Ribbon operators in topologically ordered 2D spin systems

Jacob C. Bridgeman, Eric Huang, and Steven T. Flammia

Centre for Engineered Quantum Systems, School of Physics,

The University of Sydney, Sydney, Australia

David Poulin

Department of Physics, Université de Sherbrooke, Sherbrooke, Québec, Canada

Many interesting models are thought to exhibit topological order, including the Toric code phase, the fractional quantum Hall effect[1] and the antiferromagnet on the Kagome lattice[2, 3]. Understanding these models away from exactly solvable points has proven challenging.

The Toric code has a pair of logical operators, corresponding to topologically nontrivial strings of operators. These are symmetries of the Hamiltonian, mapping ground states to ground states. If these strings do not close, they cease to be symmetry operators, and instead define anyonic excitations at their ends. If we add a small local perturbation, for example a local field or spin interaction, we do not expect a phase change, however finding the string operators, and so understanding the anyons, is a difficult task[4, 5]. More generally, given some Hamiltonian, diagnosing the presence and class of topological order it exhibits is a demanding task. Even if we can compute the topological entanglement entropy, the behaviour of the anyons is unknown. Understanding this is crucial if we wish to utilise this system in a topological quantum computation situation[6].

In this work, we present a numerical algorithm for finding ribbon operators; the analogue of the strings in non-exactly solvable models. The algorithm utilises tensor network techniques to construct these extended operators without requiring any understanding of the ground space of the Hamiltonian in question. We apply this to find ribbons for the Honeycomb model in the nonperturbative regime.

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