Quantum information and black holes



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Quantum entanglement in the 21st century

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arXiv papers with "entanglement" in the title



arXiv papers with "entanglement" in the title



Quantum information and gravitation

Quantum information concepts, such as quantum entanglement and quantum error correction, have become increasingly prominent in recent discussions of quantum gravitational phenomena, especially regarding black holes and quantum cosmology.

Recently [Almheri, Marolf, Polchinski, Sully (AMPS) 2012, Mathur 2009, Braunstein 2009], such considerations have precipitated a crisis which has not yet been resolved, the black hole "firewall puzzle."

An uncomfortable tension between central tenets of quantum mechanics

and general relativity (unitarity and the equivalence principle), has cast doubt on long held beliefs about black hole geometry.





Why should QIPers care about gravitation?

Quantum Church-Turing Thesis: any process occurring in Nature can be simulated efficiently by a general purpose universal quantum computer.

The thesis is challenged by:

Quantum field theory, which has an infinite number of degrees of freedom, per unit volume (We need to include energy E in the resource accounting ... At a fixed energy very-short-wavelength degrees of freedom are inaccessible.)

Quantum gravity, in which spacetime geometry is (presumably) emergent and hence interactions are not geometrically local.

If we can simulate strongly-coupled quantum gravity with a quantum computer that will be an important application (to physicists)!

If not, the quantum circuit model does not fully capture the computational power inherent in the laws of Nature!

Thinking about simulating quantum gravity with a quantum computer may lead to deep physics insights.



PARADOX!

When the theories we use to describe Nature lead to unacceptable or self-contradictory conclusions, we are faced with a great challenge and a great opportunity....

Planck 1900

Hawking 1975



"The ultraviolet catastrophe."

Classical electromagnetic theory and statistical mechanics predict that in thermal equilibrium the electromagnetic field in a cavity stores an infinite amount of energy.

The end of classical physics!

"The information loss puzzle."

Classical gravitation theory and quantum field theory on curved spacetime predict that the formation and subsequent complete evaporation of a black hole cannot be unitary.

The end of relativistic causality?

Black Hole



Classically, a black hole is a remarkably simple object (it has "no hair") composed of *pure spacetime geometry.*

If Alice crosses the event horizon of a black hole, she will never be able to return to or communicate with Bob, who remains outside.

time

Light Cone

To understand the event horizon better, consider the concept of a light cone. Imagine a light source that emits a flash (the spacetime event *P*). The flash travels outward as a spherical shell expanding at light speed. Plotted as a function of time, the expanding shell defines a cone, the future light cone of *P*. All events that can be influenced by event *P* lie inside its future lightcone.



Tipping of the light cones

We find by solving the Einstein field equations that the lightcones tip inward as one approaches the black hole. The future light cone of a point inside the horizon lies entirely inside the horizon. Any signal emitted from a point inside the horizon necessarily travels more deeply into the black hole.

The unfortunate astronaut who enters the black hole is unavoidably drawn toward the singularity, where enormous gravitational forces tear him apart.



Penrose diagram



Another way to represent the spacetime geometry of the black hole is often convenient and illuminating.

Each point represents a two-sphere, infalling and outgoing light rays (null geodesics) are lines tilted at 45 degrees.

The forward light cone of a point behind the horizon meets the singularity.

A conformal mapping makes past and future null infinity appear to be a finite affine distance away.

Black hole radiance

Classically, nothing can escape from a black hole, but quantumly, black holes *radiate*.

Quantum fluctuations in the vacuum continually create pairs of virtual particles, which then reannihilate. But if one member of the pair ducks behind the event horizon, the other escapes.

To an observer far away, the black hole seems to be a source of featureless thermal radiation with wavelength comparable to the black hole radius:

 $k_B T_{\text{black hole}} = \hbar c / 4\pi R_{\text{black hole}}$

Since the radiation really arises from quantum fluctuations just outside the horizon, its properties don't depend on how the black hole was formed.



Vacuum entanglement

The vacuum state of a quantum field theory is highly entangled. If we divide space in half, field fluctuations on the the left side are correlated with fluctuations on the right side.

> A uniformly accelerated observer in flat space sees a thermal bath of quanta, with typical wavelength comparable to proper distance to the event horizon.

$$z(\tau) = \frac{1}{a}\cosh(a\tau), \quad t(\tau) = \frac{1}{a}\sinh(a\tau).$$

Field fluctuations are periodic in imaginary time, with period equal to inverse temperature (Unruh temperature): $e^{-i\tau H} \sim e^{-\beta H}$

accelerated

observer

forward

light

cone

Left

> 7

spatial slice

norizon

$$k_{B}T = \frac{\hbar}{2\pi c}a \approx (10^{-20} \,\mathrm{K} \,\mathrm{for}\,a = 1\,g).$$

Black hole "thermal atmosphere"

A static observer at a fixed proper distance from the black hole horizon is uniformly accelerated (with larger acceleration closer to the horizon), and hence sees a thermal radiation bath (which is hotter closer to the horizon).

This acceleration, when red shifted to infinite distance from the black hole, is the black hole's "surface gravity":

Correspondingly, the thermal radiation detected by an observer at infinity has temperature :

Thermal wavelength comparable to black hole 's size. (Cold for large black hole!)

"thermal

atmosphere"

thermal bath not seen by freely falling observers

 $k_{\rm R}T_{\rm RH} = \hbar c / 4\pi R_{\rm BH}$

 $\approx 10^{-7} K$ (solar mass)



simple (have no hair), and yet also very complex (have enormous entropy, e.g., 10⁷⁸ for a solar mass).



Black hole evaporation

Suppose we prepare a quantum state, encoding some information, as pressureless dust on the brink of gravitational collapse.



It collapses, and begins to emit Hawking radiation. This radiation is featureless, not dependent on the information encoded in the original collapsing body.

Eventually, all the mass is radiated away, and the black hole disappears. What happened to the information?

Other hot bodies emit thermal radiation. Such processes are *thermodynamically* irreversible but not *microscopically* irreversible.

But a black hole is different than other hot bodies, because it has an event horizon. Does that mean that this process is microscopically irreversible, that the information is lost not just in practice but in principle?

Information Puzzle: Is a black hole a quantum cloner?

Suppose that the collapsing body's quantum information is encoded in the emitted Hawking radiation; the information is *thermalized*, not destroyed.

The green time slice crosses both the collapsing body behind the horizon and nearly all of the radiation outside the horizon. *Thus the same (quantum) information is in two places at the same time.*

A quantum cloning machine has operated, which is not allowed by the linearity of quantum mechanics.

We're stuck: either information is destroyed or cloning occurs. Either way, quantum physics needs revision.



Black hole as quantum cloner



The "nice slice" shown in green can be chosen to cross both the collapsing body behind the horizon and 99% of the escaping Hawking radiation outside the horizon.

Yet the slice only occupies regions of low curvature, where we would normally expect semiclassical physics to be reliable.

The same quantum information is in two different places at the same time.

"Black hole complementarity"

Perhaps the lesson is that, for mysterious reasons that should be elucidated by a complete theory of quantum gravity, it is wrong to think of the "outside" and "inside" portions of the time slice as two separate subsystems of a composite system.

 $\mathcal{H} \neq \mathcal{H}_{in} \otimes \mathcal{H}_{out}$

Rather, the inside and outside are merely complementary descriptions of the same system. Which description is appropriate depends on whether the observer enters the black hole or stays outside (Susskind, 1993).



"No-cloning" lower bound on the information retention time

singularity

Bob

Alice

Let's demand that verifiable cloning does not occur. Then the proper time during which Alice can send her qubits to Bob cannot be larger than O(1) in Planck units:

$$\tau_{\text{proper}}^{(\text{Alice})} \approx r_s \exp(-\Delta t_s / r_s) \le O(1) \times r_{\text{Planck}}$$

and therefore

$$\Delta t_{S} \geq O\left(r_{S} \log r_{S}\right)$$

(where r_s is measured in Planck units). If Alice's quantum information were revealed in the Hawking radiation faster than this, then Alice and Bob would be able to verify that Alice's quantum information is in two places at once, in violation of the no-cloning principle.

Entropy radiated by an evaporating black hole (Page 1993)

The "conventional wisdom" is that information absorbed by a black hole is revealed after a time

$$\Delta t_{S} = O(r_{S}^{3})$$
mparable to the black hole's
poration time), which is

(CO)S eva comfortably longer than the "nocloning" lower bound.



Consider the black hole and the radiation it has emitted as two subsystems of a composite system, where the composite system's state is pure. If we suppose that the radiation subsystem is chosen uniformly at random (with respect to Haar measure), then the smaller subsystem is very nearly maximally entangled with the larger subsystem. Information about the black hole's internal state starts to be revealed only after the "halfway point" where half of the black hole's initial entropy is radiated away. It takes Schwarzschild time of order r_s^3 to reach the halfway point.

The information retention time reconsidered

The "conventional wisdom" is that information absorbed by a black hole is revealed after a time

$$\Delta t_{S} = O\left(r_{S}^{3}\right)$$

(comparable to the black hole's evaporation time), which is comfortably longer than the "nocloning" lower bound.



Hayden and I (2007) reexamined the black hole information retention time using tools from quantum Shannon theory and the theory of (approximate) unitary *t*-designs. We argued that the actual value of the information retention time may be

$$\Delta t_{S} = O(r_{S} \log r_{S})$$

i.e., barely compatible with the no-cloning lower bound. Part of the argument was mathematically precise, but part of it was rather tentative and speculative (as might be expected for a claim about quantum gravity).

"Black holes as mirrors"

Alice throws *k* qubits (maximally entangled with reference system *N*) into an "old" black hole. As radiation *R* escapes, the correlation of *N* with *B*′ decays. Eventually, *N* is nearly uncorrelated with *B*′ and nearly maximally entangled with a subsystem of *ER* --- at that stage, Bob can decode Alice's quantum message with high fidelity (Hayden-Preskill, 2007).



 $\int_{\text{Haar}} \left(dV^B \right) \left\| \rho^{NB'}(V^B) - \rho^N \otimes \rho^{B'}_{\max} \right\|_1 \leq \frac{|N|}{|R|} = \frac{2^k}{2^{k+c}} = 2^{-c}$ Bob can decode with high fidelity after receiving only *k+c* qubits of Hawking radiation, where *c* is a constant, if the mixing unitary *V^B* is Haar random, or even if it is a typical unitary realized by a small quantum circuit (depth ~log *r_s*).

Fast Scrambling Conjecture

Conjecture: black holes are Nature's fastest scramblers of quantum information. -- Sekino and Susskind 2008

The (geometrically nonlocal) Hamiltonian evolution dictated by quantum gravity destroys the correlation between black hole B' and the reference system N in Schwarzschild time t = $O(\beta \log S)$ where S is black hole entropy, and β its inverse temperature.



One expects that random (geometrically nonlocal) quantum circuits with depth O(log n) are also good scramblers (approximate unitary 2-designs), though this has been proven only for depth O(log³ n) – Brown and Fawzi 2013.

In contrast the black hole evolution requires no input randomness. It efficiently encodes a quantum error-correcting code that protects against erasure of a constant fraction of the qubits (according to the conjecture).

Counting states

There is a compelling candidate, still far from completely understood, for a quantum theory of gravity. In this theory, the fundamental dynamical objects are not (only) pointlike particles, but extended objects of various dimensionalities. Formerly know as *string theory*, it is now often called *M theory* [*M* = mystery, mother, membrane, matrix, ...] to emphasize that strings are not the only fundamental entiities.



Among the extended objects are *Dbranes*, on which open strings can terminate. A *D*-brane provides a stringtheoretic description of a black hole horizon, and its microscopic states can be counted. The state counting, in cases that can be analyzed, is consistent with the known black hole entropy.

Polchinski 95; Strominger and Vafa 96



Maldacena 97

AdS-CFT Correspondence

Anti-de Sitter (AdS) space is a spacetime of constant negative curvature. Although the spatial slices are infinite, the spacetime has a boundary in the sense that light rays can reach infinity and return in a finite proper time.



There is persuasive evidence that string theory on (d+1)dimensional AdS space can be described *exactly* using a ddimensional (conformal) field theory (CFT) defined on the boundary of the spacetime --- a "holographic" description!

Furthermore, the AdS-CFT correspondence involves a remarkable *ultraviolet/infrared connection*: short distances on the "boundary" correspond to long distance in the "bulk". (Timelike geodesics that probe deeply into the bulk connect points that are far apart on the boundary.) The CFT spawns an emergent extra dimension corresponding to *renormalization group flow*.

AdS-CFT Black hole

Formation and complete evaporation of a black hole in de Sitter space has a dual description in terms of *thermalization* in the dual field theory.

In this dual description the evolution is manifestly unitary --it is just Schrodinger evolution governed by the CFT Hamiltonian; no information is destroyed.



Even so, the mechanism by which information can escape from behind a putative event horizon remains murky.

The semiclassical causal structure of the black hole is somehow misleading ... quantum gravity allows information to propagate beyond the apparent light cone.



Black hole complementarity challenged

Three reasonable beliefs, not all true!

[Almheri, Marolf, Polchinski, Sully (AMPS) 2012, Mathur 2009, Braunstein 2009]:

- (1) The black hole "scrambles" information, but does not destroy it.
- (2) An observer who falls through the black hole horizon sees nothing unusual (at least for a while).
- (3) An observer who stays outside the black hole sees nothing unusual.

"Conservative" resolution: A "firewall" at the horizon, rather than (2).









AMPS experiment

Now a single infalling agent, when still a safe distance from the singularity, can be informed that both the AB and BR entanglement have been confirmed, hence verifying a violation of the monogamy of entanglement.

In contrast to the cloning experiment described earlier, there is no need for super-Planckian signals, because the infaller need not wait for information to be radiated before crossing the horizon.

What happens when this experiment is attempted?

Quantum Frontiers

A blog by the Institute for Quantum Information and Matter @ Caltech



Posted on August 29, 2013 by preskill

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What's inside a black hole?

**** 0 26 Votes

I have a multiple choice question for you.

What's inside a black hole?

(A) An unlimited amount of stuff.

(B) Nothing at all.

(C) A huge but finite amount of stuff, which is also outside the black hole.

(D) None of the above.

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What's inside a black hole?



A. An unlimited amount of stuff.



Information is lost!

"There is all that stuff that fell in and it crashed into the singularity and that's it. Bye-bye." – Bill Unruh

But ...

-- Why S = Area / 4?

-- What about AdS/CFT duality?

B. Nothing at all.



Firewalls!

"It is time to constrain and construct the dynamics of firewalls." – Raphael Bousso

But ...

-- "Curtains for the equivalence principle?" (Sam Braunstein, 2009)

C. A huge but finite amount of stuff, which is also outside the black hole.



Complementarity!

B (recent radiation) can be entangled with both A (behind the horizon) and R (early radiation), because A and R are two descriptions of the *same* system.

Complementarity rescued, but ...

-- R could be far, far away from the black hole. And what happens when the AMPS experiment is attempted?

What's inside a black hole?

A. An unlimited amount of stuff.

B. Nothing at all.

C. A huge but finite amount of stuff, which is also outside the black hole.

D. None of the above.

Holographic entanglement entropy



To compute entropy of region A in the boundary field theory, find minimal area of the bulk surface with the same boundary:

$$S(A) = \frac{1}{4G_N} \min_{\partial m = \partial A} \operatorname{area}(m) + \cdots$$

Ryu and Takayanagi 2006

Recover, for example, in 1+1 dimensional conformal field theory:

$$S(A(L)) = \frac{c}{3}\log(L/a) + \cdots$$

Strong subadditivity from holography



Tripartite Info: $I(A;B) + I(A;C) \le I(A;BC)$

("extensivity" of mutual information for disjoint A, B, C). True for holographic theories, not in general.

Hayden, Headrick, Maloney, 2011

Building spacetime from quantum entanglement





Ei

$$\sum_{i} e^{-\beta E_i/2} \mid E_i \rangle \otimes \mid E_i \rangle$$

A connected geometry is constructed as a superposition of disconnected geometries. The entangled state becomes a product state as the neck pinches off and the geometry becomes disconnected. (Van Raamsdonk 2010).





Quantum Frontiers

A blog by the Institute for Quantum Information and Matter @ Caltech

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Posted on June 7, 2013 by preskill

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Entanglement = Wormholes

★★★★★ **0** 37 Votes

One of the most enjoyable and inspiring physics papers I have read in recent years is this one by Mark Van Raamsdonk. Building on earlier observations by Maldacena and by Ryu and Takayanagi. Van Raamsdonk proposed that quantum entanglement is the fundamental ingredient underlying spacetime geometry.* Since my first encounter with this provocative paper, I have often mused that it might be a Good Thing for someone to take Van Raamsdonk's idea really seriously.

Now someone has.

Love in a wormhole throat



Alice and Bob are in different galaxies, but each lives near a black hole, and their black holes are connected by a wormhole. If both jump into their black holes, they can enjoy each other's company for a while before meeting a tragic end.

C. A huge but finite amount of stuff, which is also outside the black hole.



A black hole wormhole-connected to the Hawking radiation it has emitted (Maldacena and Susskind 2013).

Complementarity!

B (recent radiation) can be entangled with both A (behind the horizon) and R (early radiation), because A and R are two descriptions of the *same* system.

Complementarity rescued, perhaps by identifying nontraversable wormholes with entanglement (ER = EPR).

But ...

-- R could be far, far away from the black hole. Even so, measuring R can affect A.

Horowitz-Maldacena Proposal (2003)



HM Proposal: Quantum information escapes from a black hole via *postselected teleportation*. The black hole S-matrix is unitary if the "Unruh vacuum" at the horizon is maximally entangled and the postselected final state at the horizon is also maximally entangled. Monogamy of entanglement and no-cloning are (temporarily) violated, allowing smoothness of the horizon to be reconciled with unitarity. (Lloyd and Preskill, 2013).

Horowitz-Maldacena Proposal (2003)



HM Proposal: Quantum information escapes from a black hole via *postselected teleportation*. The black hole S-matrix is unitary if the "Unruh vacuum" at the horizon is maximally entangled and the postselected final state at the horizon is also maximally entangled. Monogamy of entanglement and no-cloning are (temporarily) violated, allowing smoothness of the horizon to be reconciled with unitarity. (Lloyd and Preskill, 2013).

Generic final state



Consider dividing the infalling matter into a relatively small subsystem M_1 (matter that collapses quickly) and a larger subsystem M_2 (which collapses slowly).

If M_2 is initially in a fixed (vacuum) state, then a generic final state boundary condition, will project onto a very nearly maximally entangled state of M_1 and the outgoing radiation; hence the black hole Smatrix will be very nearly unitary.

For Haar random *U*, L1 -norm deviation from unitarity:

$$\left(\frac{|\mathcal{H}_{M_1}|}{|\mathcal{H}_{in}|}\right)^{1/2} \approx \exp\left(-S_{BH} / 2 + O(m^{3/2})\right)$$

Such a small violation of unitarity may be an artifact of the semiclassical framework used in the analysis, as nonperturbative quantum gravity corrections of that order are expected.

Hidden postselection



The infalling observer has limited time before reaching the singularity, and therefore can access only the subsystem A_1 of the infalling Hilbert space.

The matter system M_1 is nearly uncorrelated with A_1 ; the information "reflected" at the singularity escapes the infaller's notice.

L1 norm deviation of $M_I A_I$ from a product state is :

$$\left(\frac{|\mathcal{H}_{M_1}|\cdot|\mathcal{H}_{\text{in},1}|}{|\mathcal{H}_{\text{in},2}|}\right)^{1/2} \approx \exp(-S_{BH}/2+\ldots)$$

The infaller sees no evidence that he is about to be subjected to a final state boundary condition.

Monogamy violation by postselection



For a fixed state of the infalling matter, the infalling radiation is projected onto a particular state.

H (the Unruh partner of R), and A (the Unruh partner of B) are projected onto a postselected entangled state, establishing the entanglement between R and B.

When the AMPS experiment is attempted, the verification of the BR entanglement may interfere with the AB entanglement, preventing the infalling observer from detecting the violation of monogamy of entanglement.



Decoding complexity

Can the BR entanglement be verified in time to confirm that monogamy of entanglement is violated?

Harlow and Hayden 2013 argued that decoding the Hawking radiation is hard when the black hole is only partially evaporated, even if the black hole S matrix is known and efficiently computable. The decoder solves a QSZK-hard problem, and hence presumably requires time exponential in the entropy of the remaining black hole.

Aaronson: if the decoding is easy, then no injective 1-way functions are secure against quantum attacks.

A novel application of complexity theory to fundamental physics! Is complementarity rescued? (Oppenheim and Unruh 2014 suggested simplifying the decoding by precomputing the black hole unitary, but their proposal requires a very delicate (possibly unphysical?) procedure for creating a prescribed black hole.)

Entanglement Renormalization and Holography



In AdS/CFT, the emergent dimension of space can be regarded as a renomalization scale.

Entanglement renormalization, run backwards, prepares a region of length L in circuit depth O(log L).

View the bulk space as a prescription for building up the boundary state (Swingle, 2009).

Think of a growing tensor network as a model of an evolving bulk spatial slice. The slice expands, corresponding to adding additional layers to the network.

Firewalls: What now?

-- We're confused because we don't have a complete theory. We thought we did, but AdS/CFT does not seem to be telling us about what is behind the horizon.

-- The system may be huge, the curvature small. Yet, if firewalls exist the quantum "corrections" are dramatic. The black hole has no inside!

-- A sharp paradox should always be welcomed; it's resolution may lead to great advances! In this case, we hope for a deeper understanding of how spacetime emerges (or does not, in the case of the black hole interior).

-- We are trying simultaneously to determine both what the theory predicts and what the theory *is*, without guidance from experiment. Are we smart enough to figure it out? (I don't see why not ...)

-- The stakes are high, including implications for the quantum Church-Turing thesis and quantum cosmology.

-- Quantum informationists have much to contribute to the debate! Especially if quantum entanglement is really the foundation of spacetime geometry.