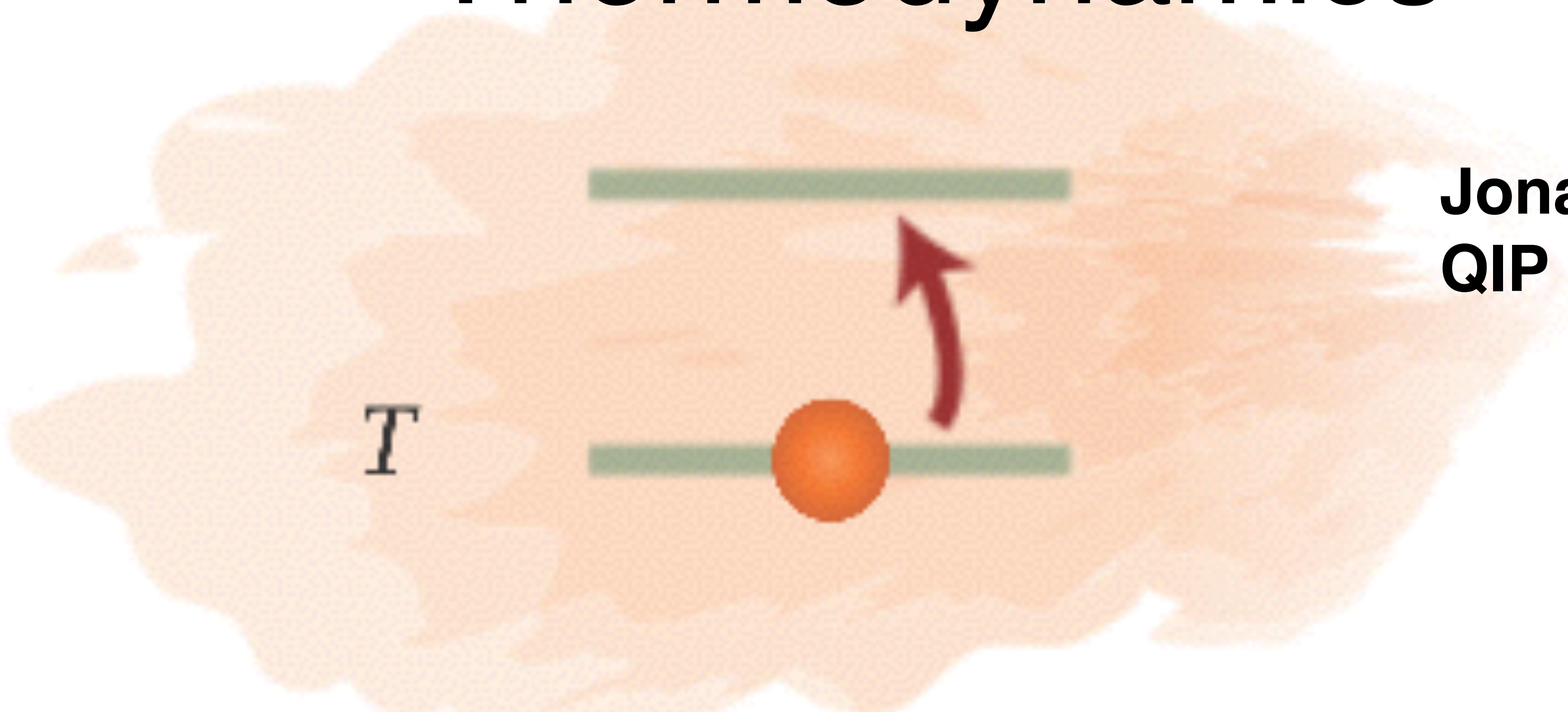
The 2nd laws of Quantum Thermodynamics



Jonathan Oppenheim (UCL)
QIP 2014

1305.5278 Brandao, Horodecki, Ng, J.O., Wehner

1111.3834 Horodecki, JO; (Nature Comms)

1111.3882 Brandao, Horodecki, JO, Renes, Spekkens (PRL)

0212019 Horodecki², JO (PRA)

Outline

- 2^{nd} law: allowable state transitions $(\rho, H) \to (\sigma, H')$
- Many 2^{nd} laws: $F \longrightarrow F_\alpha$
- Many families of 2^{nd} laws depending on "how cyclic" the process is
- 0th, 1st law: class of operations
- Tools from QI:
  - resource theories
  - catalytic majorization
  - quantum Renyi-divergences
  - entanglement embezzling

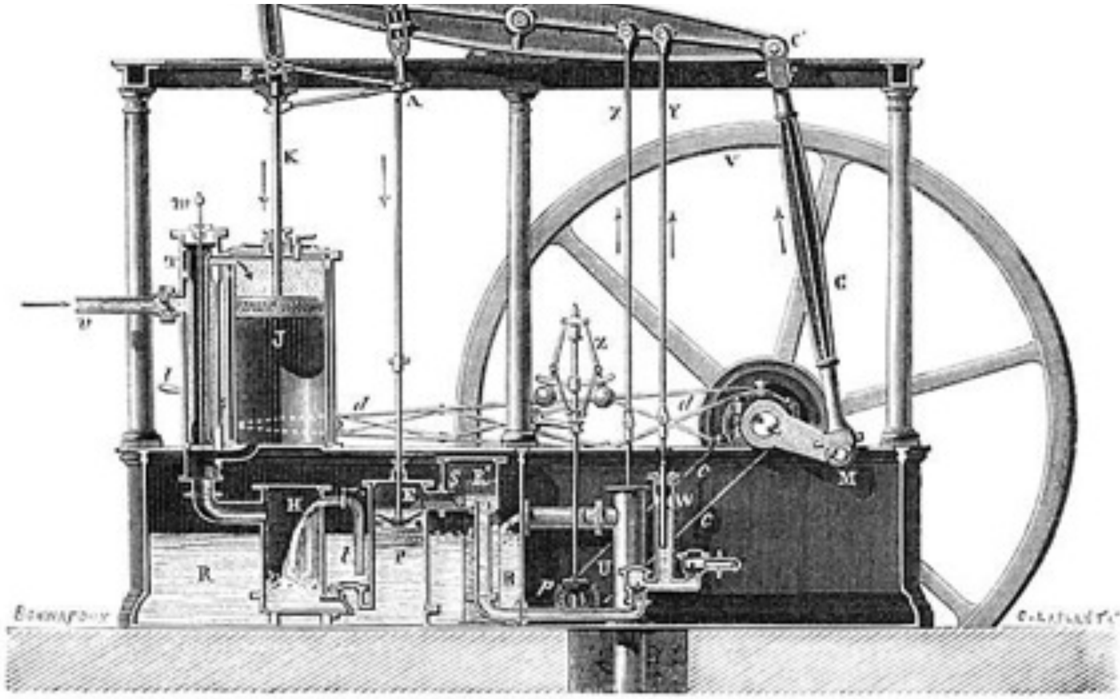
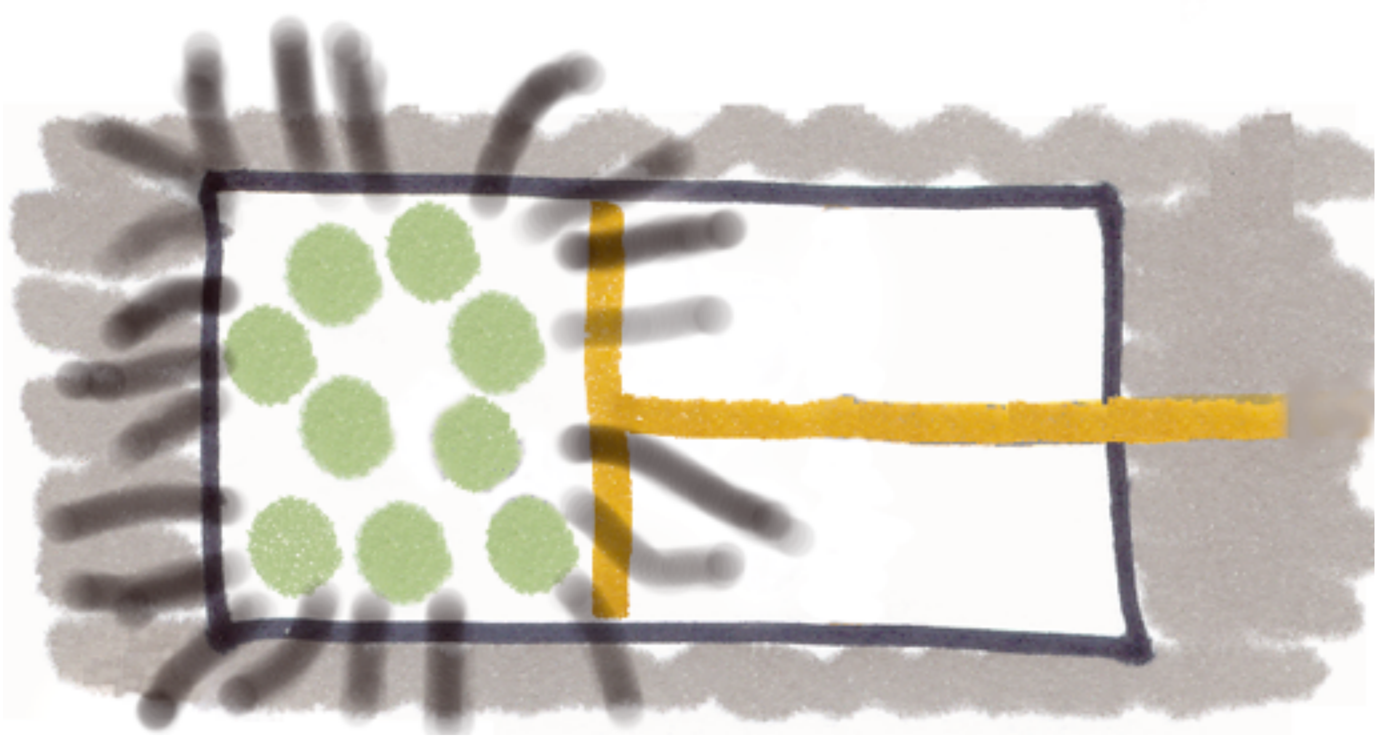


Fig. 59. — Machine à balancier de Watt.

v. Tuyau de prise de vapeur; T, tiroir; J, cylindre; H, condenseur; PE pompe d'épuisement; WY pompe alimentaire de la chaudière UX pompe d'alimentation de la bâche R; p Z régulateur; dd excentrique; ABCD parallélogramme; GM bielle et manivelle; V volant.

1st wave
(phenomenology)
Carnot (1824)
Joule (1843)
Kelvin (1849)
Clausius (1854)



2nd wave
(stat mech)
Maxwell (1871)
Boltzman (1875)
Gibbs (1876)

Quantum Thermodynamics

- **Gibbs state with full information**
  - Gemmer, Michel, Mahler (2005), Popescu, Short, Winter (2006)
- **Meaning of Negative Entropy, conditional erasure**
  - Del Rio et al. (2011), Faist et al. (2013)
- **Smallest possible fridges**
  - Linden, Popescu, Skrzypczyk (2010)
- **Deterministic work extraction**
  - HHO (2003), Dahlston et al. (2010), HO (2011), Aaberg (2011), Egloff et al. (2012)
- **Average work extraction**
  - Brandao et. al. (2011), Skrzypczyk et. al. (2013)
- **Non-ideal heat baths, correlations, entanglement**
  - Reeb, Wolf (2013); Gallego et. al. (2013), Hovhannisyan et. al. (2013)
- **Thermalisation times**
- **Micro engines & machines**
  - Scovil & Schultz-Dubois (1959), Howard (1997)
  - Rousselet et al. (1994), Faucheux et al. (1995), Scully (2002)
- **Prehistory**
  - Ruch and Mead (75), Janzig et. al. (2000)

Thermodynamics of quantum systems

- No thermodynamic limit!
- Coherences in energy basis
- More precise control?
- Rigorous theory?

3 laws of classical thermodynamics

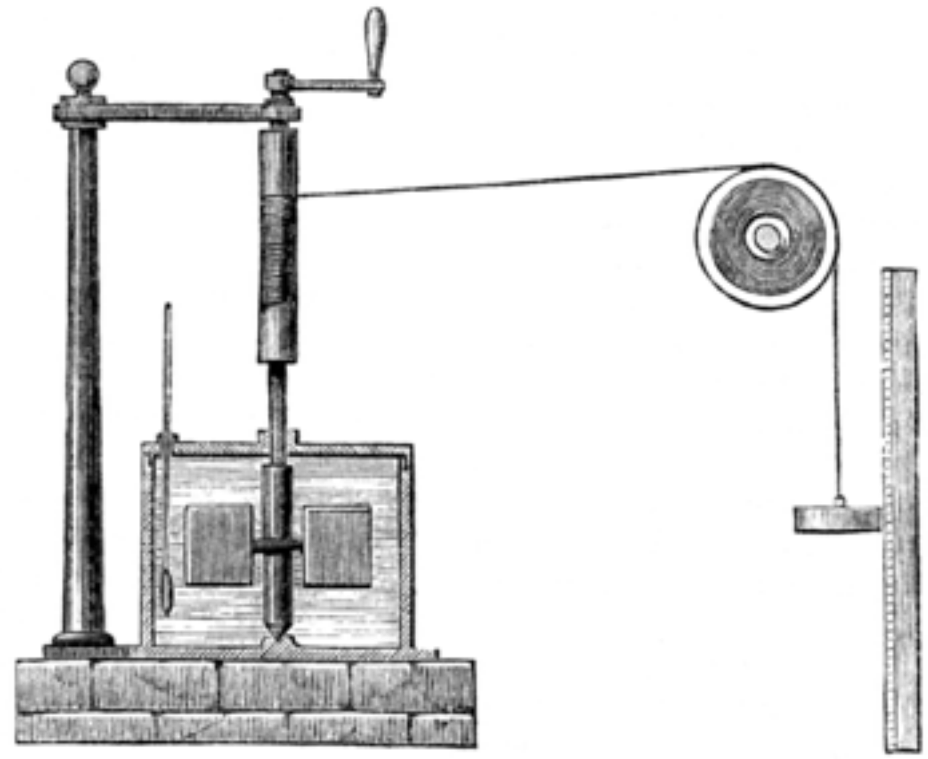
0) If R_1 is in equilibrium with R_2 and R_3 then R_2 is in equilibrium with R_3

1) $dE = dQ - dW$ (energy conservation)

2) Heat can never pass from a colder body to a warmer body without some other change occurring. – Clausius

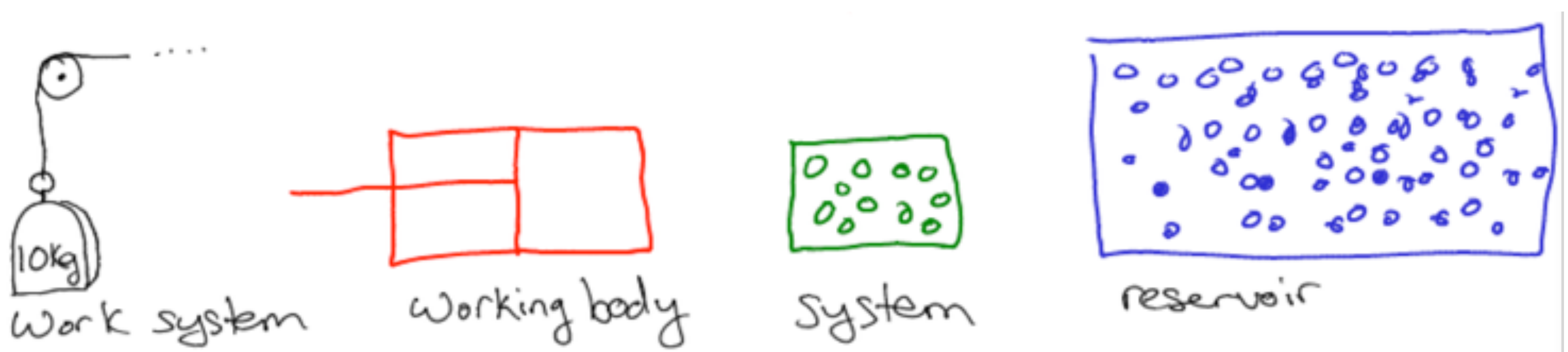
3) One can never attain $S(\rho) = C$ in a finite number of steps

1) $dE = dQ - dW$ (energy conservation)



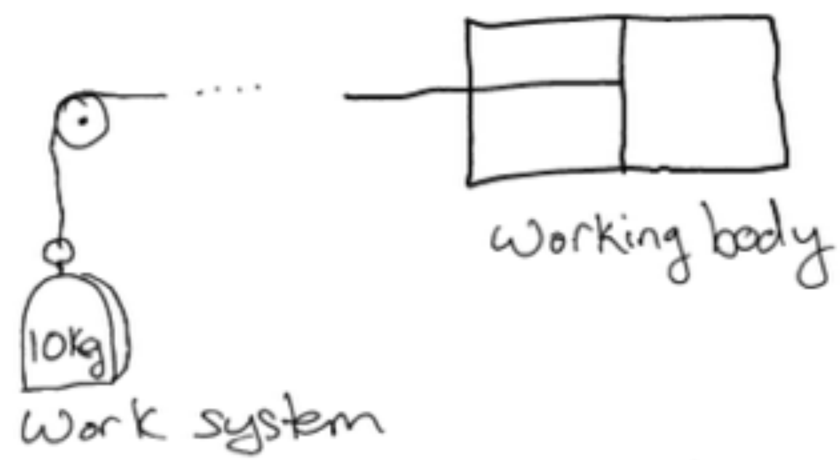
Joule (1843)

Not a consequence, but part of the class of operations



The second law

Heat can never pass from a colder body to a warmer body without some other change occurring. – Clausius



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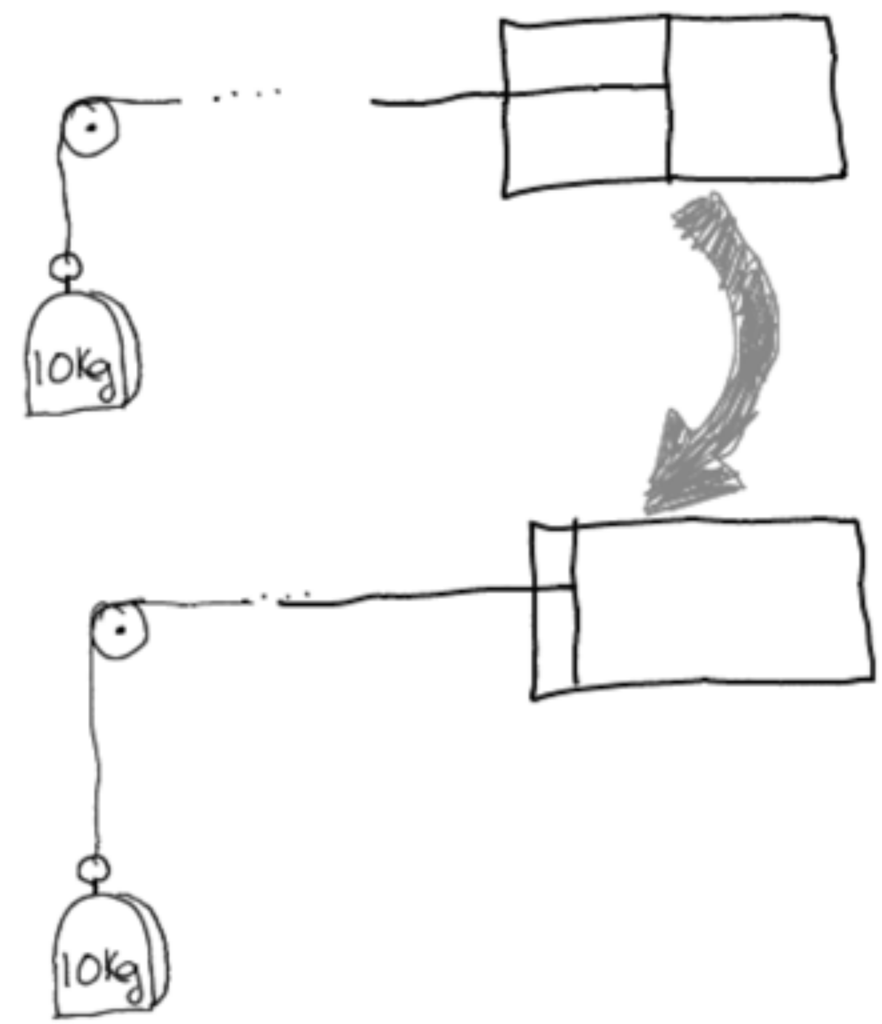

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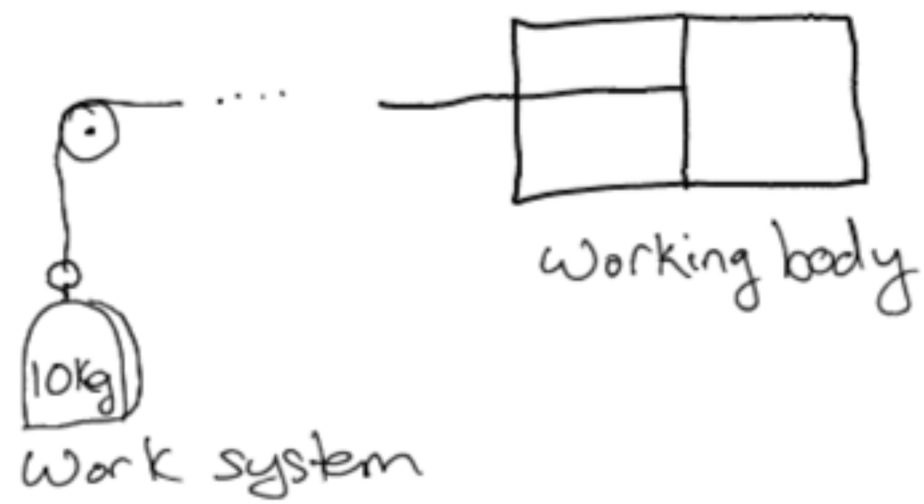
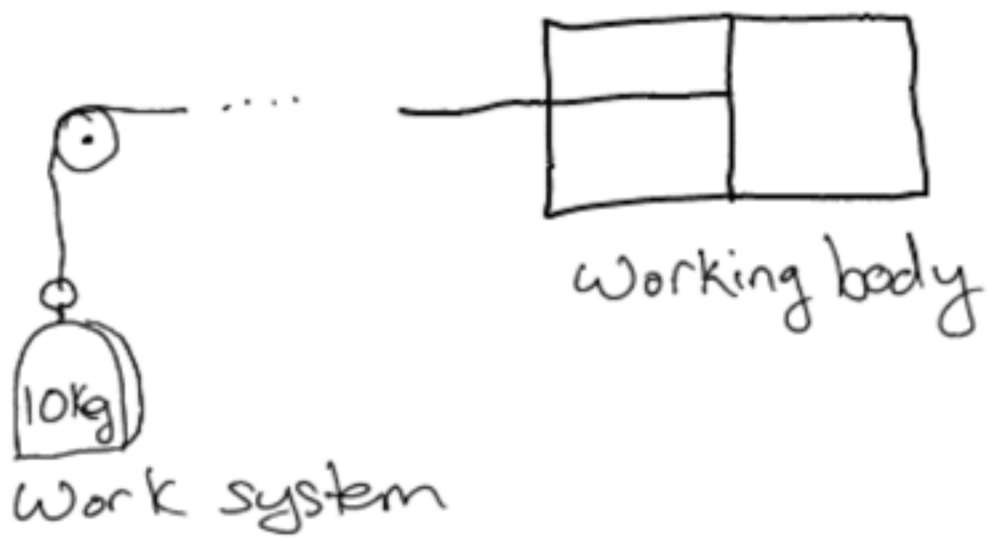
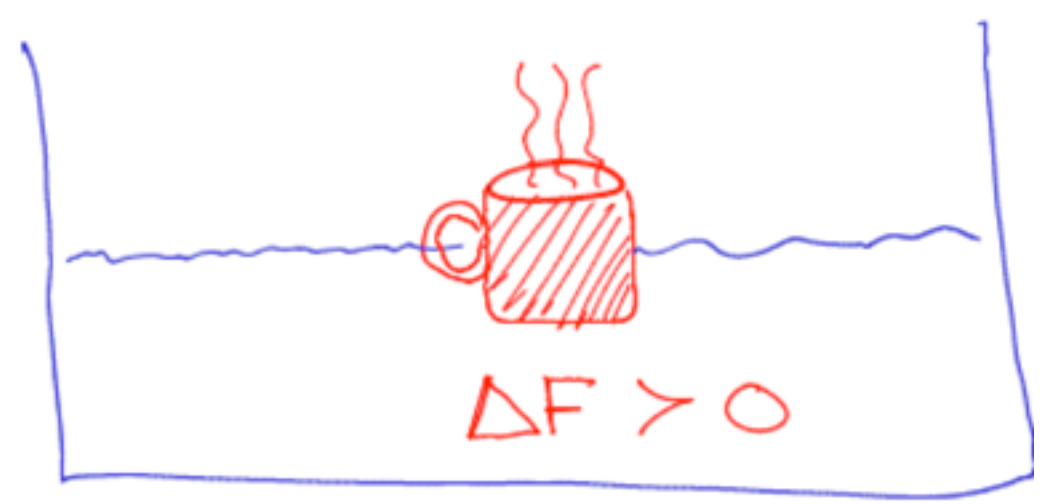


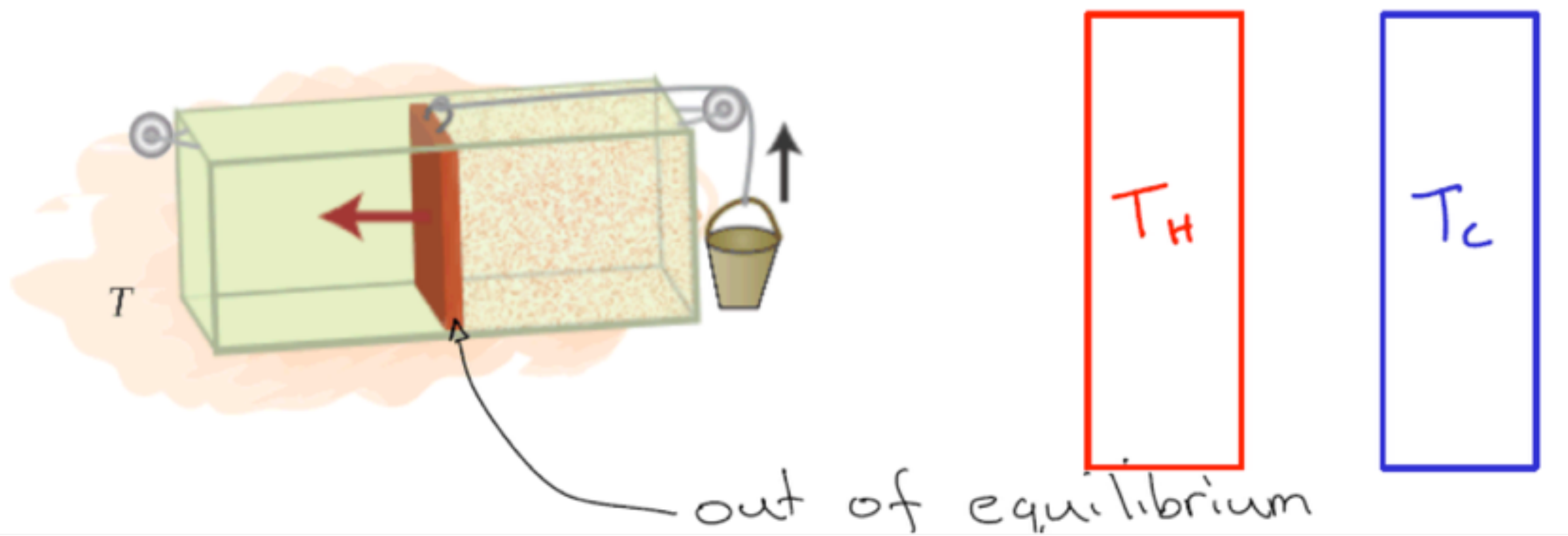
The second law
(+ first law)

In any cyclic process, the free energy of a system can only decrease.





Free Energy



$$F = E - TS$$

$$W_{\text{gain}} = F(\rho_{\text{initial}}) - \mathrm{F}(\rho_{\text{final}})$$

$$\rho_{\text{initial}} \rightarrow \rho_{\text{final}} \quad \text{only if} \quad \Delta F \geq 0$$

Catalytic Thermal Operations $\wedge_\tau$

- (ρ_s, H_s)
  - ρ_s = resource
  - H_s = Hamiltonian
- adding arbitrary many copies of state τ_R
- borrowing ancillas (working body) and returning them in the "same" state σ_c
- energy conserving unitaries U
  (1st law) $[U, H_s + H_R + H_c] = 0$
- tracing out (trash)

Thermal Operations

work is

or in the micro - regime

work bit ("wit")

1 law of quantum thermodynamics

Class of operations:

0) The only free state τ which doesn't enable arbitrary transitions is the thermal state $\tau = e^{-\beta H_R}/Z$

1) Energy conserving unitaires

Thermo-monotones:

2)* The state ρ_s must get closer to τ_s in terms of free energy type distances $F_a(\rho_s \,||\, \tau_s)$ $\forall a \leq 0$

Zeroeth Law

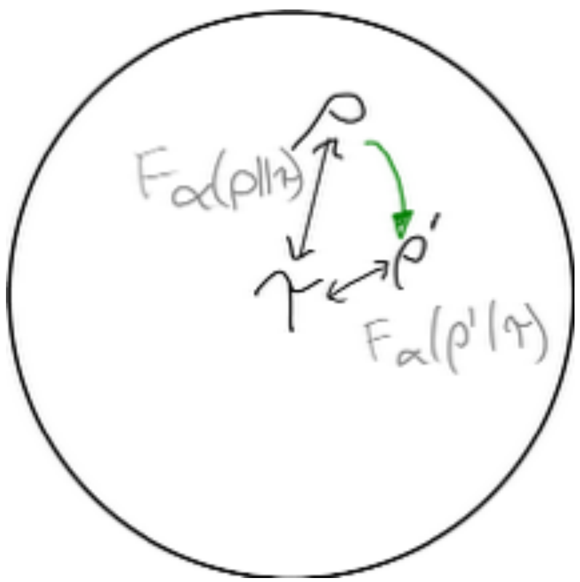
After decohering in the energy eigenbasis, one can extract work from many copies of any state which is not passive ($p_i \leq p_j$ iff $E_i \geq E_j$)

[just swap levels i and j, while raising the weight, and repeat over many blocks]

Many copies of any state except the thermal state results in a state which is not passive after decohering (Pusz and Woronowicz (78), Brandao et. al. 2011).

This gives us an equivalence class, of allowed free states (τ_β, H_R) labelled by β. Any other free state allows arbitrary transitions.

The second laws (psuedo-classical)



Thermal monotones:

$$F_\alpha(\rho||\tau) = \frac{kT}{1-\alpha}\log\ tr\,\rho^\alpha\tau^{1-\alpha} - kT\log Z \quad \alpha \geq 0$$

$$F_1(\rho||\tau) = F(\rho)$$

- Ordinary 2[nd] law is one of many
- In macroscopic limit, weak interactions, all $F_\alpha \simeq F$
- For ρ block diagonal, 2[nd] law is necessary and sufficient

Quantum Second Laws

$$\hat{F}_\alpha(\rho||\tau) := \frac{kT}{\alpha - 1}\log\left(tr\left(\tau^{\frac{1-\alpha}{2\alpha}}\rho\ \tau^{\frac{1-\alpha}{2\alpha}}\right)^{\alpha}\right) - kT\log Z$$

Müller–Lennert et al.
Wilde et al.
Jaksic et al. (2013)

or

$$\tilde{F}_\alpha(\rho||\tau) := kT\,\frac{\mathrm{sgn}(\alpha)}{\alpha - 1}\log\ tr\ \rho^{\alpha}\ \tau^{1-\alpha} - kT\log Z$$

Petz (86)

$\tilde{F}_\alpha$ monotonic under CPT maps for $\alpha \in [0, 1]$

$\hat{F}_\alpha$ monotonic for $\alpha \geq \frac{1}{2}$

Frank and Lieb (2013)

$$\tilde{F}(\rho||\tau) \geq \tilde{F}(\wedge_\tau(\rho)||\wedge_\tau(\tau)) = \tilde{F}(\rho'||\tau)$$

Quantum Second Laws

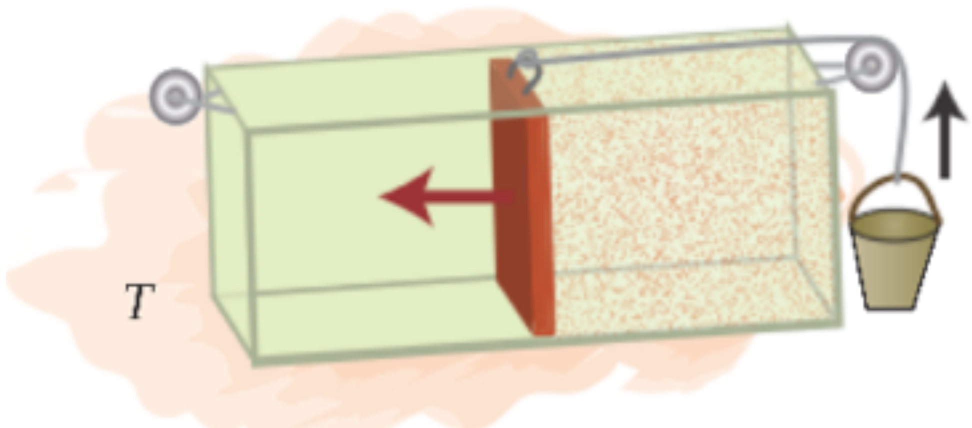
$$\rho \xrightarrow{\text{C.T.O.}} \rho' \qquad \text{only if}$$

$$\hat{F}_\alpha(\rho||\tau) \geq \hat{F}(\rho'||\tau) \qquad \alpha \geq \frac{1}{2}$$

$$\hat{F}_\alpha(\tau||\rho) \geq \hat{F}_\alpha(\tau||\rho) \qquad \frac{1}{2} \leq \alpha \leq 1$$

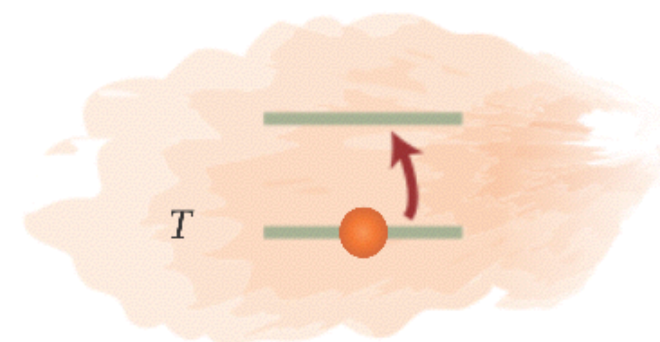
$$\hat{F}_\alpha(\rho||\tau) \geq \hat{F}_\alpha(\rho'||\tau) \qquad 0 \leq \alpha \leq 2$$

Macro



weight

Micro



wit

$\cancel{F=E-TS}$

$$F_\infty = kT \log \min\{\lambda : \rho \leq \lambda \tau\} - kT ln Z$$

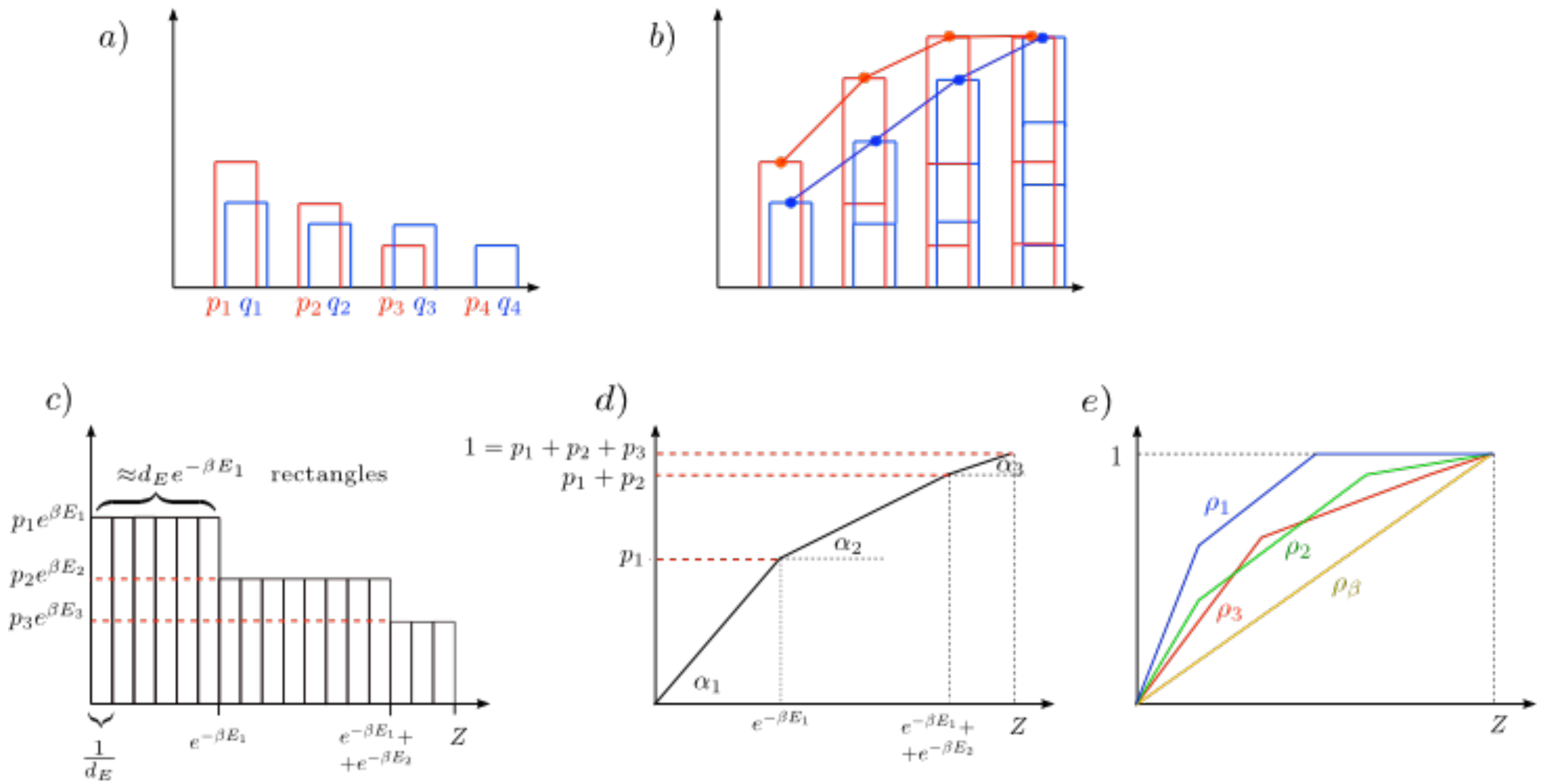
$$F_0 = kT \log \sum h(\omega, g, E_i) e^{-\beta E_i}$$

$$W_{land} = kT \log(1 + e^{-\beta E_e})$$

Quasi-classical 2nd laws

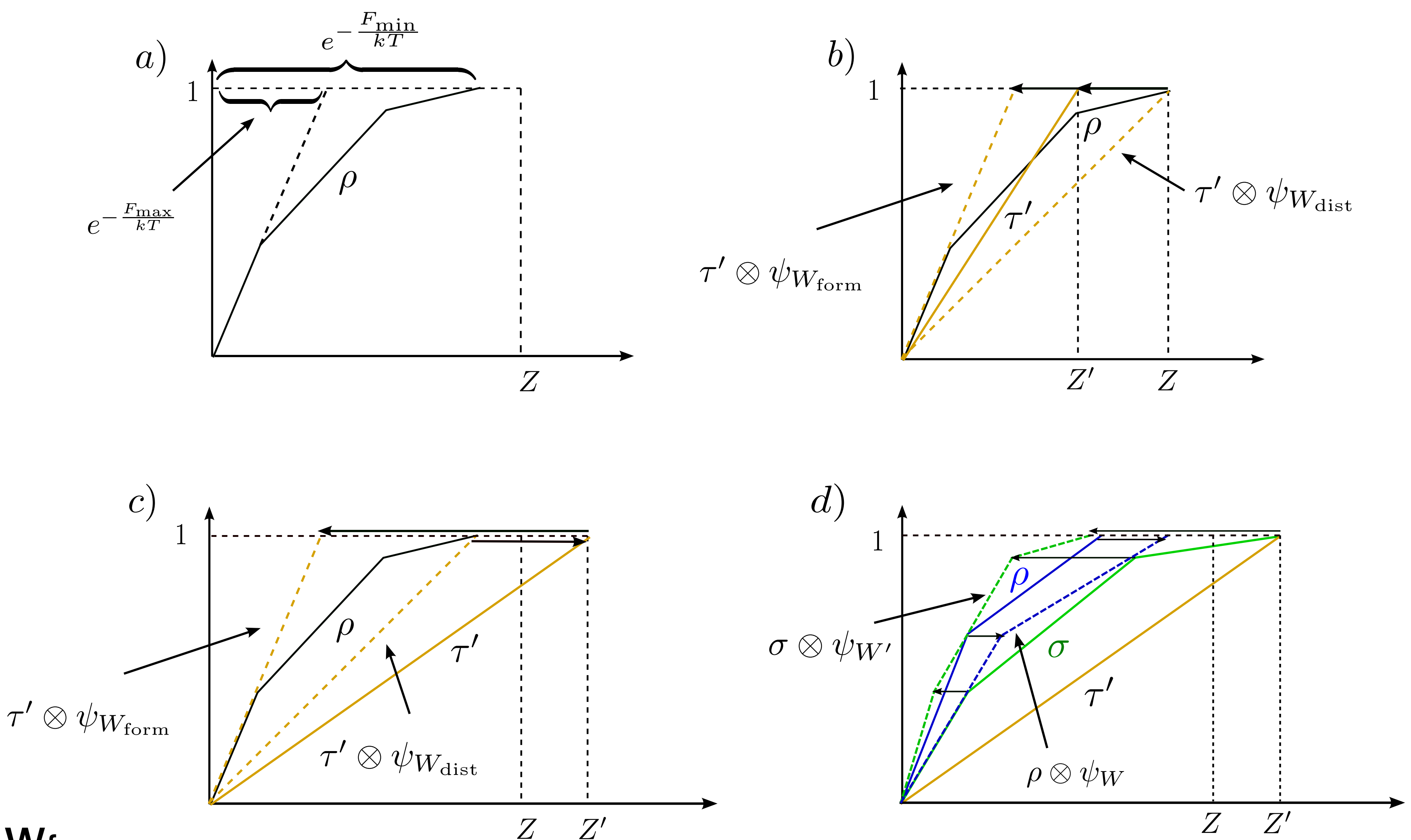
For degenerate energy levels

$$x \xrightarrow{\text{N.O.}} y \quad \text{iff } x > y \text{ Horodecki, JO (2003)}$$



$P(E_1, g_1)e^{\beta E_1} \geq P(E_2, g_2)e^{\beta E_2} \geq \ldots \beta$ - ordered conjecture of Ruch & Mead (75)

Work of $(\rho, H) \to (\sigma, H')$



$$\Delta F \begin{cases} \xrightarrow{W_{\text{form}}} F_\infty(\rho \| \tau) \\ \xrightarrow{W_{\text{dist}}} F_o(\omega \| \tau) \end{cases}$$

Strengthens Dahlston et al. for $H = 0$

c.f. Ahlberg (2011)

Thermo-majorization

Majorization within energy blocks

$\tau_R \otimes \rho_S$ Thermo-majorization

$E = E_R + E_S$ fixed i.e. $P_E\, \tau_R \otimes \rho_S\, P_E$

For each E_S, $g_R(E_R) = g_R(E - E_S) = e^{-\beta E_S} g_R(E)$

eigen values of $P_E\, \tau_R \otimes \rho_S\, P_E$ are $\dfrac{e^{\beta E_S}}{g_R(E)} P(E, g)$

with multiplicity $g_R(E) e^{-\beta E_S}$

Catalytic transformations

Working body, clock, etc.

$$x \xrightarrow{\text{C.N.O.}} y \quad \ni \quad z \text{ s.t. } x \otimes z > y \otimes z$$

“trumping” e.g. Klimesh - Turgut

$$x = \left(\frac{1}{2}, \frac{1}{4}, \frac{1}{4}, 0\right) \qquad y = \left(\frac{4}{10}, \frac{4}{10}, \frac{1}{10}, \frac{1}{10}\right) \qquad z = \left(\frac{6}{10}, \frac{4}{10}\right)$$

$$x \not> y \qquad x \otimes z > y \otimes z$$

$$D_\alpha(x \,||\, \eta) \geq D_\alpha(y \,||\, \eta) \qquad \forall\, \alpha \in (-\infty, \infty)$$

$$\eta = \left(\frac{1}{k}, \ldots \frac{1}{k}\right)$$

How cyclic?

$\sigma_{\mathrm{in}} \otimes \rho_S \rightarrow \sigma_{\mathrm{out}} \otimes \rho_s'$ $\qquad \| \sigma_{\mathrm{in}} - \sigma_{\mathrm{out}} \|_1 \leq \epsilon$

Embezzlement

$$\sigma_{\mathrm{in}} = \sum_i \frac{1}{i} |i><i|$$

c.f. van Dam, Hayden (2002)

$\|\sigma_{\mathrm{out}} - \sigma_{\mathrm{in}}\|_1 \leq \epsilon$ $\qquad \sigma_{\mathrm{in}} \otimes \frac{\mathbb{I}}{2} \longrightarrow \sigma_{\mathrm{out}} \otimes |0><0|$

2nd laws

1) All transformations are possible

$||\sigma_{in} - \sigma_{out}||_{,} \leq \epsilon$

2) Ordinary 2nd law: $F_1(\rho \,||\, \tau)$ goes down

$||\sigma_{in} - \sigma_{out}||_{,} \leq \epsilon / \log d_{catalyst}$

3) $F_\alpha(\rho \,||\, \tau)$ must go down $\quad \alpha \geq 0$

Small work distance

$$D_{work}\left(\sigma^{in} > \sigma^{out}\right) := \mathrm{kT} \inf_{\alpha > 0} \left[F_\alpha(\sigma_{in} || \tau) - F_\alpha(\sigma_{out} || \tau)\right]$$

Resource Theories

Theory	Entanglement	Purity	Asymmetry
Class of Operations	LOCC	Noisy $\frac{\mathbb{I}}{d}, u$	$T(g) 0 \, \varepsilon \, 0 T^t(g) = \varepsilon$
Free States	Seperable σ	$\mathbb{I}/d$	$\sigma = T(g) \sigma \, T^t(g)$
Monotones	$\inf_\sigma R(\rho \Vert \sigma)$	$R(\rho \vert \frac{\mathbb{I}}{\mathrm{d}}) = N - S(\rho)$	$\inf_\sigma R(\rho \Vert \sigma)$

Horodecki, JO (2002, 2012)

“Quantumness” Thermodynamics

Conclusions

Laws of thermodynamics 1st law

- Thermal Operations (U_E, τ, tr)

0^{th} : τ must be thermal for non-trivial theory

2^{nd} * : $F_\alpha(\rho \,||\, \tau)$ must go down

* : embezzlement, how cyclic $\forall a \leq 0$

Doesn't depend on how much control one has!

quasi-classical states: 2nd laws are necessary and sufficient

Many free energies $\longrightarrow$ irreversibility

Limitations due to finite size, quantumness