

When Does Nonlocality Distillation Outperform Entanglement Concentration?

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1 Motivation and Impact

Assuming access to only a few copies of a noisy entangled quantum state, we ask the question whether an optimal entanglement concentration protocol also serves as an optimal nonlocality distillation protocol. In the commonly considered framework for entanglement concentration, it is assumed that many identical copies of a noisy entangled state are readily available. If the number of copies is large enough, multiple copies of perfect Bell states can be obtained from the entanglement concentration protocol. As a consequence, the issue of comparing the distillation rate of entanglement and nonlocality concentration protocols does not come up. If the number of copies we have available are so small that not even a single perfect Bell state can be deterministically obtained, then it is not known which of the two; entanglement or nonlocality distillation, maximizes the CHSH [3] violation.

Much of the recent work on nonlocality distillation has focused on the box world framework [5], [1], [7], [6]. Nonlocality distillation for noisy entangled states on the other hand is a more practical scenario and considered initially under the notion of collective measurements by Peres [10] and improved upon by Liang and Doherty [9]. It is not known whether the proposed protocols are optimal. We utilize the formulation of nonlocality distillation as a semi-definite optimization problem based on Tsirelson's vectorization [12] to determine whether the optimal nonlocality distillation protocol is an optimal entanglement concentration protocol. If the answer turns out to be negative then it would reinforce the notion that entanglement and nonlocality are different resources [2]. If we have only a few copies of an entangled state in the lab then considering which protocol to apply would depend on the task at hand.

The main difficulty we encounter is that the SDP obtained via Tsirelson's vectorization is state independent. We need to construct additional constraints, similar to the SDP for quantum nonlocal boxes [8], to obtain the feasible region corresponding to the specific quantum state being considered. Given this new complexity, why should we expect our SDP approach to work, when those employed previously [9] did not yield an optimality proof? Given that we can construct the dual for our primal SDP allows us to numerically verify whether an optimal solution has been obtained. This information can then be used to obtain an analytic solution, similar to our earlier approach [8]. In addition, unlike the situation in the box world, the resulting

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SDP for shared entanglement encapsulates adaptive protocols as well. This is because while in the latter case it is possible to initialize the players with n copies of an identical state and move the adaptive structure of the protocol into the observables, the operation of the boxes in the former case cannot be captured by local observables.

2 Framework

Consider Alice and Bob in their spatially separated labs, with only a few copies of noisy entangled states between them. Let the states $|\psi\rangle$ and $|\phi\rangle$ be the Bell states given by,

$$|\psi\rangle = \frac{1}{\sqrt{2}} (|00\rangle + |11\rangle) \quad \text{and} \quad |\phi\rangle = \frac{1}{\sqrt{2}} (|01\rangle + |10\rangle).$$

For the moment we consider nonlocality distillation for the mixed state ρ given by

$$\rho = p|\psi\rangle\langle\psi| + (1-p)|\phi\rangle\langle\phi|.$$

For a single mixed state we define six vectors, one vector for each of Alice's two observables A_0 and A_1 , two for Bob's observable B_0 , and two vectors for Bob's observable B_1 ,

$$\begin{aligned} x_0 &= (A_0 \otimes \mathbb{1})|\psi\rangle & y_0 &= (\mathbb{1} \otimes B_0)|\psi\rangle & z_0 &= (\mathbb{1} \otimes B_1)|\psi\rangle \\ x_1 &= (A_1 \otimes \mathbb{1})|\psi\rangle & y_1 &= (\mathbb{1} \otimes \sigma_x B_0 \sigma_x)|\psi\rangle & z_1 &= (\mathbb{1} \otimes \sigma_x B_1 \sigma_x)|\psi\rangle. \end{aligned}$$

Let $G = [g_{ij}]$ be the Gram Matrix of the six vectors $\{x_0, x_1, y_0, y_1, z_0, z_1\}$,

$$G = \begin{pmatrix} x_0 \cdot x_0 & x_0 \cdot x_1 & x_0 \cdot y_0 & x_0 \cdot y_1 & x_0 \cdot z_0 & x_0 \cdot z_1 \\ x_1 \cdot x_0 & x_1 \cdot x_1 & x_1 \cdot y_0 & x_1 \cdot y_1 & x_1 \cdot z_0 & x_1 \cdot z_1 \\ y_0 \cdot x_0 & y_0 \cdot x_1 & y_0 \cdot y_0 & y_0 \cdot y_1 & y_0 \cdot z_0 & y_0 \cdot z_1 \\ y_1 \cdot x_0 & y_1 \cdot x_1 & y_1 \cdot y_0 & y_1 \cdot y_1 & y_1 \cdot z_0 & y_1 \cdot z_1 \\ z_0 \cdot x_0 & z_0 \cdot x_1 & z_0 \cdot y_0 & z_0 \cdot y_1 & z_0 \cdot z_0 & z_0 \cdot z_1 \\ z_1 \cdot x_0 & z_1 \cdot x_1 & z_1 \cdot y_0 & z_1 \cdot y_1 & z_1 \cdot z_0 & z_1 \cdot z_1 \end{pmatrix},$$

and set W to be the weight matrix

$$W = \begin{pmatrix} 0 & 0 & p & q & p & q \\ 0 & 0 & p & q & -p & -q \\ p & p & 0 & 0 & 0 & 0 \\ q & q & 0 & 0 & 0 & 0 \\ p & -p & 0 & 0 & 0 & 0 \\ q & -q & 0 & 0 & 0 & 0 \end{pmatrix}.$$

The simple but key distinction from the primal SDP for quantum nonlocal boxes is that we do not need to flip the entries in matrix W corresponding to input 11 i.e., the entries with a minus sign. This results in simpler solution matrices G . We may state the primal SDP as follows.

$$\begin{aligned} & \max_G \frac{1}{2} \text{Tr}(GW) \\ & \text{subject to } G \succeq 0 \\ & g_{ii} = 1 \text{ for all } i \in \{1, \dots, 6\} \\ & y_{|s|} \cdot z_{|t|} = y_{|s'|} \cdot z_{|t'|} \text{ if and only if } s \oplus t = s' \oplus t', \end{aligned} \tag{1}$$

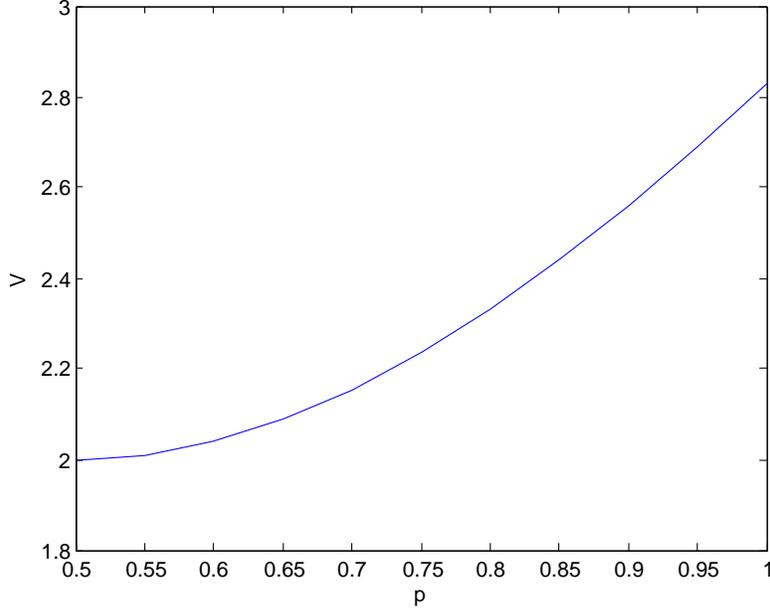


Figure 1: Plot of primal solutions values for $\frac{1}{2} \leq p \leq 1$ and $n = 1, 2$. The expression for the curve is given by $2\sqrt{1 + (2p - 1)^2}$

The inner product constraints for the single copy correspond to the requirement that $y_0 \cdot z_0 = y_1 \cdot z_1$ and $y_0 \cdot z_1 = y_1 \cdot z_0$. *These constraints are not sufficient to obtain a valid quantum solution.* Using cvx to solve the above primal results in the solution value $V = 2\sqrt{2}$, regardless of the noise value p we choose for the quantum state. *We need more constraints!*

3 Work in Progress

At first glance it appears that we need to add perhaps more constraints of the same type as the existing ones, i.e., equivalence between two entries in the G matrix. The new constraints however must apply to the two entries in the matrix G that have been left unrestricted, i.e., $y_0 \cdot y_1$ and $z_0 \cdot z_1$. We can restrict them using constraints such as Dieks inequality [4]. These however are quadratic inequalities and result in a complicated dual. A simpler approach is to utilize the constraints derived by Popescu and Rohrlich [11] to obtain $y_0 \cdot y_1 = -z_0 \cdot z_1$. It appears that this constraint is sufficient in the sense that the cvx solutions obtained using it match the optimal value we obtain by numerical optimization (Figure 1). Furthermore, such a linear constraint does not lead to a complicated dual. We still need to work out how these constraints generalize as we increase the number of copies.

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