# Outcome strategies in geometric Bell inequalities

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The Bell theorem demonstrates a conflict between local hidden variable theories and quantum mechanics. The theorem also leads to advantage of certain quantum information processing tasks. However, in realistic situations one must take into account various losses and hence find a version of the theorem, which is possibly the most robust against various experimental imperfections. For a collection of $N$ qubits in a GHZ state,

$$|GHZ\rangle = \frac{1}{\sqrt{2}}(|0\rangle^{\otimes N} + |1\rangle^{\otimes N}, \tag{1}$$

Nagata *et al.* have [1] showed that the greatest resistance against white noise admixture is achevied with geometrical Bell inequalities, which can utilize any number of local measurement settings. They treat the correlation function parametrized by measurement angles as a multidimensional vector. Then we take set $A$ of vectors , which are correlation functions obeying the assumpions of local realism. Then, if we have $\vec{V}\cdot\vec{V} > \vec{V}\cdot\vec{A}$, $\vec{A}$ being any element of $A$, we are certain that $A$, which represents the quantum mechanical statistics, is not an element of $S$. The scalar product can be defined as a sum or an integral over local measurement settigs, for example

$$E_{QM}\cdot E_{LR} = \int_0^{2\pi} ... \int_0^{2\pi} E_{QM}(\alpha_1,...,\alpha_N)E_{LR}(\alpha_1,...,\alpha_N)d\alpha_1...d\alpha_N, \tag{2}$$

with local observables parametrized by $a(\alpha_i) = \cos\alpha_i\sigma_x + \sin\alpha_i\sigma_y$.

More recently, this type of inequalities was generalized to collections of spins-1 [2]. As observables we have used squared spin-1 components,

$$A(\alpha_i) = (\vec{S}\cdot\vec{n})^2 - 2/3, \tag{3}$$

where $\vec{S}$ is a vector of spin-1 operator and $\vec{n} = \frac{1}{\sqrt{3}}\left(\sqrt{2}\cos\alpha_i, \sqrt{2}\sin\alpha_i, 1\right)$. The inequalities constructed like that are violated by a biased GHZ state, and the violation factor grows exponentially with the number of particles. However, the growth is weaker than in case of qubits. The violation ratio for infinitively many measuring settings behaves like $\left(\frac{2\pi}{3\sqrt{3}}\right)$, rather than $\left(\frac{\pi}{2}\right)$.

It is therefore interesting to find a way to generalize the geometrical Bell inequalities to higher-dimensional systems. Our aim is to compare what strategies of outcome treatment We consider $N$ qudits, which are in a GHZ state,

$$|GHZ_{N,d}\rangle = \frac{1}{\sqrt{d}}\sum_{j=0}^{d-1}|j\rangle^{\otimes N}, \tag{4}$$

and all the observables have their eigenvectors in the following forms

$$|\alpha\rangle = \text{Diag}\left(1, e^{i\alpha}, e^{2i\alpha}, ...\right) F|0\rangle, \tag{5}$$

where $F$ is the Fourier transformation. Note that $\left\{|\alpha\rangle, \left|\alpha+\frac{2\pi}{d}\right\rangle, ...\right\}$ form an orthogonal basis, and we shall give them a label $j$, such that $\alpha_j = \alpha + \frac{2\pi j}{d}$. We investigate various interpretations of local detection of one of these states. One will assign a dichotomic scalar $-(d-1)/d$ for $|\alpha_0\rangle$ and $-1/d$ otherwise. These values are then multiplied. In the second strategy, label $j$ is translated into a complex number $e^{2\pi ij/d}$, and in the other two, the labels are summed modulo $d$ and the value of the sum is associated with either a scalar or a $(d-1)$-dimensional vector. All these strategies lead to a firm violation of Bell inequalities, which drops, however, as $d$ grows. Multiplying complex numbers provides violation ratios lower than in other strategies, which are equivalent to each other.

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