Fault-tolerant Quantum Computation in Multi-qubit Large Block Codes

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Quantum computers are extremely vulnerable to noise during computation. They are more sensitive to error than classical computers because quantum information is subject to more types of errors and because thermodynamic means to suppress errors, widely used in classical computers, cannot be used in quantum computers without destroying the superpositions and interference effects on which quantum algorithms depend. Theory has shown that if errors of each type are sufficiently local, and their rates are small enough to fall below a threshold, it is possible to carry out quantum computations of arbitrary size with arbitrarily small error, by so-called fault-tolerant methods [1, 2]. Since the various threshold theorems have been established, a number of fault-tolerant schemes have been proposed, including those introduced in [3, 4, 5]. These fault-tolerant schemes protect quantum information from decoherence by encoding it in quantum error-correcting codes [3], and replacing the quantum gates of an ideal quantum circuit by fault-tolerant circuits that effect the desired quantum gates without ever decoding the quantum information.

Most of these schemes fall into two broad categories: *concatenated codes* and *topological codes*. In the first group, logical qubits are encoded in separate code blocks, generally constructed by concatenating fairly small one-qubit codes. The size of these code blocks grows exponentially with the levels of concatenation, but so does their minimum distance; provided that the underlying physical error rate is sufficiently low below the error threshold, the probability of an uncorrectable error falls off doubly exponentially. This means that asymptotically the overhead for such schemes scales only modestly with the size of the computation, but the minimum overhead can be very large.

In the second group of fault-tolerant schemes, logical qubits are encoded into topological codes that encode multiple logical qubits into one or more code blocks laid out as a lattice (usually two-dimensional) of qubits. The minimum distance of these codes typically scales with the linear size of the lattice. These codes have topological properties that make them robust against local errors.

All schemes require relatively low rates of error, though the ability to tolerate errors has slowly been improved by a long string of theoretical developments. Most also require a large amount of overhead–in some cases, a very large amount. A logical qubit is encoded in hundreds or thousands of physical qubits, if not more. It was long ago observed (in particular, by Steane [6]) that block codes encoding multiple qubits can achieve significantly higher rates for the same level of protection from errors. But performing logical gates in such codes is quite difficult. Surface codes have special

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properties to avoid this problem, and can encoded multiple logical qubits in a single block; these schemes are very promising. But the topological structure of the codes means the logical qubits are spatially separated in a way that keeps the code rate low.

Herein, we propose a new scheme that exploits the advantages of large block codes which encode multiple qubits to store quantum information during quantum computation, and these code blocks are called *memory blocks*. Our scheme also involves several *processor blocks*, which are quantum codes suitable for the implementation of different logical gates. All these quantum codes would be Calderbank-Shor-Steane (CSS) codes [7, 8] corrected by Steane syndrome extraction [9]. By varying the particular choice of ancilla states used in the procedure, it is possible to at the same time measure the logical operators of the code. Sequences of such logical operator measurements enable the performance of arbitrary Clifford gates, and also allow teleportation of logical qubits between code blocks, even between different codes. Universal quantum computation can be achieved by teleporting logical qubits from the memory blocks into processor blocks that allow an encoded T gate by a transversal circuit. This process is illustrated in Fig. 1.

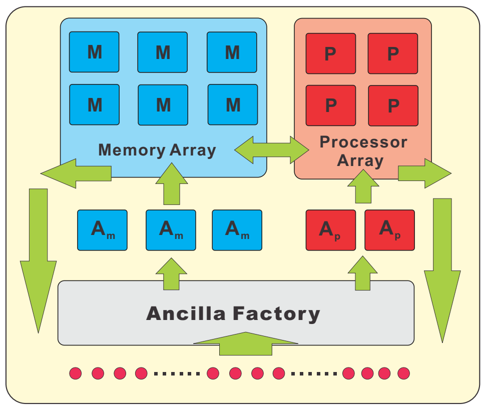

Figure 1: Architecture of Teleporation-based FTQC. Ancilla factory continuously products fresh and clean block of ancilla used for the process of Steane error extraction, logical state measurement and logical state teleporatation.

The third type of building blocks in our scheme are the *ancilla factories* that produce the logical ancilla states needed for logical teleportation [10]. Because these ancillas are stabilizer states, this distillation procedure is more like entanglement distillation [11] than like magic state distillation [12]. Magic state distillation must generically be applied iteratively, and states where the distillation step fails must be discarded; distilling stabilizer states can be done in one step, and errors in the states can be corrected. So the yield of such a procedure is reasonably high. Because it is most efficient to distill ancilla states in group in the ancilla factories and distribute them to the codewords, it might be most efficient to restrict the number of different types of ancillas that are used, even at cost of somewhat increasing the complexity of the logical circuit.

At a first-step research, we consider independent depolarizing errors in the quantum circuits and the types of errors under consideration are: memory errors in the memory code blocks, errors during gate operations, errors in ancilla preparation, and the measurement errors. Moreover, we derive effective error models that greatly simplify the process of simulation, given errors in a block are not correlated. For Steane syndrome extraction and logical state measurement, the errors will have effects only on the data block, as shown in Fig. 2.

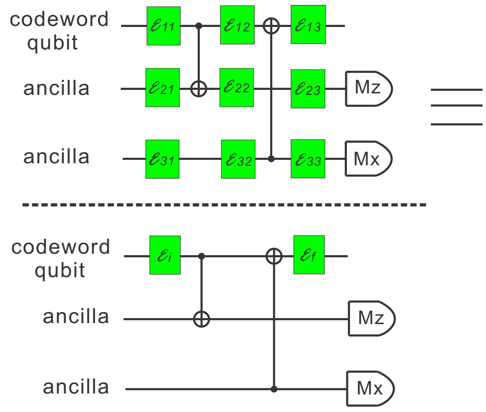

Figure 2: Effective errors of the building block of the scheme.

For demonstration, we simulate two kinds of memory blocks: the $[[2047, 23, 77]]$ and $[[2921, 57, 77]]$ CSS codes, obtained by concatenating the $[[23, 1, 7]]$ quantum Golay code with the $[[89, 23, 11]]$ and $[[127, 57, 11]]$ quantum BCH codes [13], respectively. As for the processor blocks, we choose the concatenated $[[15, 1, 3]]$ truncated Reed-Muller code that allows a transversal T gate [14]. As shown in the following two figures, the logical error rates drop to below 10^{-10} around effective error rate $p_{eff} = 0.01$, or physical error rate 0.001, which is good enough for typical quantum algorithms. Therefore, our scheme marks a promising direction for fault-tolerant quantum computation. We also discuss other variations of the scheme and possible future work.

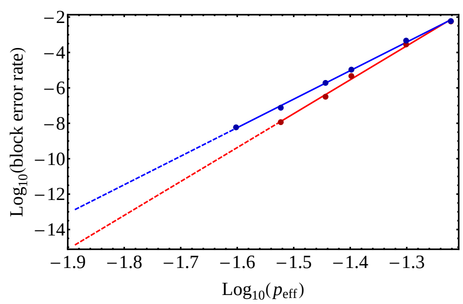

Figure 3: Logical error rates of the [[2047,23,77]] code (blue) and [[2921,57,77]] codes (red) versus p_{eff}.

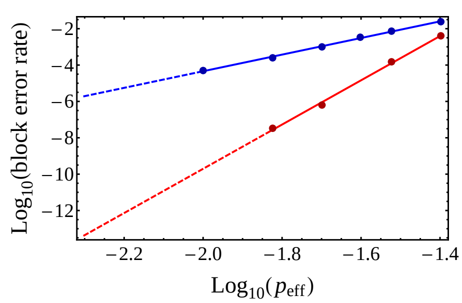

Figure 4: Logical error rates of the concatenated $[[15, 1, 3]]$ code of two levels (blue) and three levels [[15,1,3]] code (red) using soft-decision decoding.

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