Globalness of separable maps characterized by classical correlations without globally causal structure

Seiseki Akibue
Department of Physics Graduate School of Science
The University of Tokyo, Tokyo 113-0033, Japan
akibue@eve.phys.s.u-tokyo.ac.jp

Masaki Owari Go Kato
NTT Communication Science Laboratories
NTT corporation, Kanagawa, 243-0198, JAPAN

Mio Murao
Department of Physics Graduate School of Science, The University of Tokyo, Tokyo 113-0033, Japan

1 Introduction and motivation

When we perform a quantum information processing task between two spatially separated parties, transmission of quantum states (quantum communication) between the parties are necessary in general. However, reliable quantum communications is much harder than reliable classical communication due to decoherence. Therefore we want to reduce quantum communications as much as possible by using a class of quantum maps implementable without quantum communication. *Local operations and classical communication* (LOCC) is such a class of maps, which is implementable by a sequence of conditional local quantum measurements and classical communications of measurement outcomes.

In spite of its clear operational meaning, analysis of quantum information processing tasks under the restriction of LOCC is hard in general since the mathematical structure of LOCC is highly complicated [1]. A slightly larger class of quantum maps with a simpler mathematical structure called *separable maps* is often used instead of LOCC. Elements of an quantum instrument representing a separable map are given by products of local operations. For some quantum information processing tasks, e.g. convertibility of pure bipartite states [8], separable maps achieving the tasks are also LOCC. On the other hands, the gap between these two classes has been observed in quantum information tasks such as state discrimination [9, 10, 11] and entanglement distillation [12]. It is also shown that even for LOCC with unbounded rounds of communications, this gap exists [1].

The gap between LOCC and separable maps can be considered as "*globalness*" of separable maps not included in LOCC and can be investigated in terms of additional resources required to perform separable maps on top of LOCC. Entanglement, which is defined as non-local quantum correlation non-increasing under LOCC [2, 3, 4], has been frequently analyzed as the resource to achieve global quantum maps together with LOCC [5, 6, 7]. This approach is useful to characterize the difference between LOCC and separable maps. For the state discrimination tasks, the globalness of separable maps discriminating a set of orthogonal product states can be characterized by the amount of entanglement required in an entanglement assisted LOCC implementation of the separable map. However, the minimum amount of entanglement required for this type of tasks had not been known and whether the number of rounds of classical communications can reduce the amount of entanglement resources or not had not been well understood either. Further, the difference between the separable map and non-separable maps cannot be characterized simply by the amount of entanglement. Thus, another approach is required for fully characterizing the globalness of separable maps.

Submitted to:
QIP 2015

In this contribution, we propose a totally new approach to characterize the globalness of separable maps by introducing the (fictitious) classical correlations not obeying globally causal structure as an additional resource to perform global quantum maps on top of local operations, instead of entanglement or quantum communications. We refer to this type of correlations as *acausal classical correlations*. The idea of considering acausal classical correlations originates from an observation of globally causal structure of LOCC. In an LOCC protocol, parties communicate each other's measurement outcome at each step. The effects of these communications can be considered as a subclass of correlations having globally causal structure. We consider a generalization of communications described by acausal classical correlations.

We define a new class of maps called *local operation and acausal communication* (LOCC*) and analyze the properties of LOCC*. Although LOCC* is not guaranteed to be deterministically implementable within quantum mechanics in general, there are a class quantum maps which can be implementable by LOCC*. We show that the class of LOCC* implementable in quantum mechanics is closely related to a class of separable maps and also to the framework of the *quantum process* introduced by [16] for analyzing *quantum* correlations with *indefinite* causal order. We note that more general acausal correlations not restricted in classical correlations have been considered as resources for information processing in the context of closed time-like curves (CTCs) predicted by general relativity. For example, *post-selected* closed time-like curves (pCTCs) [13, 14, 15] are shown to remarkably enhance the performance of quantum computation [15].

2 Results

We present a new formulation and analysis of globalness of separable maps in terms of acausal classical correlations, and also entanglement necessary to discrimination of product basis, and derive the following results:

1. We prove that the intersection of the class of LOCC* and that of completely positive trace preserving (CPTP) maps is equivalent to that of separable maps shown in Fig. 1. We also show that LOCC* can be simulated by *local post-selection* without communication with constant probability. This result gives a new characterization of separable maps in terms of acausal classical correlations.
2. It was shown that in quantum mechanics, correlations with indefinite globally causal structure between parties are allowed [16] in contrast to classical mechanics. We analyze when acausal classical correlations are implementable in quantum mechanics. For this purpose, we investigate a class of acausal classical correlations, which we call *valid channels*, that are consistent with quantum mechanics. We define valid channels as acausal classical correlations with which all local instruments generate CPTP maps, and prove the one-to-one correspondence between a valid channel and a classical process [16]. We further prove that a valid channel only generates one-way LOCC in a bipartite system. This result indicates that acausal classical correlations required to generate two-way LOCC or separable maps in bipartite systems without entanglement also generates at least one non-CPTP map. Note that acausal classical correlations generating two-way LOCC are not necessary to be valid channels under the definition of the acausal classical correlations where local instruments are performed only once.
3. Separable maps are implementable by using local operations and particular types of acausal classical correlations (LOCC*) without using shared entanglement. On the other hand, shared entanglement is necessary if we implement separable maps by using local operations and classical

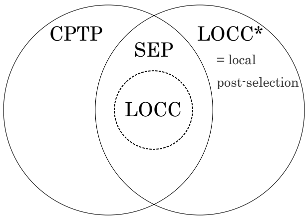

Figure 1: The intersection of LOCC* and the set of CPTP maps is equivalent to the set of separable maps (SEP). The set of LOCC is strictly smaller than SEP. Normalized LOCC* is equivalent to the set of local post-selections.

correlations with globally causal structure (LOCC) or by using local operations and one-way classical communications (one-way LOCC). There is a tradeoff between the restrictions on classical temporal correlations and the requirements on quantum spatial correlations. As an analysis of entanglement resources, we derive an amount of the optimal entanglement resource that is necessary to perfectly discriminate orthonormal basis states by one-way LOCC. We construct a two-way LOCC protocol which discriminates product basis states and consumes less amount of entanglement resources than the best one-way LOCC protocol. This result indicates that there is a trade-off relationship between the amount of necessary entanglement resources and the round of classical communications.

References

[1] E. Chitambar, D. Leung, L. Mancinska, M. Ozols, A. Winter, *Everything You Always Wanted to Know About LOCC (But Were Afraid to Ask)*, Commun. Math. Phys. 328, 303 (2014)

[2] V. Vedral, M. B. Plenio, M. A. Rippin, and P. L. Knight, *Quantifying Entanglement*, Phys. Rev. Lett. 78, 2275, (1997)

[3] Martin B. Plenio and Shashank Virmani, *An introduction to entanglement measures*, Quant. Inf. Comp. vol. 7, 1-51, (2007)

[4] Michal Horodecki, Pawel Horodecki, and Ryszard Horodecki, *Limits for Entanglement Measures*, Phys. Rev. Lett. 84, 2014, (2000)

[5] W. Dür, G. Vidal, and J. I. Cirac, *Optimal Conversion of Nonlocal Unitary Operations*, Phys. Rev. Lett. 89, 057901, (2002)

[6] A. Soeda, P. S. Turner, and M. Murao, *Entanglement cost of implementing controlled-unitary operations*, Phys. Rev. Lett. 107, 180501 (2011)

[7] D. Stahlke and R. B. Griffiths, *Entanglement requirements for implementing bipartite unitary operations*, Phys. Rev. A 84, 032316 (2011)

[8] Vlad Gheorghiu and Robert B. Griffiths, *Separable operations on pure states*, Phys. Rev. A 78, 020304 (2008)

[9] C. H. Bennett et.al., *Quantum nonlocality without entanglement*, Phys. Rev. A 59, 1070-1091 (1999)

[10] J. Niset and N. J. Cerf, *Multipartite nonlocality without entanglement in many dimensions*, Phys. Rev. A 74, 52103 (2006)

[11] D. P. DiVincenzo, T. Mor, P. W. Shor, J. A. Smolin, B. M. Terhal, *Unextendible Product Bases, Uncompletable Product Bases and Bound Entanglement*, Comm. Math. Phys. 238 (2003)

[12] E. Chitambar and R. Duan, *Nonlocal Entanglement Transformations Achievable by Separable Operations*, Phy. Rev. Lett. 103, 110502 (2009)

[13] C. H. Bennett. talk at QUPON, Vienna, Austria, May (2005)

[14] S. Lloyd, L. Maccone, R. Garcia-Patron, V. Giovannetti, and Y. Shikano, *Quantum mechanics of time travel through post-selected teleportation*, Phys. Rev. D 84, 025007 (2011)

[15] S. Lloyd, L. Maccone, R. Garcia-Patron, V. Giovannetti, Y. Shikano, S. Pirandola, L. A. Rozema, A. Darabi, Y. Soudagar, L. K. Shalm, and A. M. Steinberg, *Closed Timelike Curves via Postselection: Theory and Experimental Test of Consistency*, Phys. Rev. Lett. 106, 040403 (2011)

[16] O. Oreshkov, F. Costa, C. Brukner, *Quantum correlations with no causal order*, Nature communications 3, 1092 (2012).

3 Acknowledgements

This work is supported by Project for Developing Innovation Systems of MEXT, Japan and JSPS by KAKENHI (Grant No. 23540463, No. 26330006). We also gratefully acknowledge to the ELC project (Grant-in-Aid for Scientific Research on Innovative Areas MEXT KAKENHI (Grant No. 24106009)) for encouraging this research.