Proposal for Quantum Spectrum Estimation with Trapped Atoms

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work with: Alagic, Haah, Campbell, Rey, Gorshkov
PLAN OF TALK

◦ Small Quantum Computers
◦ Spectrum Estimation
◦ Young Diagram Spectrum Estimation
  ◦ using Quantum Computer
  ◦ using High-symmetry Hamiltonian
◦ Summary
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Fully functioning quantum computers:

- scalable
- fault-tolerant
- universal

Small quantum computers:

- may not scale
- not fault-tolerant
- special purpose

achievable with current technology
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SPECTRUM ESTIMATION

Density matrix \( \rho \)

\( \rho = \sum_{i=1}^{d} p_i |\psi_i\rangle \langle \psi_i| \)

(eigenvalue) spectrum \( p = (p_1, p_2, \ldots, p_d) \)

POVM \( \{\Pi_{q_1}, \Pi_{q_2}, \ldots\} \)

each \( \Pi_q \) acts in full Hilbert space \( (\mathbb{C}_d)^{\otimes n} \)

best estimate of spectrum: \( q \)

\( \Pr(q|\rho, n) = \text{Tr}(\Pi_q \rho^{\otimes n}) \)

centrated for good strategy
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Suppose have strategy \( \{ \Pi_q \} \) 

eigenvalues basis indep.

\[
\text{spec}(\rho) = \text{spec}(V \rho V^\dagger) \quad V \in SU(d)
\]

\[
\{ V \otimes^n \Pi_q V^\dagger \otimes^n \} \quad \text{equally “good” strategy}
\]

permutation of copies

\[
\text{Tr}(\Pi_q \rho \otimes^n) = \text{Tr}(\sigma \Pi_q \sigma^{-1} \rho \otimes^n) \quad \sigma \in S_n
\]

\[
\{ \sigma \Pi_q \sigma^{-1} \} \quad \text{equally “good” strategy}
\]

together:

\[
\{ g \Pi_q g^{-1} \} \\
g \in \langle \sigma, V \otimes^n \rangle \cong S_n \times SU(d)
\]
“Young Diagram” strategy (Keyl-Werner 2001)

Invariant under symmetries:

\[ g \Pi_\lambda g^{-1} = \Pi_\lambda \quad \forall g \in S_n \times SU(d) \]

\( \Pi_\lambda \) projects onto irrep \( \lambda \) of \( S_n \times SU(d) \)

\( \{ \Pi_\lambda \} \) optimal measurements of spectrum

Young Diagrams \( \lambda \)

\( S_n \times SU(d) \) irreps

\( n \) boxes at most \( d \) rows
YOUNG DIAGRAM SPECTRUM ESTIMATION

\[(\mathbb{C}_d)^{\otimes n}\] contains one copy of each irrep

\[(\mathbb{C}_3)^{\otimes 5}\]

Pr\((\lambda | \rho, n) = \text{Tr}(\Pi_\lambda \rho^{\otimes n})\) concentrated at \(\lambda / n \rightarrow p\)

\[\rho = 0.7 |0\rangle \langle 0| + 0.2 |1\rangle \langle 1| + 0.1 |3\rangle \langle 3|\]
Are these highly entangled, non-local measurements physical?

same measurements analyzed in great merged talk by Wright and Haah as first step of full tomography
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USING A QUANTUM COMPUTER

Circuit: (Inverse) Quantum Schur Transform

in

ρ
ρ
ρ
ρ

Transforms irrep basis to computational basis

gate count: \( n \ poly(d, \log n, \log 1/\epsilon) \)

out

measures \( \{\Pi_{\lambda}\} \) locally

Drawback: need a fully operational quantum computer!

Efficient Quantum circuits for Schur and Clebsch-Gordan Transforms Bacon, Chuang, Harrow (PRL)
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Aim: engineer $H$ with $S_n \times SU(d)$ symmetry

$$H = \sum E(\lambda) \Pi_\lambda$$ irreps = energy spaces

Could infer $p$ if:
- can prepare state $\rho^\otimes n$
- can measure energy
- $E(\lambda)$ invertible function

$$\langle E \rangle = \text{Tr}(\rho^\otimes n H) = \sum \text{Pr}(\lambda|n, \rho) E(\lambda)$$

peaked near $\frac{\lambda}{n} = p$  \quad $\langle E \rangle \rightarrow E(\lambda = np)$
USING A HIGH SYMMETRY HAMILTONIAN

\[ H = \hbar \sum_{j<k} [1 - (j, k)] \] swap

“all-to-all” swaps

\[ S_n \times SU(d) \]

1D square trap

n atoms ↔ n sites

\[ H \] interactions

sites:

spatial modes

levels:

nuclear spin

\[ |i_3, i_2, \ldots, i_n\rangle \]

\[ i = 1, 2, \ldots, d \]

\[ (1, 3)|i_1, i_2, i_3, i_4\rangle \]

\[ = |i_3, i_2, i_1, i_4\rangle \]

\[ d = 2I + 1 \]
USING A HIGH SYMMETRY HAMILTONIAN

Physical system requirements

alkaline-earth atoms
square trap

\[ H = \hbar \sum [1 - (j, k)] \]
\[ S_n \times SU(d) \]

\[ \hbar \sum [1 - O_{jk}] \]
breaks \( SU(d) \)

\[ \sum h_{jk} [1 - (j, k)] \]
breaks \( S_n \)
USING A HIGH SYMMETRY HAMILTONIAN

Physical system requirements (ctd…)

Hilbert space \((\mathbb{C}_d)^\otimes n\)

Hamiltonian \(H = \hbar \sum [1 - (j, k)]\)

requires atoms ground electronic state \(|g\rangle\)

if allow first excited state \(|e\rangle\)

\[
\begin{align*}
(\mathbb{C}_d \otimes \mathbb{C}_2)^\otimes n \\
H = \hbar \sum [1 - (j, k)] \Pi^g + \delta \sum n_j^e
\end{align*}
\]

(later) feature not a bug!
Aim: engineer $H$ with $S_n \times SU(d)$ symmetry

**Could infer $p$ if:**
- ✓ can prepare state $\rho^\otimes n$
- x can measure energy
- x $E(\lambda)$ invertible function

**But we show**

**Could infer $p$ if:**
- ✓ can prepare state $(\rho \otimes |g\rangle\langle g|)^\otimes n$
- ✓ can apply pulse $|e\rangle \leftrightarrow |g\rangle$
- ✓ can measure # e atoms $n^e$
USING A HIGH SYMMETRY HAMILTONIAN (OUR APPROACH)

\[ H = \hbar \sum [1 - (j, k)] \prod^g + \delta \sum n^e_j \]

Protocol:

1. prepare state \((\rho \otimes |g\rangle \langle g|)^\otimes n\)
2. apply pulse \(V_\beta\) to \(|e\rangle \leftrightarrow |g\rangle\)
3. evolve under \(H\) for time \(t\)
4. invert pulse \(V_\beta^\dagger\)
5. measure number atoms in \(|e\rangle\)

\[ V_\beta = \begin{pmatrix} \cos \beta & i \sin \beta \\ i \sin \beta & \cos \beta \end{pmatrix} \]

operationally:

\[ \langle n^e(t) \rangle = \text{Tr}[n^e V_\beta^\dagger e^{-itH} V_\beta (\rho \otimes |g\rangle \langle g|)^\otimes n V_\beta^\dagger e^{itH} V_\beta] \]
USING A HIGH SYMMETRY HAMILTONIAN (OUR APPROACH)

repeat for multiple times

\[ \langle n^e(t) \rangle = \text{Tr}[n^e V_\beta^\dagger e^{-itH} V_\beta (\rho \otimes |g\rangle \langle g|) \otimes n V_\beta^\dagger e^{itH} V_\beta] \]

\[ = \frac{\sin^2 \beta}{2} \left[ 1 - \sum_{i=1}^{\sqrt{n}} p_i \cos(\omega_i t) \right] + O\left( \frac{1}{\sqrt{n}} \right) \]

(representation-theoretic calculation)

fit against data to estimate \( p \)
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CONCLUSIONS

◦ experimentally accessible (coauthor G. Campbell preparing to implement)
◦ spectrum estimation useful in this system e.g., entanglement of simulators

OPEN QUESTIONS

◦ same system for general Schur transform?
◦ natural systems for other small quantum computing tasks?
◦ physical systems for special-purpose fault tolerant scalable tasks?