Polar codes in network quantum information theory

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Abstract

Polar coding is a method for communication over noisy classical channels which is provably capacity-achieving and has an efficient encoding and decoding. Recently, this method has been generalized to the realm of quantum information processing, for tasks such as classical communication, private classical communication, and quantum communication. In the present work, we apply the polar coding method to network quantum information theory, by making use of recent advances for related classical tasks. In particular, we consider problems such as the compound multiple access channel and the quantum interference channel. The main result of our work is that it is possible to achieve the best known inner bounds on the achievable rate regions for these tasks, without requiring a so-called quantum simultaneous decoder. Thus, our work paves the way for developing network quantum information theory further without requiring a quantum simultaneous decoder.

1 Introduction

One of key features distinguishing the theory of quantum physics from its classical counterpart is that of the problem of simultaneous measurement of non-commuting observables. Indeed, the uncertainly principle captures one of the most profound characteristics of quantum mechanics, that is the impossibility to simultaneously measure non-commuting operators to arbitrary precision, and the principle itself is considered a cornerstone of modern physics.

In quantum communication theory, the problem of simultaneous measurement arises for multi-user communication models when one needs to simultaneously measure, or decode, two or more possibly non-commuting output states of a quantum channel in order to achieve the maximum rate of communication. The problem of simultaneous decoding in the quantum setting manifests itself in particular in the difficulty in constructing a measurement operator achieving this task and its existence has remained a conjecture.

In contrast to simultaneous measurement, building decoders based on the measuring outputs of two or more users of a channel successively, i.e. successive decoding, has been successfully implemented in the quantum setting and moreover a coding strategy based in successive decoding has been shown to achieve the maximum communication rate region for the classical-quantum multiple access channel (cq-MAC), with the help of the gentle measurement lemma [1] bounding the measurement disturbance of quantum states.

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Despite this success with successive decoding and the multiple access channel, achieving the rate region for other multi-user quantum channels has remained elusive. An important example of such a channel is provided by the interference channel. In this model two or more senders wish to communicate information simultaneously, and solely with their intended corresponding receiver by means of a noisy channel modeled by cross-talk, or interference. In the classical setting the capacity for interference channel is known exactly only in the case of very strong interference [2].

The best known achievable rate region for the two-user classical interference channel is given by the Han-Kobayashi region [3]. The coding strategy which achieves this region relies on the simultaneous decoding of two 3-user multiple access channels. The Han-Kobayashi rate region has also been shown to be achievable for the classical-quantum interference channel, that is an interference channel with classical inputs and quantum outputs, based on the conjectured existence of a three-sender simultaneous decoder [4, 5], and it was shown in [6] that the region can be achieved using a specialized 3-user quantum simultaneous decoder. This result raised the question of whether this region can be achieved using a successive decoder [7].

In this article we address this question by exploiting the recently introduced polar coding technique for the classical symmetric binary-input memoryless channel [8]. Indeed, polar codes have attracted a great deal of attention as the first *constructive* capacity-achieving codes and the first known codes to achieve capacity with an efficient encoding and decoding. In the quantum setting efficiency has only been shown in general for encoding of classical-quantum communication [9]. However an efficient encoding and decoding scheme has been shown for certain quantum channels in the case of quantum communication [10, 11].

Recently the polar coding technique has been applied to a variety of multi-user classical channels including the multiple access channel [12, 13, 14, 15], broadcast channels [16, 17], interference networks [18] and for the task of source coding [19] and universal coding for compound channels [20, 21, 18]. In the quantum setting polar coding has been generalized to the case of single user quantum channels for the task of sending classical [22, 23] and quantum information [24, 25].

In this work we show that polar coding can also be applied to the cq-MAC to achieve every point in the known achievable rate region [1] and also that an approach for universal polar codes from [20] can be used to obtain achievable rates for the compound cq-MACs. Indeed compound channels form a class of channels with so-called channel uncertainty. In this model, a channel is chosen from a set of possible channels and used to transmit the information, thus generalizing the traditional setting where both sender and receiver have full knowledge of the channel before choosing their code. The classical and quantum capacites have been studied in [26, 27], respectively. Here we apply the results obtained for compound cq-MACs in a way similar to [18] to achieve the Han-Kobayashi rate region for the two-user classical quantum interference channel using a successive cancelation decoder.

In particular, we emphasize that this is achieved without the use of a quantum simultaneous decoder. The interference channel model forms a basis from which other multi-user channels can be built. This result and the wide range of problems for which polar coding has been applied in classical information theory suggest that we can generalize a wide range of problems to the classical-quantum setting using a successive decoder and in particular without the need of a quantum simultaneous decoder.

2 Summary of results

Polar codes form the first constructive capacity achieving codes and since their introduction [8] they have been successfully applied to various communication models including discrete memoryless channels for the transmission of classical [8], classical-quantum [23] and quantum information [24]. In this work we apply the technique to network quantum information theory and in particular to compound classical-quantum multiple access and interference channels. Indeed, with the use of polar coding we obtain optimal transmission rates for the compound classical-quantum multiple access channel. Moreover, we exploit this coding technique and, in particular, the use of a successive cancellation decoder to obtain the best known achievable rate region for the two-user classical-quantum interference channel given by the Han-Kobayashi rate region. In particular we achieve this result without the use of a simultaneous decoder. Indeed the lack of a quantum simultaneous decoder is considered a major hindrance to the development of network quantum information theory, with many problems in the classical setting requiring a simultaneous decoder. In showing that a simultaneous decoder is not required for the interference channel we have opened up the possibility for outstanding problems in network quantum information theory to be solved.

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