Certifying quantumness: Benchmarks for the optimal processing of generalized coherent states[1]

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1 Introduction

Quantum technology promises revolutionary advantages in information processing and transmission compared to classical technology; however, determining which specific resources are needed to surpass the capabilities of "semi-classical machines" (see Fig. 1) remains often a nontrivial problem. To address such a problem, one first needs to establish the best classical solutions, which set benchmarks to be beaten by any implementation claiming to harness non-local quantum features for enhanced performance.

In this work[1] we develop a general formalism to obtain benchmarks for the probabilistic quantum protocols with input states generated by a group action, which are called the Gilmore-Perelomov generalized coherent states[2, 3, 4]. We explore the possible application of the general benchmark for a large variety of quantum systems, covering discrete variable systems and continuous variable (Gaussian and non-Gaussian) systems. The benchmarks derived are probabilistic[5, 6], which means that we give Alice and Bob, who are implementing a classical measure-and-prepare procedure to emulate the action of a particular quantum channel on an input set of states, the freedom to discard some unfavourable trials. (See Fig. 1 for the scenario.)

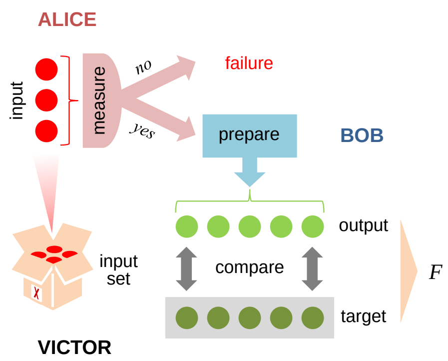

Figure 1: "Semi-classical machines". Defined as states transform process without non-local quantum operations, namely Alice and Bob perform a measure-and-prepare strategy to fulfill their task, where Alice can perform any measurement (arbitrary POVM) on the input state and informs Bob by classical communication. Note that the machines we consider are probabilistic, i.e. Alice is allowed to announce inconclusive measurement outcomes.

The benchmarks set the ultimate bounds that need to be surpassed by quantum implemen-

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tations claiming to exploit authentic quantum resources to attain classically unreachable performances. Consequently, this work provides straightforward criteria to detect genuine non-local quantum operations in a wide of experiments, varying from the teleportation of finite dimensional quantum systems, to the amplification of weak optical signals.

2 Benchmarks for the probabilistic processing of generalized coherent states

The Gilmore-Perelomov coherent state $|\phi_g\rangle$[2, 3, 4], defined by acting the irreducible unitary representation U_g of a group G on a unit vector $|\phi\rangle$ in some Hilbert space $\mathcal{H}$, covers a broad class of quantum states, among which the most known case is the coherent state of the harmonic oscillator $|\alpha\rangle := \hat{D}(\alpha)|0\rangle$, where $\hat{D}(\alpha)$ is the displacement operator. We consider the task of transforming N copies of an input coherent state $|\phi_g\rangle \in \mathcal{H}$ into M copies of another, possibly different coherent state $|\psi_g\rangle \in \mathcal{K}$. Meanwhile we assume that the input state is priorly distributed according to a probability distribution $p_\gamma(g)$ defined in the certain form

$$p_\gamma(g) = d_\gamma |\langle\phi_\gamma|\phi_{\gamma,g}\rangle|^2 \tag{1}$$

where $|\phi_{\gamma,g}\rangle := U_{\gamma,g}|\phi_\gamma\rangle$ is a coherent state in some Hilbert space $\mathcal{H}_\gamma$ and d_γ is the normalization constant. As a figure of merit, we consider the maximization of the fidelity between the output state produced by the device and the target state $|\psi_g\rangle^{\otimes M}$, precisely maximizing

$$F_{\mathscr{N}} = \frac{\int \mathrm{d}g\; p_\gamma(g)\,\mathrm{Tr}[(|\psi_g\rangle\langle\psi_g|)^{\otimes M}\,\mathscr{N}(|\phi_g\rangle\langle\phi_g|^{\otimes N})]}{\mathrm{Tr}[\mathscr{N}(\bar{\rho})]} \qquad \bar{\rho} := \int \mathrm{d}g\, p_\gamma(g)\,(|\phi_g\rangle\langle\phi_g|)^{\otimes N}$$

over all possible non-deterministic measure-and-prepare protocols $\mathscr{N}(\rho) := \int_{\hat{g}\in\mathsf{G}_{\mathrm{yes}}} \mathrm{d}\hat{g}\,\mathrm{Tr}[P_{\hat{g}}\rho]\,\sigma_{\hat{g}}$, $\mathsf{G}_{\mathrm{yes}} \subset \mathsf{G}$ ($\{P_{\hat{g}}\}_{\hat{g}\in\mathsf{G}}$ stands for the measurement).

Based on previous works [5, 6], we derived the following benchmark for the probabilistic processing of generalized coherent states.

Theorem 1. *The probabilistic benchmark for the transformation $|\phi_g\rangle^{\otimes N} \mapsto |\psi_g\rangle^{\otimes M}$ is given by:*

$$F_c(\gamma) = \frac{\int \mathrm{d}g\, p_\gamma(g)\, |\langle\psi|\psi_g\rangle|^{2M} |\langle\phi|\phi_g\rangle|^{2N}}{\int \mathrm{d}g\, p_\gamma(g)\, |\langle\phi|\phi_g\rangle|^{2N}}. \tag{2}$$

Here $p_\gamma(g)$ is the prior distribution of the input state, given in Eq. (1).

The general benchmark (2) has a wide range of applications from the storage or teleportation of qubit states to the amplification of the coherent light. We will show them in sequence for the following sections.

3 Benchmarks for qudit states

We begin introducing the application of the general benchmark (2) from the finite dimensional quantum ensembles. Consider benchmarking devices that transform N identical d-level quantum systems, which are represented by qudit states $|\psi_g\rangle$, $g \in SU(d)$, into M approximate copies. The probabilistic benchmark for this task can be derived from Eq. (2) as

$$F_{\mathrm{c}}^{(d)}(\beta) = \frac{\binom{N+\beta+d-1}{d-1}}{\binom{M+N+\beta+d-1}{d-1}}. \tag{3}$$

Notice that $\binom{k}{d} \equiv \frac{\Gamma(k+1)}{\Gamma(k-d+1)\Gamma(d+1)}$ and β is a non-negative real parameter for the prior distribution $p_\beta(g) \propto |\langle\psi|\psi_g\rangle|^{2\beta}$ of the input state. This benchmark provides important criteria in a wide range of setups. For instance, taking $d = 2$ in Eq. (3) we see the benchmark for qubit state tele-cloning, which reads $F_{\mathrm{c}}^{(\mathrm{tele})}(\beta) = (N+\beta+1)/(N+M+\beta+1)$. When $\beta = 0$ this reproduces the benchmark for both deterministic and probabilistic tele-cloning without prior knowledge known in the literature [7, 8].

4 Benchmarks for single-mode Gaussian states

Next we explore the benchmarks for continuous variable states. In this section we present the benchmark for the single-mode Gaussian state. A general pure single-mode Gaussian state is a displaced squeezed state, written in the form $|\alpha, \xi\rangle := \hat{D}(\alpha)\hat{S}(\xi)|0\rangle$, where $\hat{D}(\alpha) := \exp(\alpha\hat{a}^\dagger - \alpha^*\hat{a})$ is the displacement operator and $\hat{S}(\xi) := \exp\left[\frac{1}{2}(\xi\hat{a}^{\dagger 2} - \xi^*\hat{a}^2)\right]$ is the squeezing operator. The single-mode Gaussian state is fully characterized by the displacement $\alpha \in \mathbb{C}$ and $\xi = se^{i\theta} \in \mathbb{C}$ where $s \in \mathbb{R}^+$ is the squeezing degree and $\theta \in [0, 2\pi)$ is the squeezing phase.

Consider the prior distribution $p_{\lambda,\beta}(\alpha, \xi) := \frac{\lambda\beta}{2\pi^2} \frac{e^{-\lambda|\alpha|^2 + \lambda\Re(e^{\{-i\theta\}}\alpha^2)\tanh s} \sinh s}{(\cosh s)^{\beta+2}} \propto |\langle 0|\lambda\alpha, \xi\rangle|^2 |\langle 0|\xi\rangle|^{2(4+\beta)}$, which follows the general prescription of Eq. (1), the benchmark of the single-mode Gaussian state is

$$F_{\mathrm{c}}^{(1cs)}(\lambda, \beta) = \frac{(N+\lambda)(N+\beta)}{(N+M+\lambda)(N+M+\beta)}. \tag{4}$$

This benchmark helps to detect genuine non-local quantum operations in experiments concerning single-mode continuous variable resources, and it can be straightfowardly reduced to the benchmarks for the single-mode coherent state and the single-mode squeezed vacuum state.

5 Benchmarks for non-Gaussian squeezed states

The benchmark has applications not only in the probabilistic processing of discrete variable or Gaussian states, as described above, but also in the non-Gaussian state processing. Specifically, the benchmark (2) can be used to derive the benchmark for a family of squeezed states.

Among this family of states there is the squeezed single-photon state $\hat{S}(\xi)|1\rangle$, which can be achieved by squeezing the single-photon Fock state $|1\rangle$. Consider the probabilistic processing of the squeezed single-photon state distributed according to the prior $p_\beta(\xi) \propto |\langle 1|\hat{S}(\xi)|1\rangle|^{2(\beta+2)}$, the benchmark reads

$$F_{\mathrm{c}}^{(\mathrm{SSP})} = \frac{3N+\beta}{3(N+M)+\beta}.$$

When the amount of squeezing is low, the squeezed single-photon state is a good approximation of the odd Schrödinger cat state $|\psi_{\mathrm{odd\ cat}}(\alpha)\rangle := \frac{|\alpha\rangle - |-\alpha\rangle}{\sqrt{2\left(1-e^{-2|\alpha|^2}\right)}}$[9, 10, 11]. Hence, when the prior distribution of the squeezing parameter is concentrated in a small region around $\xi = 0$, the benchmark derived for single-photon squeezed states is close to the corresponding benchmark for odd cat states.

Another set of non-Gaussian states that can be benchmarked by Eq. (2) is the unbalanced two-mode squeezed states $S^{(2)}(\xi)|m\rangle|0\rangle$, which is obtained through a two-mode squeezing process $S^{(2)}(\xi) = \exp[-\xi^* a_1 a_2 + \xi a_1^\dagger a_2^\dagger]$ on the two-mode Fock state $|m\rangle|0\rangle$. The probabilistic benchmark for the unbalanced two-mode squeezed state $S^{(2)}(\xi)|m\rangle|0\rangle$ is

$$F_{\mathrm{c}}^{(\mathrm{UTMS})} = \frac{(m+1)N+\beta}{(m+1)(N+M)+\beta}.$$

The unbalanced two-mode squeezed state is regarded as a valuable non-Gaussian resource when one plan to generate arbitrary continuos variable states using Gaussian processes[12]. Consequently, the ability to provide the benchmark for this class of continuous variable states will shed light on the study in the processing for arbitrary two-mode continuos variable states.

6 Conclusions

In this work we derive a general and useful technique to provide benchmark for probabilistic quantum processes, including teleportation, cloning and amplification. Specifically, our technique can be applied to all Gilmore-Perelomov generalized coherent states[2, 3, 4], covering the qudit state, the one-mode Gaussian state and some non-Gaussian squeezed states.

References

[1] Yuxiang Yang, Giulio Chiribella, and Gerardo Adesso. Certifying quantumness: Benchmarks for the optimal processing of generalized coherent and squeezed states. *Phys. Rev. A*, 90:042319, Oct 2014.

[2] A. M. Perelomov. Coherent states for arbitrary lie group. *Comm. Math. Phys.*, 26:222, 1972.

[3] Askol'd Mikhailovich Perelomov. *Generalized coherent states and their applications.* Springer-Verlag, Heidelberg, 1986.

[4] R. Gilmore. Geometry of symmetrized states. *Ann. Phys.*, 74:391, 1972.

[5] Giulio Chiribella and Jinyu Xie. Optimal design and quantum benchmarks for coherent state amplifiers. *Phys. Rev. Lett.*, 110:213602, May 2013.

[6] Giulio Chiribella and Gerardo Adesso. Quantum benchmarks for pure single-mode gaussian states. *Phys. Rev. Lett.*, 112:010501, 2014.

[7] Dagmar Bruss and Chiara Macchiavello. Optimal state estimation for d-dimensional quantum systems. *Phys. Lett. A*, 253:249, 1999.

[8] S. Massar and S. Popescu. Optimal extraction of information from finite quantum ensembles. *Phys. Rev. Lett.*, 74:1259–1263, Feb 1995.

[9] Alexei Ourjoumtsev, Rosa Tualle-Brouri, Julien Laurat, and Philippe Grangier. Generating optical schrödinger kittens for quantum information processing. *Science*, 312(5770):83–86, 2006.

[10] AP Lund, H Jeong, TC Ralph, and MS Kim. Conditional production of superpositions of coherent states with inefficient photon detection. *Physical Review A*, 70(2):020101, 2004.

[11] Agata M Branczyk and TC Ralph. Teleportation using squeezed single photons. *Physical Review A*, 78(5):052304–1, 2008.

[12] David Menzies and Radim Filip. Gaussian-optimized preparation of non-gaussian pure states. *Phys. Rev. A*, 79:012313, Jan 2009.