Proposal for Quantum Spectrum Estimation with Trapped Atoms

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work with: Alagic, Haah, Campbell, Rey, Gorshkov

PLAN OF TALK

- Small Quantum Computers
- Spectrum Estimation
- Young Diagram Spectrum Estimation
  - using Quantum Computer
  - using High-symmetry Hamiltonian
- Summary

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Fully functioning quantum computers:

- scalable
- fault-tolerant
- universal

Small quantum computers:

- may not scale
- not fault-tolerant
- special purpose

} achievable with current technology

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SPECTRUM ESTIMATION

ρ Density matrix $\rho = \sum_{i=1}^{d} p_i |\psi_i\rangle\langle\psi_i|$

(eingenvalue) spectrum $p = (p_1, p_2, \ldots, p_d)$

ρ ρ ρ ••• ρ measure (n) copies

POVM $\{\Pi_{q_1}, \Pi_{q_2}, \ldots\}$

each Π_q acts in full Hilbert space $(\mathbb{C}_d)^{\otimes n}$

best estimate of spectrum: q

$\mathrm{Pr}(q|\rho, n) = \mathrm{Tr}(\Pi_q \rho^{\otimes n})$ concentrated for good strategy

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SPECTRUM ESTIMATION - SYMMETRIES

Suppose have strategy $\{\Pi_q\}$

eigenvalues basis indep.

$$\mathrm{spec}(\rho) = \mathrm{spec}(V\rho V^\dagger) \qquad V \in SU(d)$$

$\longrightarrow \{V^{\otimes n}\Pi_q V^{\dagger\otimes n}\}$ equally "good" strategy

permutation of copies

$$\mathrm{Tr}(\Pi_q\rho^{\otimes n}) = \mathrm{Tr}(\sigma\Pi_q\sigma^{-1}\rho^{\otimes n}) \qquad \sigma \in S_n$$

$\longrightarrow \{\sigma\Pi_q\sigma^{-1}\}$ equally "good" strategy

together: $\longrightarrow \{g\Pi_q g^{-1}\}$

$$g \in \langle \sigma, V^{\otimes n}\rangle \cong S_n \times SU(d)$$

YOUNG DIAGRAM SPECTRUM ESTIMATION

“Young Diagram” strategy (Keyl-Werner 2001)

Invariant under symmetries:

$$g\Pi_\lambda g^{-1} = \Pi_\lambda \qquad \forall g \in S_n \times SU(d)$$

Π_λ projects onto irrep λ of $S_n \times SU(d)$

Young Diagrams λ

$S_n \times SU(d)$ irreps $\longleftrightarrow$

n boxes

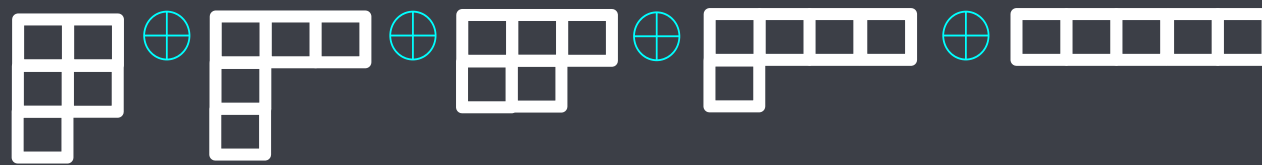
at most d rows

$\{\Pi_\lambda\}$ optimal measurements of spectrum

YOUNG DIAGRAM SPECTRUM ESTIMATION

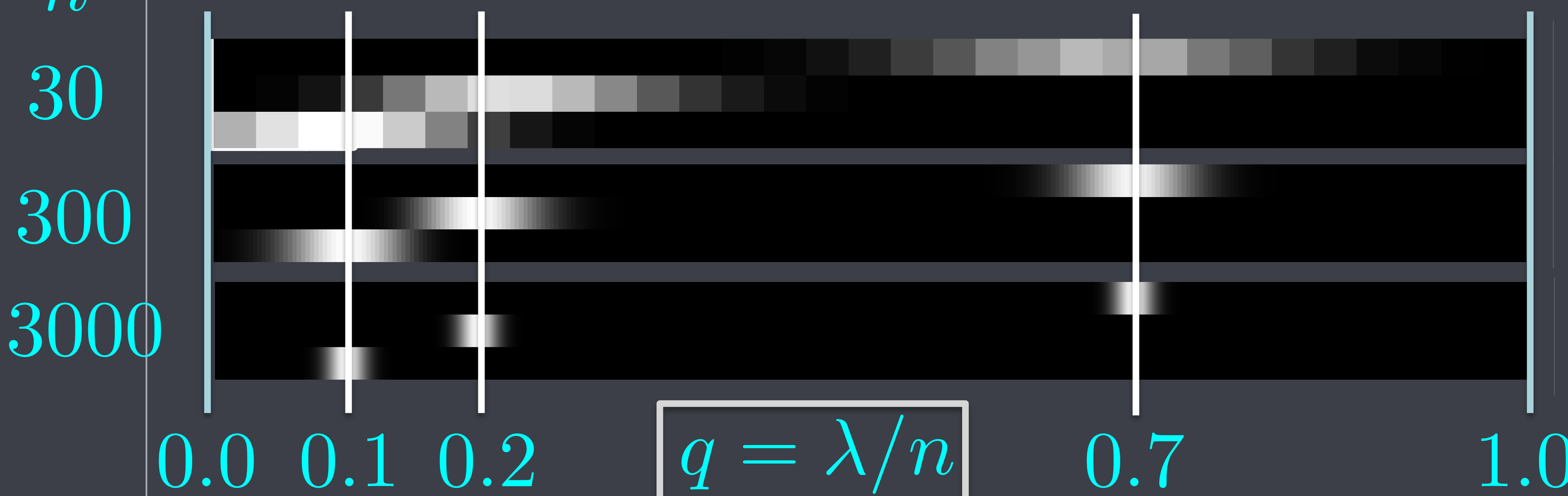
$(\mathbb{C}_d)^{\otimes n}$ contains one copy of each irrep

$(\mathbb{C}_3)^{\otimes 5} \cong$



$\Pr(\lambda|\rho, n) = \mathrm{Tr}(\Pi_\lambda \rho^{\otimes n})$ concentrated at $\lambda/n \to p$

$$\rho = 0.7|0\rangle\langle 0| + 0.2|1\rangle\langle 1| + 0.1|3\rangle\langle 3|$$



YOUNG DIAGRAM SPECTRUM ESTIMATION

same measurements analyzed in great merged talk by Wright and Haah as first step of full tomography

Are these highly entangled, non-local measurements physical?

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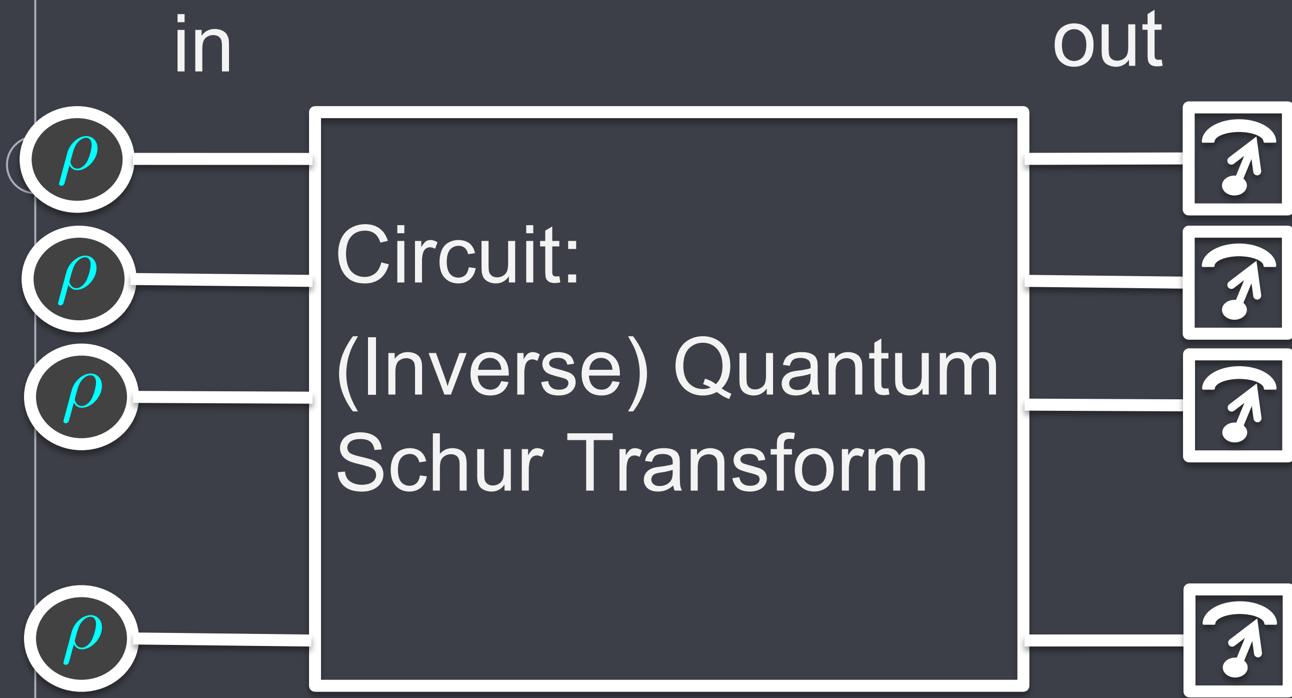
…USING A QUANTUM COMPUTER



measures $\{\Pi_\lambda\}$ locally

Transforms irrep basis to computational basis

Drawback: need a fully operational quantum computer!

gate count:
$n \ \mathrm{poly}(d, \log n, \log 1/\epsilon)$

◦*Efficient Quantum circuits for Schur and Clebsch-Gordan Transforms* Bacon, Chuang, Harrow (PRL)

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USING A HIGH SYMMETRY HAMILTONIAN

Aim: engineer H with $S_n \times SU(d)$ symmetry

⟶ $H = \sum E(\lambda)\Pi_\lambda$ irreps = energy spaces

Could infer p if:

- can prepare state $\rho^{\otimes n}$
- can measure energy
- $E(\lambda)$ invertible function

$$\langle E \rangle = \mathrm{Tr}(\rho^{\otimes n} H) = \sum_\lambda \mathrm{Pr}(\lambda | n, \rho) E(\lambda)$$

peaked near $\frac{\lambda}{n} = p$ $\langle E \rangle \longrightarrow E(\lambda = np)$

USING A HIGH SYMMETRY HAMILTONIAN

$$H = h \sum_{j<k} [1 - (j,k)]$$

swap

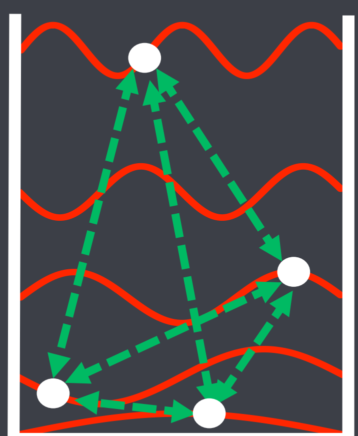
$$|i_3, i_2, \ldots i_n\rangle$$

$$i = 1, 2, \ldots, d$$

"all-to-all" swaps

$$S_n \times SU(d)$$

$$(1,3)|i_1, i_2, i_3, i_4\rangle = |i_3, i_2, i_1, i_4\rangle$$



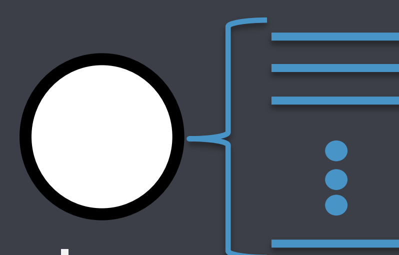
1D square trap

n atoms ⟷ n sites

H interactions

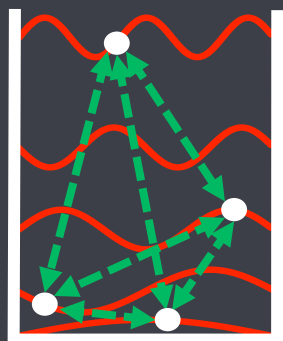
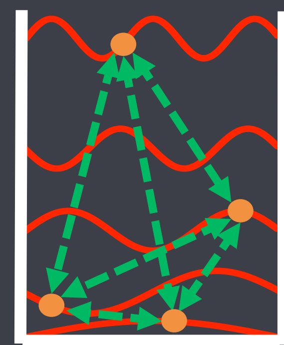
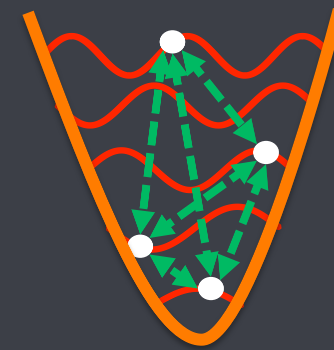
sites:
spatial modes

$$d = 2I + 1$$



levels:
nuclear spin

Physical system requirements

alkaline-earth atoms	✗	✓
square trap	✓	✗

$H = h\sum[1-(j,k)]$	$h\sum[1-O_{jk}]$	$\sum h_{jk}[1-(j,k)]$
$S_n \times SU(d)$	breaks $SU(d)$	breaks S_n

USING A HIGH SYMMETRY HAMILTONIAN

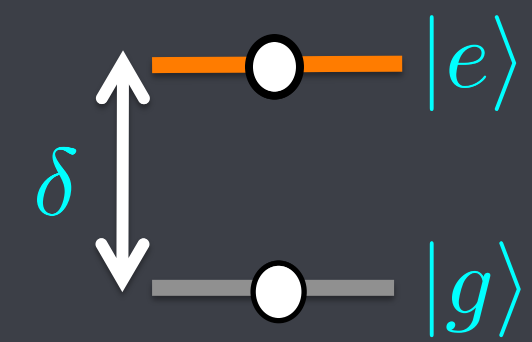
Physical system requirements (ctd…)

Hilbert space $(\mathbb{C}_d)^{\otimes n}$

Hamiltonian $H = h\sum[1-(j,k)]$

requires atoms ground electronic state $|g\rangle$

if allow first excited state $|e\rangle$



$\longrightarrow$ $(\mathbb{C}_d \otimes \mathbb{C}_2)^{\otimes n}$

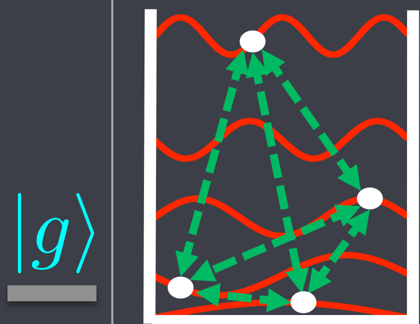
$\longrightarrow$ $H = h\sum[1-(j,k)]\Pi^g + \delta\sum n_j^e$

(later) feature not a bug!

USING A HIGH SYMMETRY HAMILTONIAN (OUR APPROACH)

Aim: engineer H with $S_n \times SU(d)$ symmetry

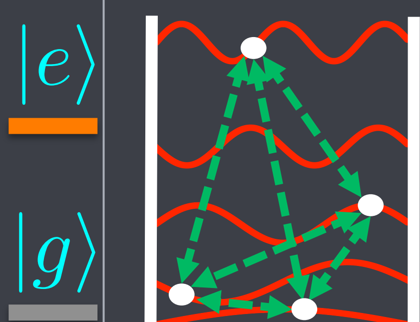
Could infer p if:



- ✓ ∘ can prepare state $\rho^{\otimes n}$
- ✗ ∘ can measure energy
- ✗ ∘ $E(\lambda)$ invertible function

BUT we show

Could infer p if:

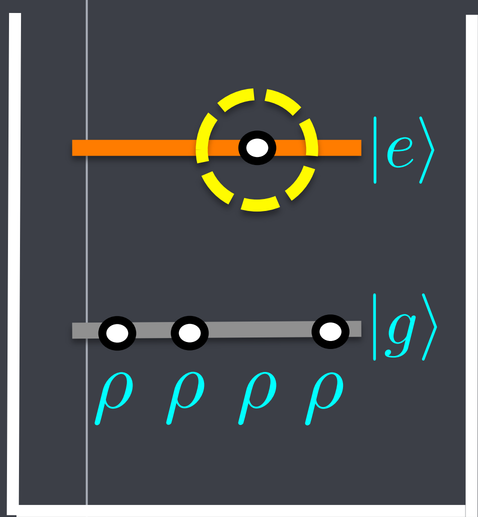


- ✓ ∘ can prepare state $(\rho \otimes |g\rangle\langle g|)^{\otimes n}$
- ✓ ∘ can apply pulse $|e\rangle \leftrightarrow |g\rangle$
- ✓ ∘ can measure # e atoms n^e

USING A HIGH SYMMETRY HAMILTONIAN (OUR APPROACH)

$$H = h\sum[1-(j,k)]\Pi^g + \delta\sum n_j^e$$

Protocol:



- prepare state $(\rho \otimes |g\rangle\langle g|)^{\otimes n}$ 1
- apply pulse V_β to $|e\rangle \leftrightarrow |g\rangle$ 2
- evolve under H for time t 3
- invert pulse $V_\beta^\dagger$ 4
- measure number atoms in $|e\rangle$ 5

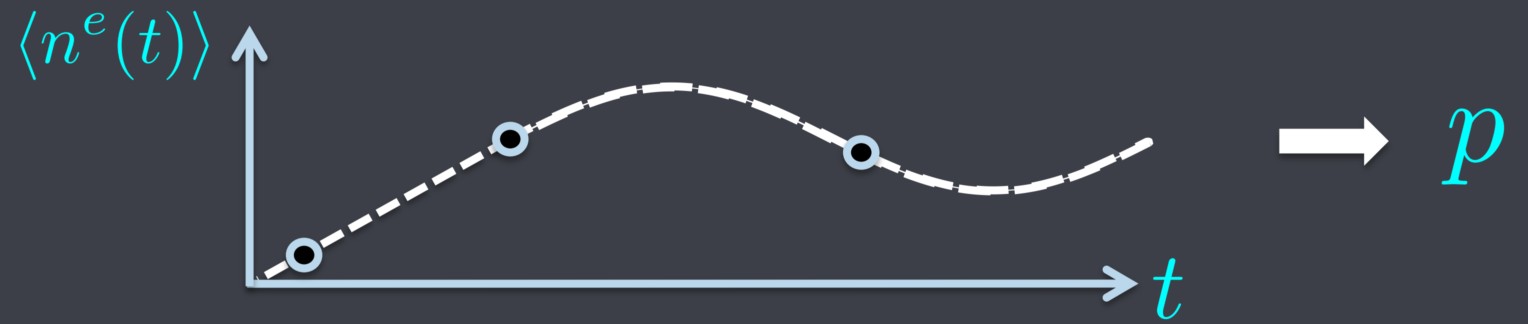
$$V_\beta = \begin{pmatrix} \cos\beta & i\sin\beta \\ i\sin\beta & \cos\beta \end{pmatrix}$$

operationally:

$$\langle n^e(t)\rangle = \mathrm{Tr}[n^e V_\beta^\dagger \mathrm{e}^{-itH} V_\beta (\rho \otimes |g\rangle\langle g|)^{\otimes n} V_\beta^\dagger \mathrm{e}^{itH} V_\beta]$$

USING A HIGH SYMMETRY HAMILTONIAN (OUR APPROACH)

repeat for multiple times



$$\langle n^e(t)\rangle = \mathrm{Tr}[n^e V_\beta^\dagger \mathrm{e}^{-itH} V_\beta (\rho \otimes |g\rangle\langle g|)^{\otimes n} V_\beta^\dagger \mathrm{e}^{itH} V_\beta]$$
$$= \frac{\sin^2\beta}{2}\left[1 - \sum_{i=1} p_i \cos(\omega_i t)\right] + \mathcal{O}(\frac{1}{\sqrt{n}})$$

(representation-theoretic calculation)

fit against data to estimate p

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CONCLUSIONS

- experimentally accessible (coauthor G. Campbell preparing to implement)
- spectrum estimation useful in this system e.g., entanglement of simulators

OPEN QUESTIONS

- same system for general Schur transform?
- natural systems for other small quantum computing tasks?
- physical systems for special-purpose fault tolerant scalable tasks?