Interaction Picture for Braiding and Fusion in TQFT's

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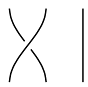
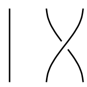
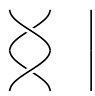
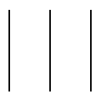
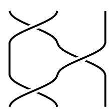
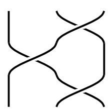
Topological quantum field theories give rise to the non-abelian anyons considered in such work as [2], [3], and [1]. In this poster we clarify the importance of the underlying topological objects, and find that the interaction picture is a natural way of calculating fusion products in anyon systems.

In two spacial dimensions, particle exchange is captured by the braid group. For example, three particle statistics are captured by the braid group on three strands: B_3. This group is generated by σ_1, σ_2 which can be represented pictorially as

$\sigma_1 =$  $\sigma_2 =$ 

Various braid group relations are made clear using the pictorial notation:

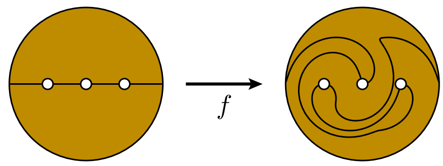
$\sigma_1\sigma_1^{-1} =$  $=$ 

$\sigma_1\sigma_2\sigma_1 =$  $=$  $= \sigma_2\sigma_1\sigma_2$

The *mapping class group* of a manifold M with boundary ∂M, will be denoted $MCG(M)$ and defined as the set of homeomorphisms $f : M \to M$ that act as the identity on ∂M, modulo continuous deformations (isotopy).

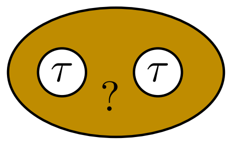
The manifolds of interest will be punctured discs, $D_n = D - Q_n$ where D is the closed unit disc and Q_n some n point subset of the interior of D. We define elements of $MCG(D_n)$ to preserve ∂D, but not necessarily Q_n.

The action of an element of $MCG(D_3)$ can be seen by drawing a line across D_3:



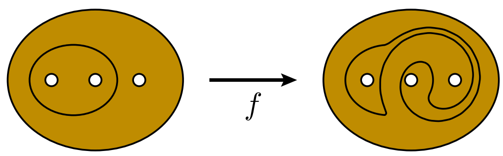
Remarkably, there is a group isomorphism between B_n and $MCG(D_n)$. The above picture corresponds to the braid element $\sigma_2\sigma_1^{-1}$.

The *observables* of the system are charge measurements within a region bounded by a circle. For example, in a Fibonacci anyon system, the total charge of two anyons (here marked with τ):

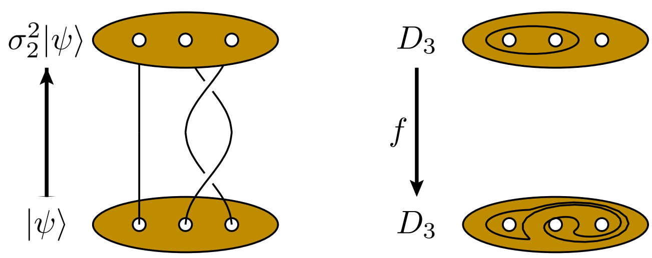


can be either vacuum or τ. So this state lives in a two dimensional Hilbert space.

The importance of the mapping class group is that it is the natural way to map between different observables that act on the same charges:



The interaction picture allows us to evolve a *state* forward in time using braid operations, or alternatively, we evolve an *observable* backwards in time using the mapping class group:



Braid group acts on states: "Schrodinger picture"

Mapping class group acts on observables: "Heisenberg picture"

[1] M. E. Beverland, R. Knig, F. Pastawski, J. Preskill, and S. Sijher. Protected gates for topological quantum field theories. 2014.

[2] C. G. Brell, S. Burton, G. Dauphinais, S. T. Flammia, and D. Poulin. Thermalization, Error-Correction, and Memory Lifetime for Ising Anyon Systems. 2013.

[3] R. N. C. Pfeifer. Measures of entanglement in non-Abelian anyonic systems. *Phys. Rev. B*, 89:035105, 2014.