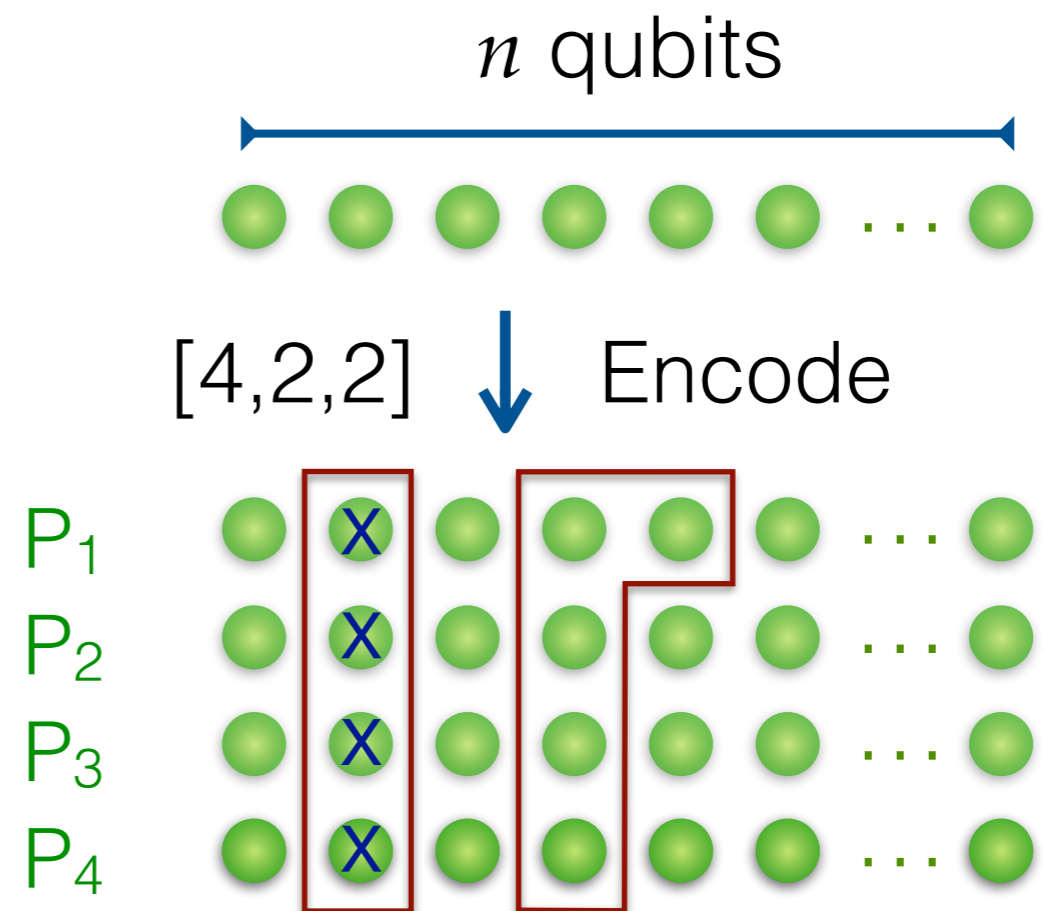
# Multi-qubit stabilizer game

- For both types of the encoding checks, the verifier plays the corresponding stabilizer game



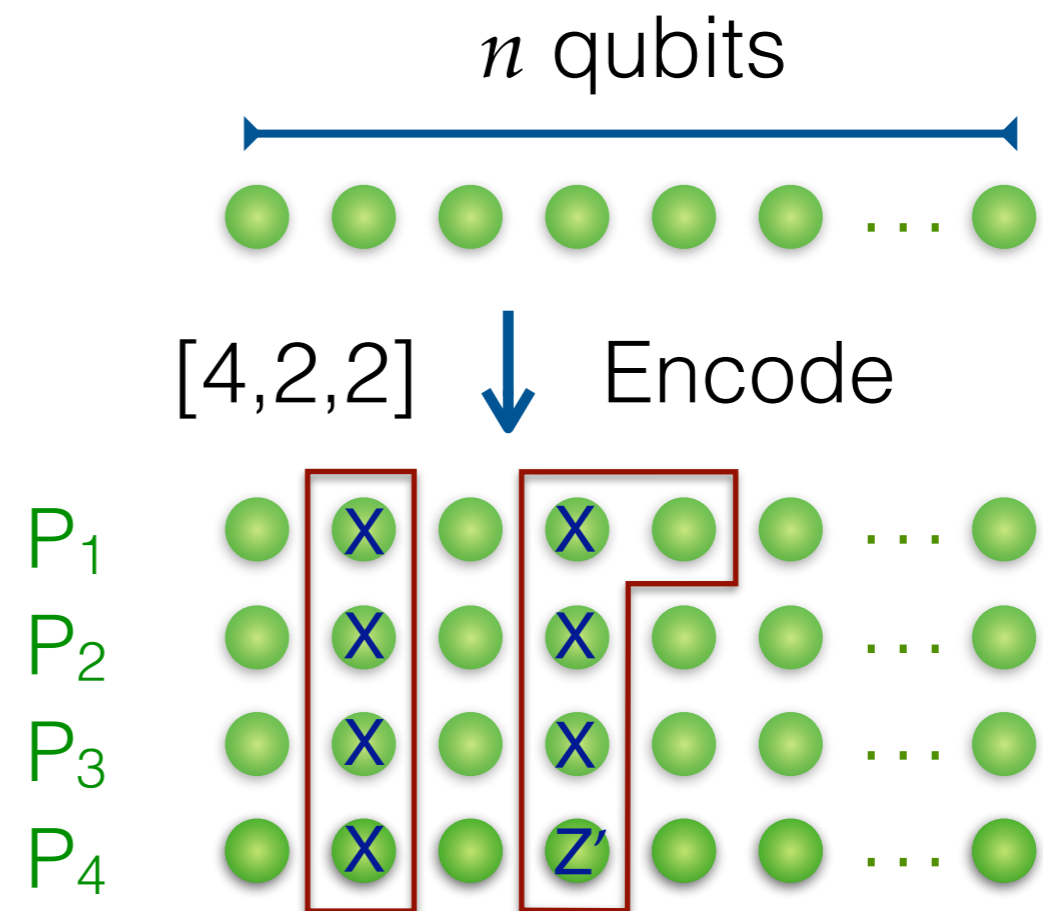
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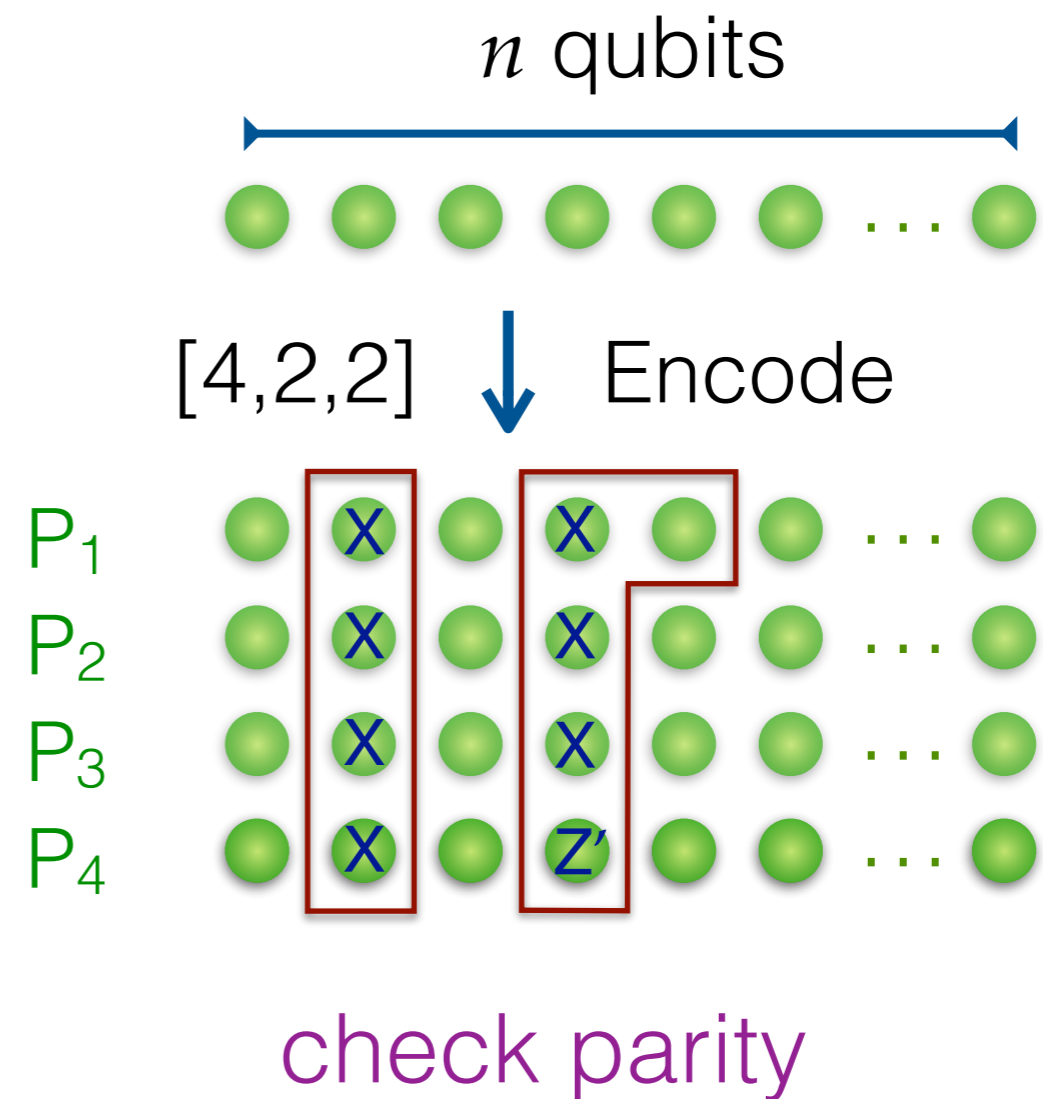
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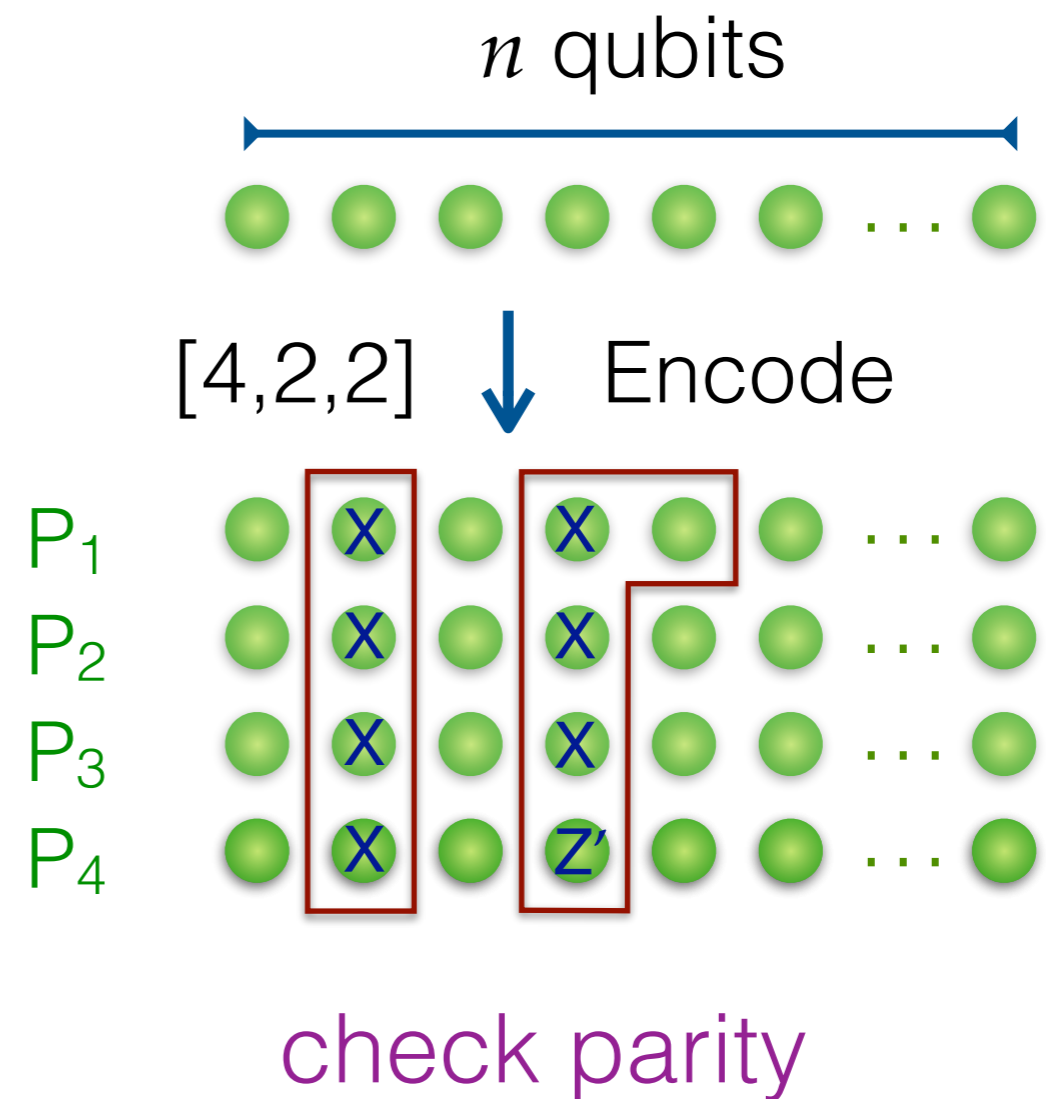
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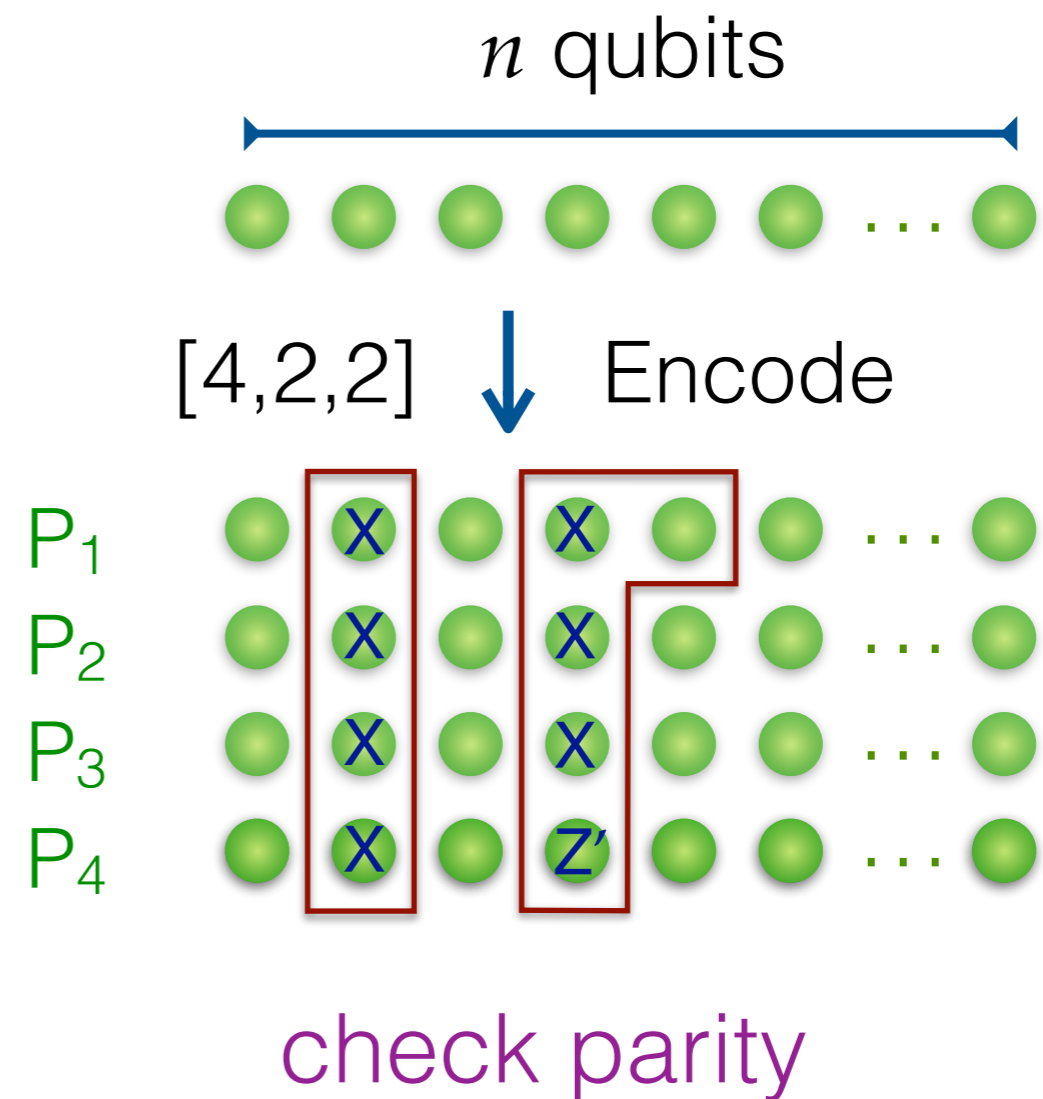
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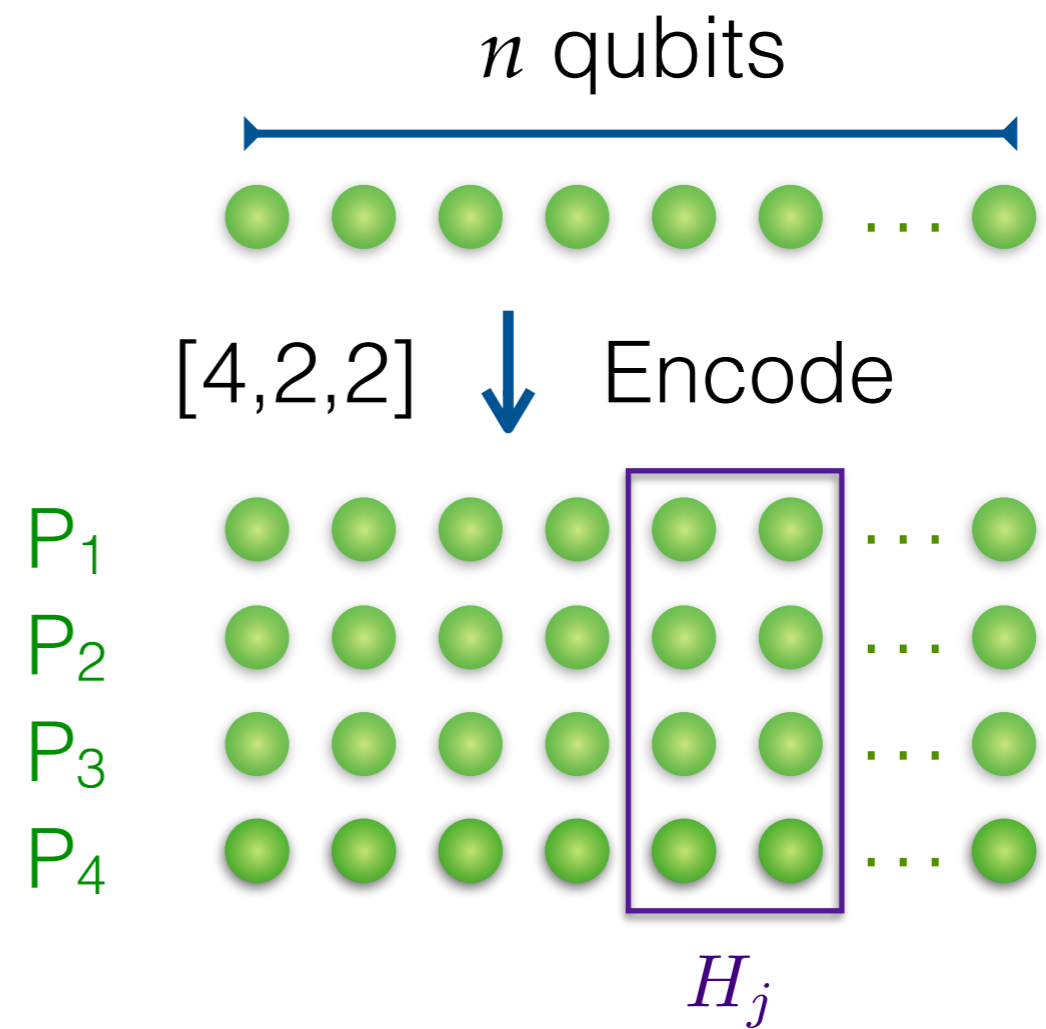


# Multi-qubit stabilizer game

- For both types of the encoding checks, the verifier plays the corresponding stabilizer game
- Full rigidity
- “Locates” the  $n$  qubits in a sequential way



# Energy measurement

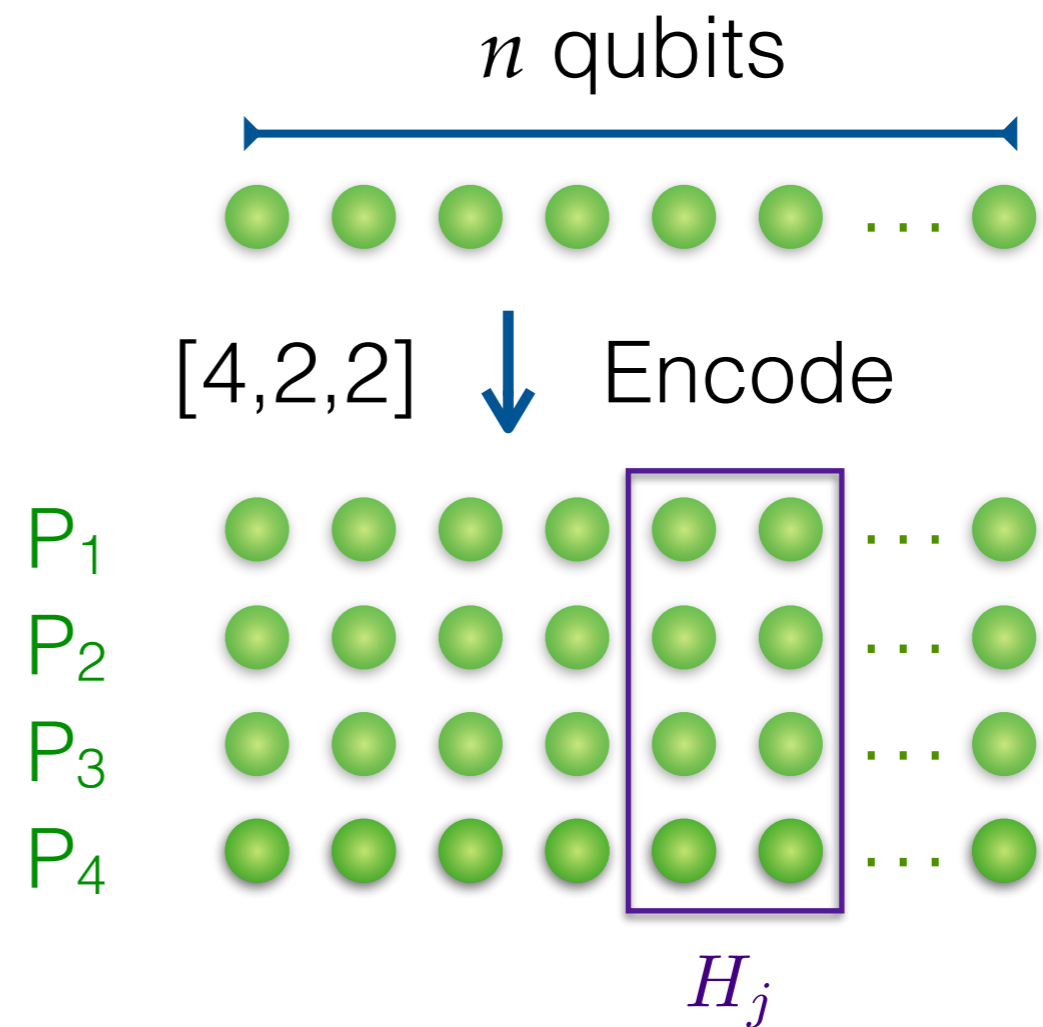




# Energy measurement

- Hamiltonians with XZ interactions remain **QMA**-complete

[Cubitt, Montanaro 14]

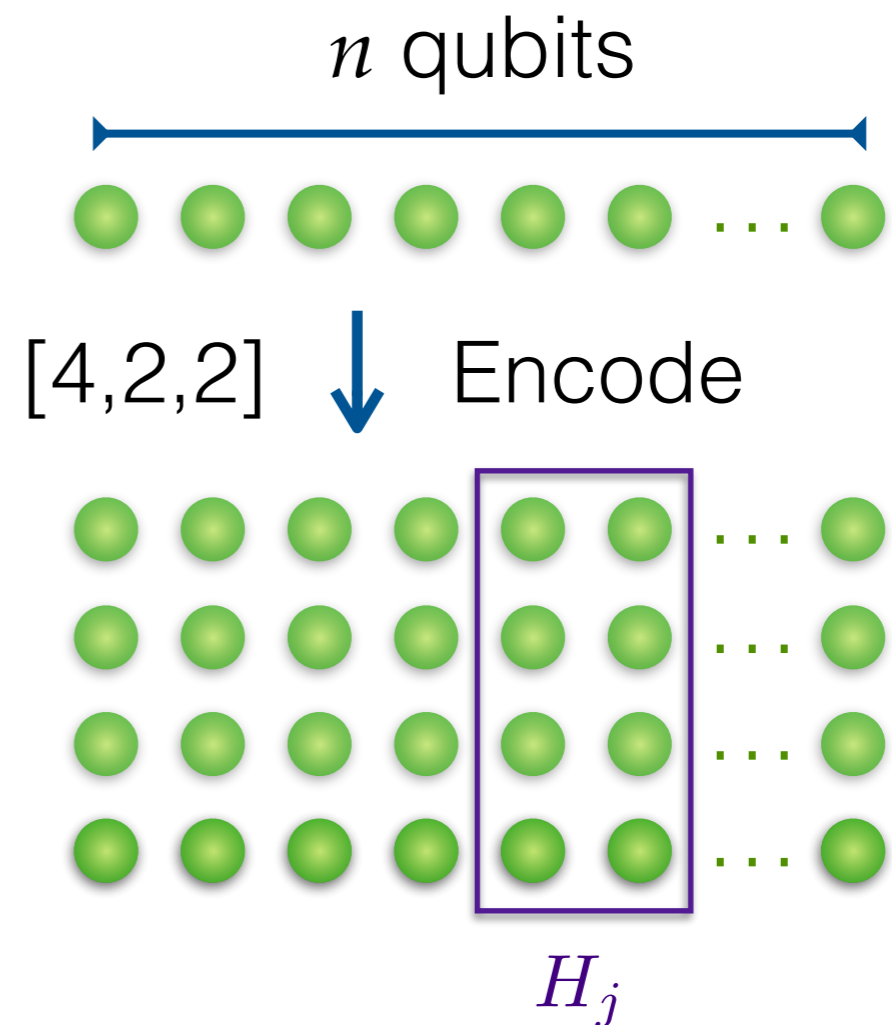


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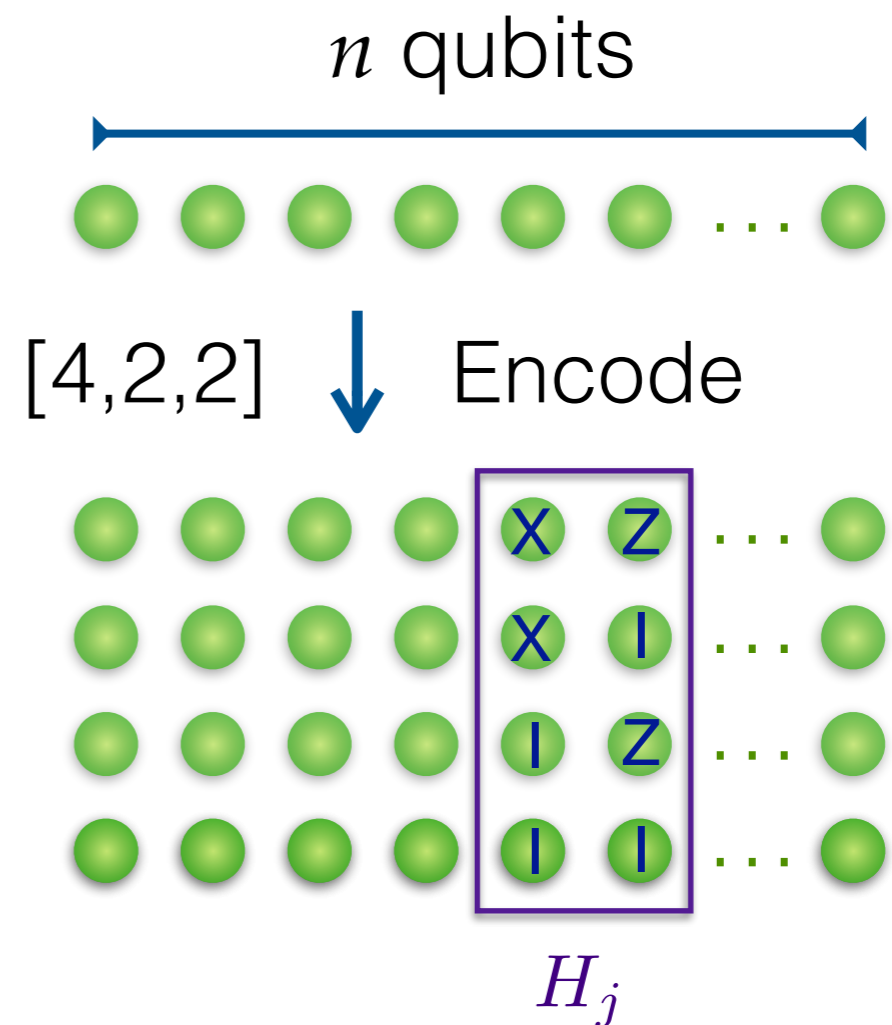
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X	X	I	I
Z	I	Z	I



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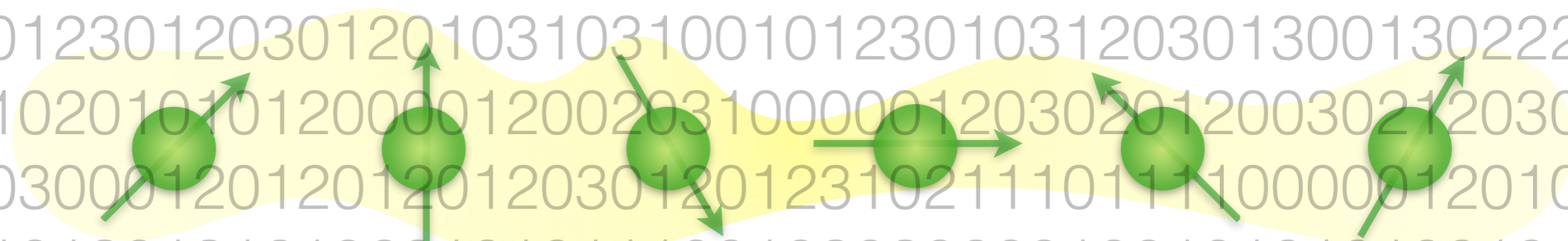
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- Beyond **QMA-hardness**?



001000020302012003021203012030300012012012012030  
120120021203123030231231023123033023321020303032  
10020303022010311101201201101032031012021201201030  
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