# Resonating valence bond states in the PEPS formalism

# **RNTHAACHEN UNIVERSITY**

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joint work with David Pérez-García, Ignacio Cirac, and Didier Poilblanc

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• I'll try my best to keep Toby happy throughout most of the talk:





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• Relation to Hamiltonians? Unique or topological ground state?

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 $\Rightarrow \mathsf{PEPS} |\Psi\rangle \text{ unique ground state of local "parent Hamiltonian" } H' = \sum h',$  $h' = (\mathcal{P}^{-1} \otimes \mathcal{P}^{-1})^{\dagger} h(\mathcal{P}^{-1} \otimes \mathcal{P}^{-1}) : h' \ge 0, \qquad h' |\Psi\rangle = 0$ 

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• e.g.: Toric Code  $\leftrightarrow \mathbb{Z}_2$  symmetry:

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- G-injectivity ⇒ PEPS is ground state of local parent Hamiltonian with topological ground space degeneracy (Proof: e.g. for Z<sub>2</sub>-injectivity → reversible mapping to Toric Code state + Hamiltonian)
- structure of ground space etc. is inherited from Toric Code



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• Triangles have **no or one singlet**:

 $|0\rangle, |1\rangle$  : spin- $\frac{1}{2}$  subspace  $|2\rangle$  : "no singlet" tag

$$|\epsilon\rangle = \frac{1}{\sqrt{2}} \sum \epsilon_{ijk} |ijk\rangle + |222\rangle = \frac{1}{\sqrt{2}} (|01\rangle - |10\rangle) |2\rangle + \text{perm.} + |222\rangle$$





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- Hamiltonian for Toric Code  $\Rightarrow$  Ham. for dimer state  $\Rightarrow$  Hamiltonian for RVB
- Parent Hamiltonian with topological ground space structure!

- **Resonating Valence Bond** state (RVB): superposition of all **singlet coverings** of lattice
- RVB: ground state of local parent Hamiltonian with topologically degenerate ground space (4-fold on torus)
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- Also, we still need to figure out if there is long-range order ...

numerical study on cylindrical geometry



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 $N_h$ 



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- no divergence along interpolation ⇒ RVB is in same phase as Toric Code!

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