Optimal purification of displaced thermal states

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A central goal of quantum communication networks is the implementation of high fidelity transmission, manipulation, and storage of quantum states [1]. On the way to this goal, the presence of noise represents the main hurdle, affecting the distance over which quantum communication is reliable, the accuracy of gate operations, and the lifetime of quantum memories. Many techniques have been developed in order to reduce the detrimental effects of noise, both for qubit systems and for continuous variable states of light and atoms [2–5]. One such technique is purification [2, 3], which attempts to reduce the noise in a teleportation channel by transforming multiple copies of a bipartite mixed state into a smaller number of copies of a state that is closer to an EPR state. Thanks to purification, one can increase the transmission fidelity and potentially build a larger and more robust system.

Purification can be applied not only to the entangled states used as a resource for teleportation, but also to the states that emerge from a noisy communication channel. For example, one can imagine the scenario where multiple qubits pointing in the same direction are sent through a depolarizing channel and the goal of purification is to produce a smaller number of qubits pointing in the same direction, but with a longer Bloch vector [4, 5].

In this work we focus on the purification of displaced thermal states, which are generated by thermal noise acting on input coherent states. Assuming that the input states are Gaussian-modulated, we establish the ultimate quantum limit for the purification fidelity, treating separately the case of deterministic protocols and that of protocols using postselection. We show that, in a suitable regime, postselection can give significant improvements in the quality of purification, at the price, of course, of a reduction of the probability of success of the protocol. The advantage of postselection is a genuine consequence of the finite width of the Gaussian modulation—in other words, it is a consequence of the availability of a non-trivial prior information about the state that one wants to purify. In the limit of infinite modulation (i.e. for uniformly distributed displacement), the fidelity gap between deterministic and probabilistic protocols disappears.

In addition to purification, we consider also the possibility of amplifying the signal carried by the input states, identifying the ultimate fidelity and the optimal quantum protocols for this task. With respect to previous works [6–8], a key improvement consists in the ability to deal with a non-uniform (Gaussian) distribution over the displacement, which, besides being more realistic, allows us to observe the difference between deterministic and probabilistic protocols.

Finally, we provide quantum benchmarks that can be used to certify that a realistic implementation of a purification protocol cannot be simulated by performing a measurement on the input thermal state and subsequently preparing a pure state. Interestingly, unlike the optimal quantum value, the value of the fidelity that has to be achieved by an experiment in order to pass the benchmark is the same both for deterministic and probabilistic processes.

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