Autonomous thermal operations

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Abstract

We consider a completely isolated quantum thermal machine, where an explicit clock is used to implement time-dependent transformations on a system, bath, and work storage device. Based on previously known results from the case without a clock system, we construct a new framework for this scenario in which the first and second laws of thermodynamics hold, and optimal work extraction and state transformations are possible. We thus show that including a clock explicitly need not entail an intrinsic thermodynamic cost, nor an ability to violate thermodynamic laws.

Motivation. Recently there has been a great deal of interest in the application of thermodynamics to individual quantum systems, which may be composed of just a few atoms or qubits. Given that thermodynamics was invented before quantum theory was even envisaged, and typically applies to macroscopic objects, it is perhaps surprising how close an analogy can be drawn between the quantum and classical case. In [1], for instance, thermal machines are constructed out of quantum mechanical parts, with an explicit system and thermal bath in which the first and second laws of thermodynamics are obeyed, and optimal work extraction is achievable. In [2], the external work reservoir is replaced with a quantum work storage system represented by a weight. In other approaches [3, 4], the machine is a system with externally-controlled Hamiltonian and access to a thermal bath.

So far, these frameworks all involve the external application of a unitary operation, or a sequence of discrete transformations, to the involved systems. An interesting open question which has been raised for instance in [2, 4] is whether this external control should carry a thermodynamic cost, and how to include this control explicitly in the framework. Here, we address this issue by extending the previous frameworks to include an explicit quantum clock by which the evolution of the system, bath and weight is controlled, and allowing the whole protocol to be carried out via a time-independent global Hamiltonian.

Methods. Consider quantum system s on which a transformation is to be performed (see Fig. 1 a). We use a similar setting to the one used in [2] with an explicit thermal bath, b, at temperature T and a suspended weight, w, that extracts work from or supplies work to the system. This way we can keep track of all the thermodynamic resources, such as heat and work, that are used during the process. In addition we introduce a clock c. This is a quantum system with a continuous-spectrum Hamiltonian that may interact with system, bath and weight, and specifies time in terms of its position $|x\rangle_c$. Initially the different subsystems are in a product state. Furthermore, the clock's initial state must not be infinitely broad in position (Fig. 1 b).

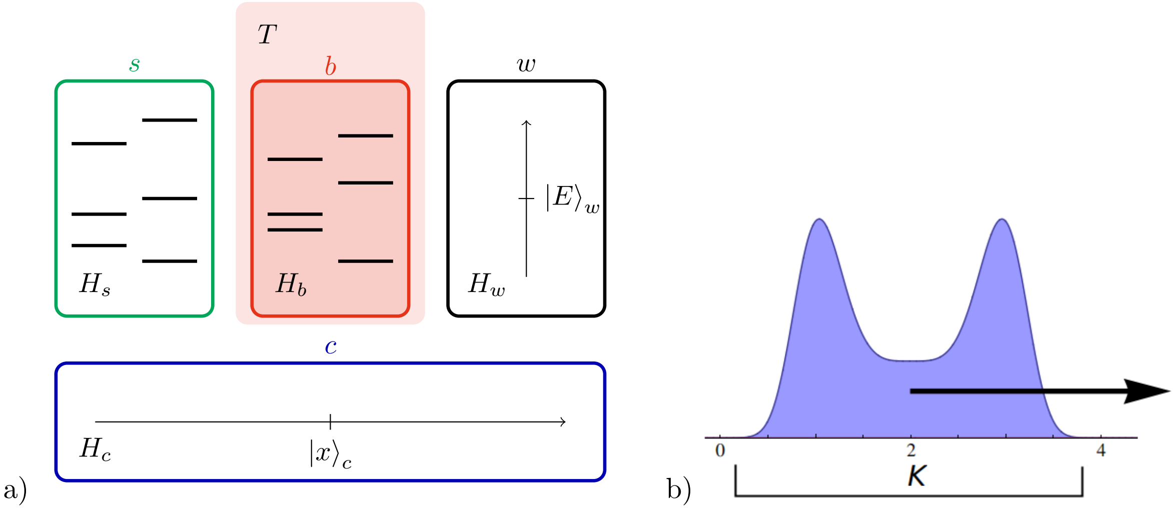

Fig. 1: a) System s, bath b, weight w and clock c with the corresponding Hamiltonians. Initially the bath is thermalized at temperature T. The energy eigenstates of the weight are denoted by $|E\rangle_w$ and the position eigenstates of the clock, which we use to specify time, are denoted by $|x\rangle_c$. b) For a general quantum state of the clock ρ_c we define the support in position by the support of the function $f(x) = {}_c\langle x|\rho_c|x\rangle_c$. The condition on the initial state of the clock is that $\mathrm{supp}(f) \subset [x_0, x_0 + K]$ for some $K > 0$ and x_0. This is simply a sanity assumption as one cannot expect to control the interaction of other systems over time by a clock that has an infinitely wide distribution.

The four different parts s, b, w and c may interact via a global time-independent interaction Hamiltonian. The time evolution is then described by the Schrödinger equation with the time-independent total Hamiltonian composed of the unperturbed Hamiltonians of the subsystems together with the interaction Hamiltonian.

Regarding the interaction Hamiltonian we make a few assumptions: (a) it must be energy conserving with respect to the Hamiltonians of system, bath and weight; (b) it must act on the weight in a translational-invariant way, in particular, the protocol must not depend on the initial energy stored in the weight; (c) the interaction must not disturb the the clock's state. Using these assumptions we are able to show that the first and second laws of thermodynamics hold in our framework.

With the established framework we investigate what state transformations on s are possible, and what their work cost is. Furthermore, we ask in how far the state of the clock is degraded after using it to induce time-varying interactions on $s \otimes b \otimes w$.

Results and Conclusions. We show that our new framework still obeys the first and second laws of thermodynamics, and allows for optimal work extraction without 'degrading' the state of the clock. This is, the final state of the clock is the shifted initial state, where the shift is determined by the execution time of the protocol.

Furthermore, if the weight's initial state has a sufficiently narrow momentum distribution one can achieve any state transformation on s to arbitrary accuracy with a work cost arbitrarily close the free energy difference of initial an final state. This includes transformations involving initial and final states that are not diagonal in the energy basis. Importantly, the momentum distribution on the weight can be shown not to change in the process. Hence, such transformations are able to use the coherences in the weight in a catalytic way – a consequence of the result recently presented in [5].

To conclude, our work shows that including a clock explicitly to induce time-varying interactions on a thermodynamic system need not entail additional consumption of resources. In the proposed framework it is possible to implement the desired time-dependent interaction Hamiltonian using the clock as well as coherences in the weight catalytically.

References

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