Quantum Algorithms for Quantum Field Theories

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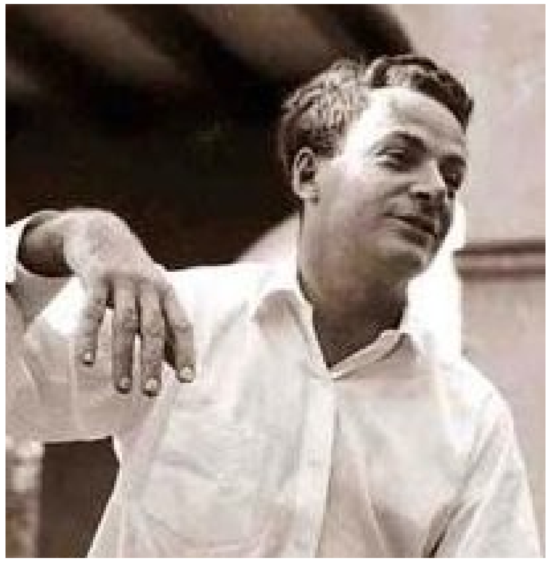
Joint work with
Keith Lee
John Preskill

Science, 336:1130 (2012)

NIST

Jan 24, 2012

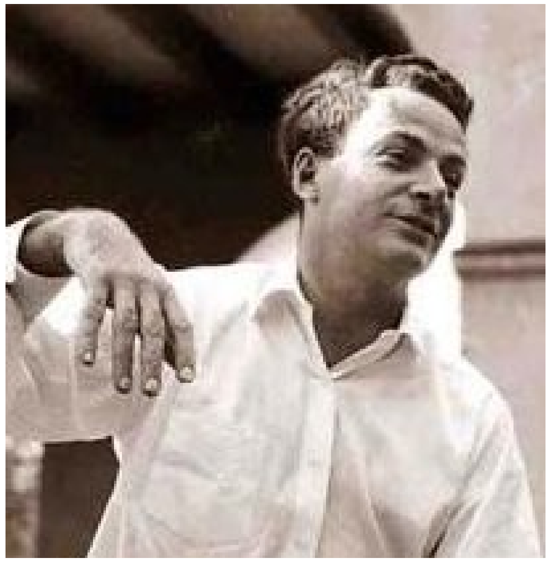


The full description of quantum mechanics for a large system with R particles has too many variables. It cannot be simulated with a normal computer with a number of elements proportional to R.

-Richard Feynman, 1982

An n-bit integer can be factored on a quantum computer in $O(n^2)$ time.

-Peter Shor, 1994



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Are there any systems that remain hard to simulate even with quantum computers?

Quantum Simulation

Condensed-matter lattice models:

[Lloyd, 1996]

[Abrams, Lloyd, 1997]

[Berry, Childs, 2012]

Many-particle Schrödinger and Dirac Equations:

[Meyer, 1996]

[Zalka, 1998]

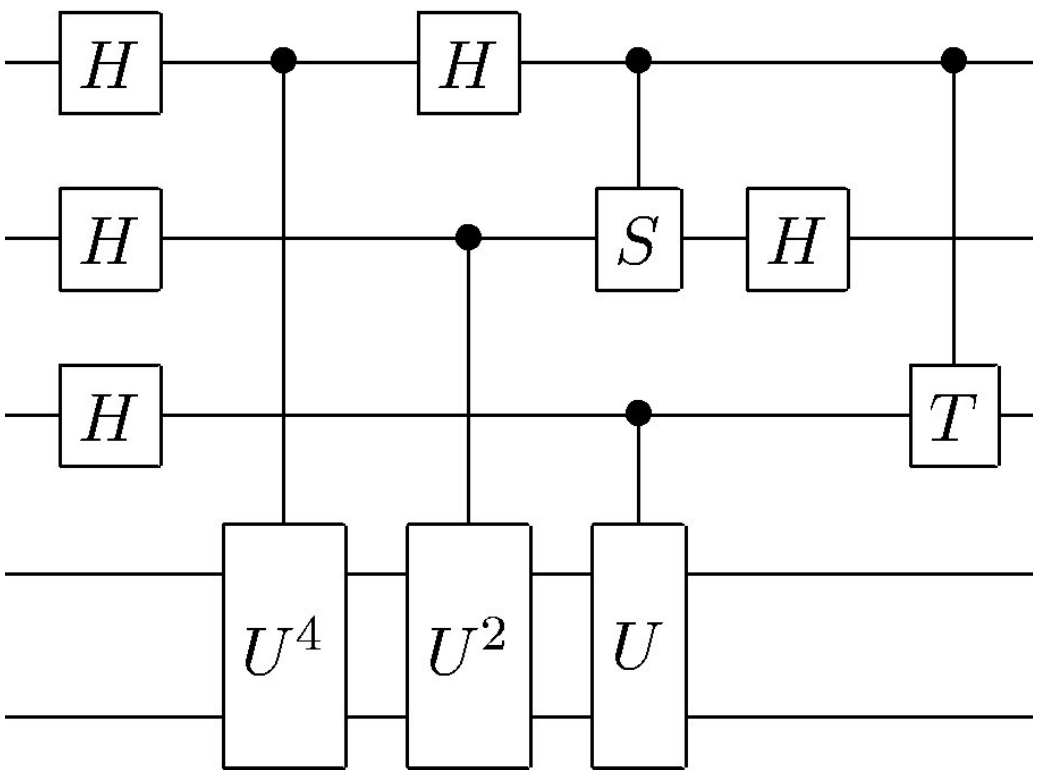
[Taylor, Boghosian, 1998]

[Kassal, S.J., Love, Mohseni, Aspuru-Guzik, 2008]

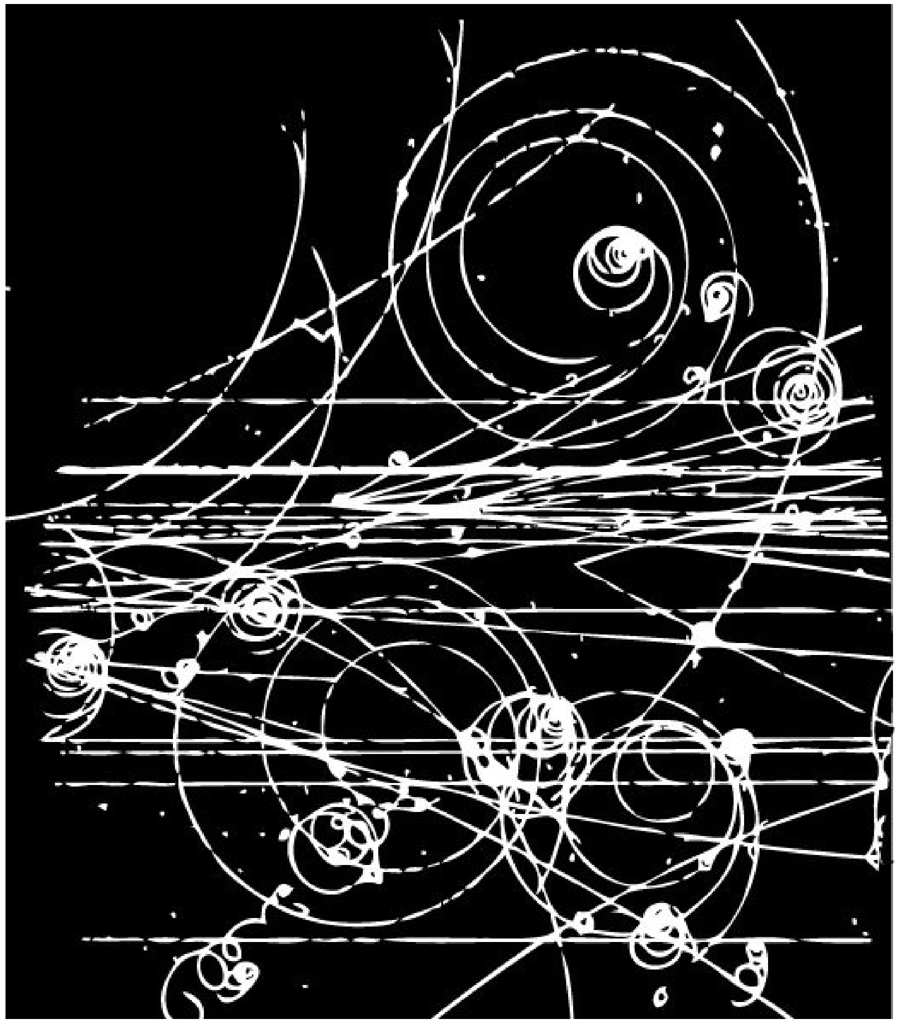
Quantum Field Theory

- Much is known about using quantum computers to simulate quantum systems.
- Why might QFT be different?
  - Field has infinitely many degrees of freedom
  - Relativistic
  - Particle number not conserved
  - Formalism looks different

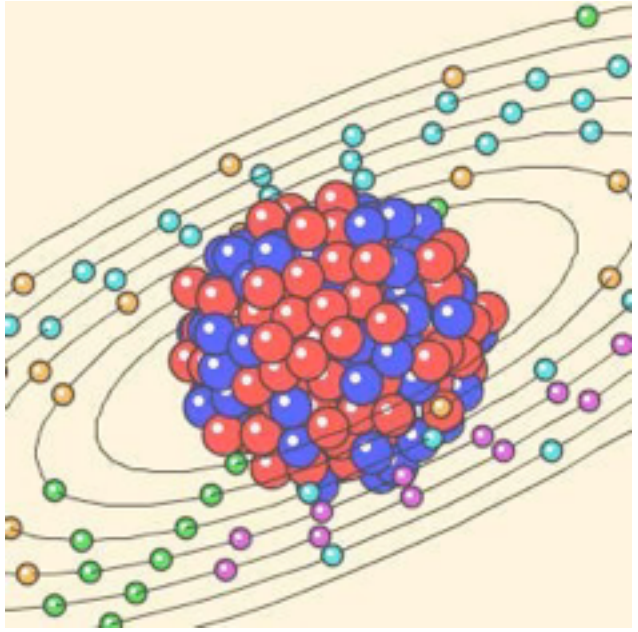
What is the computational power of our universe?



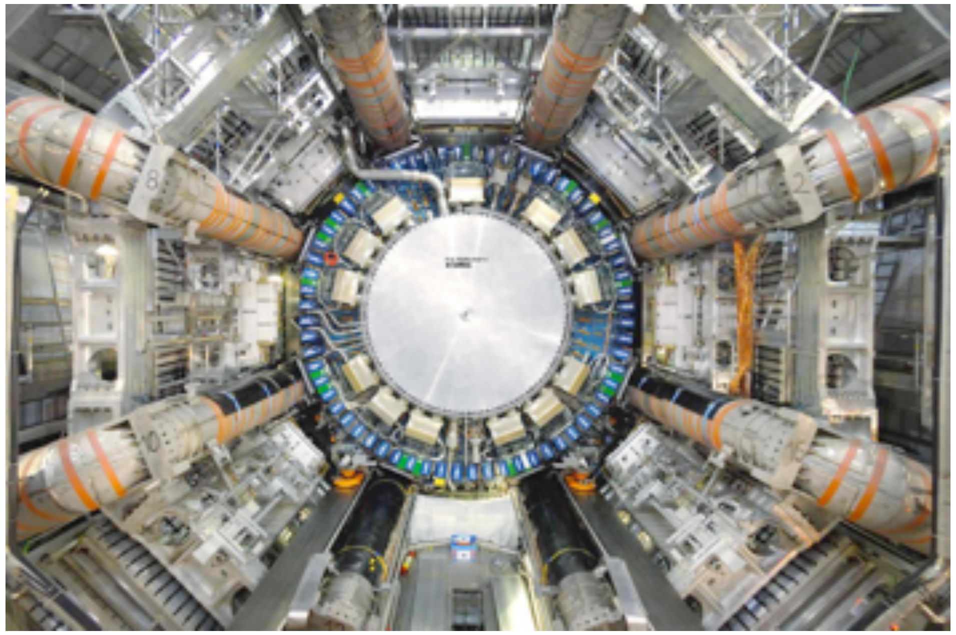
simulate



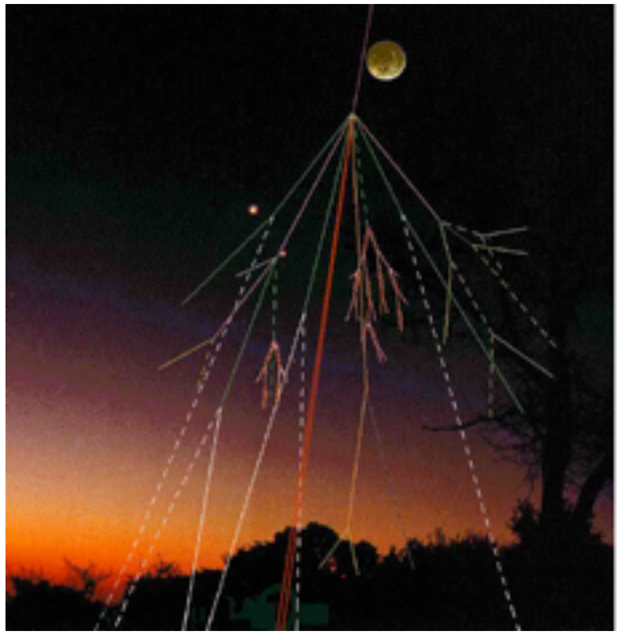
When do we need QFT?



Nuclear Physics



Accelerator Experiments

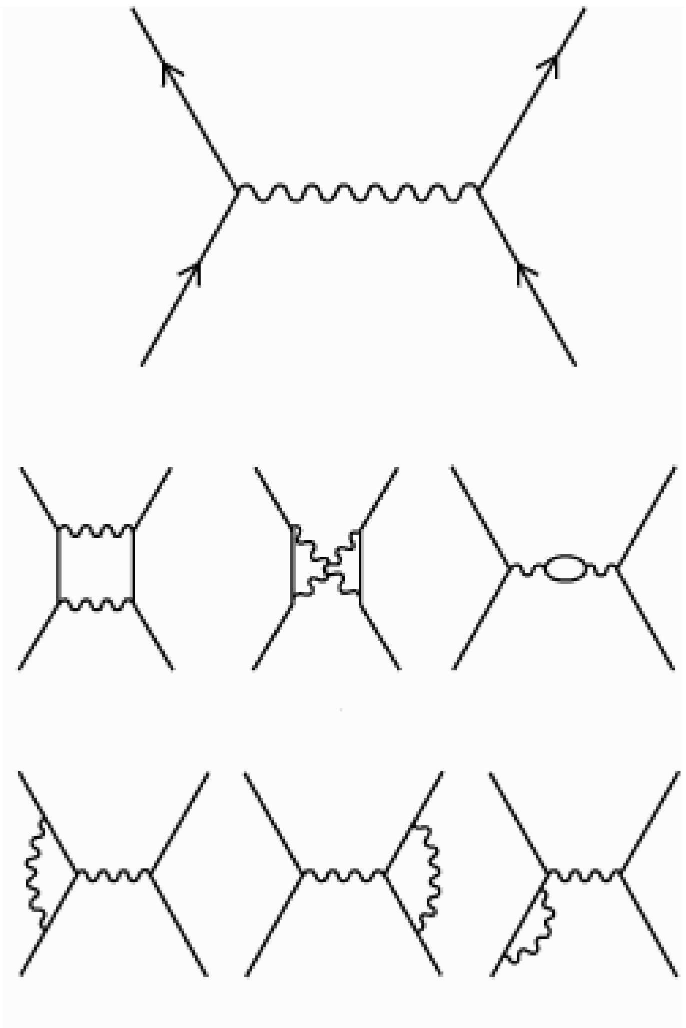


Cosmic Rays

➔Whenever quantum mechanical and relativistic effects are both significant.

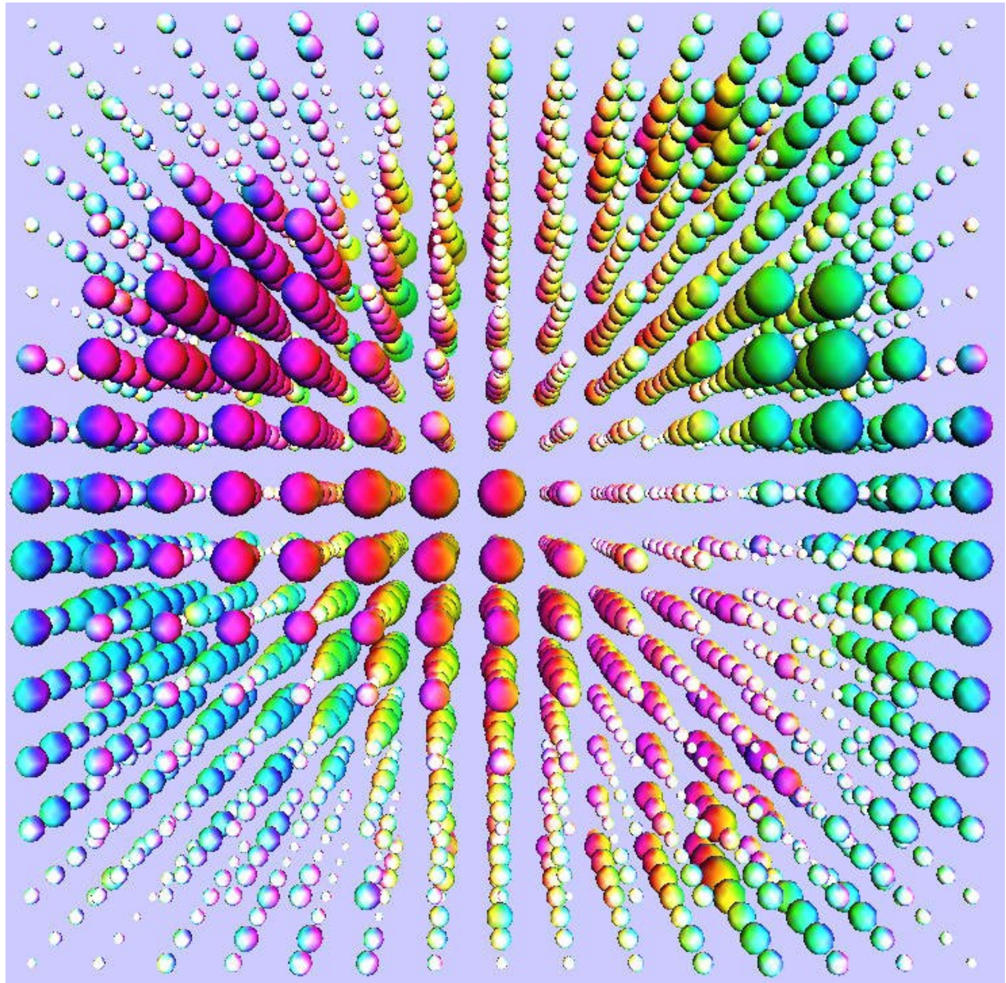
Classical Algorithms

Feynman diagrams



Break down at strong coupling or high precision

Lattice methods

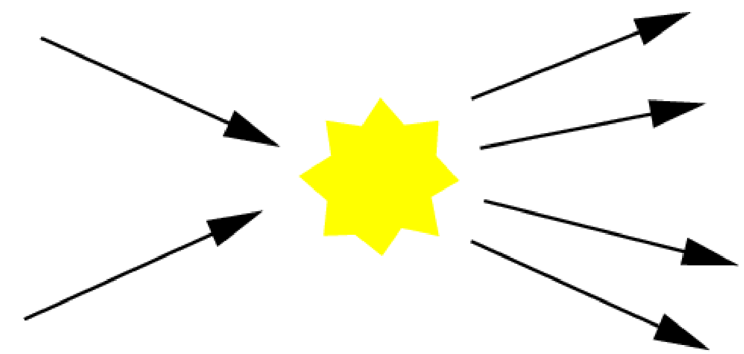


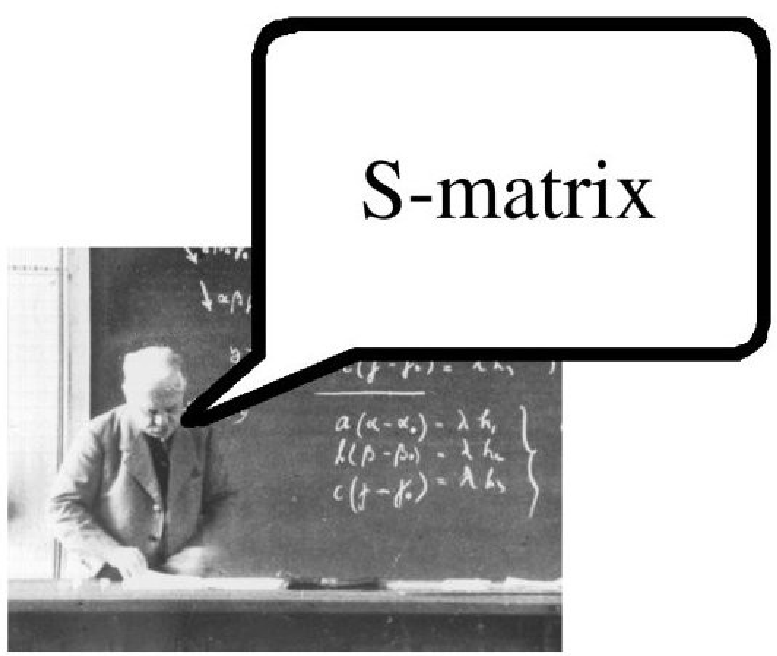
Cannot calculate scattering amplitudes

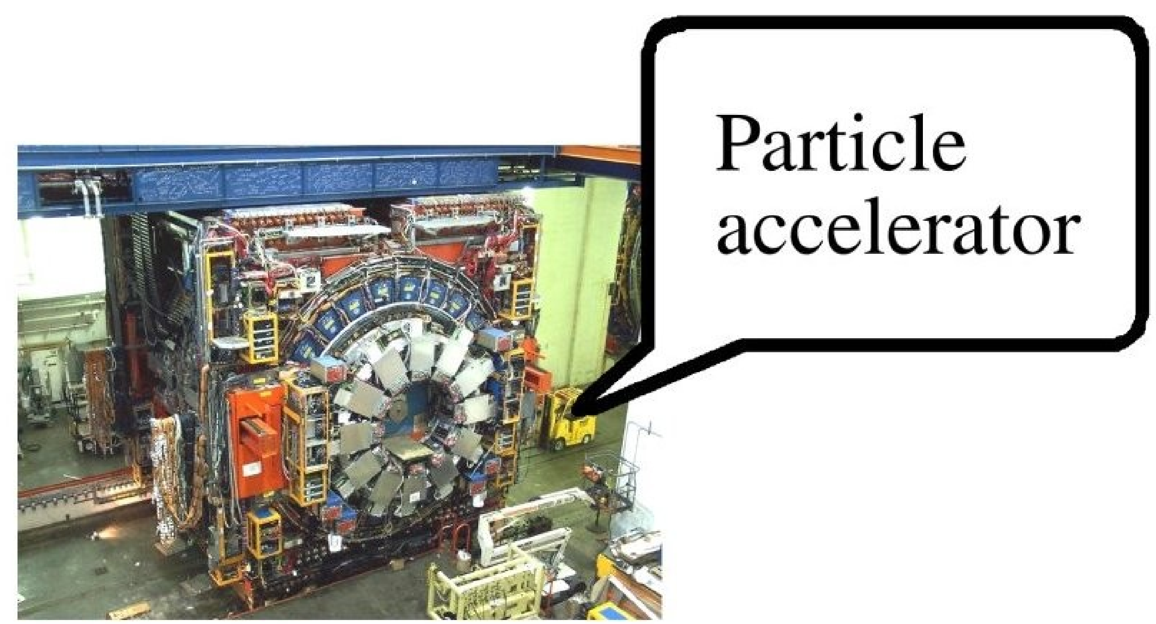
A QFT Computational Problem

Input: a list of momenta of incoming particles

Output: a list of momenta of outgoing particles







I will present a polynomial-time quantum algorithms to compute scattering probabilities in the ϕ^4 and Gross-Neveu models with nonzero mass

These are simple models that illustrates some of the main difficulties in simulating a QFT:

- Discretizing spacetime
- Preparing initial states

ϕ^4-theory

Lagrangian density

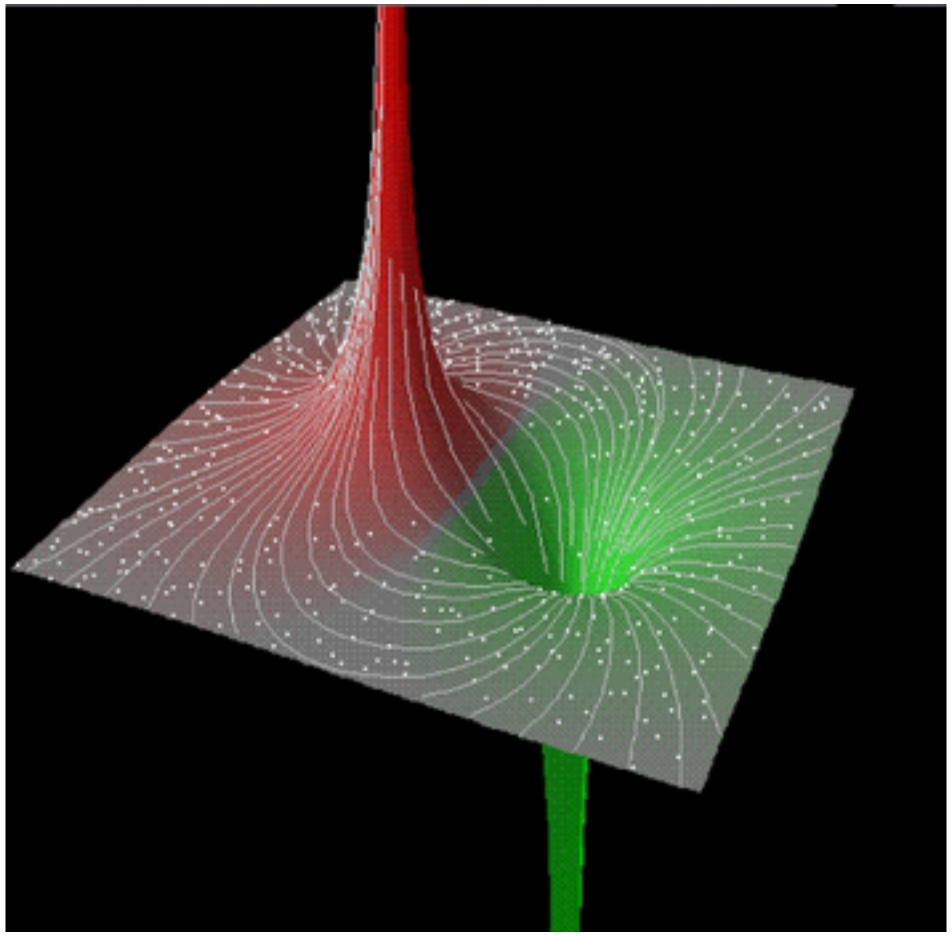
$$\mathcal{L} = \frac{1}{2}\partial^\mu \phi \partial_\mu \phi - m^2\phi^2 - \frac{\lambda}{4!}\phi^4$$

For quantum simulation we prefer Hamiltonian formulation (equivalent)

$$H = \int d^d x \left[\pi^2 + (\nabla\phi)^2 + m^2\phi^2 + \lambda\phi^4\right]$$

$$[\phi(x), \pi(y)] = i\delta^{(d)}(x-y)$$

Quantum Fields



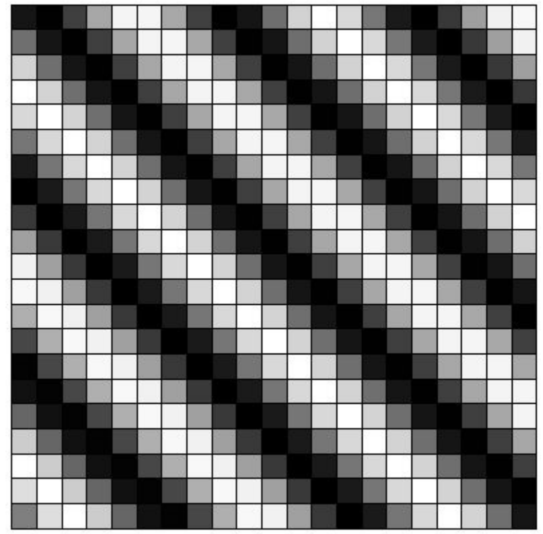
A classical field is described by its value at every point in space.

$$E(r) = \frac{1}{4\pi\epsilon_0}\frac{q}{r^2}$$

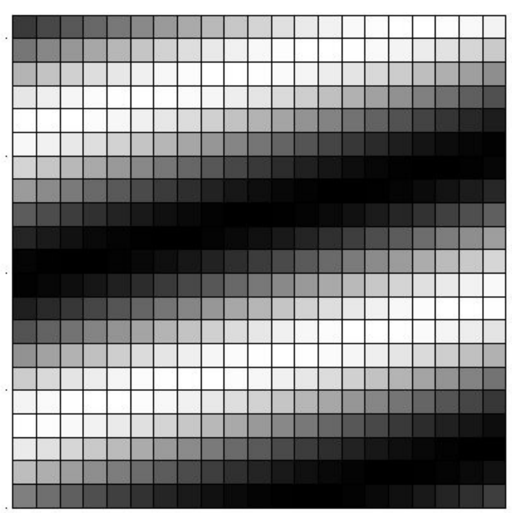
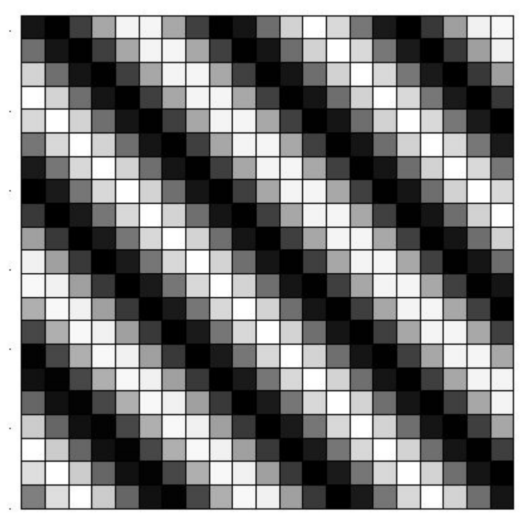
A quantum field is a superposition of classical field configurations.

$$|\Psi\rangle = \int \mathcal{D}[E]\Psi[E]\ |E\rangle$$

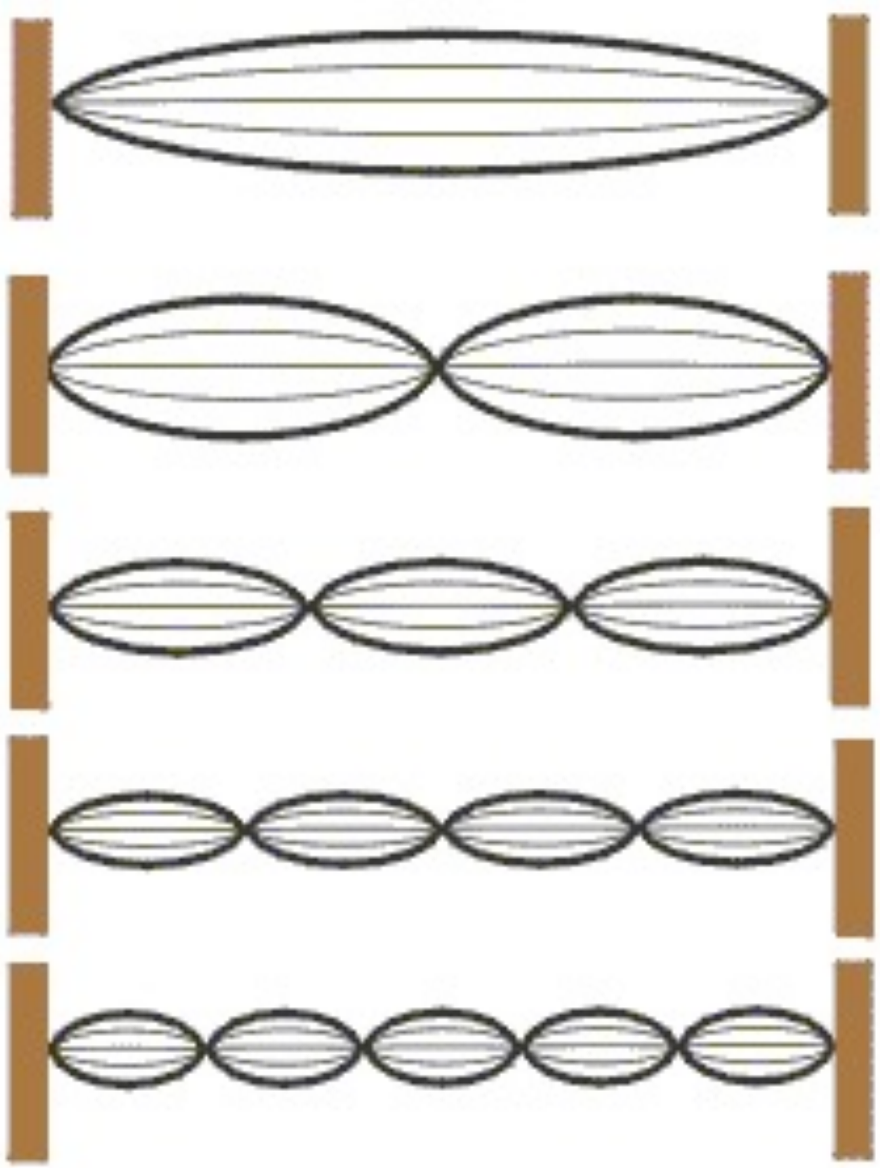
A configuration of the field is a list of field values, one for each lattice site.



A quantum field can be in a superposition of different field configurations.

$\frac{1}{\sqrt{2}}\Big|$  $\Big\rangle - \frac{i}{\sqrt{2}}\Big|$  $\Big\rangle$

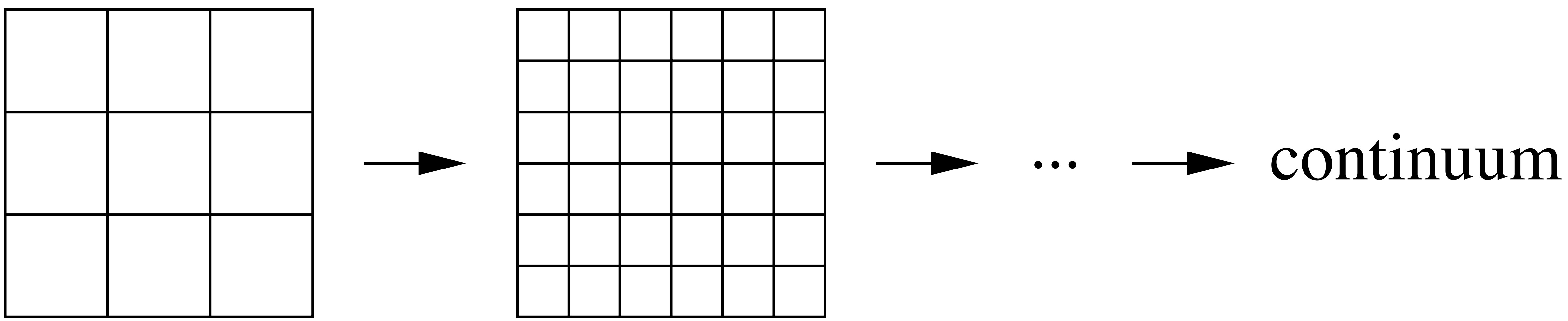
Particles Emerge from Fields

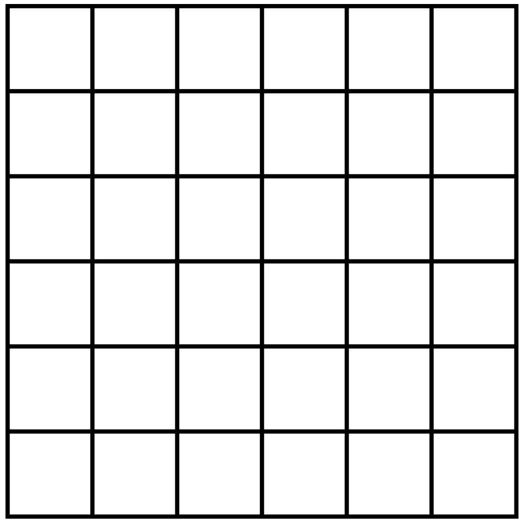


Particles of different energy are different resonant excitations of the field.

Lattice cutoff

Continuum QFT = limit of a sequence of theories on successively finer lattices

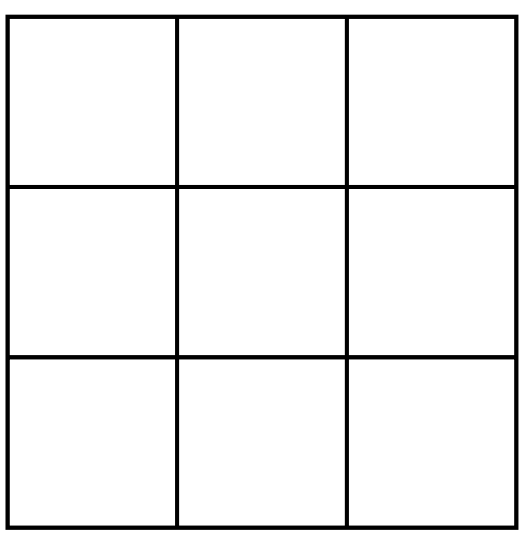




Mass: m

Interaction strength: λ

Coarse grain

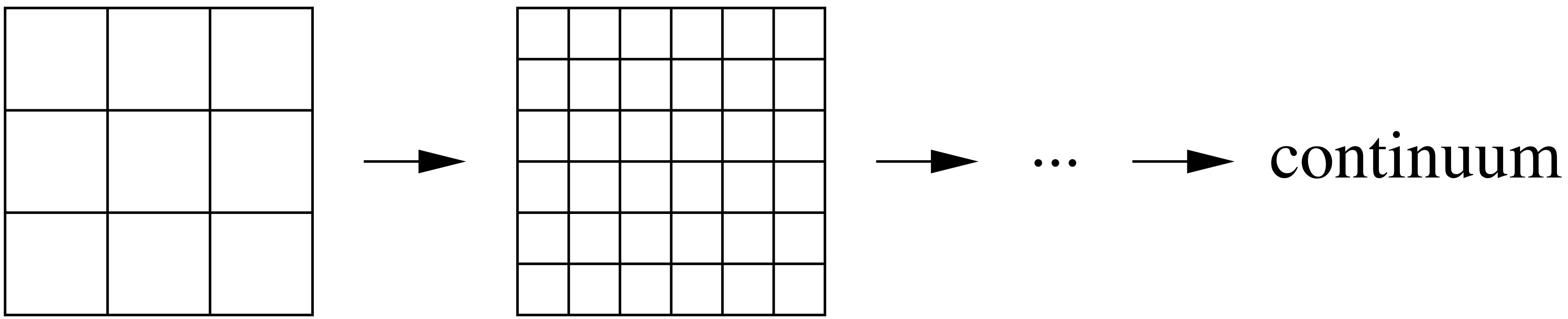


Mass: m'

Interaction strength: λ'

Lattice cutoff

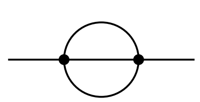
Continuum QFT = limit of a sequence of theories on successively finer lattices



m and λ are functions of lattice spacing!

Discretization Errors

- Renormalization of m and λ make discretization tricky to analyze
- In ϕ^4-theory, in d=1,2,3, discretization errors scale as a^2

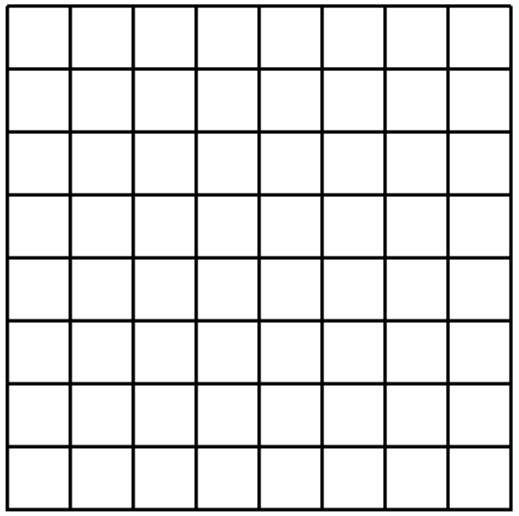


$$= \frac{(-i\lambda_0)^2}{6} \iint \frac{d^D k}{(2\pi)^D} \frac{d^D q}{(2\pi)^D} \frac{i}{(k^0)^2 - \sum_i \frac{4}{a^2}\sin^2\left(\frac{ak^i}{2}\right) - m^2} \; \frac{i}{(q^0)^2 - \sum_i \frac{4}{a^2}\sin^2\left(\frac{aq^i}{2}\right) - m^2}$$

$$\times \frac{i}{(p^0+k^0+q^0)^2 - \sum_i \frac{4}{a^2}\sin^2\left(\frac{a(p^i+k^i+q^i)}{2}\right) - m^2} \tag{207}$$

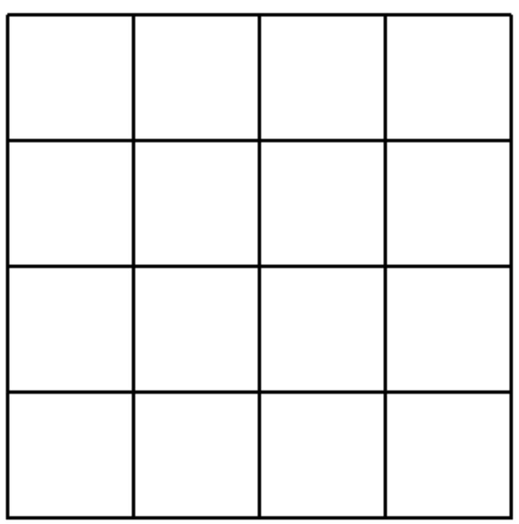
$$= \frac{i\lambda_0^2}{3} \int_0^1\int_0^1\int_0^1 dx\,dy\,dz\,\delta(x+y+z-1) \iint \frac{d^D k}{(2\pi)^D} \frac{d^D q}{(2\pi)^D} \frac{1}{\mathrm{D}^3}\,, \tag{208}$$

...it's complicated



$$H = \frac{1}{2} \sum_{x \in \Omega} a^d \left[\pi^2 + (\nabla \phi)^2 + m^2 \phi^2 + \lambda \phi^4 \right]$$

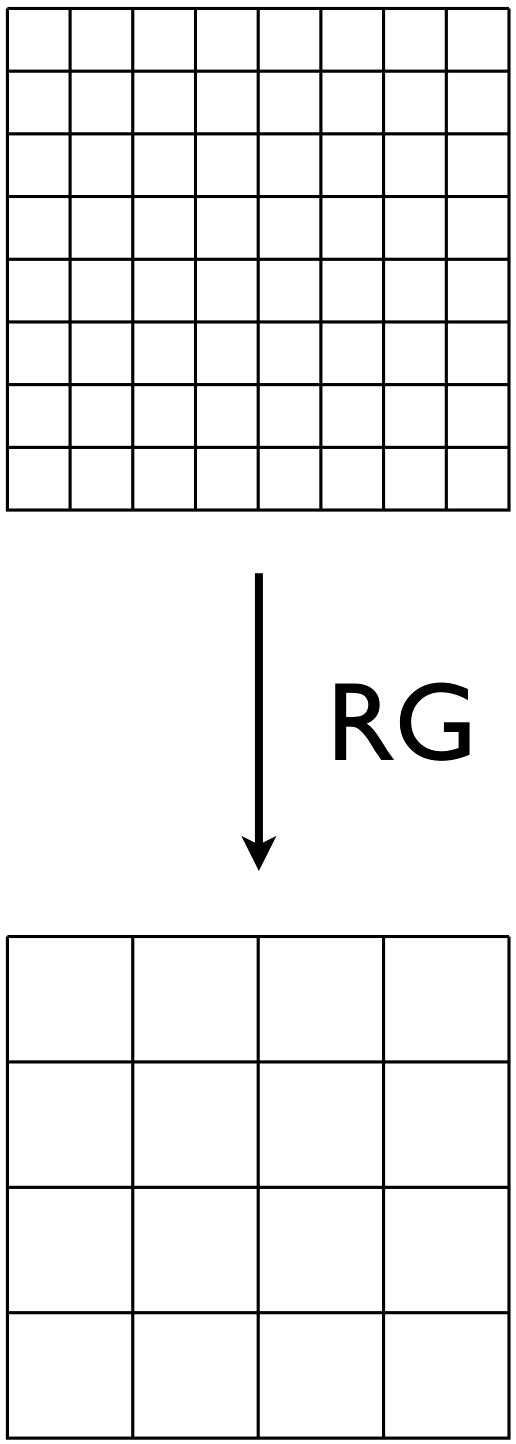
Coarse grain

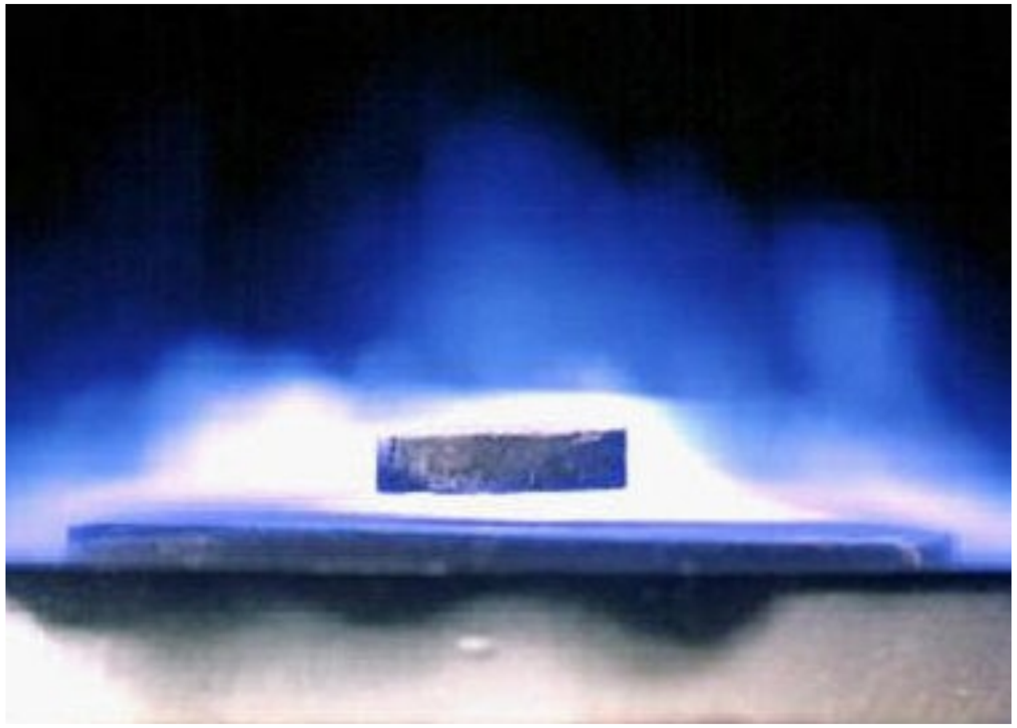


$$H_{\text{eff}} = \frac{1}{2} \sum_{x \in \Omega'} (2a)^d \left[\pi^2 + (\nabla' \phi)^2 + m_{\text{eff}}^2 \phi^2 + \lambda_{\text{eff}} \phi^4 + g\phi^6 + \ldots \right]$$

Simulation converges as a^2

Condensed Matter





There is a fundamental lattice spacing.

But:

We may save qubits by simulating a coarse-grained theory.

After imposing a spatial lattice we have a many-body quantum system with a local Hamiltonian

Simulating the time evolution in polynomial time is a solved problem

Standard methods scale as N^2. We can do N .

- Discretizing spacetime
- Preparing initial states

With particle interactions turned off, the model is exactly solvable.

1. Prepare non-interacting vacuum (Gaussian)

2. Prepare wavepackets of the non-interacting theory

3. Adiabatically turn on interactions

4. Scatter

5. Adiabatically return to non-interacting theory to make measurements

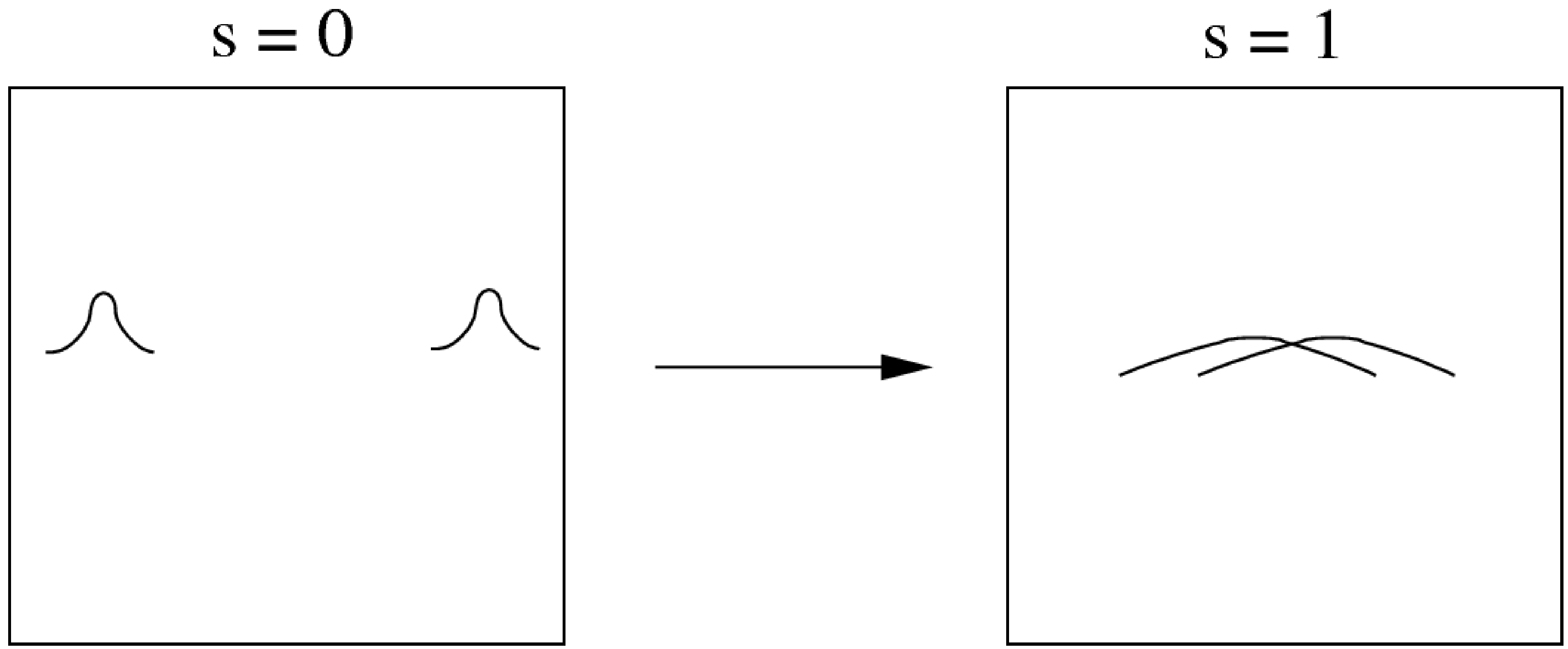
Adiabatically Turn on Interaction

$$H(s) = \sum_{x \in \Omega} \left[\pi^2 + (\nabla \phi)^2 + m^2 \phi^2 + s \lambda \phi^4 \right]$$

Use standard techniques to simulate H(s) with s slowly varying from 0 to 1

This almost works....

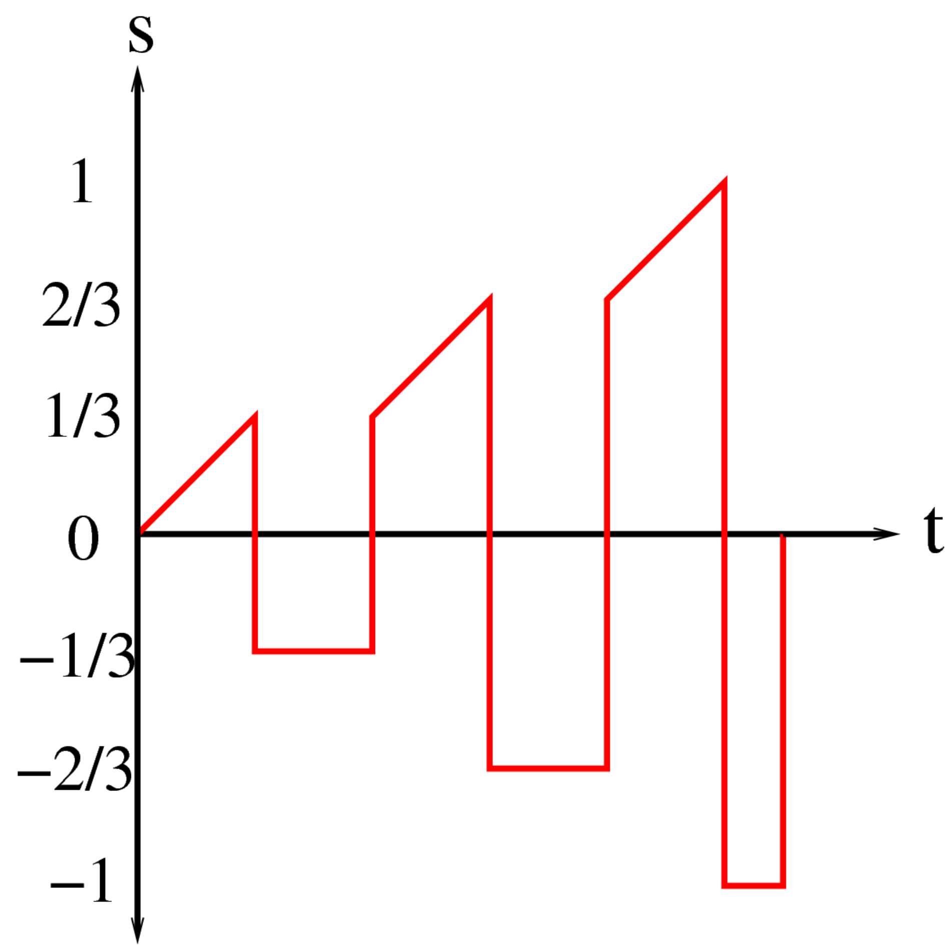
During the slow time evolution



The wavepackets propagate and broaden.

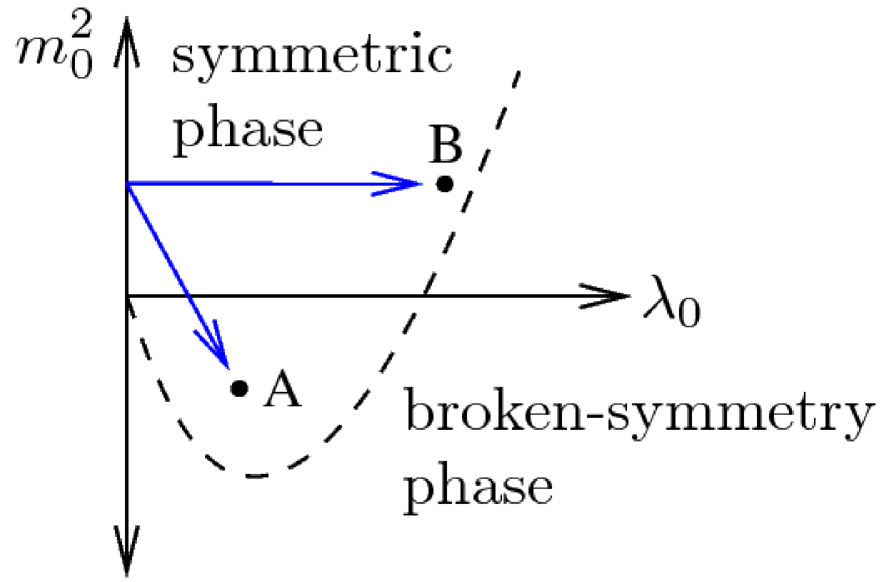
Solution: intersperse backward time evolutions with time-independent Hamiltonians

This winds back the dynamical phase on each eigenstate, without undoing the adiabatic change of eigenbasis



Strong Coupling

ϕ^4-theory in 1+1 and 2+1 dimensions has a quantum phase transition in which the $\phi \to -\phi$ symmetry is spontaneously broken



Near the phase transition perturbation theory fails and the gap vanishes.

$$m_{\text{phys}} \sim (\lambda_c - \lambda_0)^\nu \qquad \nu = \begin{cases} 1 & d = 1 \\ 0.63\ldots & d = 2 \end{cases}$$

Complexity

Weak Coupling:

$d = 1$	$(1/\epsilon)^{1.5}$
$d = 2$	$(1/\epsilon)^{2.376}$
$d = 3$	$(1/\epsilon)^{5.5}$

Strong Coupling:

	$\lambda_c - \lambda_0$	p	n_{out}
$d = 1$	$\left(\frac{1}{\lambda_c - \lambda_0}\right)^9$	p^4	n_{out}^5
$d = 2$	$\left(\frac{1}{\lambda_c - \lambda_0}\right)^{6.3}$	p^6	$n_{\text{out}}^{7.128}$

Fermions:

- Fermion doubling problem
- Free vacuum different from Bosonic case

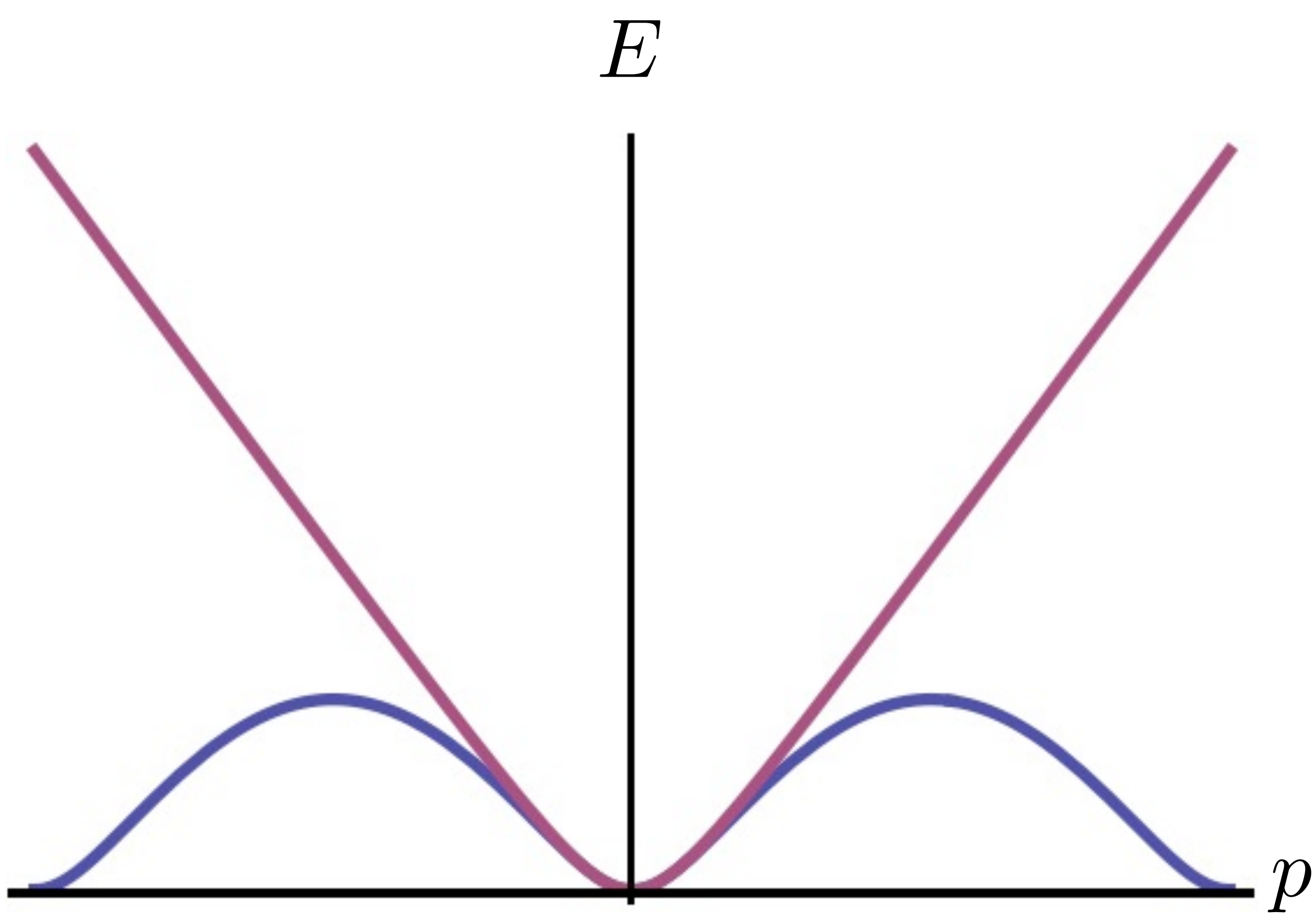
Gross-Neveu:

$$H = \int dx \left[\sum_{j=1}^{N} \bar{\psi}_j \left(m_0 - i\gamma^1 \frac{d}{dx} \right) \psi_j + \frac{g^2}{2} \left(\sum_{j=1}^{N} \bar{\psi}_j \psi_j \right)^2 \right]$$

Fermion Doubling Problem

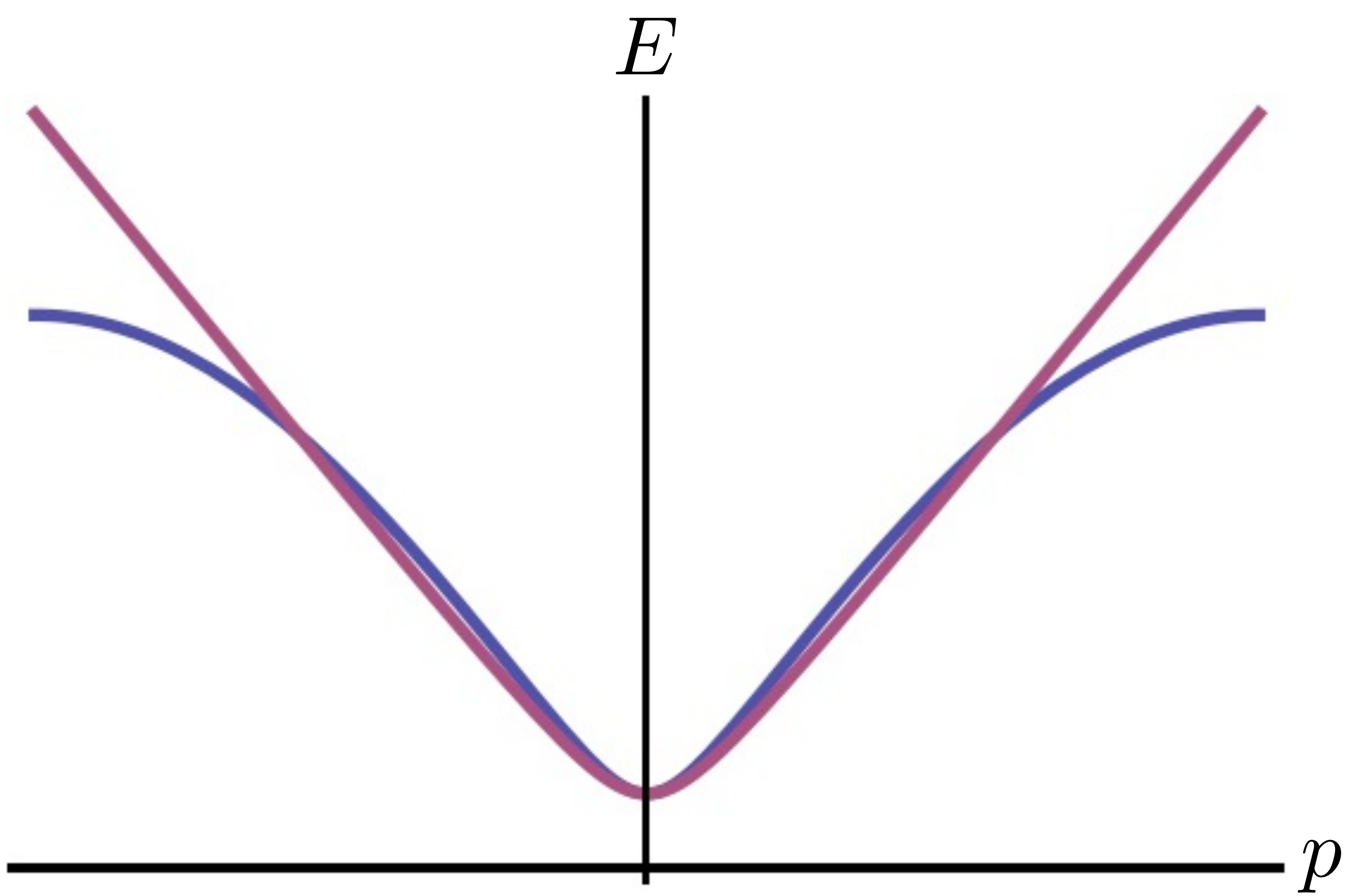
$$\frac{d\psi}{dx} \to \frac{\psi(x+a) - \psi(x-a)}{2a}$$

$$\sqrt{p^2 + m^2} \to \sqrt{\sin^2 p + m^2}$$



Wilson Term

$$H \to H - \frac{r}{2a} \sum_x \bar{\psi} \left(\psi(x+a) - 2\psi(x) + \psi(x-a) \right)$$



Preparing Fermionic Vacuum

$$H = \sum_x \bar{\psi}(x) m \psi(x)$$

↓ Adiabatic

$$H = \sum_x \bar{\psi}(x) \left(m + \frac{d}{dx} \right) \psi(x)$$

↓ Adiabatic

$$H = \sum_x \bar{\psi}(x) \left(m + \frac{d}{dx} \right) \psi(x) + \frac{g^2}{2} \left(\bar{\psi}(x) \psi(x) \right)^2$$

Eventual goal:

Simulate the standard model in polynomial time with quantum circuits.

Solved problems:

ϕ^4-theory [*Science*, 336:1130 (2012)]

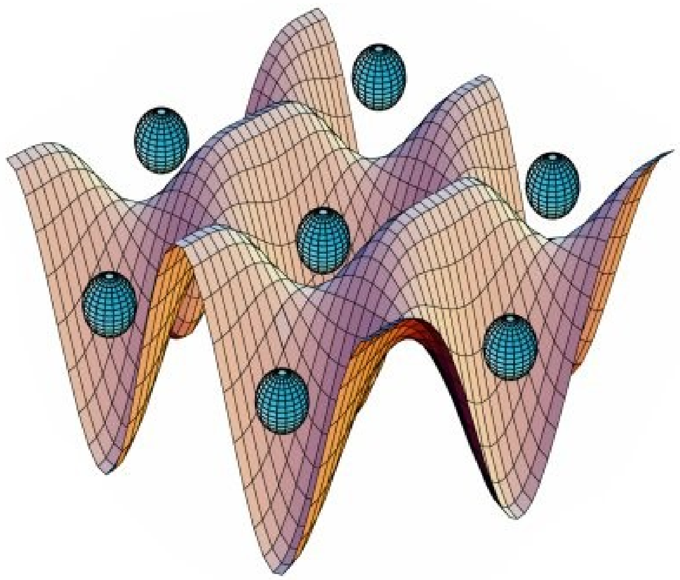
Gross-Neveu [S.J., Lee, Preskill, *in preparation*]

Open problems:

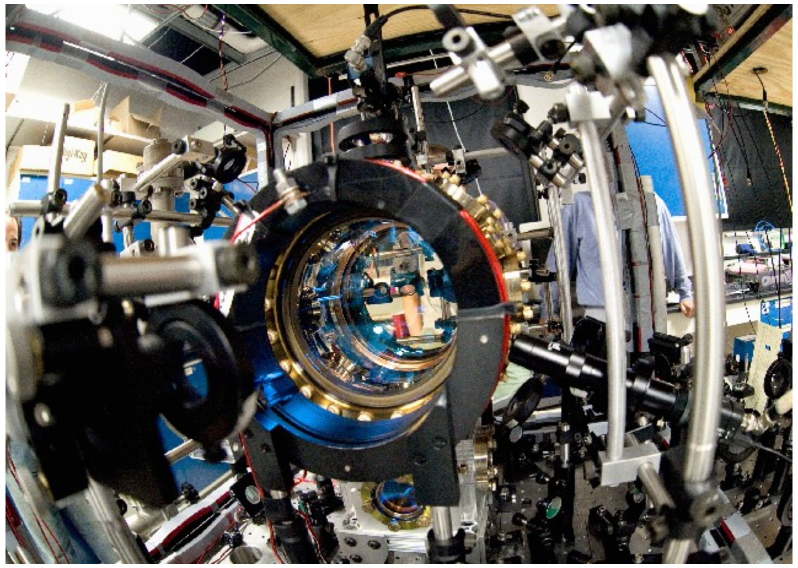
gauge symmetries, massless particles,
spontaneous symmetry breaking,
bound states, confinement, chiral fermions

Analog Simulation

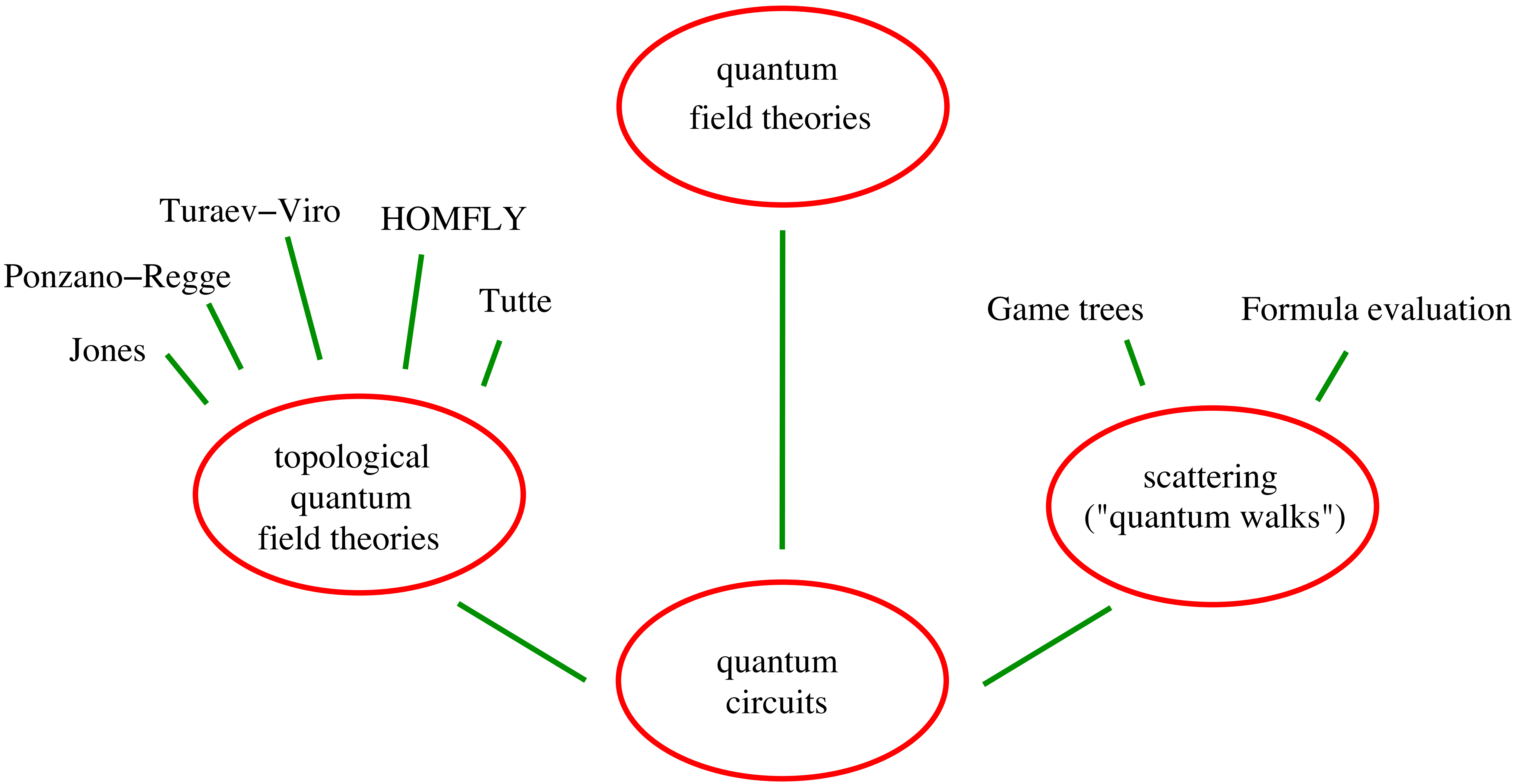
- No gates: just implement a Hamiltonian and let it time-evolve

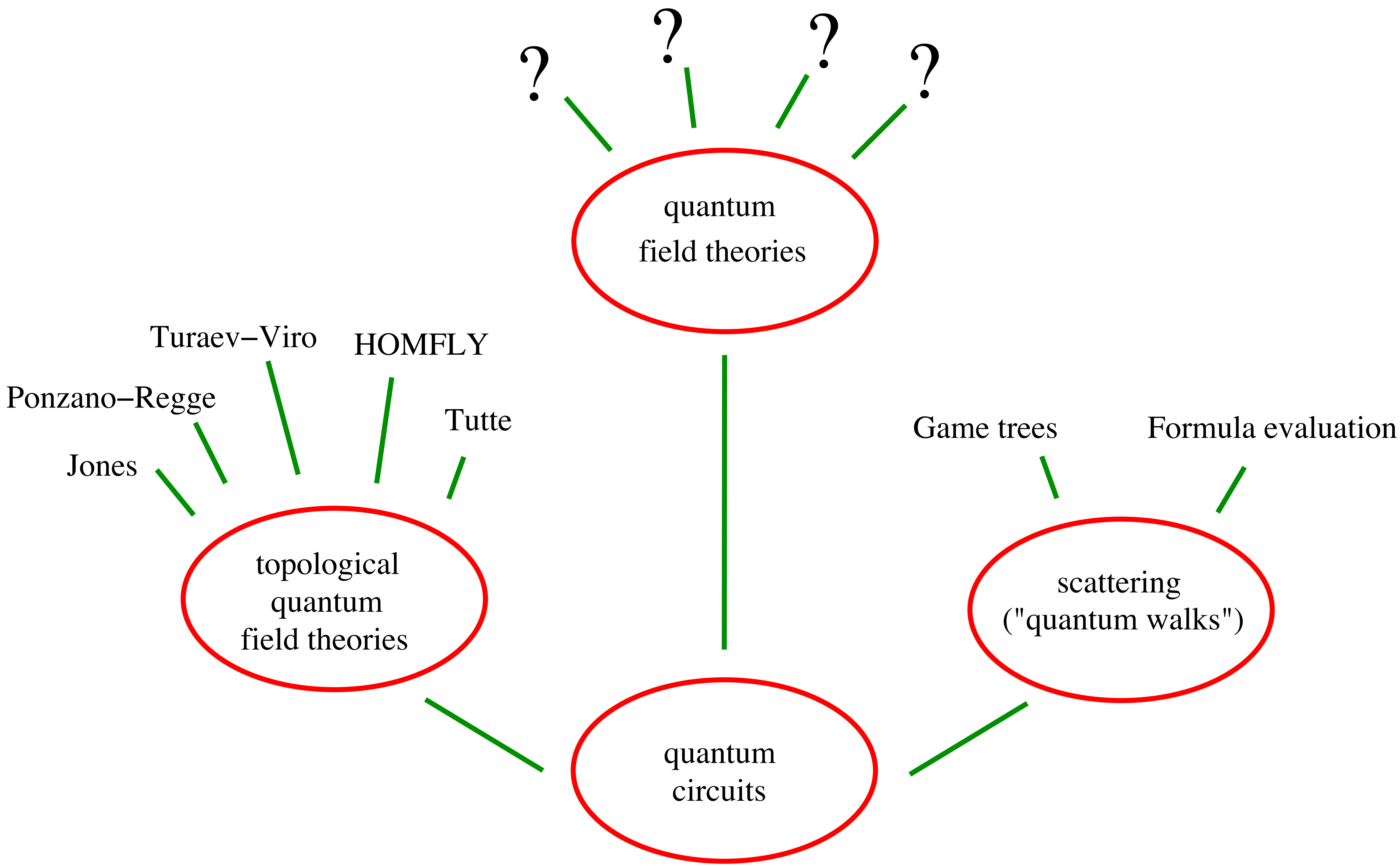


- Current experiments do this!



Broader Context

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quantum
field theories
Turaev−Viro
HOMFLY
Ponzano−Regge
Tutte
Jones
Game trees
Formula evaluation
topological
quantum
field theories
scattering
("quantum walks")
quantum
circuits

What I'm trying to do is get you people who think about computer simulation to see if you can't invent a different point of view than the physicists have.

-Richard Feynman, 1981

In thinking and trying out ideas about "what is a field theory" I found it very helpful to demand that a correctly formulated field theory should be soluble by computer... It was clear, in the '60s, that no such computing power was available in practice.

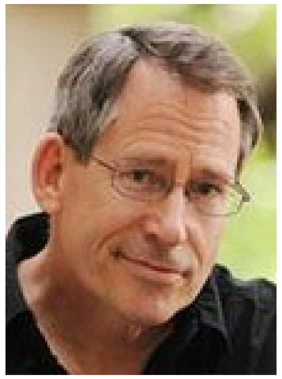
-Kenneth Wilson, 1982

Conclusion

Quantum computers can simulate scattering in ϕ^4-theory and the Gross-Neveu model.

There are many exciting prospects for quantum computation and quantum field theory to contribute to each other's progress.

I thank my collaborators:



Thank you for your attention.