Experimental multipartite entanglement without multipartite correlations

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Non-classical correlations between measurement results make entanglement the essence of quantum physics and the main resource for quantum information applications. However, there are n-particle states which do not exhibit n-partite correlations at all but still are genuinely n-partite entangled. We introduce a general construction principle for such states, implement them in a multi-photon experiment and analyze their properties in detail. Remarkably, even without n-partite correlations, these states do violate Bell inequalities showing that there is no local realistic model describing their properties.

Correlations between measurement results are the most prominent feature of entanglement. Correlations made Einstein, Podolski and Rosen [1] to question the completeness of quantum mechanics, and are nowadays the main ingredient for the many applications of quantum information like entanglement based quantum key distribution [2] or quantum teleportation [3].

For example, when observing two maximally entangled qubits, correlations enable us to use the measurement result observed on the first system to infer exactly the measurement result on the second system (for the corresponding basis). In this scenario the two particle correlations are formally given by the expectation value of the product of the measurement results obtained by the two observers. Note, the single particle correlation, i.e. the expectation value of the results for one or the other particle are zero in this case, that means we cannot predict anything about the individual results. When studying the entanglement between n particles a natural extension is to consider n-partite correlations, i.e. the expectation value of the product of n measurement results. Such n-partite correlation functions are frequently used in classical statistics and signal analysis [4, 5] and are omnipresent in multi-party entanglement witness [6–11] and Bell inequalities [12–22]. The quantum mechanical correlation function is defined as

$$T_{j_1 \cdots j_n} = \langle r_1 \dots r_n \rangle = \mathrm{Tr}(\rho \; \sigma_{j_1} \otimes \cdots \otimes \sigma_{j_n}),$$

where r_k is the outcome of the local measurement of the k-th observer, parametrized by the Pauli operator σ_{j_k}.

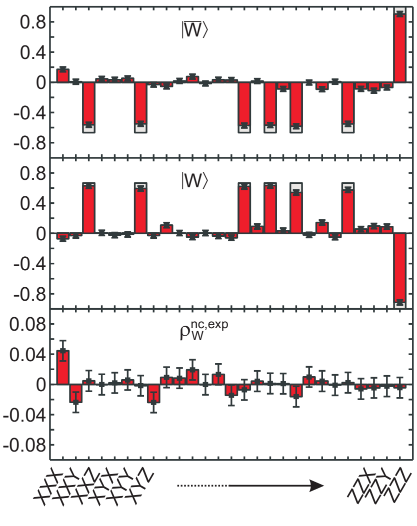

FIG. 1. Experimental tripartite correlations for $|W\rangle$ and $|\overline{W}\rangle$ and $\rho_W^{nc,exp}$ (red) in comparison to the theoretically expected values (grey). The plot presents measured values of $T_{j_1 j_2 j_3}$ for the observables listed below the plot. Obviously, the states $|W\rangle$ and $|\overline{W}\rangle$ have opposite tripartite correlations canceling each other when the states are mixed with equal weights. Note, that the the scale for the state $\rho_W^{nc,exp}$ is magnified by a factor of 10.

Recently, Kaszlikowski *et al.* [23] pointed at a particular quantum state with vanishing multiparty correlations which, however, is genuinely multipartite entangled. This discovery, of course, prompted vivid discussions on the differences between classical and quantum correlations [23–31]. It seems that for entangled states the standard, n-partite correlation function is not sufficient to fully describe the many features of entangled multi-party states [26]. Still, the question arises, what makes up such states without multipartite correlations and also how nonclassical they can be, i.e., whether they are not only entangled but whether their entanglement suffices to even violate a Bell inequality.

Here we generalize, highlight and experimentally test such remarkable quantum states. Starting from the state given in [23]:

$$\rho_W^{nc} = \frac{1}{2}|W\rangle\langle W| + \frac{1}{2}|\bar{W}\rangle\langle\bar{W}|, \tag{1}$$

where $|W\rangle = (|001\rangle + |010\rangle + |100\rangle)/\sqrt{3}$ is the well known three qubit W state, we describe a simple principle how to construct quantum states without n-partite correlations for odd n, and show that, indeed, they can be genuinely n-partite entangled. To this end, we introduce a general concept of "anti"-states. We implement such states in a multi-photon experiment and analyze the obtained measurement results, also in perspective to the comments raised recently. We find that even if these states do not exhibit

n-partite correlations, the existence of correlations between a smaller number of particles enables their unique properties. Finally, using our recently introduced method to design multi-party Bell inequalities [32] we can show that these states, despite not having full correlations, can violate Bell inequalities.

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