Hybrid quantum information processing with single photons and coherent states

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A hybrid approach to linear-optical quantum information processing was recently proposed [1], which combines advantages of two well known previous schemes: One, referred as the linear optical quantum computation (LOQC), employs single photons to construct the logical qubits [2], typically based on the horizontal and vertical polarization states, $|H\rangle$ and $|V\rangle$. The other well-known scheme, referred as the coherent-state quantum computation (CSQC) [3, 4], uses optical coherent states, $|\alpha\rangle$ and $|-\alpha\rangle$, as the qubit basis. In LOQC, single qubit operations are deterministic, while two-qubit operations are difficult to realize. On the other hand, two qubit operation with CSQC is near-deterministic [5, 6], while Z rotations are highly nontrivial [7]. In hybrid approach, each of these advantages of LOQC and CSQC can be combined to implement near-deterministic universal gate operations.

The logical basis of hybrid qubit is defined as

$$\big\{|0_L\rangle = |+\rangle|\alpha\rangle, \quad |1_L\rangle = |-\rangle|-\alpha\rangle\big\},$$

where $|\pm\rangle = (|H\rangle \pm |V\rangle)/\sqrt{2}$ with photon polarization states [1]. The Pauli X operation can be performed by bit flip operations on each of two modes, $|+\rangle \leftrightarrow |-\rangle$ and $|\alpha\rangle \leftrightarrow |-\alpha\rangle$, which can be realized by a polarization rotator and a π phase shifter, respectively. An arbitrary Z rotation ($\hat{Z}_\theta$) is performed by applying the phase shift operation only on the single-photon mode: $\{|+\rangle,\ |-\rangle\} \to \{|+\rangle,\ e^{i\theta}|-\rangle\}$. The Hadamard and CZ operations can be also implemented in a simple and near-deterministic manner, by using the gate teleportation scheme. The Bell measurement required in the teleportation protocol can be performed using two smaller Bell measurement components, B_α and B_s: i) B_α discriminates four Bell states in the coherent-state representation with a success probability $\exp(-2|\alpha|^2)$, using a 50:50 beam splitter and two photon number parity measurements [5, 6], ii) B_s identifies two out of four Bell states in the single-photon encoding with success probability 1/2 [8], using three polarizing beam splitters and four on-off photodetectors. The logical Bell measurement succeeds (so does the teleportation process) when at least one of B_α and B_II succeeds, which leads to the success probability of $P_s = 1 - \exp(-2|\alpha|^2)/2$. For example, 99%

success rate of teleportation is achieved by the logical qubit encoding with $\alpha = 1.4$. It outperforms the previous schemes that requires massive overheads with repetitive applications of teleporters [2, 7]. The details of the hybrid teleportation protocol is described in Ref. [1].

For realizations, efficient generation of the hybrid qubit is essential. It is well known that strong nonlinearity *e.g.* using cross-Kerr media in principle enables to generate the entanglement between the single photons and coherent states. However, it is practically very hard to attain such a strong nonlinearity experimentally [9]. We have recently proposed an alternative scheme to generate a hybrid qubit by using photon-addition technique on different spatial mode [10]. The hybrid qubit generated in this scheme is

$$|\psi(\alpha)\rangle = \frac{1}{\sqrt{2}}(|0\rangle|\alpha\rangle + |1\rangle|-\alpha\rangle), \tag{1}$$

where $|0\rangle$ and $|1\rangle$ are the vacuum and single photon states, respectively. Here we consider a photon-added coherent state, obtained by applying a photon creation operator onto a coherent state, as a good approximation of a coherent state with a larger amplitude, $\mathcal{N}^{-1/2}\hat{a}^\dagger|\alpha\rangle \approx |g\alpha\rangle$. With an initial state $|0\rangle \otimes |\alpha\rangle$, photon addition takes place either in the first mode, generating a single-photon state $|1\rangle$ while leaving $|\alpha\rangle$ in the second mode, or in the second mode, producing a photon-added coherent state in the second as leaving the vacuum in the first mode. By removing the which way information of these two photon addition events (conditioned on a single click after beam splitter) [10], the resulting state is

$$|\Psi\rangle = \frac{1}{\sqrt{2}}(|1\rangle|\alpha\rangle + |0\rangle|g\alpha\rangle). \tag{2}$$

With a displacement operation $D(-\frac{\alpha+g\alpha}{2})$, the final state

$$|\Psi\rangle = \frac{1}{\sqrt{2}}(|0\rangle|\alpha'\rangle + |1\rangle|-\alpha'\rangle), \tag{3}$$

can be obtained, where $\alpha' = (g\alpha - \alpha)/2$.

The generated hybrid qubit in Eq. (3) has a different form with the one in (1). The single photon mode in Eq. (3) is constructed based on the vacuum and single photon Fock states, while the one in Eq. (1) is based on the photon polarization states. It is notable that in principle one can generate the state (1) using two ideal states of (3) probabilistically. However, in a realistic model, there exist many factors to reduce the fidelity of output state in the generation process. Therefore, it is worth considering the quantum information processing with the type of hybrid qubits in (3) itself. In fact, the type in (3) is more robust to photon loss errors than the one in (1). This is because photon loss in the single photon mode $|1\rangle \to |0\rangle$ can be regarded as a bit-flip error in this approach, while it is a leakage from the qubit-space for the type (1). In this work, we also investigate the performance of quantum information processing by comparing these two types of hybrid qubits. Our approach paves an alternative new way to the realization of practical quantum information processing with linear optics.

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