Classical Weak Values and what they tell us about the Quantum Weak Value

Angela Karanjai[1], Stephen Bartlett[1] and Terry Rudolph[2]

[1] *School of Physics, The University of Sydney, Sydney, New South Wales 2006, Australia*
[2] *Controlled Quantum Dynamics Theory, Imperial College London, London SW7 2BW, United Kingdom*

One of the most distinctive features of quantum mechanics is collapse of the wave function due to measurement. This phenomenon puts restrictions on the types of measurements that can be performed on the same state of a particle. Weak measurements, first introduced by Y.Aharonov et al [1], is a measurement procedure in which it is possible to measure the average value of an ensemble of particles, all prepared in the same initial state, with minimal disturbance to the individual particles. Hence, it is possible to measure the average value of a variable 'weakly' and then perform a subsequent measurement on the undisturbed state.

There are two ways in which a measurement can be made weak; firstly, each measurement device can be coupled very weakly to each particle, alternatively, the uncertainty in the initial momentum of each measurement device can be required to be very small. In both these limits the disturbance to each particle, caused due to measurement, is very small. However in these limits, the information acquired from each measurement is also very small. Therefore, this measurement must be carried out on a large number of particles, each prepared in the same initial state, to calculate the average value of a variable A. The average value can be calculated with arbitrary accuracy as the number of particles becomes arbitrarily large.

After the particles in the ensemble are weakly measured, a second measurement of variable B can be performed on the particles (which are still approximately in the same initial state). The second measurement is a projective measurement and hence collapses the state of each particle into an eigenstate of B. One can now select a particular subset of the particles which have all collapsed to a specific eigenstate $|b\rangle$. For these states, we know the value of the B, and we can also calculate the average shift in the measurement devices used to 'weakly' measure A. When a weak measurement is followed by this kind of post selection, then the average shift in the measurement devices is proportional to the real and imaginary parts of a quantity known as the weak value [4].

Since its conception, weak values have been the topic of much debate, owing to the fact that it seeks to analyse the property of particles between two measurements, a situation previously considered counterfactual in quantum mechanics. It is thus useful to study the weak measurement protocol classically, to home in on which effects are quantum and which can be reproduced classically. The aim is to better understand the statistical effects of post selection and to shed light on measurement disturbance. We analyse weak values within epistemically restricted Liouville (ERL) mechanics [2], which has a classical ontology and where particles evolve under classical dynamics. It is particularly useful model to use as it is operationally equivalent to gaussian quantum mechanics.

We first show how we can recover the same information-disturbance tradeoff using the epistemically restricted gaussian phase space distributions evolving under classical dynamics. We demonstrate how each of the two limits manifests classically. We show that despite having the same general effect of 'weakening' the measurement, the reason for weakness in each of the limits is different classically. In the limit of the small coupling the weakness is due to each measurement device interacting weakly with each particle, whereas in the limit of a small uncertainty in the momentum of the device, the interaction between each measurement device and particle is strong, however, the large uncertainty in our information of each measurement is what makes the measurement 'weak'. We find that this difference, which becomes apparent from treating the system classically, is also reflected mathematically in the quantum framework.

We then show that results of weak measurements and the weak value is reproduced classically using ERL mechanics. We find that the real part of the weak value is equal to the conditional expectation value and the shift in the measurement device, proportional to the imaginary part of the weak value is a measure of disturbance to the state and vanishes in the limit of no disturbance. Finally we calculate an upper bound to the coupling strength and the momentum uncertainty for the weak approximation for gaussian states. For gaussian states, this is a much tighter upper bound than the one calculated for all quantum states in [3].

References

[1] Yakir Aharnov, David Z. Albert, and Lev Vaidman. How the result of a measurement on a component of the spin of a spin 1/2 particle can turn out to be a 100. *Physical Review Letters*, 60(14), 1988.

[2] Stephen Bartlett, Terry Rudolph, and Robert W. Spekkens. Reconstruction of gaussian quantum mechanics from liouville mechanics with an epistemic restriction. *arxiv:1111.5057v2*, 2012.

[3] I.M. Duck, P.M Stevenson, and E.C.G. Sudarshan. The sense in which a ”weak measurement” of a spin 1/2 particle's spin component yields a value of 100. *Physical Review D*, 40(6):2112, 1989.

[4] Richard Jozsa. Complex weak values in quantum measurement. *Physical Review A*, 76:044103, 2007.