Complexity Classification of Commuting Two-qubit Hamiltonians

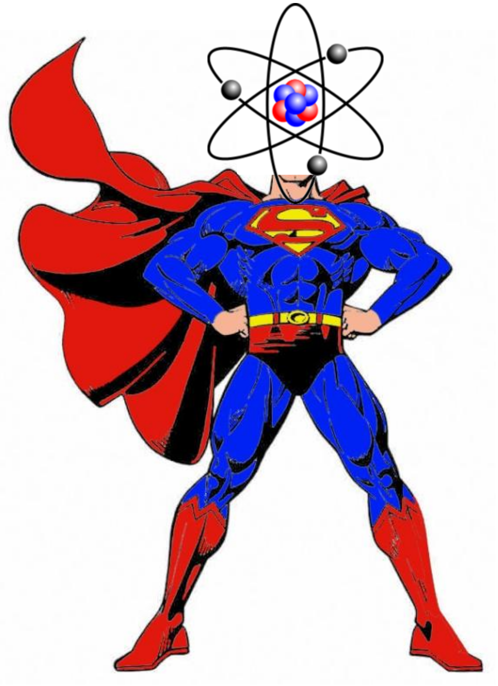
Adam Bouland

Based on joint work with Laura Mančinska and Xue (Lucy) Zhang

arXiv: coming soon!

Establishing Quantum Advantage

- Decision Problems
  - Shor's algorithm
- Sampling Problems
  - Boson Sampling [Aaronson Arkhipov]
  - IQP [Bremner Jozsa Montanaro Shepherd]
  - Many others [Knill LaFlamme]
    [Morimae Fuji Fitzsimons][Fefferman Umans]…



This work: Classify when you get quantum supremacy for sampling

Model

$$H = \begin{pmatrix} a & b & c & d \\ b^* & e & f & g \\ c^* & f^* & h & j \\ d^* & g^* & j^* & k \end{pmatrix}$$

$$|0\rangle^{\otimes n/2}|1\rangle^{\otimes n/2} \longrightarrow$$

Apply +/-H to arbitrary pairs of qubits

$\downarrow$

Measure in standard basis

Goal: Classify which H give you advantage over classical computation

Universality

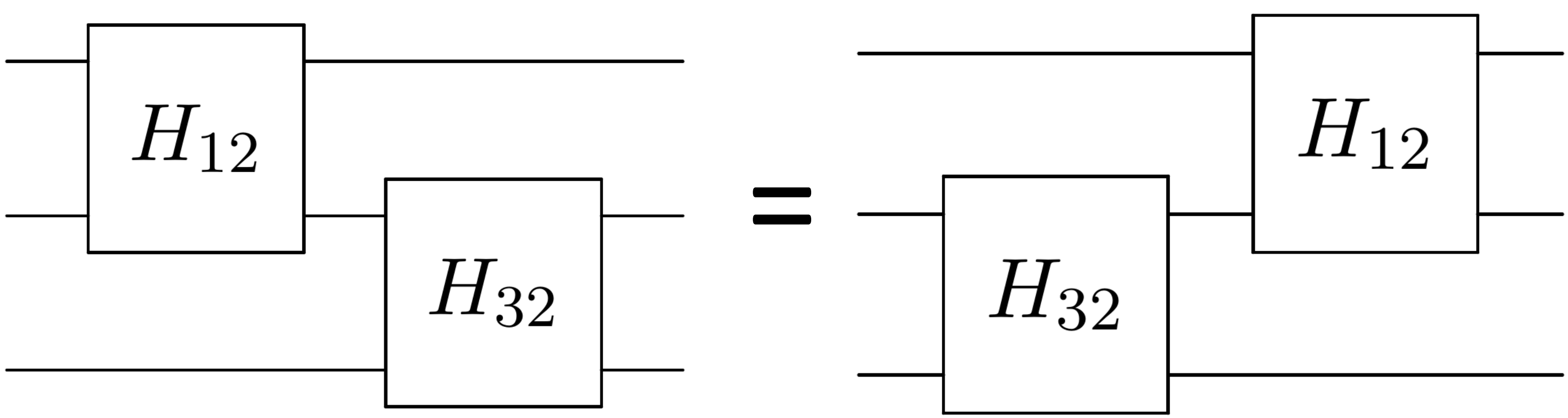
We don’t even know which 2-qubit H are universal for quantum computing!

This work

Classical Simulability

We classify the power of **commuting** 2-qubit Hamiltonians

$$[H_{ij}, H_{kl}] = 0$$

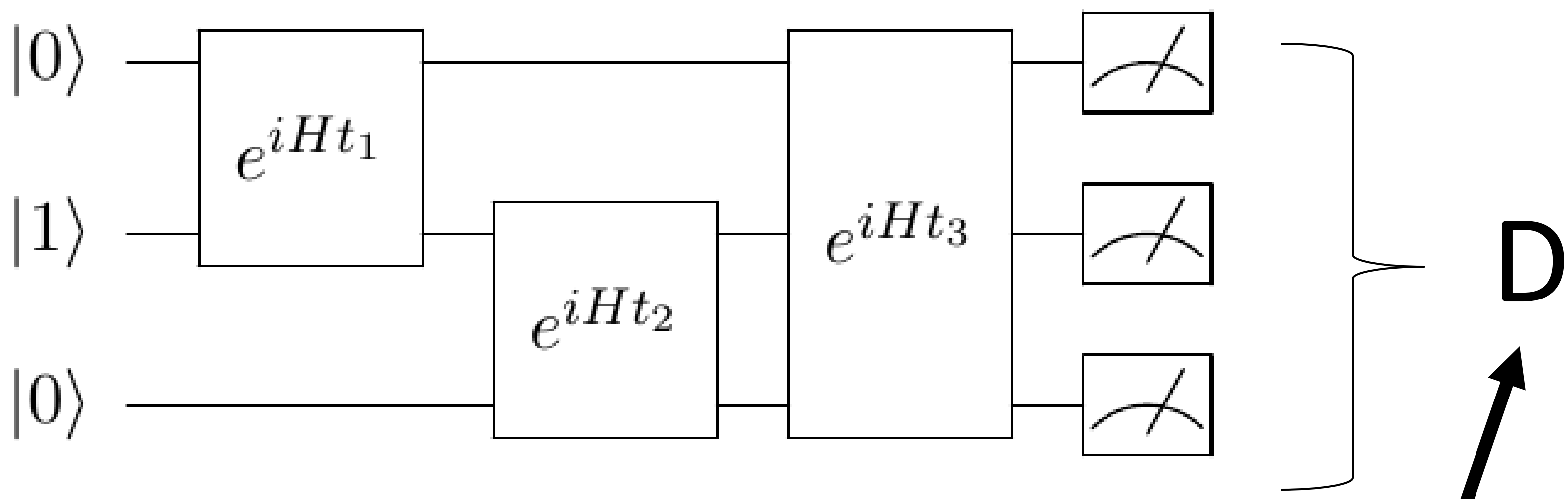


Main Result:
Dichotomy + Classification

For any 2-qubit commuting H:

- **If H generates entanglement, then it allows to you perform hard sampling problems**
- Otherwise, H is efficiently classically simulable

Hard Sampling Task



Hard to sample from classically

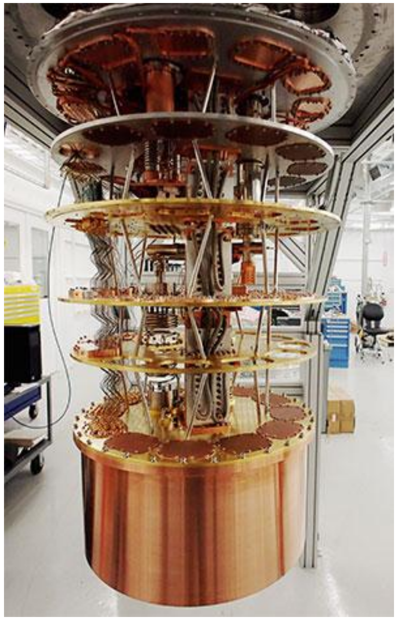
Hard to Sample

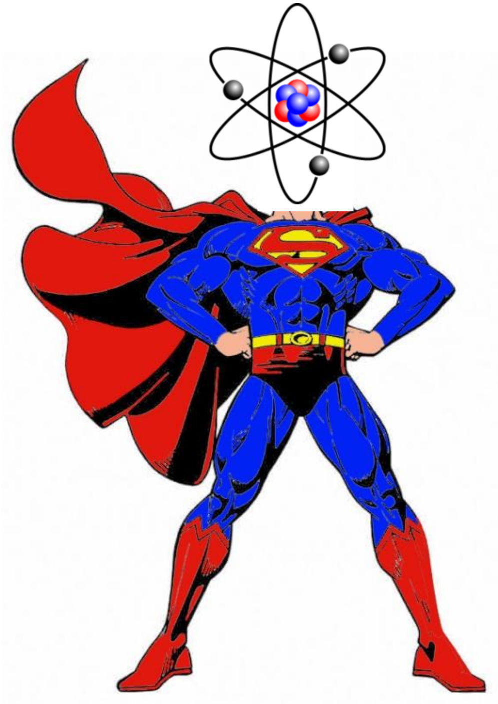
There does not exist a randomized classical algorithm M satisfying

$$\frac{1}{\sqrt{2}} \Pr[M \text{ outputs } y] \leq \mathcal{D}(y) \leq \sqrt{2}\, \Pr[M \text{ outputs } y]$$

Assumption: The polynomial hierarchy doesn't collapse

Why this matters





Lower fault-tolerance thresholds
[Aliferis et al. '09]

Relation to prior work

Previously: Knew **some** commuting H allow you to perform difficult sampling tasks


This work: **All** commuting H (other than non-entangling ones) allow you to perform difficult sampling tasks

Proof Outline

Technique: Show **postselected** circuits with H are universal for Quantum Computing

$\Rightarrow$ hardness of sampling by known techniques

Proof Outline

- Postselected commuting circuits = BQP

  -> Postselected commuting circuits = PostBQP
- If you can simulate
  - Postselected simulation can solve PostBQP
  - BUT PostBPP != PostBQP

    [Stockmeyer ‘83, Toda ‘91, Aaronson ‘05,
    Bremner Jozsa Shepherd ’11, Aaronson & Arkhipov ‘13]

Not possible to simulate □

Goal: Postselected Universality

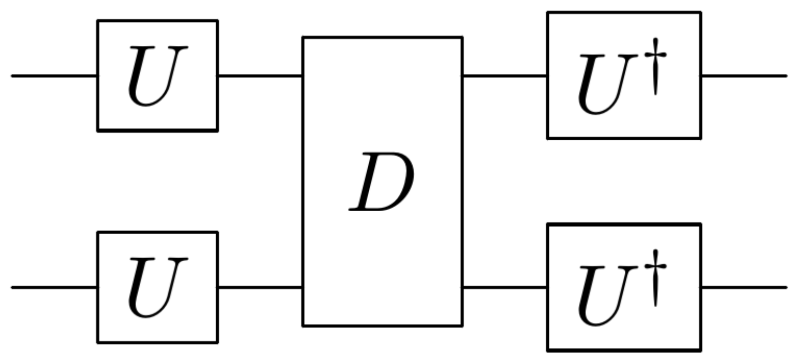
- 1-qubit gates + any entangling Hamiltonian is universal [Dodd et al. '02, Bremner et al. '02]

To complete proof: Get all 1-qubit gates under postselection

Goal: 1 qubit gates

If H is commuting, then

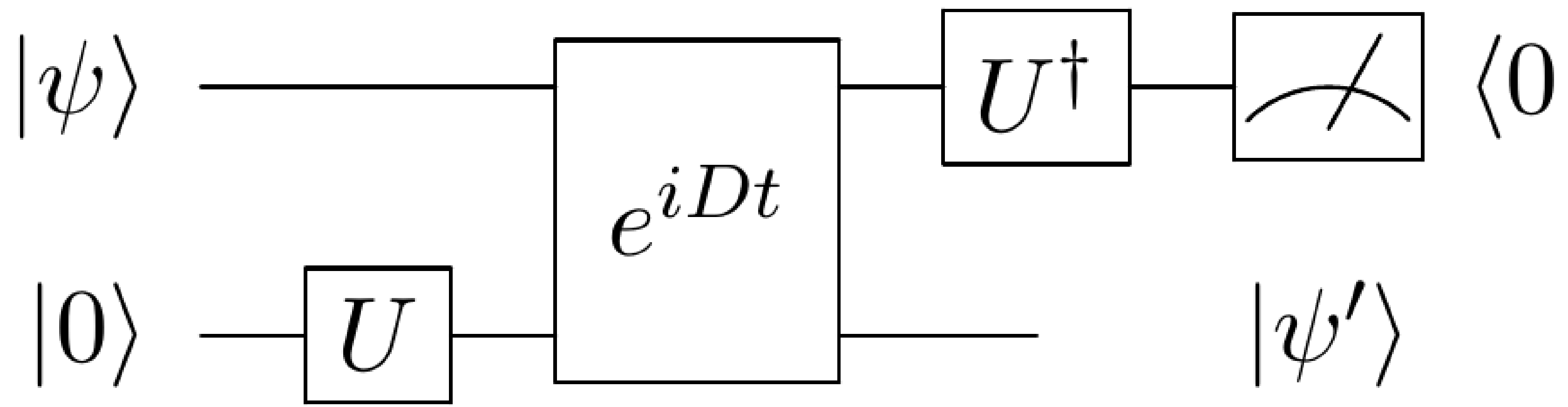
H=



$$U = \begin{pmatrix} \alpha & \beta \\ -\beta^* & \alpha^* \end{pmatrix} \qquad D = \begin{pmatrix} a & 0 & 0 & 0 \\ 0 & b & 0 & 0 \\ 0 & 0 & c & 0 \\ 0 & 0 & 0 & d \end{pmatrix}$$

Goal: 1 qubit gates

Postselection gadget:



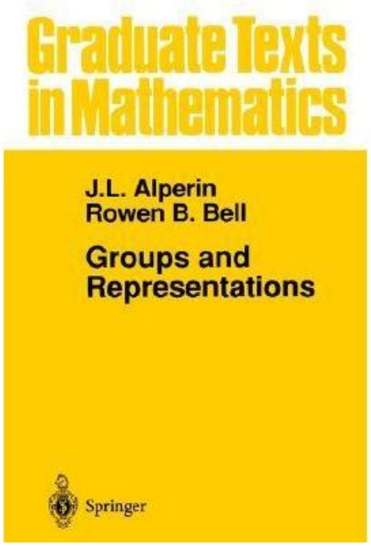
$$L(t) = \frac{1}{|\alpha||\beta|\sqrt{-2i\sin(2t)}} \begin{pmatrix} |\alpha|^2 e^{ia't} & \alpha\beta^* e^{it} \\ \alpha^*\beta e^{it} & |\beta|^2 e^{id't)} \end{pmatrix}$$

Goal: 1 qubit gates

Suffices to show can perform all 1-qubit operations using products of L(t)'s

$$S = \overline{< L(t)'s >}$$

S is a group



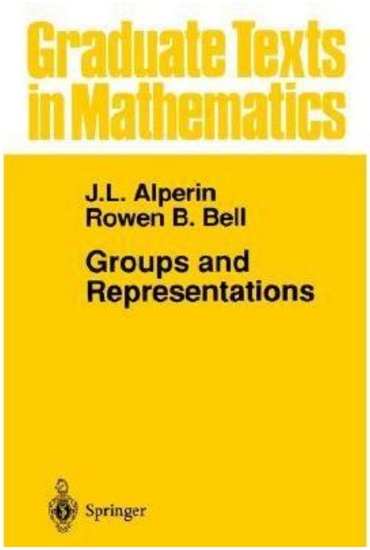
Goal: 1 qubit gates

Suffices to show can perform all 1-qubit operations using products of L(t)’s

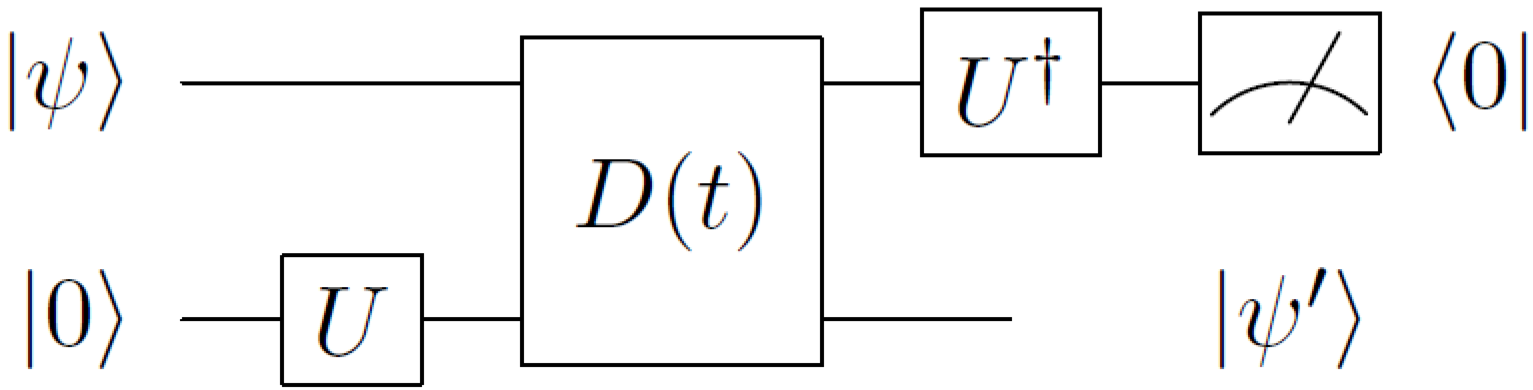
$$S = \overline{< L(t)'s >}$$

~~S is a group~~

Inverses?

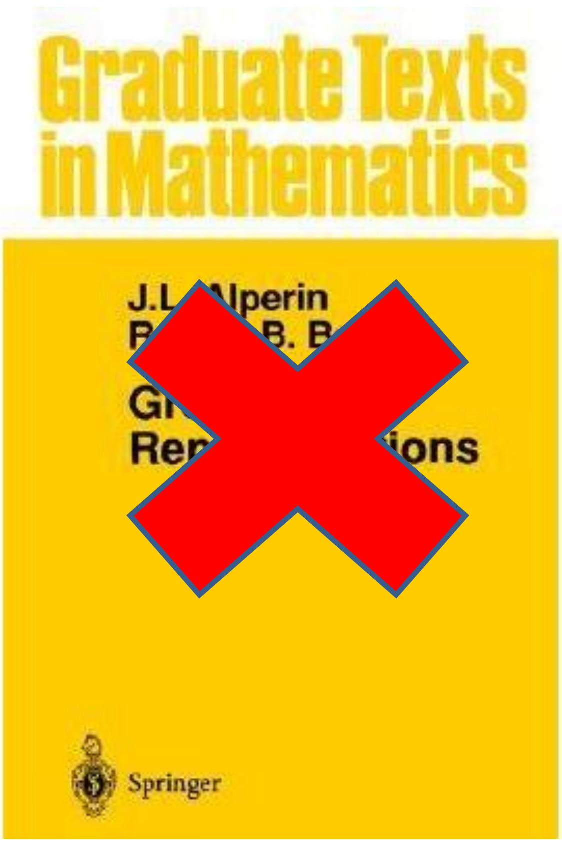


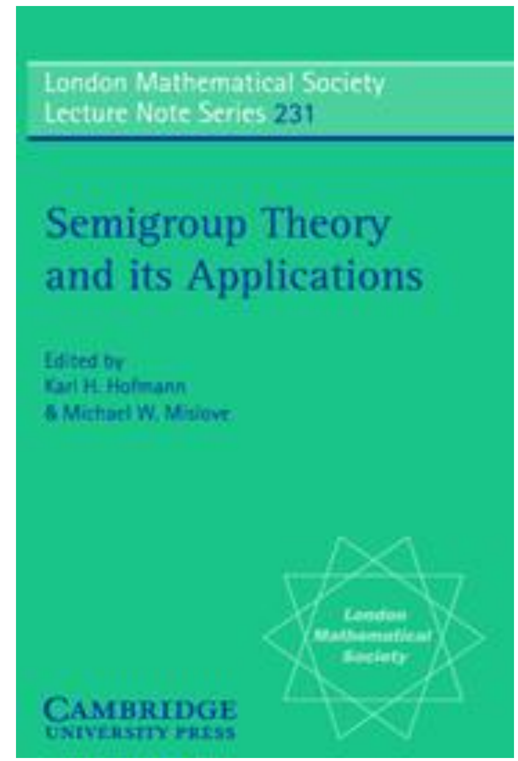
Goal: 1 qubit gates



How do you invert postselection?

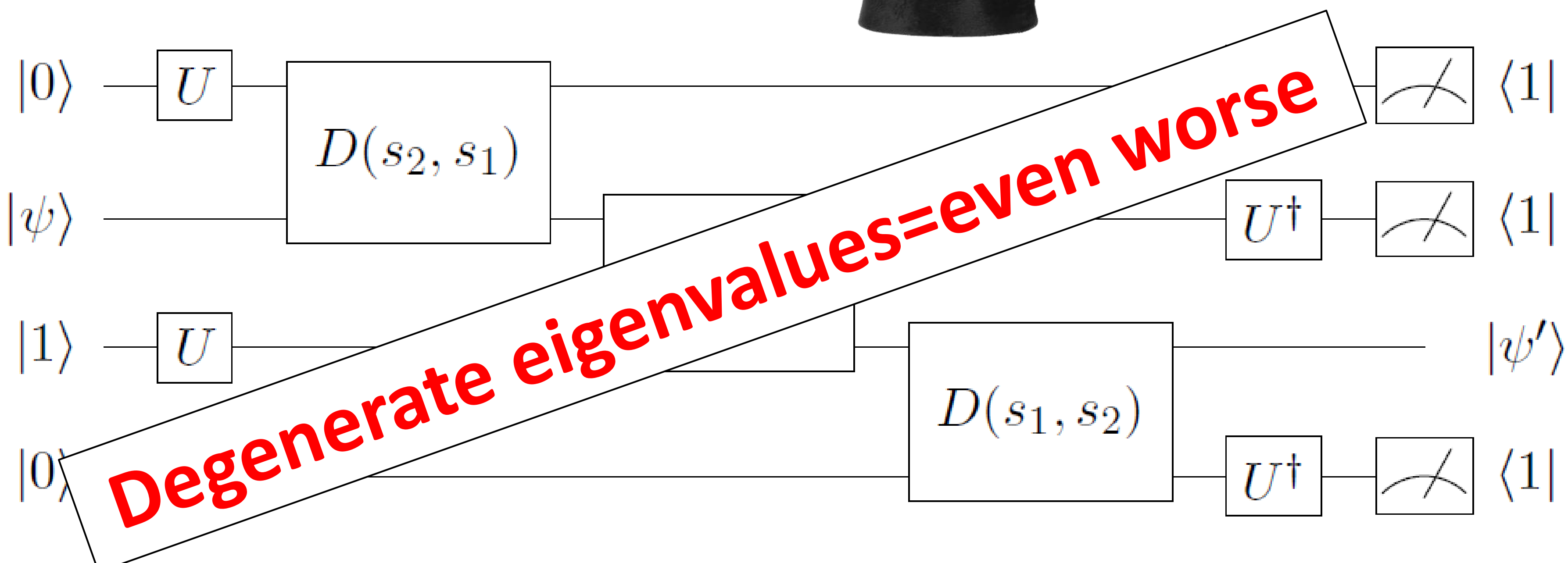
Goal: 1 qubit gates





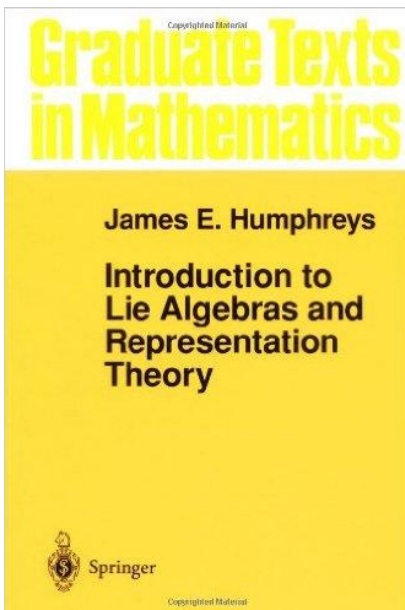
Goal: 1 qubit gates

$L(t)^{-1}$ =



Goal: 1 qubit gates

L(t)'s (and their inverses) form a group



& densely generate SL(2,C) □

Last case

This works for all entangling H **except**

a=d=-1, b=c=1, |α|=|β|

$$H \sim \sigma_x \otimes \sigma_x$$

Prior work: Hard because can embed permanents in output distribution



Open problems

- Complete the classification!
- Extend hardness to L1 error
- Classify commuting gate sets

Open: Classify subgroups of SU(8)

Open: Classify subgroups of SL(2,C)

Thanks!

Questions?