Probing an untouchable environment as a resource for quantum computing

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In the theory of open quantum systems, an environment is usually treated as a large bath, averaging out most of its dynamical details. Thus we do not see it as a quantum object that we can control actively. It is indeed near-impossible to precisely identify the quantum nature of an environment, let alone to control it at will. In this paper, we demonstrate how this formidable task can be achieved, provided the dimension of the environment can be regarded as finite within the timescale of our manipulation. The information thereby obtained will be useful not only for deeper understanding of the system dynamics under decoherence but also for exploiting the environment as a *resource* for quantum engineering, such as quantum computation.

We shall present a method to identify the Hamiltonian H_{SE}, which describes the effective interaction between the principal system S and the surrounding system E and (a part of) its internal dynamics. We shall call E the *environment* symbolically throughout the paper. The environment E is assumed to be finite dimensional, as we will formally state later, but its size is unknown a priori. The knowledge of H_{SE} is vital for controlling E as a useful system, not to mention for checking its controllability.

Setup.– Let $\mathcal{H}_S$ and $\mathcal{H}_E$ be the Hilbert spaces of the principal system (S) and its environment (E), whose dimensions are d_S and d_E, respectively. Then, the assumptions on which we base our analysis are as follows.

(i) $d_E := \dim \mathcal{H}_E$ is finite, although its value may be unknown.

(ii) Ancillary states, each of which is a maximally entangled pair, $|\Upsilon_{a_1 a_2}\rangle = \frac{1}{\sqrt{d_S}} \sum_{i=1}^{d_S} |i_{a_1} i_{a_2}\rangle$ can be provided abundantly. They will form the ancillary system A as the state-steering protocol proceeds (see below). The interaction between A and E is negligible.

(iii) The state on $\mathcal{H}_S \otimes \mathcal{H}_E$ can be initialised to a fixed (unknown) state $|\Psi_{SE}(0)\rangle$.

(iv) Any quantum operations can be applied on SA instantaneously.

State-steering protocol.– In order to make use of the *'mirroring effect'* of entanglement to identify H_{SE}, we first need to steer the state on SEA to establish maximal entanglement between SE and A. Let us describe how the state-steering protocol goes, and delineate why it works out for our purpose. We start with an initial (fixed, but unknown) state $\rho_{SE}^{(0)}$ and abundant copies of a maximally entangled state $|\Upsilon_{a_1a_2}\rangle$. At $t = 0$, $\mathcal{H}_A$ is a null space, supporting no states. The SWAP operation between S and a_1, which must be fast enough compared with the system dynamics, will be denoted as SWAP_{Sa_1}. The C-th round of the protocol proceeds as follows (C starts from zero):

Step 1: Apply SWAP_{Sa_1}, where a_1 is a subsystem of the newly provided $|\Upsilon_{a_1a_2}\rangle$, and then let A incorporate a_1 (the former S) and a_2.

Step 2: Perform state tomography of ρ_A, and apply a local filtering operation $\mathcal{F}_{\mathrm{LF}}^A$ on ρ_A. If it fails, carry out the whole protocol from the beginning.

Step 3: Let the SE system evolve for a time duration ($< \Delta t_C$) so that the functional of ρ_{SA}, ΔE_{SA}, which is defined below by Eq. (1), increases by $\epsilon_C > 0$. See Sec. IV of the supplementary material [1] as to how we should determine Δt_C and ϵ_C.

Step 4: Terminate the protocol if ΔE_{SA} is found to be non-increasing; otherwise, let the SE system evolve so that $\Delta E_{SA} \geq \epsilon_C$, and go back to Step 1.

The local filtering operation on ρ_A is written as $\mathcal{F}_{\mathrm{LF}}^A \rho_A = F_{\mathrm{LF}} \rho_A F_{\mathrm{LF}}^\dagger$, where $F_{\mathrm{LF}} = \sqrt{\lambda_{\min} \cdot \rho_A^{-1}}$ with ρ_A^{-1} the inverse of ρ_A on its support and $\lambda_{\min}$ the smallest eigenvalue of ρ_A.

The quantity ΔE_{SA} we measure in Step 3 is given as

$$\begin{aligned} \Delta E_{SA} &:= S(\rho_{SA}) - S(\rho_A) + \ln d_S, \\ &= D(\rho_{SE}||\rho_S \otimes \rho_E) + D(\rho_S||I^S/d_S). \end{aligned} \tag{1}$$

This equations implies that when $\Delta E_{SA} = 0$ there is a subsystem A_1 of A that is maximally entangled with S. Further, we expect that the remaining part A_2 of A is maximally entangled with E as a result of $\mathcal{F}_{\mathrm{LF}}^A$. This naive guess is proven in detail in the supplementary material [1].

Note, however, that the argument in the supplementary material involves some mathematical subtleties concerning multiple possibilities of the set $(d_E, |\Psi_{SEA}\rangle, H_{SE})$ that leads to identical observable dynamics on S and A. We call this set of three ingredients a *triple*. There can be equivalent classes of triples, so that all triples within a single equivalent class give rise to the identical observable dynamics on S and A, regardless of any active controls on them. For our purpose of controlling E, it suffices to identify one in the class for the observed time evolution on SA. It is possible to prove that the resulting state of the state-steering protocol is in the same equivalence class, in which $A2$ and E are maximally entangled as well as $A1$ and S are. [1].

Estimation of the Hamiltonian.– Now that we have two pairs of maximally entangled states, we move on to the Hamiltonian identification stage. One of the well-known properties of maximally entangled states is that an application of unitary operation on two subsystems at one side of the pairs is equivalent to that of its transpose on the other side. For $|\Psi_{SEA}\rangle$ that is entangled with respect to the partition between SE and A, we can write

$$U_{SE}|\Psi_{SEA}\rangle = V_A|\Psi_{SEA}\rangle, \tag{2}$$

where $V_A = U_{SE}^T$ acting on $\mathcal{H}_A$. Therefore, the unitary evolution we observe on the ancillary system $A = A_1A_2$ should contain information about the Hamiltonian $H_{SE} = i/t \ln U_{SE}(t)$. Naturally, however, simply looking at the state of A does not reveal any information on U_{SE}, but it turns out that identifying $\rho_{SA}(t)$ by quantum state tomography suffices for our purpose. A specific method of estimating the Hamiltonian H_{SE} is given in the supplementary material [1].

The Hamiltonian H_{SE} thereby estimated contains all the necessary information to characterise the observable dynamics, albeit unmodulable per se. What we can control actively is the system S. Thus, the dynamics of the entire system SE is governed by the Hamiltonian

$$H(t) = H_{SE} + \sum_n f_n(t) H_S^{(n)}, \tag{3}$$

where $H_S^{(n)}$ are independent Hamiltonians that act on S and can be modulated by $f_n(t)$. As we have already identified H_{SE}, there is sufficient information to judge the controllability of the system SE under this Hamiltonian (3). A theorem from the quantum control theory states that the set of realisable unitary operations on the system is generated by dynamical Lie algebra [2]. Therefore, our knowledge of H_{SE} allows the controllable system to encompass not only the principal system S but also (a part of) the environment E. That is, we are now able to exploit the dynamics inside E for useful quantum operations, such as quantum computing, by controlling a small system S only.

[1] M. Owari, K. Maruyama, T. Takui, and G. Kato, arXiv: 1301.2152 (2010).

[2] D. D'Alessandro, *Introduction to Quantum Control and Dynamics* (Taylor and Francis, Boca Raton, 2008).