Dynamics of nonlinear Landau-Zener tunneling and time fluctuations of a flux qubit population in a biharmonic driving field

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Recently due to the development of nanotechnology and quantum information much interest has been focused in the study of behavior in external fields "artificial atoms": quantum dots, quantum wells, ions in magnetic traps, macroscopic superconducting circuits, etc. Manipulation and control of the quantum states of these systems are extremely promising for the future development of information-telecommunication and supercomputer systems, because "artificial atoms" can be used as the logic elements (qubits) for quantum computer. However each system has its own advantages and disadvantages. Important criteria for qubit feasibility are tolerance to decoherence, efficient qubit interaction, and scalability. One of the most promising candidates for the physical embodiment of a reliable logic element for quantum computer is Josephson-junction qubits - superconducting circuits with sizes of the order of tens or hundreds of micrometers. These qubits are characterized by relatively long relaxation times (tens of microseconds) and the simple control method (by the magnetic field) of the quantum states. Note that superconducting qubits have been already applied for the construction of the first computer processor hardware (D-Wave). During the last few decades we could bear witness to an immense research activity, both in experimental and theoretical physics, aimed at understanding the detailed dynamics of superconducting qubits, however, many questions related to the nonlinear dynamic effects are unexplored. For example, this is a dynamic control qubits in strong driving fields, the study of multiphoton transitions, and the measurement of the qubit states.

We are studying a flux qubit with three embedded in a circuit Josephson junctions (3JJ-qubit) that was first proposed by Mooij et al. The qubit states can be control and readout by measuring the magnetic field using a dc-SQUID. The coherent dynamics in a resonance external fields of this qubit has been studied. However, this method of spectroscopic investigations (Rabi-spectroscopy) are not simple because stable tunable micro-wave sources not easy to produce in this range. Measurement difficulties are connected with the frequency dependence of the dispersion and damping, as well as with strict requirements to impedance control tolerances which limit the application of broadband spectroscopy. In this connection the development of amplitude spectroscopy based on the method of obtaining information by means of the response function "sweep" over the signal amplitude and some control parameter (an applied magnetic flux or a bias) aroused great interest. This method may be applied to systems with crossing energy levels between which the transitions can be realized by changing external parameters. The frequency of such a driving field can be orders of magnitude lower than the distance between levels. This means that the system evolves adiabatically except for the immediate vicinity of quasi-crossing levels, where the Landau-Zener transitions occur. The main advantage of this type of spectroscopy is that the system is investigated in wide ranges of the amplitude change.

The main goal of this work is to investigate how to control transitions between qubit states and time fluctuations (or mesoscopic fluctuations) between of the energy levels population by changing the form of superposition of electromagnetic pulses of strong amplitude using the technique of the amplitude spectroscopy. It is shown that the relative phase of the pulses is responsible for the Landau-Zener transition rate and, respectively, for the transitions sequence between the adiabatic states. Since in the coaxial lines on the qubit a random sequence of pulses is arrived whose durations are controlled with accuracy of the field's period, that leads to strong mesoscopic fluctuations of the qubit population. Using the rotating wave approximation (RWA) made it possible to find simple conditions of a system resonant excitation. The numerical simulation based on the master equation carried out confirms qualitative conclusions following from RWA without noise effects. Recent experimental investigations have demonstrated that the duration pulses can be accurately controlled on the one period of the external field due to the loss of the coaxial line. Because the influence of the relative phase of the pulses on the qubits population similar the magnetic field in the mesoscopic systems that leads to the destruction of the interference pattern of the population. We investigate the influence of the pulse duration and noise on the discovered fluctuations effects. The rate of the transition probabilities between the qubit levels and the standard deviation (mesoscopic fluctuations), taking into account the noise (dephasing and energy relaxation), are found by solving the equation for the density matrix.

In the present paper the mesoscopic fluctuations of the transition probability between the ground and excited states of a superconducting Josephson qubit excited by a superposition of two radio pulses is investigated both numerically and analytically. The Hamiltonian of the qubit is:

$$H(t) = \frac{1}{2}(\varepsilon(t)\sigma_z + \Delta\sigma_x)\ , \tag{1}$$

where $\varepsilon(t)$ is the energy bias of the qubit, and Δ is the tunnel level splitting, σ_z and σ_x are the Pauli matrices. It is assumed that initially a qubit is in the ground state, and the transitions to the exited level are induced by an external biharmonic driving field

$$\varepsilon(t) = \varepsilon_0 + A\big(\cos(\omega t + \theta) + \gamma\cos(2\omega t)\big)\,, \tag{2}$$

where ε_0 is dc and A is ac components of the driving amplitude; γ and θ - are relative amplitude and phase of mixing pulses. In the adiabatic approximation the qubit can be in states $\phi_\pm(t)$ with the energies $E_\pm = \pm\frac{1}{2}\sqrt{\varepsilon_0^2 + \Delta^2}$. When the control field is changed such a way that the anticrossing levels $E_\pm$ takes place then the Landau-Zener transitions between them may be induced. The rate of the Landau-Zener transitions can controlled by the amplitudes and relative phase of the pulses. By analogy with the theory of mesoscopic systems, we can see that the number of quasicrossings adiabatic levels (number of transitions) during the external field time is analogous to the number of s scatterers which are placed on the length of the wire. For a fixed pulse duration phase is responsible for changing the configuration of the scatterers, and the total duration of the signal behaves like a length of wire. The generator creates pulses in the form (2), but their duration can be changed advancing through the coaxial line. To account for this effect, we introduce the random arrival time of the signal on the qubit, or proper random phase $\varphi = \omega t_0$.

It is important to keep in mind that even in the case of a single qubit – two-level system – the effect of the applied field with an arbitrary amplitude cannot be accurately described. In this case only approximate solutions are obtained which allow to understand what happens to a two-level system in the non-stationary field. If the field amplitude is large and the frequency of the applied field is close to the distance between levels then it is possible to transfer to a rotating system of coordinates and to use the rotating wave approximation (RWA), which allows to introduce an effective time-independent Hamiltonian. In this approximation we calculate expression for the transition probabilities for amplitudes with the effective Hamiltonian and the resonance condition $\varepsilon_0 + (n+2m)\omega \approx 0$, at $\omega >> \Delta$. It is shown that the peaks of the resonances depend on the relative phase and amplitudes of the two harmonics driving the qubit (see, for instance, A.M. Satanin, M.V. Denisenko, *et al.* Phys. Rev. B 90, 104516 (2014) for more details).

The transition probabilities between the qubit levels, taking into account the noise (dephasing Γ_f and energy relaxation Γ_e), are found by solving the equation for the density matrix. The figure shows the results of numerical calculation rate of transition probabilities (Γ) (see Fig. 1 (a)) and the standard deviation $\sigma = \sqrt{\langle\Gamma^2\rangle - \langle\Gamma\rangle^2}$ depending on the relative phase difference θ in Fig. 1 (b). It is shown that the fluctuations of the rate of transition probabilities for the vicinity of $\theta = 0$ are maximized and the decrease of the characteristic energy relaxation time leads to a reduction of fluctuations (Fig. 1 (b)). We can say that the parameter plays a role similar to the inelastic length. However, the dephasing parameter more similar to the invers length of the elastic scattering in the conductors. It may be seen that with increasing of the rate of phase relaxation it is observed the suppression of fluctuations that is quite similar to behaviour of conductance of mesoscopic systems. Note that these fluctuations were observed in a recent experiment with a Josephson qubit (S. Gustavsson *et.al.* Phys. Rev. Lett. 110, 016603 (2013)). Figure 1 (c) shows the behavior of the maximum

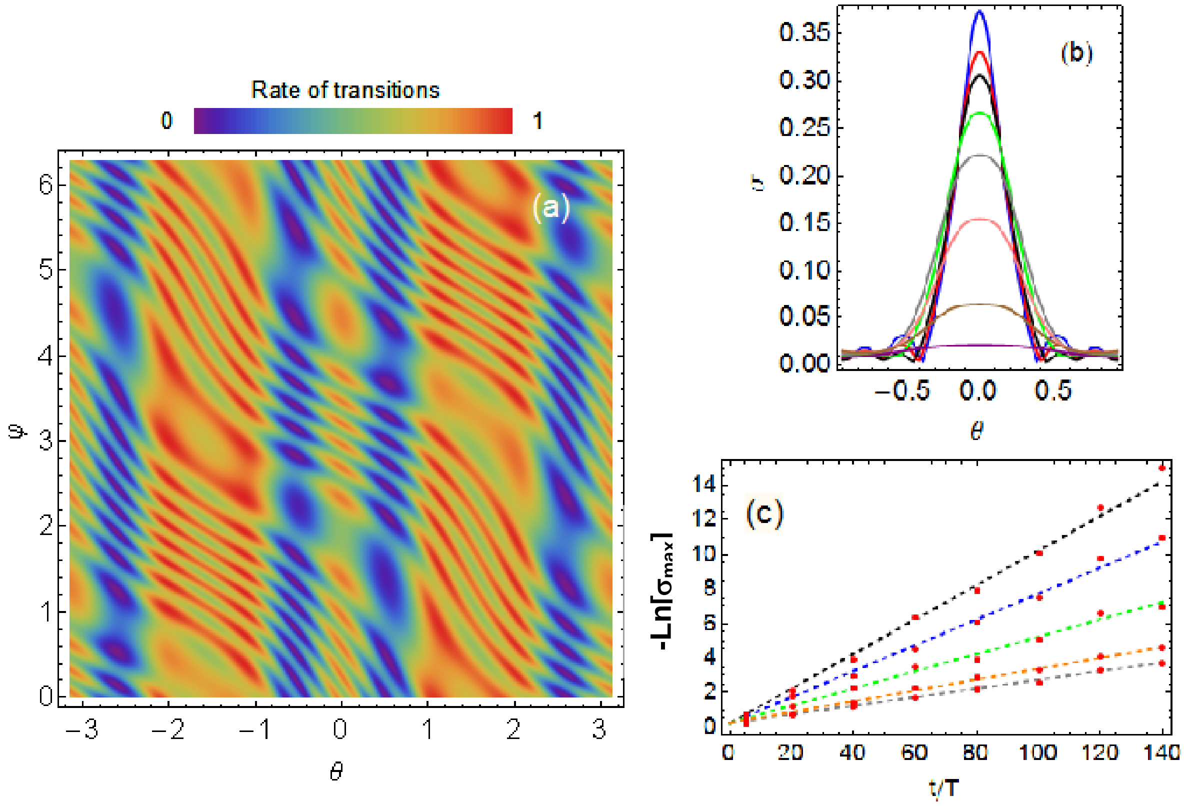

Figure 1. (a) The rate of transition probabilities the qubit levels as function the relative phase θ and random phase, $\varphi = \omega t_0$, pulses; (b) The standard deviation σ for different dephasing (from blue - $\Gamma_f = 0.001$ to purple $\Gamma_f = 0.075$); (c) Decreasing dependence of the peak of the standard deviation of the duration of the pulse. Red dots - the calculated values, and the dashed lines - qualitative law of decrease of the peak dispersion.

standard deviation σ based on the pulse width τ. Maximum magnitude decreases in the following law: $\sigma_{\max} \sim e^{-\Gamma_f t/T}$ therefore, the logarithm is a linear function of signal duration. It is shown that strong mesoscopic fluctuations occur on time scales shorter than the time dephasing, whereas at the long-time, the variance is completely suppressed. This fact enables us once again emphasize the analogy with the mesoscopic, consisting in the fact that the dependence of the fluctuations of the population of the qubit on the pulse duration is similar to the behavior of conductivity fluctuations on the length of the wire.