Exponential Decay of Correlations Implies Area Law

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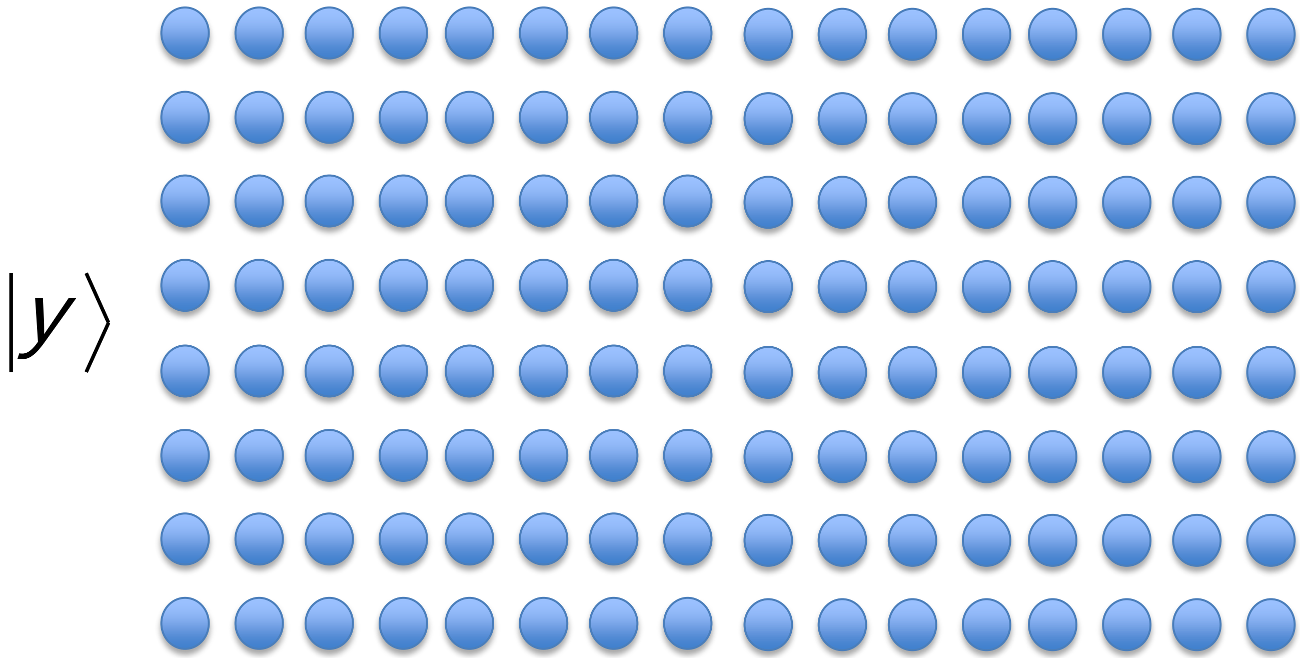
QIP 2013, Beijing

Condensed (matter) version of the talk



- Finite correlation length implies correlations are short ranged

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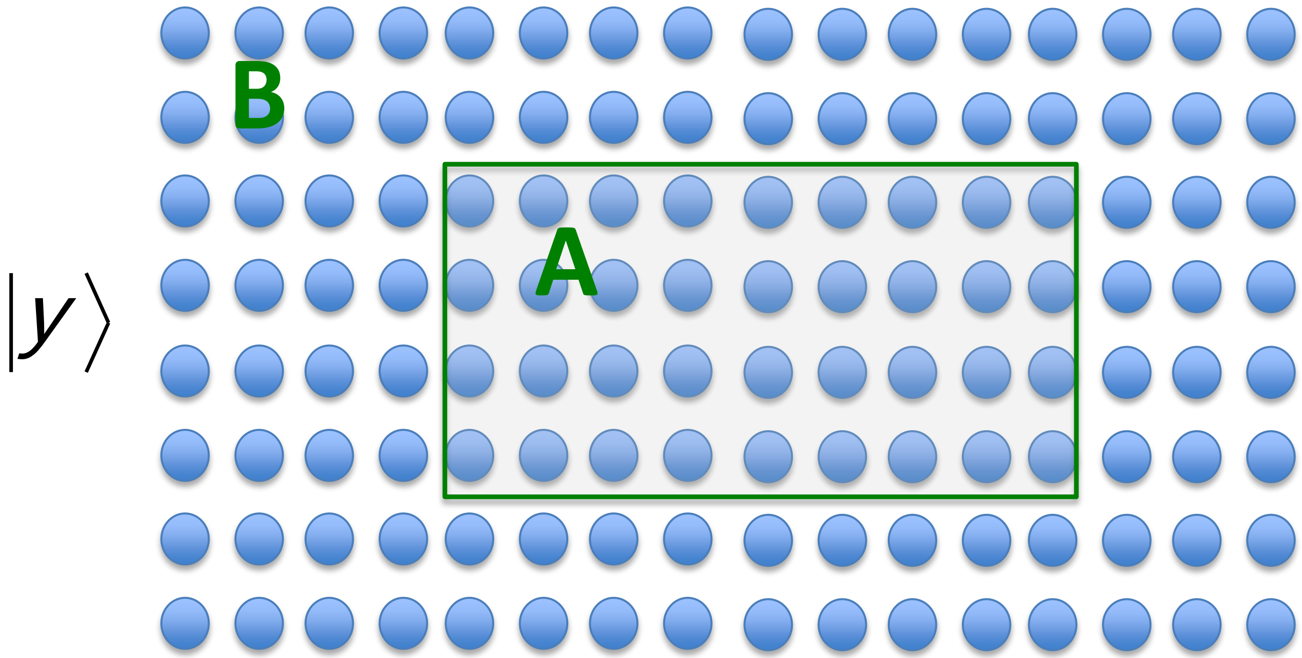

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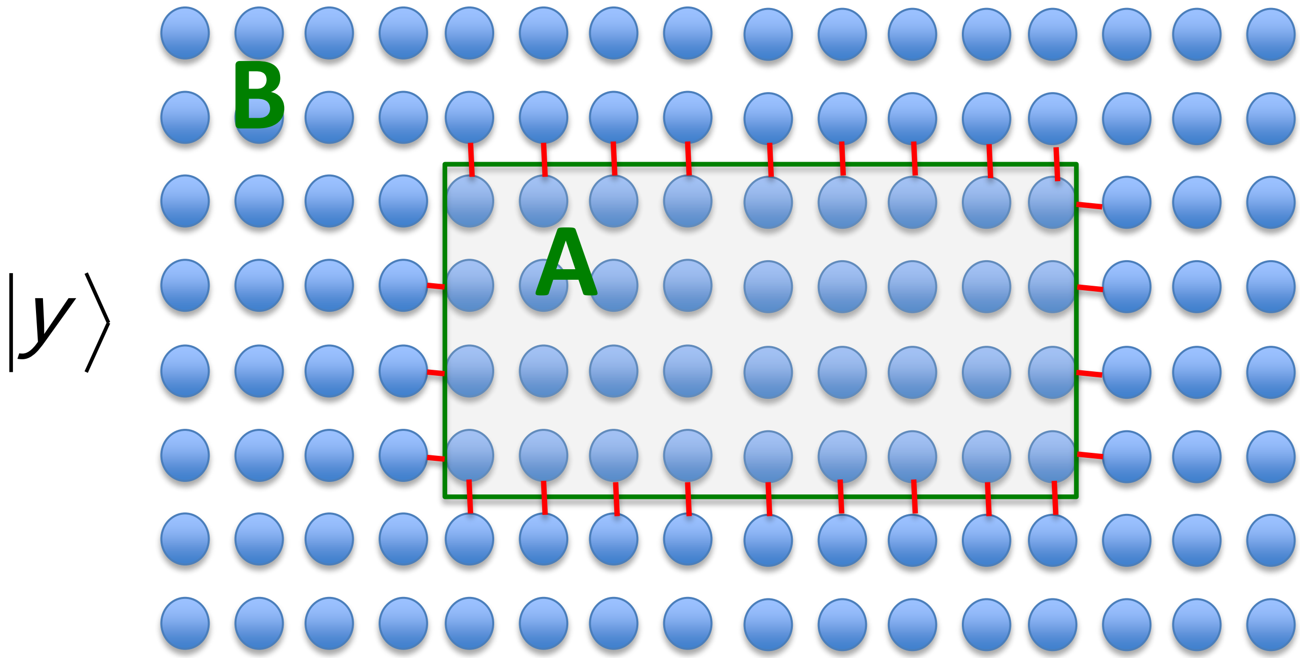

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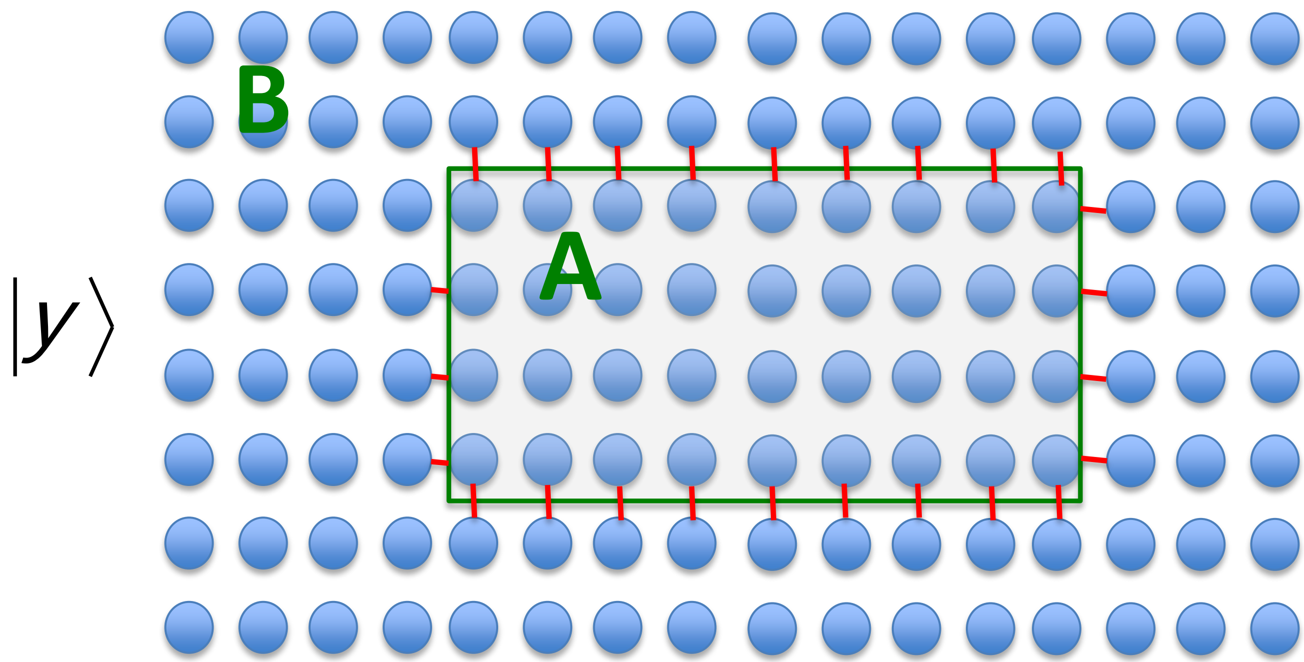

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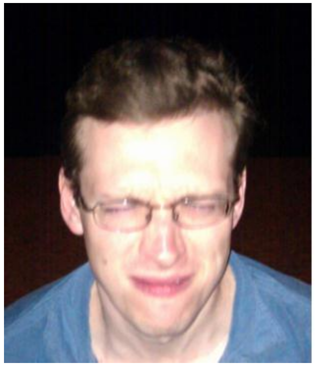
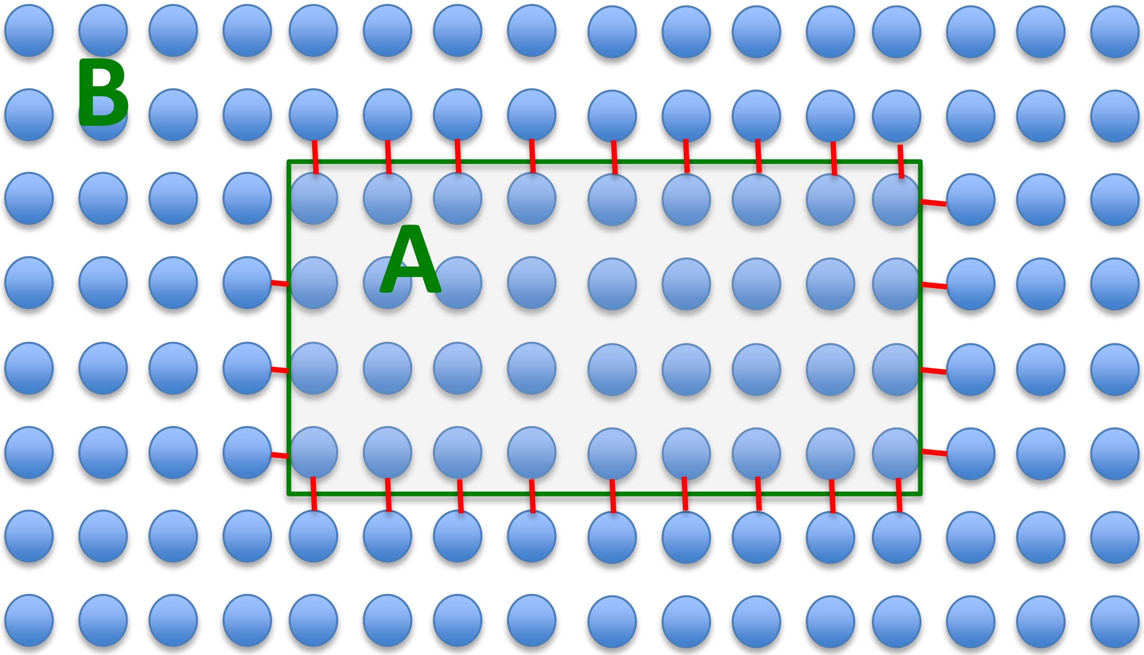


Frowny Cubitt:
Handwaving

$|y\rangle$



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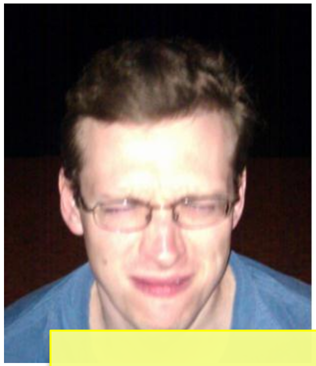
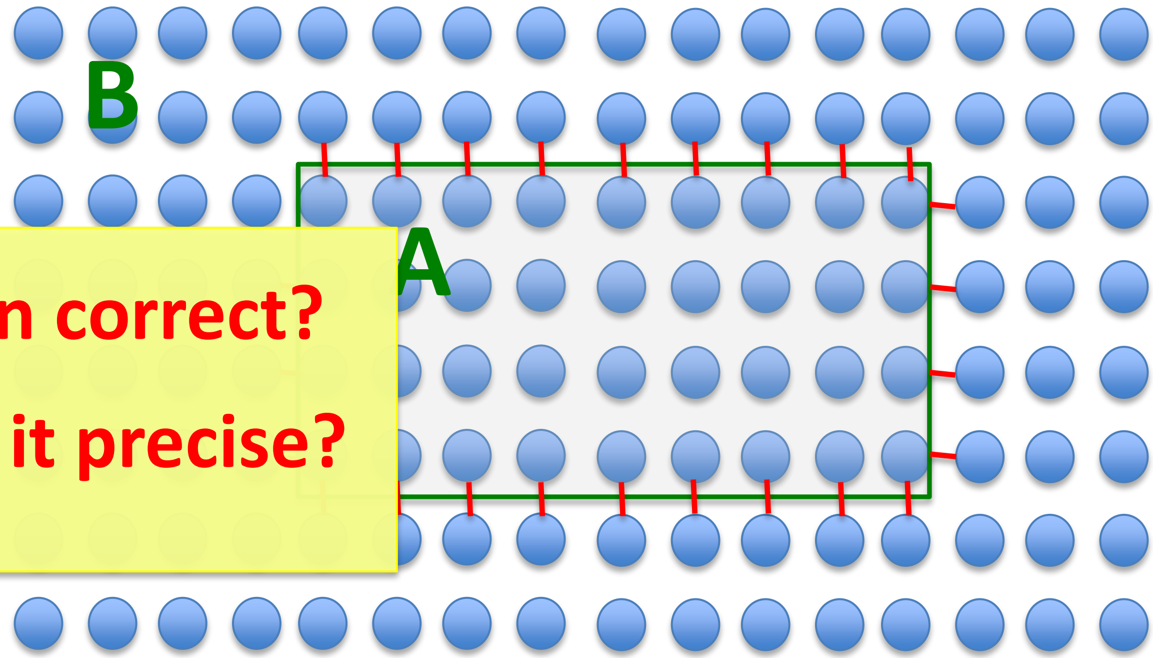




- Is the intuition correct?

- Can we make it precise?

- Finite correlation length implies correlations are short ranged
- A is only entangled with B at the boundary: **area law**

Outline

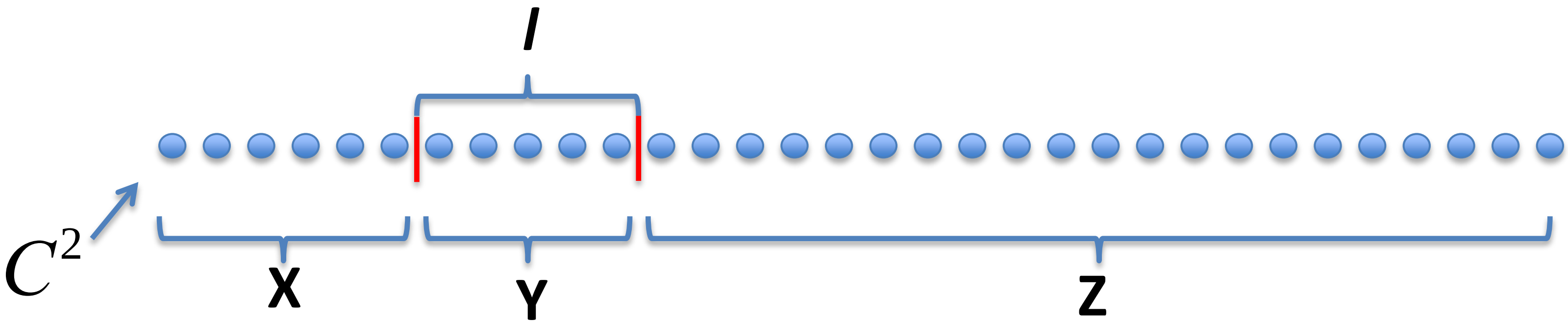
- **The Problem**
  Exponential Decay of Correlations
  Entanglement Area Law

- **Results**
  Decay of Correlations Implies Area Law
  Decay of Correlations and Quantum Computation

- **The Proof**
  Decoupling and State Merging
  Single-Shot Quantum Information Theory

Exponential Decay of Correlations

Let $|\psi\rangle_{1,\dots,n} \in (C^2)^{\otimes n}$ **be a *n*-qubit quantum state**



Correlation Function:

$$Cor(X:Z) := \max_{\|M\|,\|N\| \leq 1} \left| tr\left((M \otimes N)(\rho_{XZ} - \rho_X \otimes \rho_Z) \right) \right|$$

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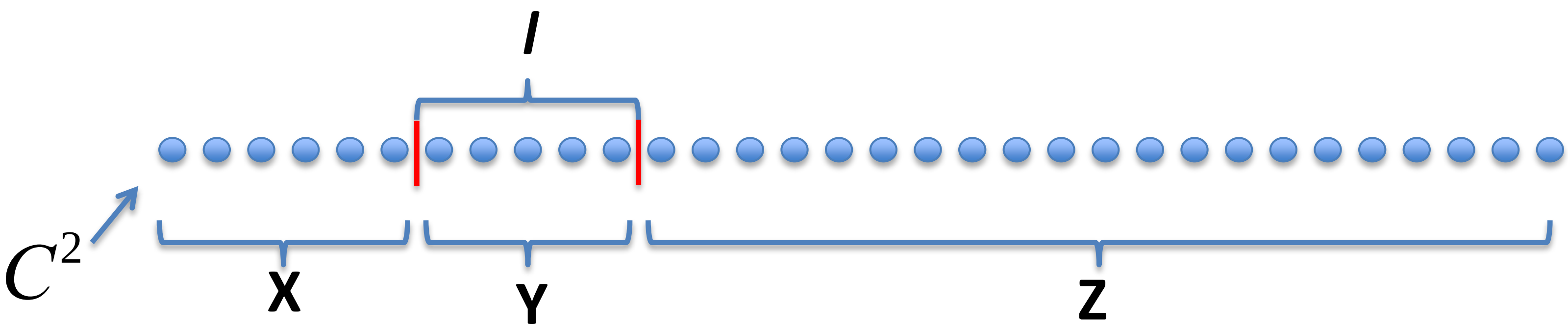

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$$= \max_{\|M\|,\|N\| \leq 1} \langle M_X N_Z \rangle_\psi - \langle M_X \rangle_\psi \langle N_Z \rangle_\psi$$

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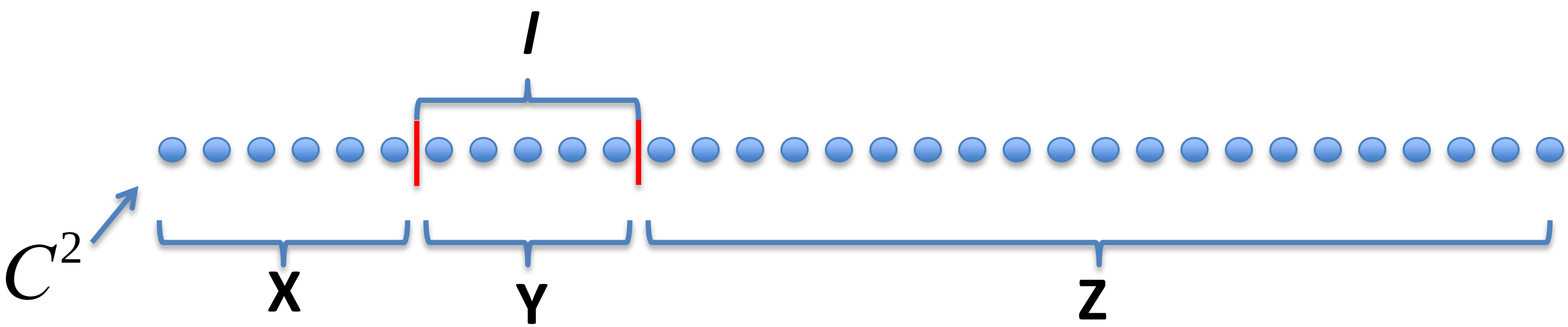

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Exponential Decay of Correlations: There is **ξ > 0** s.t. for all cuts X, Y, Z with |Y| = l

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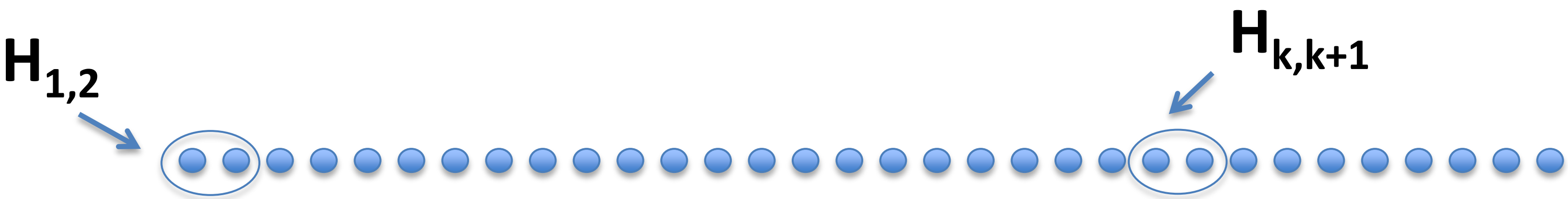
Which states exhibit exponential decay of correlations?

Example: |0, 0, …, 0> has **0**-exponential decay of cor.

Local Hamiltonians



Local Hamiltonian: $$H = \sum_k H_{k,k+1}$$

Groundstate: $$|\psi_0\rangle \;:\; H|\psi_0\rangle = E_0|\psi_0\rangle$$

Spectral Gap: $$\Delta(H) := E_1 - E_0$$

Thermal state: $$\rho_\beta := e^{-\beta H}/Z$$

States with Exponential Decay of Correlations

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(Araki, Hepp, Ruelle ’62, Fredenhagen ’85)

Groundstates in relativistic systems

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Groundstates of gapped local Hamiltonians
Analytic proof: Lieb-Robinson bounds, etc…

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(Araronov, Arad, Landau, Vazirani ‘10)
Groundstates of gapped frustration-free local Hamiltonians
Combinatorial Proof: Detectability Lemma

Exponential Decay of Correlations…

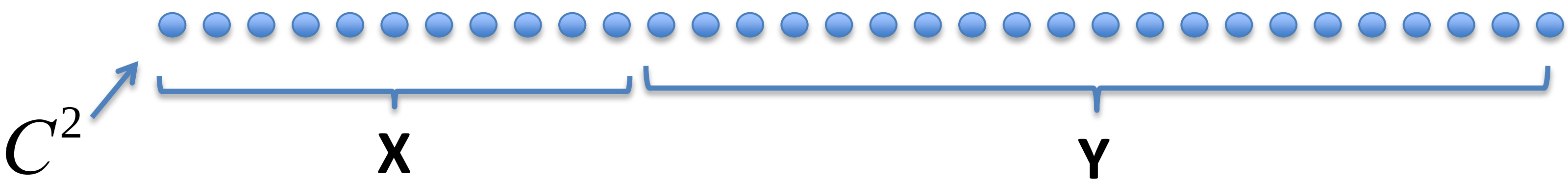
… intuitively suggests the state is *simple*, in a sense similar to a product state.

Can we make this rigorous?

But first, are there *other ways* to impose simplicity in quantum states?

Area Law in 1D

Let $|y\rangle_{1,\ldots,n}$ Î $(C^2)^{\text{Ä}\, n}$ **be a *n*-qubit quantum state**



Entanglement Entropy: $E\left(|y_{XY}\rangle\right) := S(r_X)$

Area Law: For all partitions of the chain (X, Y)

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For the majority of quantum states: $S(r_X) \text{ » } size(X) = r$

Area Law puts severe constraints on the amount of entanglement of the state

Quantifying Entanglement

Sometimes entanglement entropy is not the most convenient measure:

Max-entropy: $$S_{\max}(\rho) := \log rank(\rho)$$

Smooth max-entropy: $$S_{\max}^{\varepsilon}(\rho) := \min_{\rho_{\varepsilon} \in B_{\varepsilon}(\rho)} S_{\max}(\rho_{\varepsilon})$$

$$B_{\varepsilon}(\rho) := \{\sigma : \|\rho - \sigma\|_1 \le \varepsilon\}$$

Smooth max-entropy gives the minimum number of qubits needed to store an ε-approx. of ρ

States that satisfy Area Law

Intuition - based on concrete examples (XY model, harmonic systems, etc.) and general non-rigorous arguments:

Model	Spectral Gap	Area Law
Non-critical ⟺	Gapped ⇒	$S(X) \leq O(\text{Area}(X))$
Critical ⟺	Non-gapped ⇒	$S(X) \leq O(\text{Area}(X)\log(n))$

States that satisfy Area Law

(Aharonov *et al* ’07; Irani ’09, Irani, Gottesman ‘09)

Groundstates 1D Ham. with *volume* law **S(X) ≥ Ω(vol(X))**

Connection to QMA-hardness

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Thermal states of local Ham. **$I(X:Y) \leq O(Area(X)/\beta)$**

Proof from Jaynes' principle

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(Arad, Kitaev, Landau, Vazirani '12) **$S(X) \leq O(1/\Delta)$**

Groundstates 1D *gapped* local Ham.

Combinatorial Proof: Chebyshev polynomials, etc…

Area Law and MPS

Matrix Product State (MPS):

$$|\psi\rangle_{1,\dots,n} = \sum_{i_1=1}^{2} \dots \sum_{i_n=1}^{2} tr\left(A_{i_1}^{[1]} \dots A_{i_n}^{[n]}\right) |i_1, \dots, i_n\rangle, \quad A_j^{[l]} \in Mat(D, D)$$

D : bond dimension

- Only **nD^2** parameters.
- Local expectation values computed in **poly(D, n)** time
- Variational class of states for powerful **DMRG**

In 1D: Area Law → State has an efficient classical description MPS with D = poly(n)
(Vidal 03, Verstraete, Cirac '05, Schuch, Wolf, Verstraete, Cirac '07, Hastings '07)

Area Law vs. Decay of Correlations

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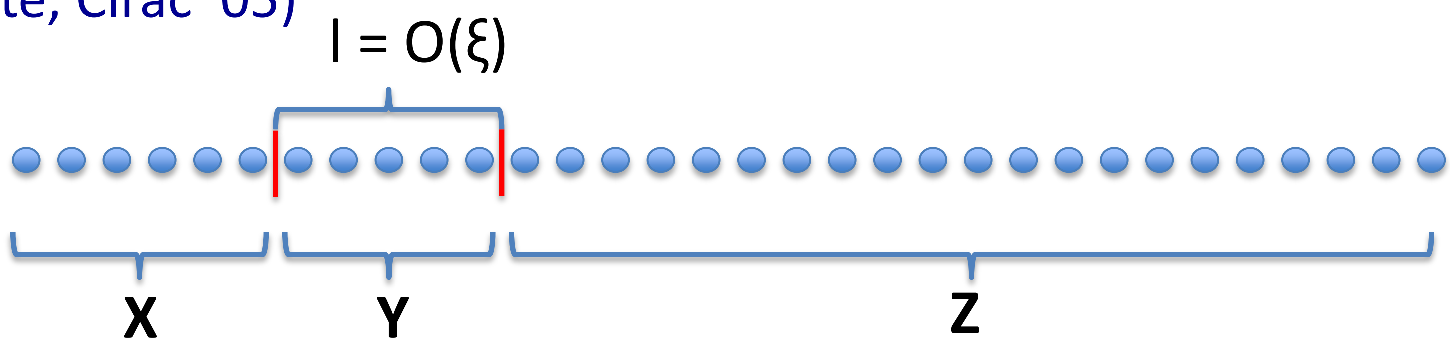

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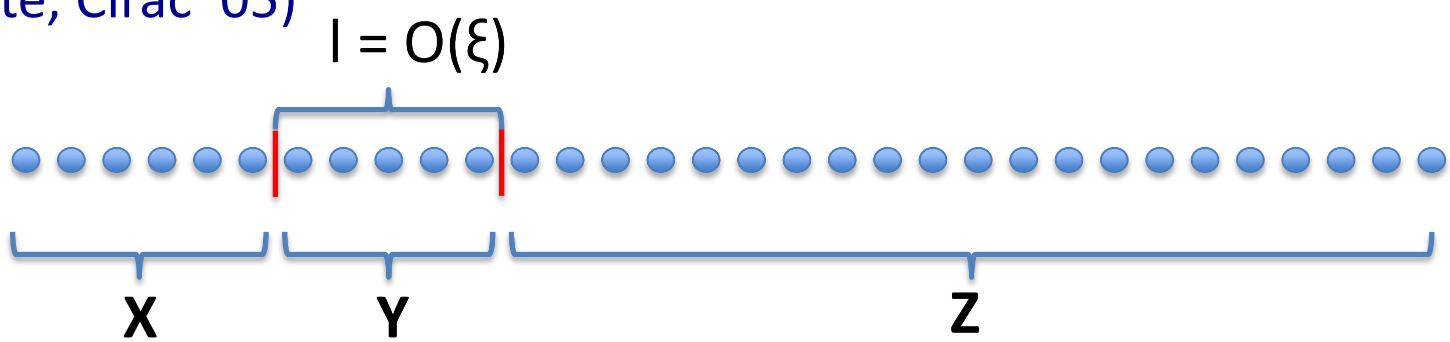

ξ-EDC implies $r_{XZ} \, »_{2^{-l/x}} \, r_X \, Ä \, r_Z$ which implies

$$\left|y\right\rangle_{XYZ} \approx_{2^{-l/x}} \left(U_{Y_1Y_2 \to Y} \otimes I_{XZ}\right)\left|p\right\rangle_{XY_1}\left|u\right\rangle_{Y_2Z}$$ (by Uhlmann’s theorem)

X is only entangled with Y!

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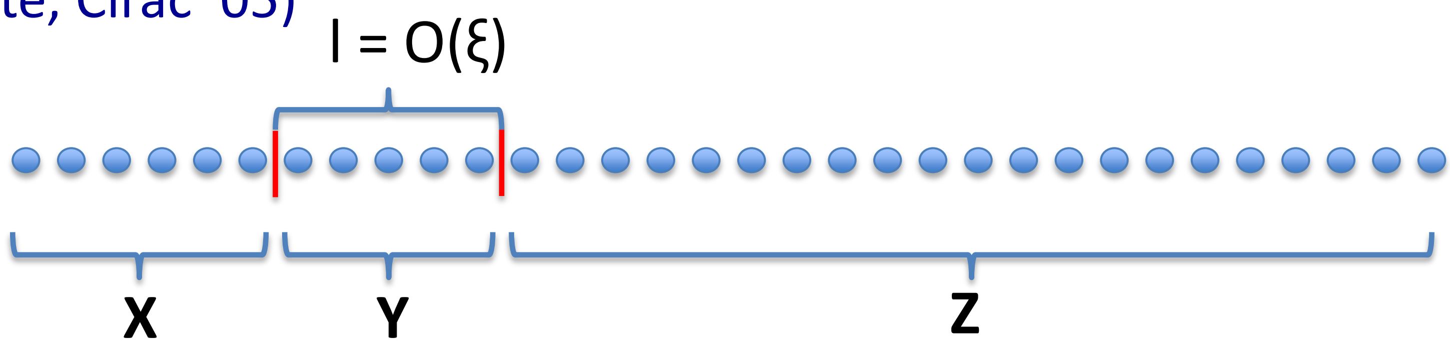

ξ-EDC implies $\rho_{XZ} \approx_{2^{-l/\xi}} \rho_X \otimes \rho_Z$ ~~which implies~~

$|\psi\rangle_{XYZ} \approx_{2^{-l/\xi}} \left(U_{Y_1Y_2 \to Y} \otimes I_{XZ}\right) |\pi\rangle_{XY_1} |\upsilon\rangle_{Y_2Z}$ (by Uhlmann's theorem)

X is only entangled with Y! Alas, the argument is *wrong*…

Reason: Quantum Data Hiding states: For random ρ_{XZ} w.h.p.

$$Cor(X:Z) \leq 2^{-\Omega(l)}, \quad \left\| \rho_{XZ} - \rho_X \otimes \rho_Z \right\|_1 = \Omega(1)$$

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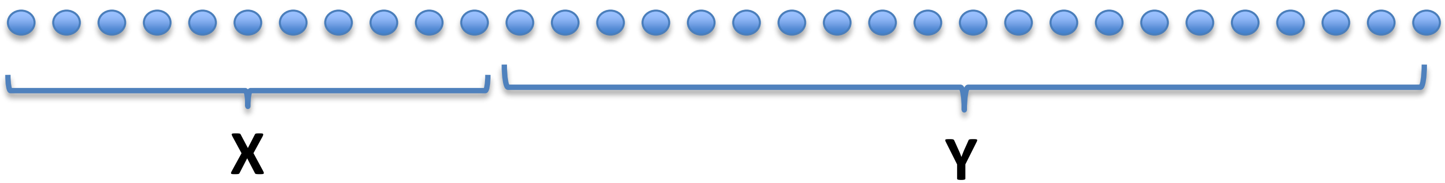
3. Cop out: data hiding states are unnatural; “physical” states are well behaved.

4. We fixed a partition; EDC gives us more…

5. It’s an interesting quantum information problem:
   How strong is data hiding in quantum states?

Exponential Decaying Correlations Imply Area Law



Thm 1 (B., Horodecki '12) If $|\psi\rangle_{1,\dots,n}$ has ξ-EDC then for every X and m,

$$S_{\max}^{2^{-\Omega(m)}}(X) \leq l_0 2^{O(\xi \log(\xi))} + m$$

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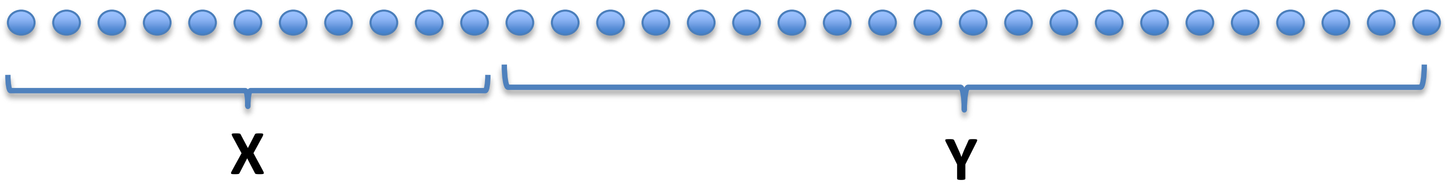


Thm 1 (B., Horodecki '12) If $|y\rangle_{1,\dots,n}$ has ξ-EDC then for every X and m,

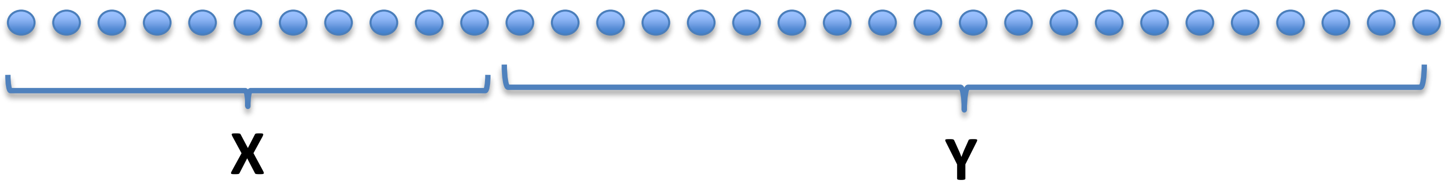
$$S_{\max}^{2^{-W(m)}}(X) \leq l_0 2^{O(x\log(x))} + m$$

Obs1: Implies $S(X) \pounds l_0 2^{O(x\log(x))}$

Obs2: Only valid in 1D…

Obs3: Reproduces bound of Hastings for GS 1D gapped Ham., using EDC in such states

Efficient Classical Description



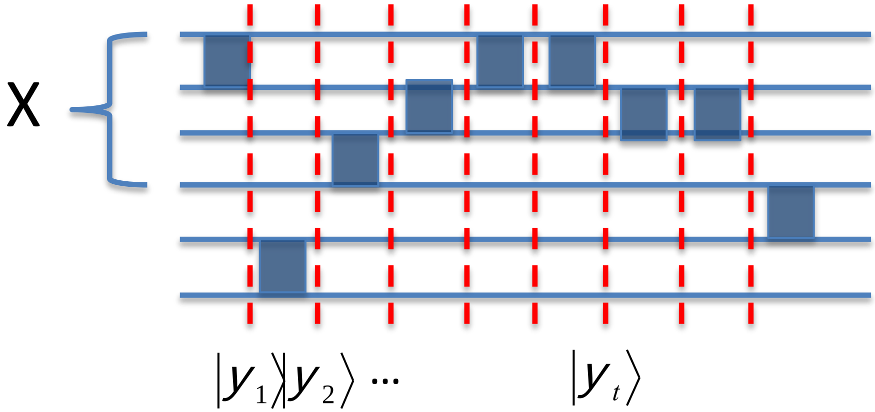
(Cor. Thm 1) If $|\psi\rangle_{1,\ldots,n}$ has ξ-EDC then for every ε>0 there is MPS $|\psi_\varepsilon\rangle$ with poly(n, 1/ε) bound dim. s.t.

$$|\langle\psi|\psi_\varepsilon\rangle| \geq 1 - \varepsilon$$

States with exponential decaying correlations are *simple* in a precise sense

Correlations in Q. Computation

What kind of correlations are necessary for exponential speed-ups?



1. (Vidal '03) Must exist *t* and X = [1,r] s.t. $S^{e}_{\max}(r_{t,X}) \geq n^{d}$

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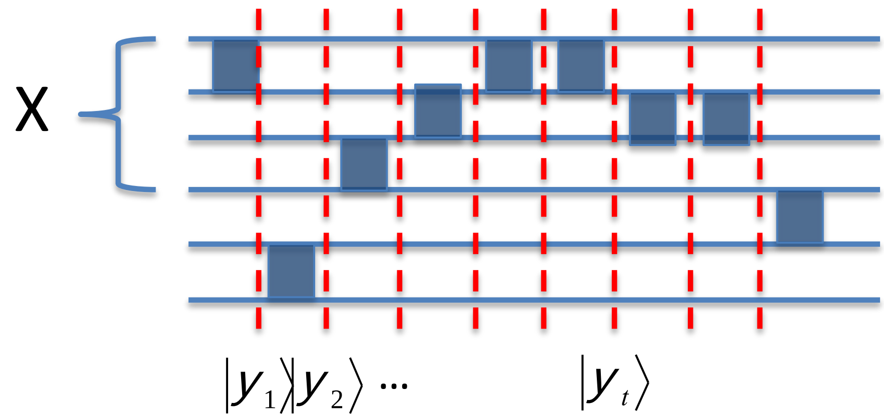
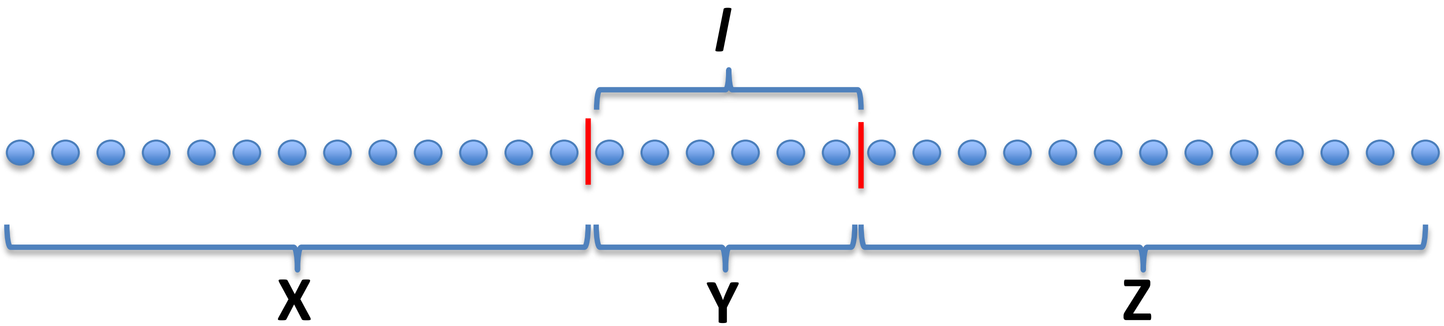


1. (Vidal '03) Must exist *t* and X = [1,r] s.t. $S^{e}_{\max}(r_{t,X})$ 3 n^{d}

2. (Cor. Thm 1) At some time step state must have long range correlations (at least algebraically decaying)
   - Quantum Computing happens in "critical phase"
   - Cannot hide information everywhere

Random States Have EDC?



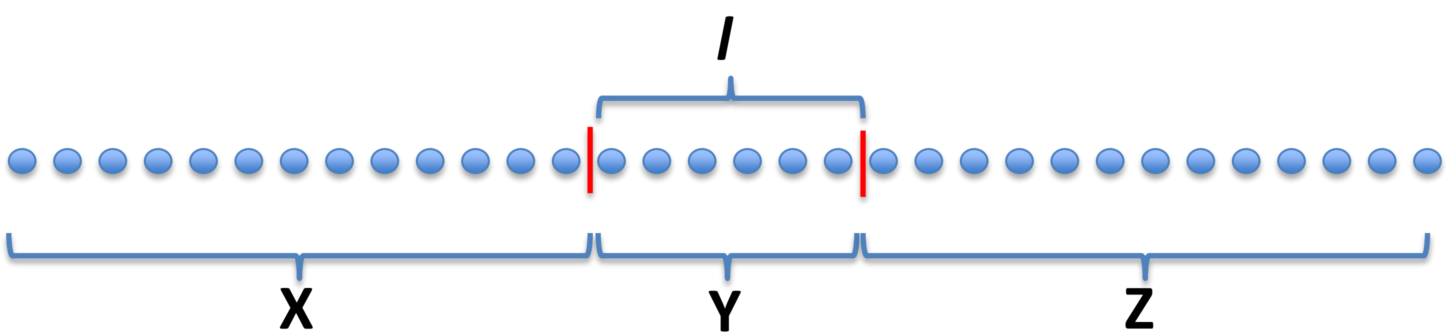
$|\psi\rangle_{XYZ}$: Drawn from Haar measure

w.h.p, if size(X) ≈ size(Z): $cor(X:Z) \leq 2^{-\Omega(l)}$

and $S(X) \approx S(Z) \approx n/2 - l$

Small correlations in a *fixed* partition do not imply area law.

Random States Have EDC?



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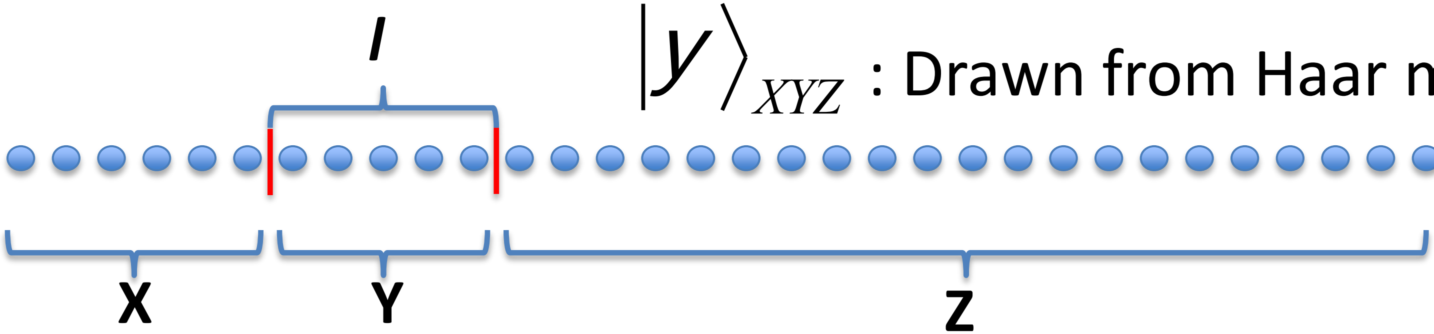
Small correlations in a *fixed* partition do not imply area law.

But we can **move the partition freely**...

Random States Have Big Correl.

$|\psi\rangle_{XYZ}$: Drawn from Haar measure



Let size(XY) < size(Z). W.h.p. $\left\| \rho_{XY} - \tau_X \otimes \tau_Y \right\|_1 \leq 2^{-\Omega(n)}, \quad \tau_X := \frac{I}{|X|}$

X is **decoupled** from *Y*.

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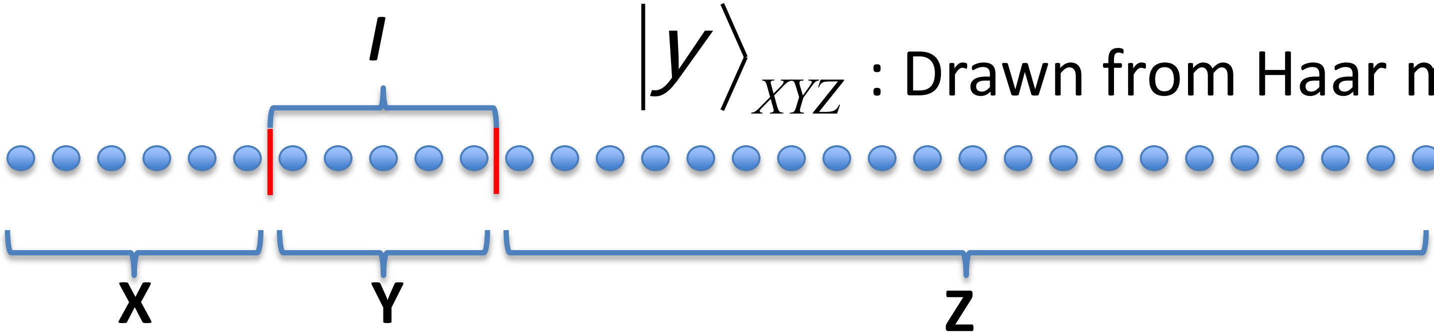

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X is **decoupled** from *Y*.

Extensive entropy, but also *large* correlations: $U_{Z\to Z_1 Z_2}|\psi\rangle_{XYZ} \approx |\Phi\rangle_{XZ_1} \otimes |\Phi\rangle_{YZ_2}$

(Uhlmann's theorem)

$|\Phi\rangle_{XZ_1}$:Maximally entangled state between XZ_1.

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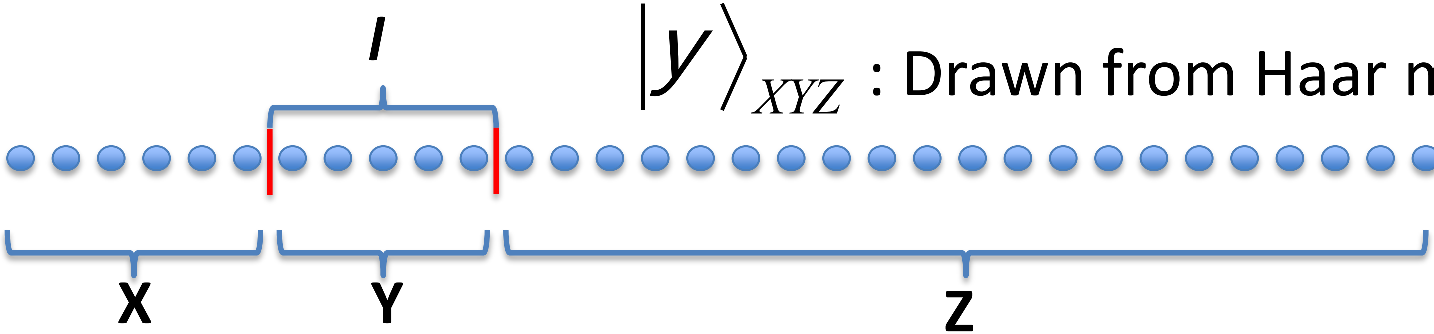

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$\mathrm{Cor}(X:Z) \geq \mathrm{Cor}(X:Z_1) = \Omega(1) >> 2^{-\Omega(n)}$: **long-range correlations!**

Random States Have Big Correl.

It was thought random states were counterexamples to area law from EDC.

Not true; reason hints at the idea of the general proof:

We'll show large entropy leads to large correlations by choosing a **random measurement that decouples *A* and *B***

X is decoupled from *Y*.

Extensive entropy, but also *large* correlations:

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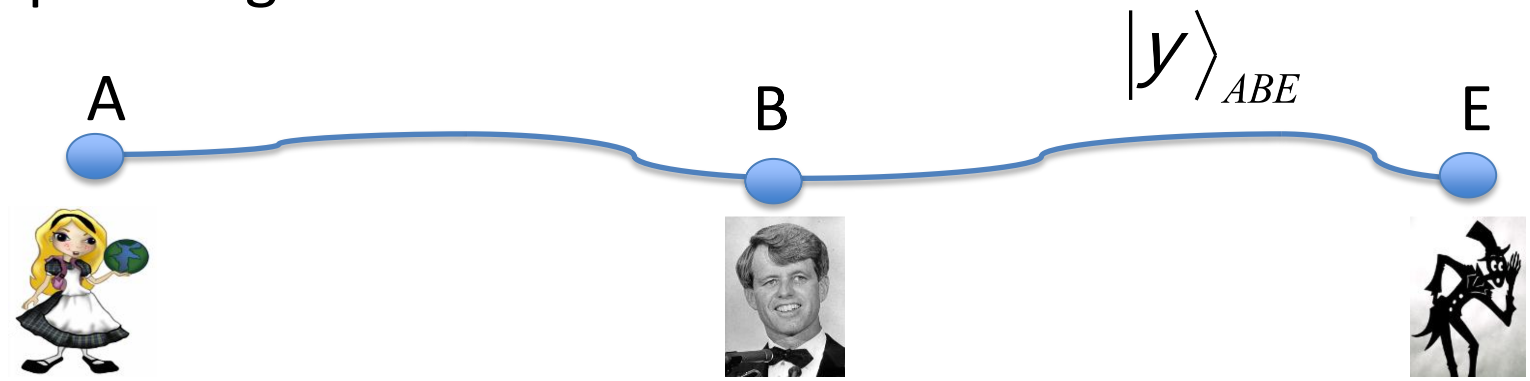
State Merging

We apply the state merging protocol to show large entropy implies large correlations



State merging protocol: Given $|y\rangle_{ABC}$ Alice can distill -S(A|B) = S(B) – S(AB) EPR pairs with Bob by making a random measurement with N≈ $2^{I(A:E)}$ elements, with I(A:E) := S(A) + S(E) – S(AE), and communicating the outcome to Bob. (Horodecki, Oppenheim, Winter ‘05)

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A B E

Disclaimer: merging only works for $|\psi\rangle_{ABE}^{\otimes n}$

Let's *cheat* for a while and pretend it works for a single copy, and later deal with this issue

State merging protocol: Given $|\psi\rangle_{ABC}$ Alice can distill $-S(A|B) = S(B) - S(AB)$ EPR pairs with Bob by making a random measurement with $N \approx 2^{I(A:E)}$ elements, with $I(A:E) := S(A) + S(E) - S(AE)$, and communicating the outcome to Bob. (Horodecki, Oppenheim, Winter '05)

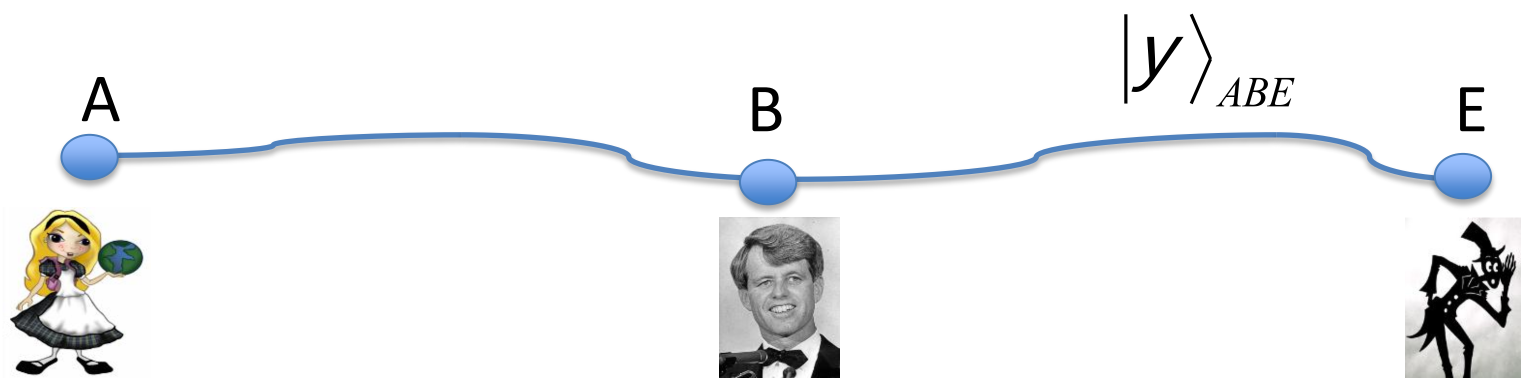
State Merging by Decoupling

State merging protocol works by applying a random measurement $\{P_k\}$ to A in order to *decouple* it from E:

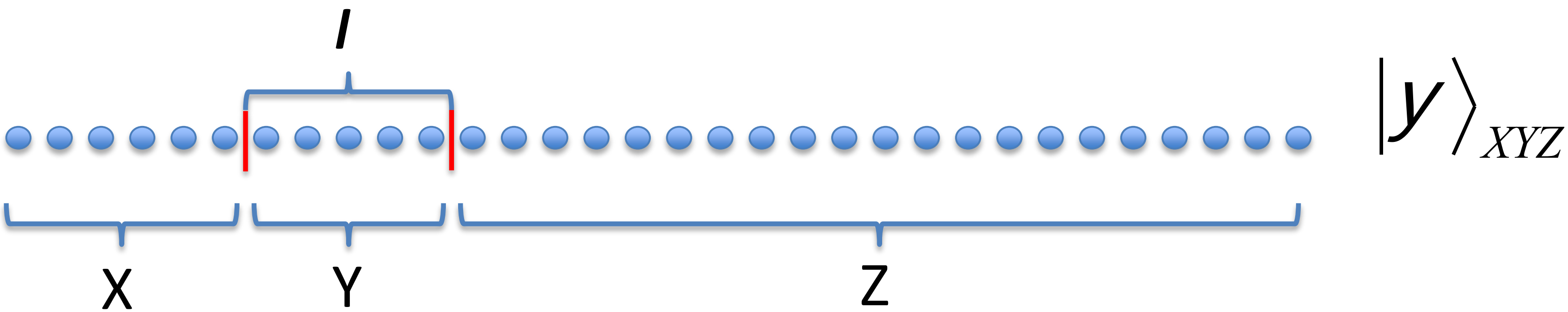
$$|\psi\rangle_{ABE} \mapsto |\varphi\rangle_{\bar{A}BE} \propto \left(P_k \otimes id_{BE}\right)|\psi\rangle_{ABE} \qquad \left\| \varphi_{\bar{X}Z} - \tau_{\bar{X}} \otimes \varphi_Z \right\|_1 \approx 0$$

log(# of P_k's) $\approx I(A:E)$

EPR pairs: $\log|\bar{X}| \approx S(B) - S(AB)$



What does state merging imply for correlations?



$$S(Z) > S(Y) \Rightarrow \quad Cor(X:Z) \geq O\left(2^{-I(X:Y)}\right)$$

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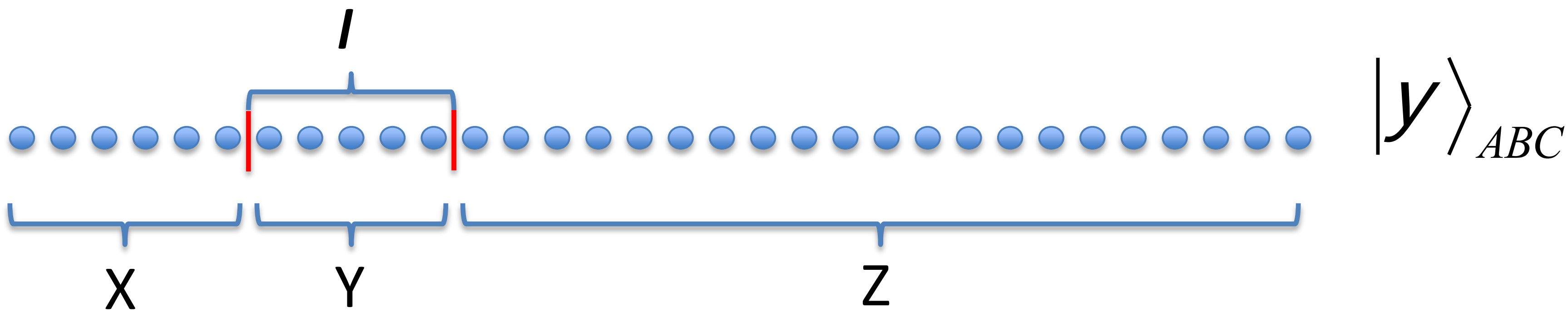
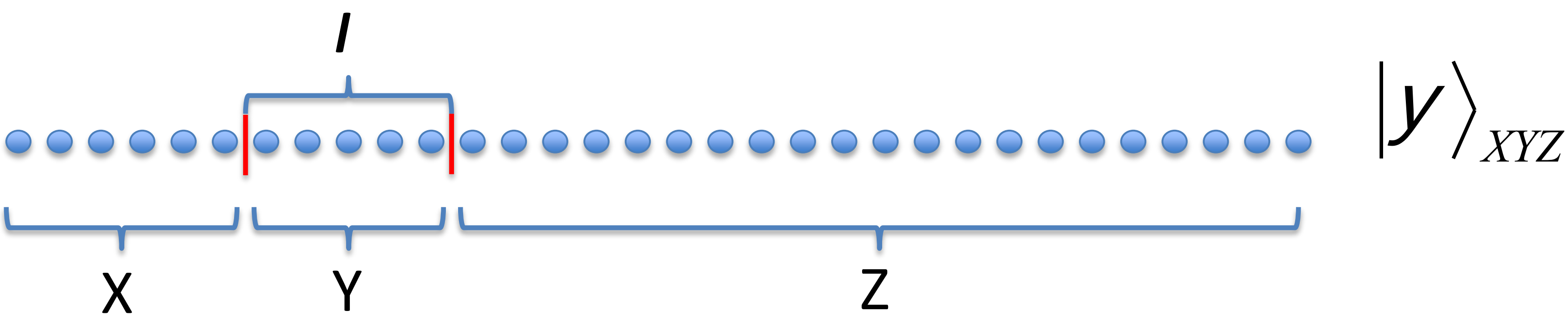


$$S(Z) > S(Y) \Rightarrow \quad Cor(X:Z) \geq O\left(2^{-I(X:Y)}\right)$$

S(Z) – S(XZ) > 0
(EPR pair distillation
by random measurement)

Prob. of getting one of the
$2^{I(X:Y)}$ outcomes in random
measurement

Area Law from Subvolume Law



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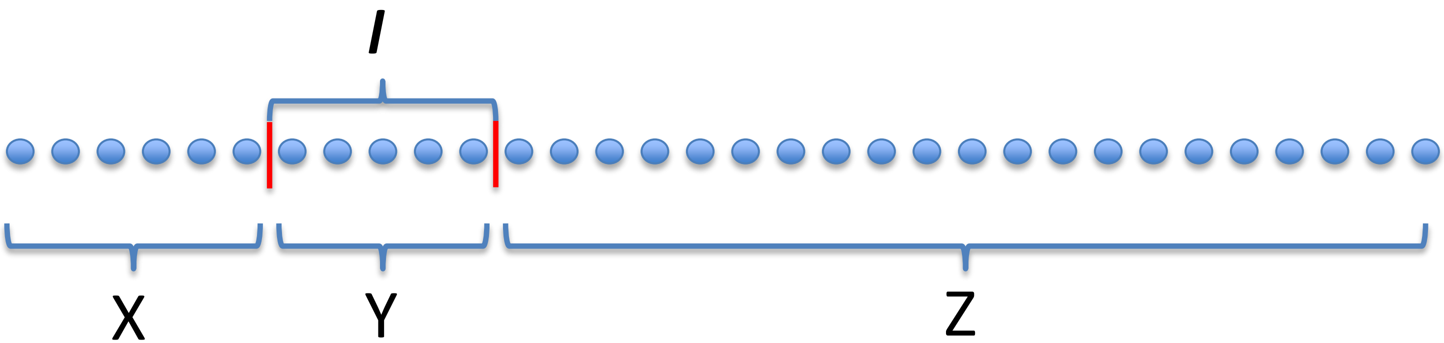
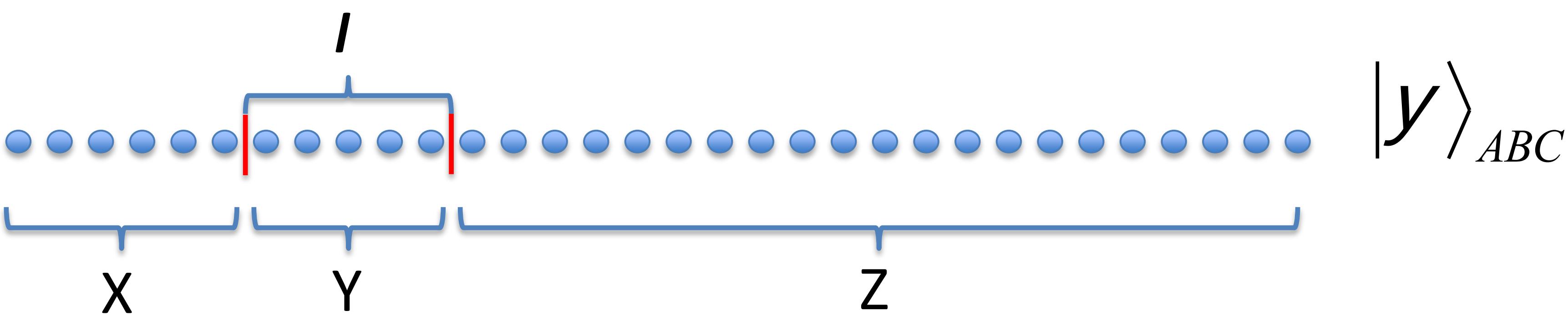


$$|\psi\rangle_{ABC}$$

$$S(Z) \leq S(Y) \;\; \Leftarrow \;\; Cor(X:Z) < O\left(2^{-I(A:B)}\right)$$

Area Law from Subvolume Law



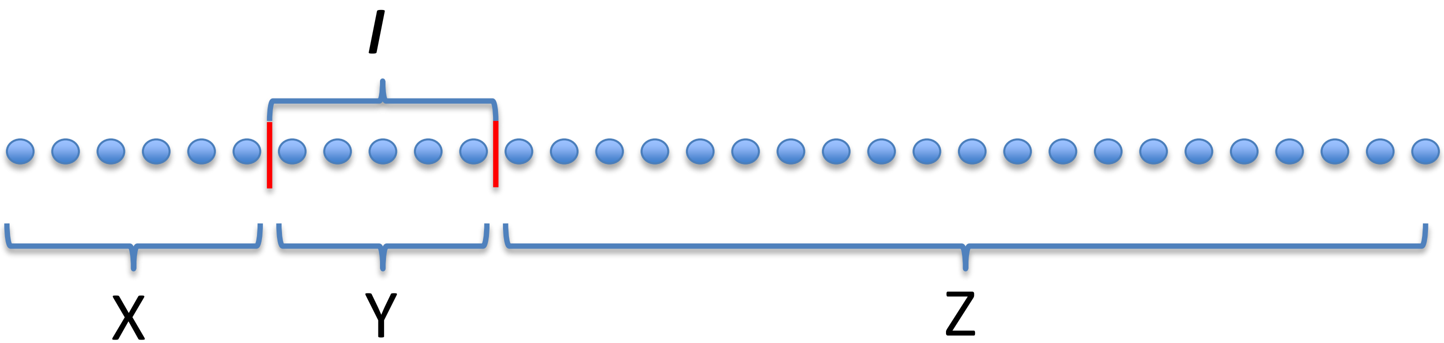
$$S(Z) \leq S(Y) \Leftarrow \quad Cor(X:Z) < O\left(2^{-I(A:B)}\right)$$

Suppose **S(Y) < l/(4ξ)** ("subvolume law")

Since **I(X:Y) < 2S(Y) < l/(2ξ)**, **ξ**-EDC implies **Cor(X:Z) < $2^{-l/ξ}$ < $2^{-I(X:Y)}$**

Thus: **S(Z) < S(Y)** : **Area Law for Z!**

Area Law from Subvolume Law



$|y\rangle_{ABC}$

$$S(Z) \leq S(Y) \;\Leftarrow\; Cor(X:Z) < O\left(2^{-I(A:B)}\right)$$

Suppose **$S(Y) < l/(4\xi)$** ("subvolume law")

Since **$I(X:Y) < 2S(Y) < l/(2\xi)$**, **$\xi$**-EDC implies **$Cor(X:Z) < 2^{-l/\xi} < 2^{-I(X:Y)}$**

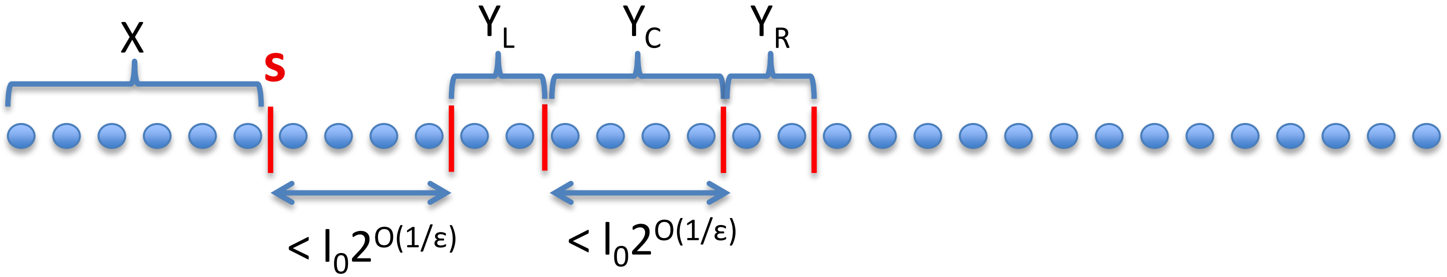
Thus: **$S(Z) < S(Y)$** : **Area Law for Z!**

It suffices to prove that nearby the boundary of Z there is a region of size **$< l_0 2^{O(\xi)}$** with entropy **$< l/(4\xi)$**

Saturation Mutual Information

Lemma (Saturation Mutual Info.) Given a site *s*, for all l_0, $\varepsilon > 0$ there is a region $Y_{2l} := Y_{L,l/2}Y_{C,l}Y_{R,l/2}$ of size $2l$ with $1 < l/l_0 < 2^{O(1/\varepsilon)}$ at a distance $< l_0 2^{O(1/\varepsilon)}$ from *s* s.t.

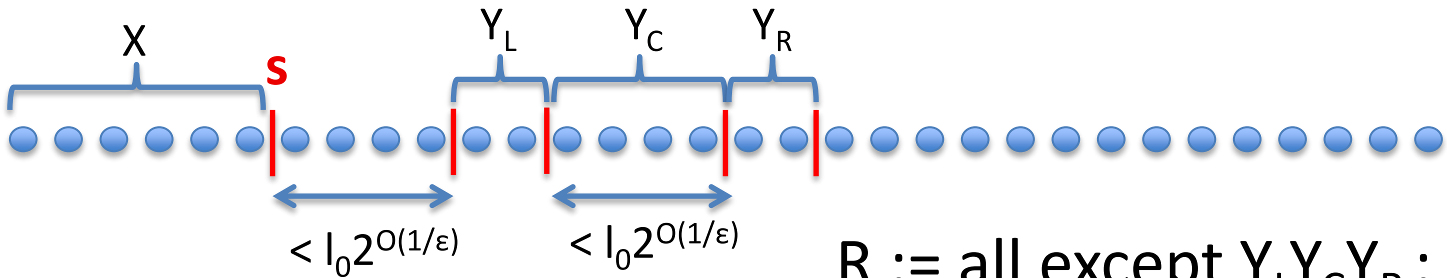
$$I(Y_{C,l}:Y_{L,l/2}Y_{R,l/2}) < \varepsilon l$$



Proof: Easy adaptation of result used by Hastings in his area law proof for gapped Hamiltonians (based on successive applications of subadditivity)

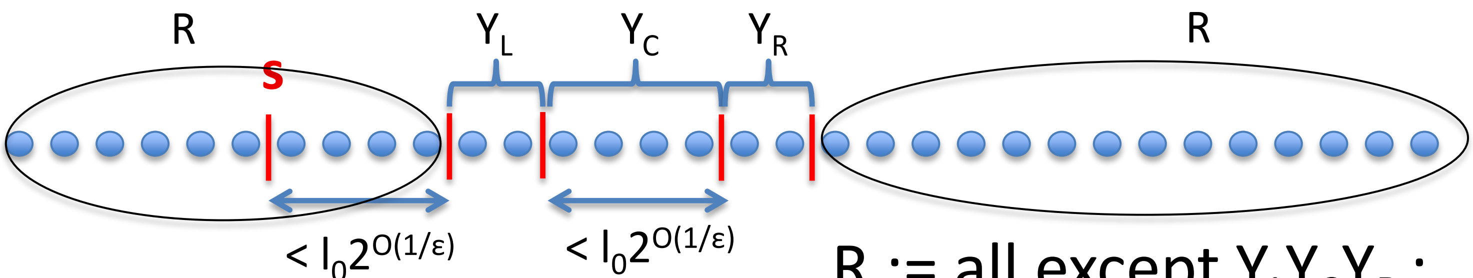
Getting subvolume law

"It suffices to prove that nearby the boundary of Z there is a region of size **$< l_0 2^{O(\xi)}$** with entropy **$< l/(4\xi)$**"



R := all except $Y_L Y_C Y_R$: $|y\rangle_{Y_L Y_C Y_R R}$

Getting subvolume law



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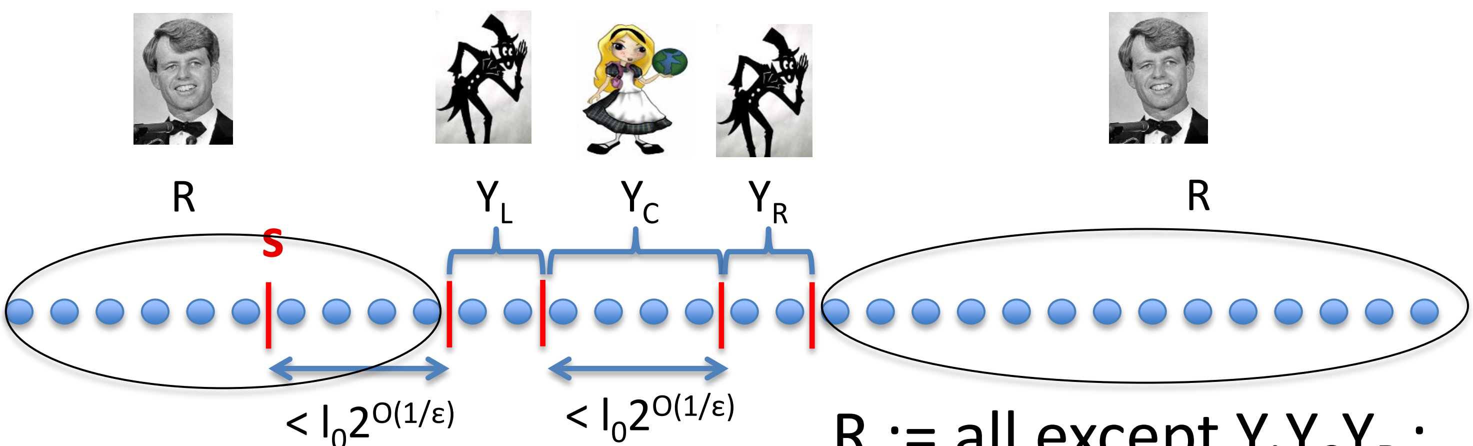
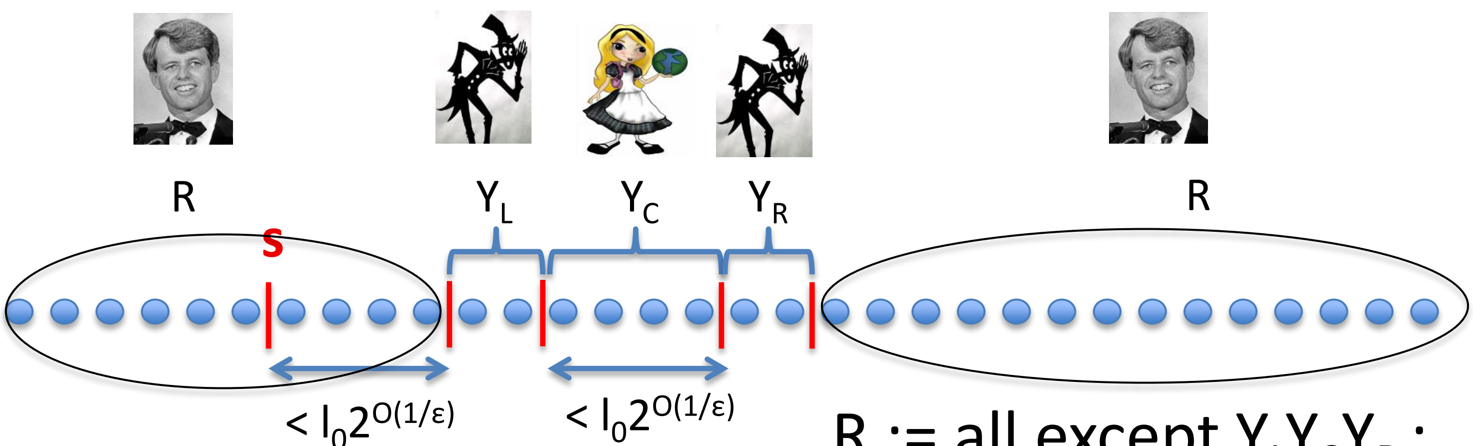


R := all except $Y_L Y_C Y_R$: $|y\rangle_{Y_L Y_C Y_R R}$

i. From state merging bound:

If $Cor(Y_C : R) £ 2^{-I(Y_C : Y_L Y_R)}$ then $S(R) £ S(Y_L Y_R)$

Getting subvolume law



R := all except $Y_L Y_C Y_R$: $|\psi\rangle_{Y_L Y_C Y_R R}$

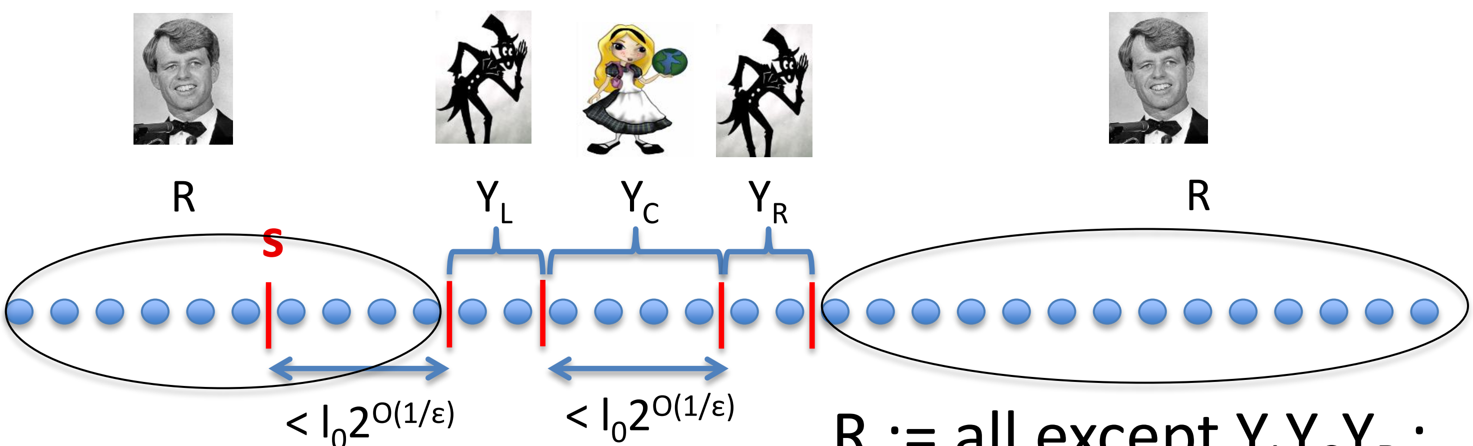
i. From state merging bound:

If $Cor(Y_C : R) \leq 2^{-I(Y_C : Y_L Y_R)}$ then $S(R) \leq S(Y_L Y_R)$

i. From saturation lemma with **ε = 1/(4ξ)**,

$$Cor(Y_C : R) \leq 2^{-l/2\xi} \leq 2^{-I(Y_C : Y_L Y_R)}$$

Getting subvolume law



R := all except $Y_LY_CY_R$: $|\psi\rangle_{Y_LY_CY_RR}$

i. From state merging bound:

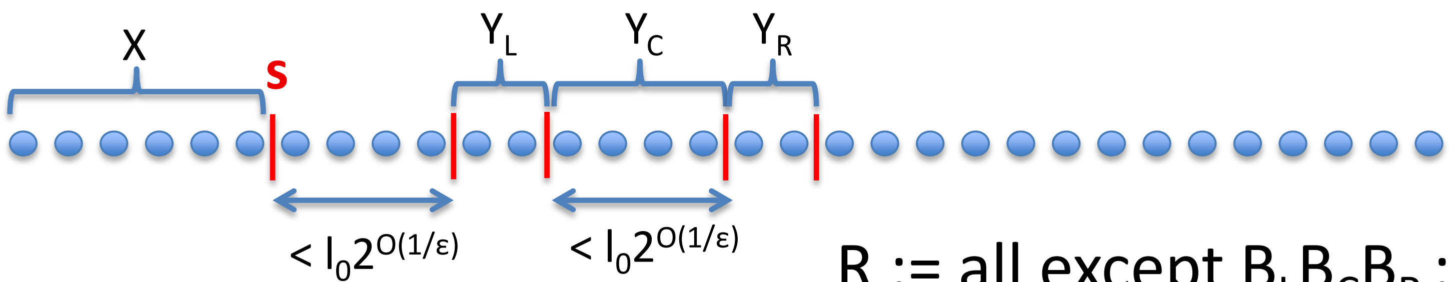
If $Cor(Y_C : R) \leq 2^{-I(Y_C:Y_LY_R)}$ then $\boxed{S(R) \leq S(Y_LY_R)}$

i. From saturation lemma with **ε = 1/(4ξ)**,

$$Cor(Y_C : R) \leq 2^{-l/2\xi} \leq 2^{-I(Y_C:Y_LY_R)}$$

Getting subvolume law

"It suffices to prove that nearby the boundary of Z there is a region of size $< l_0 2^{O(\xi)}$ with entropy $< l/(4\xi)$"



R := all except $B_L B_C B_R$: $|y\rangle_{B_L B_C B_R R}$

iii. $S(R)\,£\,S(Y_L Y_R)$ implies

$$S(Y_C) \leq S(Y_C) + S(Y_L Y_R) - S(R) = I(Y_C : Y_L Y_R)$$

$$\leq l/(4\xi) \qquad \text{(by saturation lemma)}$$

Y_C gives the region of subvolume entropy!

Making it Work

So far we have cheated, since merging only works for many copies of the state. To make the argument rigorous, we use **single-shot information theory** (Renner *et al* '03, …)

State Merging	⇨	**Single-Shot State Merging** (Dupuis, Berta, Wullschleger, Renner '10) **+ New bound on correlations by random measurements**
Saturation Mutual Info.	⇨	**Saturation max- Mutual Info.** Proof much more involved; based on - Quantum substate theorem, - Quantum equipartition property, - Min- and Max-Entropies Calculus - EDC Assumption

Overview

- Condensed Matter (CM) community always knew EDC implies area law

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- Condensed Matter (CM) community always knew EDC implies area law
- Quantum information (QI) community gave a counterexample (hiding states)
- QI community sorted out the trouble they gave themselves (this talk)
- CM community didn't notice either of these *minor* perturbations

"EDC implies Area Law" stays true!

Conclusions and Open problems

- **EDC implies Area Law and MPS parametrization** in 1D.
- **States with EDC** are simple – **MPS efficient parametrization**.
- Proof uses **state merging protocol** and **single-shot information theory**: **Tools from QIT** useful to address problem **in quantum many-body physics**.

1. Can we improve the **dependency of entropy with correlation length**?
2. Can we prove **area law for 2D systems**? **HARD!**
3. Can we decide if **EDC** alone is **enough** for **2D area law**?
4. See **arxiv:1206.2947** for more open questions

Thanks!