Modelling Quantum Fields with Detector interactions using Continuous Matrix Product States

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Summary: *We used continuous matrix product states to describe the interaction between detectors and a scalar field, allowing a more general class of interaction to be described non-perturbatively. This allowed us to derive a set of coupled partial differential equations that describe the time-evolution of the system. Significantly, this reduces the numerical complexity of non-perturbatively time evolving a detector-field interacting system. Additionally, we present a useful graphical representation for continuous matrix product states.*

Future work will be focused on solving the partial differential equations derived and using this detector-field interaction to study entanglement harvesting in different space-time geometries.

I. Introduction

Quantum field theory is one of our best theories to date in terms of its predictive power. Nevertheless, its calculational tools have traditionally been limited to perturbation theory (weak coupling), lattice gauge models (no longer a continuous field), and non-perturbative effects in a limited class of models (e.g., solitons, instantons) [1]. Recent work on continuous matrix product states [2] provides a novel and efficient way to represent states of quantum fields so that we can study them both analytically and numerically in their natural habitat of non-perturbative, continuum evolution.

One of the key conceptual breakthroughs in the study of quantum fields has been to employ a model of a local quantum system interacting with the field and acting as a particle detector [3]. This has led to many important results, such as that accelerating detectors see thermal radiation from the vacuum of a quantum field [4], now known as the Unruh effect. These studies have traditionally focussed on what is known as the Unruh-DeWitt detector [4, 5]: a two-level quantum system (qubit) with energy gap Ω interacting with the field according to the interaction Hamiltonian

$$H_I(\tau) = \eta(\tau)\phi(x(\tau))(e^{+i\Omega\tau}\sigma^+ + e^{-i\Omega\tau}\sigma^-) \tag{1}$$

The resulting evolution generated by this Hamiltonian has no 'nice' form and hence we are forced to use different methods. One of these methods is to use perturbation theory [6].

The dynamics of a detector in a scalar field has been studied in ref. [6] with the goal of extracting entanglement from the scalar field and transferring it to two detectors in order to determine properties of the curvature of the universe. This extraction of entanglement from the scalar field modes is known as entanglement harvesting and is an interaction of particular interest for producing entangled qubit pairs (as is done in ref. [7]).

In ref. [6], perturbation theory is used, which requires the assumption of weak coupling of the detector to the field, i.e. $\eta(\tau) \ll 1\ \forall\tau \in \mathbb{R}$. Additionally in ref. [6] only vacuum and thermal states of the field are studied. In ref. [7] perturbation theory is avoided by restricting the states examined to Gaussian states. Through the use of cMPS we re-examine the problem stated in ref. [6] without using perturbation theory and without having to restrict ourselves to thermal states or Gaussian states.

II. Results

We extend the time-dependent variational principle (TDVP) [8] from matrix product states (MPS) to continuous matrix product states (cMPS), which allows us to time-evolve systems described by cMPS. This extension is dependent on the cMPS tangent vectors that are introduced in ref. [2]. By applying the TDVP algorithm to the detector-field interacting system, we were able to derive a set of coupled partial differential equations that serve as the equations of motion of the system. Such equations have been well studied, and reduce the numerical complexity of the problem without having to resort to perturbative methods. We can further reduce the numerical complexity of the problem by truncating the domain of influence of the partial differential equations through the use of correlation length arguments.

Future work will involve solving these equations, which will allow us to study the back-action of a strongly-interacting detector on a field. Similar partial differential equations were derived for two detector-field interactions. Solving these equations allows for the study of entanglement harvesting.

During the course of this project, we also developed a graphical representation of cMPS, which is closely associated with its continuous product representation. An arbitrary cMPS can be represented as both

$$|\chi\rangle = \mathrm{Tr}_{\mathcal{A}}\left[B\mathcal{P}\exp\left(\int_{-L/2}^{L/2} dx(Q(x)\otimes\mathbb{I}_{\mathcal{F}} + R(x)\otimes\hat{\psi}^{\dagger}(x))\right)\right]|\Omega\rangle\,, \tag{2}$$

$$= \mathrm{Tr}_{\mathcal{A}}\left[B\prod_{-L/2}^{L/2}(\mathbb{I}_{\mathcal{A}}\otimes\mathbb{I}_{\mathcal{F}} + dx(Q(x)\otimes\mathbb{I}_{\mathcal{F}} + R(x)\otimes\hat{\psi}^{\dagger}(x)))\right]|\Omega\rangle\,. \tag{3}$$

Graphically, eq. (3) is represented in figure 1.

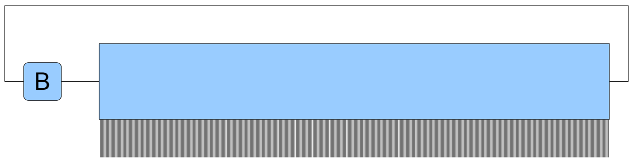

Figure 1: The graphical representation of eq. (3). Note that this is for a system of length L. If we want $L\to\infty$ we will need to rescale the diagram in order to fit the entire real line onto a finite length of paper.

This representation follows naturally from the MPS representation through a refining process (figure 2).

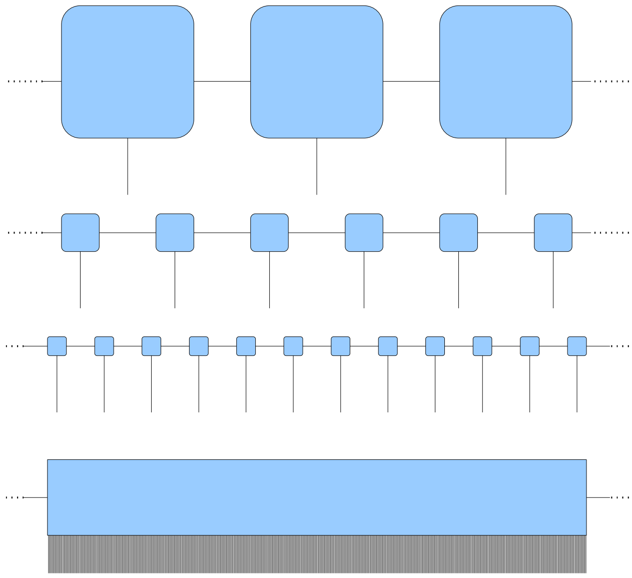

Figure 2: By refining the lattice on which a MPS is defined, we obtain the cMPS diagrams.

As an example of the usefulness of this graphical representation, the equations of motion describing the 1 detector system is expressed diagrammatically in figure 3.

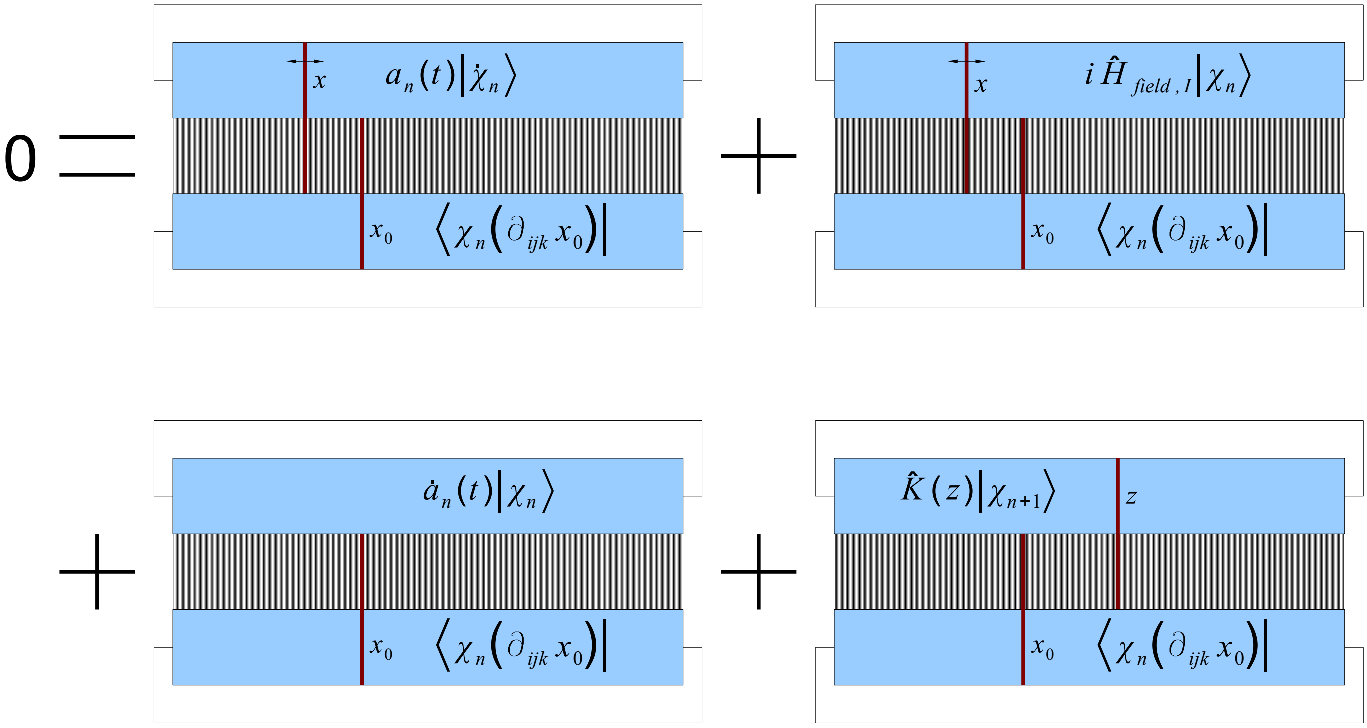

Figure 3: Equation of motion for 1 detector at z in graphical form. This represents the inner product of Schrödinger's equation with the tangent vectors of the cMPS representation. The first diagram is the field time derivative and the second diagram is the free field Hamiltonian term. The third diagram corresponds to the detector time derivative and the forth diagram corresponds to the interaction Hamiltonian. With sufficient practice, this graphical representation has many of the same advantages as tensor network diagrams with respect to MPS.

III. Significance

This is the first time a continuous tensor-network ansatz (cMPS) has been used to describe an interaction of a field with a detector. Also the cMPS formalism allows us to move away from the perturbation theory approach allowing a method subject to fewer assumptions and fewer restrictions [6, 7]. This method also has an advantage over lattice gauge theory in that we have a continuum result.

We have shown that by using cMPS to describe a 1-dimensional scalar field, we have been able to reduce the problem of time evolving a detector-field interaction into solving a set of coupled partial differential equations. Such equations have been well studied, and future work will involve solving these equations. Additionally, we have introduced continuous tensor network diagrams, and shown how they can be used to simplify the derivation of these partial differential equations.

This work has demonstrated the power of cMPS in modelling continuous 1-dimensional systems. This approach can be applied to any 1-D strongly interacting system, which is of particular interest in condensed matter physics and quantum wires. In quantum information theory, this approach can be used to study the back-action of a detector on the field and to study the effect of different space-time parameters on the entanglement that can be extracted from a relativistic quantum field.

References

[1] Michael E. Peskin and Daniel V. Schroeder. *An Introduction to Quantum Field Theory*. Westview Press, 1995.

[2] Jutho Haegeman, J. Ignacio Cirac, Tobias J. Osborne, and Frank Verstraete. Calculus of continuous matrix product states. *Phys. Rev. B*, 88:085118, Aug 2013. doi: 10.1103/PhysRevB.88.085118. URL `http://link.aps.org/doi/10.1103/PhysRevB.88.085118`.

[3] N D Birrell and P C W Davies. *Quantum Field Theory in Curved Space*. Cambridge University Press, Cambridge, England, 1982.

[4] W. G. Unruh. Notes on black-hole evaporation. *Phys. Rev. D*, 14:870–892, Aug 1976. doi: 10.1103/PhysRevD.14.870. URL `http://link.aps.org/doi/10.1103/PhysRevD.14.870`.

[5] B. S. DeWitt. *"Quantum gravity: the new synthesis," in General Relativity, An Einstein Centenary Survey*. Cambridge University Press, Cambridge, England, 1979.

[6] Greg Ver Steeg and Nicolas C. Menicucci. Entangling power of an expanding universe. *Phys. Rev. D*, 79:044027, Feb 2009. doi: 10.1103/PhysRevD.79.044027. URL `http://link.aps.org/doi/10.1103/PhysRevD.79.044027`.

[7] Eduardo Martín-Martínez, Eric G. Brown, William Donnelly, and Achim Kempf. Sustainable entanglement production from a quantum field. *Phys. Rev. A*, 88:052310, Nov 2013. doi: 10.1103/PhysRevA.88.052310. URL `http://link.aps.org/doi/10.1103/PhysRevA.88.052310`.

[8] Jutho Haegeman, J. Ignacio Cirac, Tobias J. Osborne, Iztok Pizorn, Henri Verschelde, and Frank Verstraete. Time-dependent variational principle for quantum lattices. 2011. URL `http://arxiv.org/abs/1103.0936v2`.