Non-unitary quantum gates and QMA

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The complexity class QMA [1] introduced by Kitaev is a central result in quantum computation theory, and constitutes a link between computational complexity and quantum many body physics. Variations and refinements of this construction have shed light on the hardness of a wide-range of Hamiltonian ground state energy problems [2] ,[3], [4]. In most of these works, however, a key role is played by a mapping from unitary gates to Hamiltonian terms proposed by Kitaev. Yet quantum computation need not necessarily be unitary. Measurement-based, and teleportation-based computation [5] [6] show that projective measurement can have an equally powerful role as unitary gates in quantum computation. Furthermore, renormalised projection, or equivalently, post-selection can be a powerful computational tool radically changing the computational complexity of a model [7] [8].

Here, we propose an extension [9] of Kitaev's QMA construction to non-unitary circuits via the introduction of a new Hamiltonian mapping for certain renormalised projections (or post-selection). Employing ideas from Bremner, Josza and Shepherds IQP circuits and measurement-based quantum computation, we derive a new QMA-hardness proof utilising this construction. This construction provides a new family of k-local Hamiltonians which encode a QMA-hard ground state energy problem.

This construction allows us to probe the nature of non-unitary quantum circuits and the role of post-selection in computation. The nature of the particular postselections considered are qualitatively different to their more general use. The question then naturally arises as to the class of problems verifiable by a quantum device given the ability to perform arbitrary postselection. In addition, the circuits considered are in a certain loose sense devoid of time structure. This lends a certain amount of parallelisability to the circuit, and further investigation into how this can be translated into the clock structure is being conducted.

References

[1] A. Yu. Kitaev, A. H. Shen, and M. N. Vyalyi. *Classical and Quantum Computation*, volume 47 of *Graduate Studies in Mathematics*. AMS, 2002.

[2] R. Oliveira and B. Terhal. *The complexity of quantum spin systems on a two-dimensional square lattice.* Quant. Inf. Comp. 8(10), 09000924, 2008; http://arxiv.org/abs/quant-ph/0504050.

[3] J. Kempe, A. Kitaev and O. Regev. *The complexity of the local Hamiltonian problem.* SIAM J. Comput., 35(5), pp 1070-1097, 2006.

[4] D. Nagaj. *Local Hamiltonians in quantum computation*, Ph.D. Thesis, MIT, 2008; http://arxiv.org/abs/0808.2117.

[5] R. Raussendorf, D. E. Browne, H. J. Briegel. *Measurement based quantum computation on cluster states.* Phys. Rev. A **68**, 022312 (2003).

[6] G. Brassard. *Teleportation as a quantum computation* in Physica D, 120 (1998) 43-47.

[7] S. Aaronson. *Quantum computing, post-selection and probabilistic polynomial time*, Proc. R. Soc. A **461**, 3473-3483 (2005). arXiv:quant-ph/0412.187.

[8] M. J. Bremner, R. Jozsa, and D. J. Shepherd. *Classical simulation of commuting quantum computations implies collapse of the polynomial hierarchy*, *Proc. R. Soc. A* **467**, 459 (2011).

[9] N. Usher and D. E. Browne. *In preparation.*