Boson-sampling with non-Fock states

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Boson-sampling is a highly simplified, but non-universal, approach to linear optics quantum computing (LOQC) [1]. Whilst not universal, it was shown by Aaronson & Arkhipov [2] to be a computationally hard problem. The hardness relates to the fact that each amplitude γ_S is proportional to a matrix permanent, which is computationally hard to classically calculate. This is of great practical interest as conventional approaches to LOQC might require millions of photons to implement a post-classical algorithm, whilst boson-sampling may only require approximately twenty.

The boson-sampling model has n photons prepared in $m = O(n^2)$ optical modes. The input state, $|\psi_{\text{in}}\rangle = \hat{a}_1^\dagger \ldots \hat{a}_n^\dagger |0_1, \ldots, 0_m\rangle$ is evolved via passive linear optics (beamsplitters and phase-shifters), which implements a unitary map $\hat{U}\hat{a}_i^\dagger \hat{U}^\dagger = \sum_j U_{i,j}\hat{a}_j^\dagger$. The output state is a large superposition of photon-numbers, of the form $|\psi_{\text{out}}\rangle = \sum_S \gamma_S |n_1^{(S)}, \ldots, n_m^{(S)}\rangle$, where S is a photon-number configuration, $n_i^{(S)}$ is the number of photons in the ith mode associated with configuration S, and γ_S is the associated amplitude. Finally, the output state is sampled via photodetection, which obtains the probability distribution $P(S) = |\gamma_S|^2$. For a diagrammatic representation of boson-sampling see Fig. 1.

A logical next question is 'are there other quantum states of light, which also yield computationally hard sampling problems?' Here we address this problem by showing that three other classes of quantum states of light yield computationally hard sampling problems.

The first is photon-added coherent states (PACS) [3], which are of the form $\hat{a}^\dagger|\alpha\rangle$, where $|\alpha\rangle$ is a coherent state of amplitude α. In this case, the input state to our multi-mode device is $|\psi_{\text{in}}\rangle = \hat{a}_1^\dagger \ldots \hat{a}_n^\dagger |\alpha_1, \ldots \alpha_m\rangle$. This derivation is subject to the constraint $|\alpha| < 1/\text{poly}(n)$.

The second are photon-added or subtracted squeezed vacuum (PASSV) states [4], which are of the form $\hat{a}^\dagger\hat{S}(\xi)|0\rangle$ and $\hat{a}\hat{S}(\xi)|0\rangle$ respectively. $\hat{S}(\xi)$ is the squeezing operator with squeezing parameter ξ. The input state considered is $\hat{a}_1^\dagger\hat{S}_1(\xi)\ldots\hat{a}_n^\dagger\hat{S}_n(\xi)\hat{S}_{n+1}(\xi)\ldots\hat{S}_m(\xi)|0_1\ldots 0_m\rangle$.

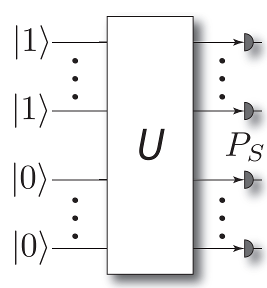

FIG. 1: The boson-sampling model for quantum computation.

The third state is 'cat states' [5] – superpositions of coherent states – which are of the form $|\text{cat}\rangle = \sum_i \lambda_i |\alpha_i\rangle$. We show that in the small amplitude limit, cat states are provably hard. In the limit of larger amplitudes we present strong evidence that the problem is computationally hard by connecting the output amplitudes to a permanent-like function. This limit is particularly interesting because these are macroscopic states, suggesting that, in general, quantum computational hardness applies to macroscopic quantum systems.

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