Witnessing the complexity of multiqubit states

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It is well known that the classical description of a generic quantum state requires exponentially many parameters in the number of parties. For instance, a generic state of n qubits

$$|\Psi\rangle = \sum_{x=0}^{2^n-1} c_x |x\rangle \tag{1}$$

requires *a priori* 2^n complex coefficients for a choice of a computational basis. Clearly, not all states possess this exponential complexity: product states, for instance, can be described with at most $2n$ complex numbers; and the class of matrix product states, which approximate well the ground state of a large class of many-body hamiltonians, requires only polynomially-many parameters. But complex states are definitely important, and not only because there are overwhelmingly many of them if one were to draw states randomly from a uniform distribution. It is notably believed that complex states are a necessary resource for quantum computers to overpower classical ones; this intuition has been validated by rigorous proofs for some models of computation and quantification of complexity [1–3].

Tree size (TS) is a measure that captures the complexity of a multiqubit state in the above sense: It can be understood as the minimal number of parameters required for describing a given state with bra-ket representation. TS is an interesting measure because not only is it in principle computable, but one can also obtain nontrivial lower bounds for it [4, 5]. In this way, it has been possible to identify explicit families of complex states whose TS scales superpolynomially in the number of qubits [3]. However, the problem of how one can test that a pure quantum state has indeed superpolynomial TS remains to be addressed. Quantum state tomography is not feasible as it requires an exponential number of measurements. We report here the finding of a complex state that can be prepared with a polynomial quantum circuit; and more importantly, its superpolynomial TS can be verified efficiently. The verification is based on measuring a witness, which requires only a polynomial number of elementary operations.

Tree-size complexity is also relevant to the "speed up" quantum computation offers over its classical counterpart. It is likely that this speed up is achieved only when the state has superpolynomial TS at some step during the computation. A remarkable result in this direction is that the state in Shor's algorithm has superpolynomial TS [4]. We find that the same holds for the states in Deutsch-Jozsa's algorithm. Moreover, we show that measurement-based quantum computation can be simulated efficiently with classical computers if the TS of the resource state is only a polynomial in the number of qubits [3].

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