

Local and global clocks in quantum mechanics - limitations to building global ordering scales out of two local quantum clocks

Sandra Ranković, Yeong-Cherng Liang, Renato Renner
Institute for Theoretical Physics, ETH Zürich, 8093 Zürich, Switzerland

Time is usually considered as a classical parameter in the laws of motion of non-relativistic quantum-mechanical systems. We take a different, operational approach to time, and towards this goal introduce concepts of local and global quantum clocks, as well as the respective ordering scales. Our two approaches investigate limitations to building global clocks and ordering scales from two spatially-separated quantum clocks, under the assumption that these clocks are locally used for ordering. First approach investigates correlations between local quantum clocks to quantify the level of their synchronisation and a possibility for the construction of a global ordering scale. Second approach, formulated as the Alternate Ticks game, allows for the synchronisation of two quantum clocks as well as the quality of these clocks to be checked by an outside, classical observer.

Ever since the development of quantum theory, time is one of the biggest unknowns of modern physics [1–7]. A notion of local and global clocks, as well as local and global causal structures, is still not well understood in quantum mechanics, and it can be at the heart of many problems involving time in quantum theory [5, 8–11].

Contribution In this work we take an operational approach to the notion of time and ordering on the local and global level in quantum mechanics. To this end, we propose a new framework in approaching the concept of time, with key features being the notions of local and global quantum clocks, as well as local and global ordering scales. As Wootters remarked [12], “coordinate time is not observable, but ‘clock time’ is observable”. We thus restrict our attention to physical systems we use as clocks, correlations between them and time scales obtained thereof. These quantities then allow us to compare the level of synchronisation between different quantum clocks, and the extent to which a global notion of time can be established between them. We construct quantum clocks by keeping only the basic properties that they need to have, but assuming the existence of ordering on the local level. We don’t assume any pre-existing ordering scale on the global level accessible to the clocks and aim to investigate limitations to building global clocks and ordering out of local quantum clocks. We construct two approaches to this problem, and our results suggest that, given only local quantum clocks and some external parametric register not accessible to these clocks, there are always limitations to building global ordering scales.

Related work During the years there have been approaches to look at time as arising from correlations between the physical systems (see e.g. [12–15]). These works have opened a path to different considerations of the notion of time in quantum mechanics. Our first approach bears some similarity to the work of Wootters [12] in that both consider correlations between quantum systems to investigate deeper the concept of time. In [12], author notices that only ‘clock time’ is observable. Author then argues that time can be replaced by correlations between systems, and that we observe a global time in our Universe because all the clocks we use are highly correlated. In our approach, we aim to obtain limitations on how well can we use separated, local quantum clocks, that are not a priori synchronised, to define notion of time and ordering on a more global level.

Our definitions of the local and global clocks and ordering structures We will approach the question of time using quantum clocks and correlations/synchronisation between them. We do not assume that a quantum clock needs to obey properties that hold for a classical clock - constant time intervals, one directional evolution, passage through succession of states - as these notions don’t have a clear operational meaning. But we do assume possibility of the ordering on

the local scale, for which we use *local clocks*. We define a **quantum clock** as a dynamical quantum system used to provide us with units of time in its reference frame (or the reference frame of the observer holding this clock) — *i.e.* with a *clock time*. These units of time are labeled by ticks of the clock, which are defined as interactions between certain parts of the clock. More precisely, a *quantum clock* consists of:

- a *dynamical part of the clock* defined on a certain (finite-dimensional) Hilbert space, that has a well-defined starting state, and evolves via a bounded quantum Hamiltonian
- a *tick system/register*, a quantum system whose change of the state by the action from the dynamical part is defined as a *tick*. The tick system is storing ticks, and can be read-out in order to obtain information about them.

We denote the state of the tick system of a certain clock, an *ordering scale* associated with this clock. This ordering scale is then used to order events occurring in the reference frame of the clock. *Local quantum clocks* are the ones that are used to provide ordering scale in their local reference frame. In contrast, by a *global quantum clock* we refer to a quantum clock whose ordering scale can be used to replace the local ordering scales arising from two or more local clocks.

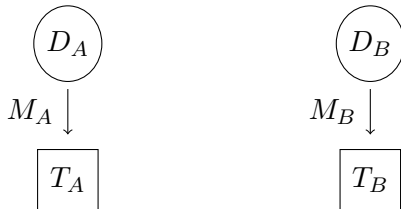


FIG. 1: Each local quantum clock consists of a dynamical part (D_A and D_B) and a register (T_A and T_B) on which the dynamical part can act by a unitary (M_A and M_B). Register is recording the ticks of a clock.

First protocol - bound on the number of ticks provided by two local quantum clocks obeying given synchronisation criterion In this approach we investigate global synchronisation of two separated, local quantum clocks by comparing their correlations, against that of two *reference* quantum systems, that are only consisting of the dynamical clock part (hence the ones whose state does not get disturb during the evolution).

After introducing some basic assumptions allowing us to consider only classical correlations, we define the following *synchronisation criterion*: conditional entropy between the states of the registers T_A and T_B can be at most $\log s$ (with $s > 0$ and $s = 1$ corresponding to the perfect synchronisation). Then, using information-theoretical tools (Technical version of the paper), we obtain limitation on the maximum number of ticks by any of the two clocks, that depends on the sizes of the dynamical parts of the clocks, and on the synchronisation criterion (*i.e.* on s). We conclude that for better synchronisation (*i.e.* smaller s) and larger number of ticks (*i.e.* longer ordering scale), larger clocks are needed. Hence, if we want to use the ordering given by two local, separated, non-ideal clocks, as a global ordering scale valid in both of their reference frames, there is a bound on the length of this scale (*i.e.* on the number of ticks it provides), and the bound depends on the size of the clocks and the extent to which these clocks are synchronised. This limitation, expressed quantitatively as a bound on N can possibly lead to the uncertainty relation in the measurement of time using quantum clocks, as well as to the more general bounds when constructing global clocks and ordering structures in quantum mechanics.

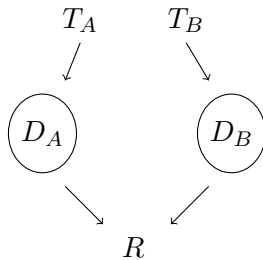


FIG. 2: Dynamical part of each clock (D_A and D_B) interacts with the photons from a light beam (that is a part of the clock register), which, depending on this interaction and the strength of the incoming beam, can then be detected by a classical referee R . As long as these signals are arriving alternately, R can use them as ticks of a global ordering scale.

Global ordering scale built by a classical observer using signals from the local quantum clocks - the Alternate Ticks game To approach the problem of a global ordering from the more practical perspective, we assume that a classical referee R wants to check the synchronisation between local quantum clocks of observers A and B , under the assumption that R only has access to the signals sent by these clocks (Fig. 2). We assume that R is able to order signals received from A and B on some parametric scale, and towards this goal R is assumed to be able to perform any map and operation needed to achieve this. One option is that R has access to some external classical parameter (e.g. Schrödinger time parameter). This parameter must be independent of A , B and R , and not accessible to players A and B . R is an external observer, and cannot change settings of A and B 's clock, nor are A and B allowed to change their clocks once they start them off. We postulate that the best thing R can do in this scenario, is to ask for the anti-synchronisation of their clocks, i.e. that they send ticks always one after the other.

Given these premises, we can represent this synchronisation set up as the following *Alternate Ticks game*. Players A and B are provided with necessary resources, and asked to construct a clock in their local reference frame each, in such a way that ticks from the clocks are always sent to R alternately (Fig. 2) and for as long as possible. Players can only communicate before the start of the game, when they are allowed to agree on a strategy. R is equidistant from the players and is detecting signals coming from their clocks. Referee stops the game as soon as two successive ticks are received from the same player, or the signals overlap. Each clock is specified by an appropriate Hilbert space, starting state and a bounded Hamiltonian (we will call it a clock Hamiltonian) defined on this space, as well as the interaction Hamiltonian between the clock and the light beam. Importantly, the clock Hamiltonian is independent of the parameter that R is using to check the ordering of the signals received from A and B .

Referee can use signals he is receiving as a new, global ordering scale to be used to order events happening in the reference frame that includes the local frames of A , B and R . One can also quantify a quality of a quantum clock by the maximum number of alternate ticks players can achieve using it. We show numerical results and bounds on the number of ticks for some scenarios in the Technical version of this paper. Optimal game scenarios could provide us with limitations on the performance of quantum clocks and hence limits of time measurements in quantum mechanics.

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