Quantum coherence, time-translation symmetry and thermodynamics

Kamil Korzekwa, Matteo Lostaglio, David Jennings and Terry Rudolph

Department of Physics, Imperial College London, London SW7 2AZ, United Kingdom

Fundamental laws of Nature often take the form of restrictions: nothing can move faster than light in vacuum, energy cannot be created from nothing, there are no perpetuum mobiles. It is due to these limitations that we can ascribe value to different objects and phenomena, e.g., energy would not be treated as a resource if we could create it for free. The mathematical framework developed to study the influence of such constraints on the possible transformations of quantum states is known under the collective name of resource theories.

Perhaps the best known example of this approach was to formalize and harness the puzzling phenomenon of quantum entanglement. However, the basic machinery developed to study entanglement is also perfectly suited to shed light on a much older subject – thermodynamics. The first and second laws are fundamental constraints in thermodynamics. These force thermodynamic processes to conserve the overall energy and forbid free conversion of thermal energy into work. Thus, a natural question to ask is: what amounts to a resource when we are restricted by these laws? This question is particularly interesting in the context of small quantum systems in the emergent field of single-shot thermodynamics [1–6].

Athermality is the property of a state of having a distribution over energy levels that is not thermal. This is a resource because, as expected from the Szilard argument, it can be converted into work, which in turn can be used to drive a system out of equilibrium. However *coherence* can be viewed as a second, independent resource in thermodynamics [7]. This stems from the fact that energy conservation implied by the first law restricts processing of coherence, and so possessing a state with coherence allows for otherwise impossible transformations. It also enforces a modification of the traditional Szilard argument: both athermality and coherence contribute to the free energy, however coherence remains "locked" and cannot be extracted as work.

Since coherence is a thermodynamic resource, an open question is what kind of coherence processing is allowed by thermodynamic means. This foundational question is of interest for future advancements in nanotechnology, as interference effects are particularly relevant [8, 9] at scales we are increasingly able to control [10–14]. Moreover, recent evidence suggests that biological systems may harness quantum coherence in relevant timescales [15–17]. Despite partial results [5, 18–22] we still lack a general framework to tackle the manipulation of coherence in thermodynamics.

The aim of this presentation is to address this problem and ask: what are the allowed transformations of quantum states that are consistent with the first and second laws of thermodynamics? The broad approach is the analysis of coherence in thermodynamics from a symmetry-based perspective. Specifically, the underlying energy-conservation within thermodynamics constrains all thermodynamic evolutions to be "symmetric" under time-translations in a precise sense. This in turn allows us to make use of harmonic analysis techniques, developed in [23], to track the evolution of coherence under thermodynamic transformations in terms of the "mode components" of the system. This constitutes a natural framework to understand coherence, allows us to separate out the constraints that stem solely from symmetry arguments from those particular to thermodynamics, and provides results that generalize recent work on coherence [21]. This approach also implies that the existing single-shot results applicable to block-diagonal results, constrained by thermo-majorization, can be viewed as particular cases of our analysis when only the zero-mode is present. Beyond this regime, every non-zero mode obeys *independent* constraints, and displays thermodynamic irreversibility similar to the zero-mode.

Exploiting these tools we arrive at the upper bounds on final coherences in the energy eigenbasis for quantum states undergoing time-translation symmetric and thermodynamic processing [24]. A rich dynamics is allowed, in which coherence can be transferred among different energy levels within each mode. We show that similarly to heat flows, coherence flows show directionality due to the limitations imposed by the second law. This new kind of irreversibility adds up to the ones identified in work extraction [3] and coherence distillation [7]. We also present a way to find the guaranteed amount of coherence that can be preserved while transforming between two states with given probability distributions over the energy levels.

---

[1] D. Janzing, P. Wocjan, R. Zeier, R. Geiss, and T. Beth, Int. J. Theor. Phys. **39**, 2717 (2000).

[2] F. G. S. L. Brandão, M. Horodecki, J. Oppenheim, J. M. Renes, and R. W. Spekkens, Phys. Rev. Lett. **111**, 250404 (2013).

[3] M. Horodecki and J. Oppenheim, Nat. Commun. **4**, 2059 (2013).

[4] J. Åberg, Nat. Commun. **4**, 1925 (2013).

[5] J. Åberg, Phys. Rev. Lett. **113**, 150402 (2014).

[6] O. Dahlsten, R. Renner, E. Rieper, and V. Vedral, New J. Phys. **13**, 053015 (2011).

[7] M. Lostaglio, D. Jennings, and T. Rudolph, arXiv:1405.2188 (2014).

[8] O. Karlström, H. Linke, G. Karlström, and A. Wacker, Phys. Rev. B **84**, 113415 (2011).

[9] H. Vazquez, R. Skouta, S. Schneebeli, M. Kamenetska, R. Breslow, L. Venkataraman, and M. Hybertsen,

Nature Nanotech. **10**, 663 (2012).

[10] D. Collin, F. Ritort, C. Jarzynski, S. B. Smith, I. Tinoco, and C. Bustamante, Nature **437**, 231 (2005).

[11] V. Serreli, C. Lee, E. Kay, and D. Leigh, Nature **445** (2006).

[12] S. Toyabe, T. Sagawa, M. Ueda, E. Muneyuki, and M. Sano, Nature Phys. **6**, 988 (2010), 1009.5287.

[13] Alemany, Anna and Ritort, Felix, Europhys. News **41**, 27 (2010).

[14] J. Cheng, S. Sreelatha, R. Hou, A. Efremov, R. Liu, J. R. C. van der Maarel, and Z. Wang, Phys. Rev. Lett. **109**, 238104 (2012).

[15] S. Lloyd, J. Phys.: Conf. Ser. **302**, 012037 (2011).

[16] N. Lambert, Y.-N. Chen, Y.-C. Cheng, C.-M. Li, G.-Y. Chen, and F. Nori, Nature Phys. **9** (2013).

[17] E. M. Gauger, E. Rieper, J. J. Morton, S. C. Benjamin, and V. Vedral, Phys. Rev. Lett. **106**, 040503 (2011).

[18] M. O. Scully, M. S. Zubairy, G. S. Agarwal, and H. Walther, Science **299**, 862 (2003).

[19] C. A. Rodríguez-Rosario, T. Frauenheim, and A. Aspuru-Guzik, arXiv 1308.1245 (2013).

[20] P. Skrzypczyk, A. J. Short, and S. Popescu, arXiv 1302.2811 (2013).

[21] P. Ćwikliński, M. Studziński, M. Horodecki, and J. Oppenheim, arXiv:1405.5029 (2014).

[22] V. Narasimhachar and G. Gour, arXiv 1409.7740 (2014).

[23] I. Marvian and R. W. Spekkens, arXiv 1312.0680 (2013).

[24] M. Lostaglio, K. Korzekwa, D. Jennings, and T. Rudolph, arXiv:1410.4572 (2014).