Entanglement–Breaking Indices

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The purpose of this work is the classification of the amount of noise introduced by a local quantum channel only by means of its action on the entanglement of a global bipartite quantum system. We examine the following scenario. Alice and Bob share a pair of entangled systems, but Alice's half of the global system suffers a noise caused by an uncontrolled interaction with an external environment and modeled by a quantum channel ϕ. Our central goal is to classify the amount of noise introduced by ϕ only by means of its action on the entanglement.

The first instance of this plan is the understanding of those channels that never break the entanglement between Alice and Bob, no matter how weak it is (provided that it exists). These channels are called *universal entanglement–preserving*. We prove that that *the only universal entanglement–preserving channels are the unitary evolutions*. This means that even if the interaction of Alice's subsystem with the surrounding world is very feeble, nevertheless it can destroy some form of weak entanglement between Alice and Bob.

From the physical point of view, it is natural to consider the *repeated applications* of a given quantum channel on Alice's half of the global system. For the purpose of classifying this noise by means of the damages its iterated action produces on the entanglement, we introduce some functionals (defined on the set of quantum channels), called *entanglement–breaking indices*. The most important ones are the *direct n–index* and the *filtered $\mathcal{N}$–index*. On one hand, the n–index associated with a quantum channel ϕ is simply the minimum number of times we have to apply ϕ in order to produce the complete destruction of the entanglement. On the other hand, Alice could play an active role against the noise repeatedly affecting her subsystem, by choosing to apply some appropriate quantum channels (called *filters*) between consecutive actions of the noise. Then, the filtered $\mathcal{N}$–index is by definition the minimum number of iterations of the noise, such that there is no filtering strategy by which Alice can hope to save her entanglement with Bob.

Since every non–unitary filter creates some entanglement between Alice's subsystem and an external environment, one could be tempted to conjecture that the optimal filtering strategy could be obtained by means of unitary operations only. However, we provide an explicit example showing that *the optimal filtering strategy is in general non–unitary*. Our example is valid in every dimension $d \geq 3$. The two–dimensional case seems to exhibit an anomalous behaviour, in the sense that our counterexample (exceptionally) does not work. In this respect, we collect a series of clues pointing out that for qubit channels the optimal filters could be always unitary evolutions. We provide some nontrivial partial proofs of this result.

Next, we turn our attention to the study of the those channels (called *entanglement–saving*) which introduce so few noise in the system, that their direct n–index takes the value $+\infty$. In other words, the complete destruction of the entanglement is never reached, regardless of the number of iterations. But also within the class of entanglement–saving channels, there are still two possibilities. Indeed, in the limit of an infinite number of applications of the channel, the amount of entanglement can tend to zero or remain well above a finite threshold. The latter case corresponds to the so-called *asymptotically entanglement–saving channels*. We completely characterize these channels. It turns out that *a quantum channel is asymptotically entanglement–saving if and only if it admits two non–commuting phase points*. A *phase point* is (by definition) an input matrix whose transformation under the action of the channel is simply the multiplication by a phase.

On the other hand, much effort is devoted to gain understanding of the entanglement–saving channels. We prove that *almost everywhere the entanglement–saving property coincides with the presence of a positive semidefinite fixed point for the channel or for some of its powers*. Moreover, we show that the restriction we pose is irrelevant for the case of qubits, and consequently we *completely characterize the entanglement–saving qubit channels*.