Merged Talk:

A Hierarchy of Information Quantities for Finite Block Length Analysis of Quantum Tasks

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Second Order Asymptotics for Quantum Hypothesis Testing

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arXiv: 1208.1400

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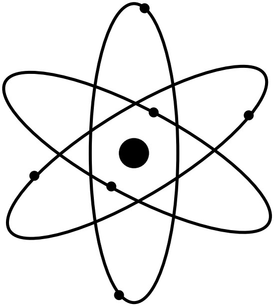
Quantum Hypothesis Testing

Theory A
(Established Theory)
Null Hypothesis

Theory B
(New Theory)
Alternate Hypothesis



Theory A predicts that
System is in state ρ.

Theory B predicts that
System is in state σ.

Quantum Hypothesis Testing

Null Hypothesis: ρ; Alternate Hypothesis: σ.

- Devise a *test*, a POVM $\{Q, 1 - Q\}$ with $0 \leq Q \leq 1$.
- If Q clicks, you accept the null hypothesis.

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- The error of the first kind, α, validates the alternate hypothesis even though the null hypothesis is correct.
  This is fatal — you will write a crackpot paper!

Quantum Hypothesis Testing

- We are interested in the minimal β that can be achieved if α is required to be smaller than a given constant, ε, i.e. the SDP

$$\beta^{\varepsilon}_{\rho,\sigma} := \min_{\substack{0 \leq Q \leq 1 \\ \alpha(Q) \leq \varepsilon}} \beta(Q) = \min_{\substack{0 \leq Q \leq 1 \\ \mathrm{tr}(\rho Q) \geq 1-\varepsilon}} \mathrm{tr}(\sigma Q).$$

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- The additive normalization $\log(1-\varepsilon)$ ensures (Dupuis+'12)

$$D^{\varepsilon}_{h}(\rho\|\sigma) \ge 0 \quad \text{and} \quad D^{\varepsilon}_{h}(\rho\|\sigma) = 0 \iff \rho = \sigma.$$

- It also satisfies data-processing, $D^{\varepsilon}_{h}(\rho\|\sigma) \ge D^{\varepsilon}_{h}\big(\mathcal{E}(\rho)\big\|\mathcal{E}(\sigma)\big)$.

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- Our goal is to investigate the second order term, $O(\sqrt{n})$.

Main Result

Theorem

For two states ρ, σ *with* $\mathrm{supp}\{\sigma\} \supseteq \mathrm{supp}\{\rho\}$, *and* $0 < \varepsilon < 1$, *we find*

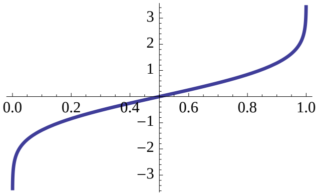
$$D_h^\varepsilon(\rho^{\otimes n}\|\sigma^{\otimes n}) \le nD(\rho\|\sigma) + \sqrt{nV(\rho\|\sigma)}\Phi^{-1}(\varepsilon) + 2\log n + O(1), \quad \textit{and}$$

$$D_h^\varepsilon(\rho^{\otimes n}\|\sigma^{\otimes n}) \ge nD(\rho\|\sigma) + \sqrt{nV(\rho\|\sigma)}\Phi^{-1}(\varepsilon) - O(1).$$

- D and V are the mean and variance of $\log\rho - \log\sigma$ under ρ, i.e.

$$V(\rho\|\sigma) := \mathrm{tr}\Big(\rho\big(\log\rho - \log\sigma - D(\rho\|\sigma)\big)^2\Big).$$

- Φ is the cumulative normal distribution function, and $\Phi^{-1}(\varepsilon)$ is



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- Classically, the result is known to hold with both logarithmic terms equal to $\frac{1}{2}\log n$ (e.g. Strassen'62,Polyanskiy,Poor&Verdú'10).
- One ingredient of both proof is the *Berry-Essèen theorem*, which quantizes the convergence of the distribution of a sum of i.i.d. random variables to a normal distribution.
- Intuitively, our results can be seen as a quantum, entropic formulation of the *central limit theorem*.

Smooth Entropies

- We also investigate the smooth min-entropy (Renner'05), where it was known (T,Colbeck&Renner'09) that

$$H_{\min}^{\varepsilon}(A^n|B^n)_{\rho^{\otimes n}} \leq nH(A|B)_\rho + O\big(\sqrt{n}\big), \quad \text{and}$$
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- Both hypothesis testing and smooth entropies have various applications in information theory, some of which we explore next.

Randomness Extraction against Side Information

- Consider a CQ random source that outputs states $\rho_{XE} = \sum_x p_x |x\rangle\langle x| \otimes \rho_E^x$.
- Investigate the amount of randomness that can be extracted from X such that it is independent of E and the random seed, S.

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- For any $0 \leq \varepsilon < 1$ and ρ_{XE} a CQ state, we define

$$\ell^{\varepsilon}(X|E) := \max\left\{\ell \in \mathbb{N} \,\middle|\, \exists\, \mathcal{P}, \sigma_E : |Z| = 2^{\ell} \wedge \tau_{ZES} \approx^{\varepsilon} 2^{-\ell} 1_Z \otimes \sigma_E \otimes \tau_S\right\}.$$

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- This quantity can be characterized in terms of the smooth min-entropy (Renner'05). We tighten this and show

Theorem

Consider an i.i.d. source $\rho_{X^n E^n} = \rho_{XE}^{\otimes n}$ and $0 < \varepsilon < 1$. Then,

$$\ell^{\varepsilon}(X^n|E^n) = nH(X|E) + \sqrt{nV(X|E)}\Phi^{-1}(\varepsilon^2) \pm O(\log n).$$

Data Compression with Side Information

- Consider a CQ random source that outputs states $\rho_{XB} = \sum_x p_x |x\rangle\langle x| \otimes \rho_B^x$.
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- For any $0 \leq \varepsilon < 1$ and ρ_{XB} a CQ state, we define

$$m^\varepsilon(X|B)_\rho := \min \left\{ m \in \mathbb{N} \,\middle|\, \exists \mathcal{P} : |M| = 2^m \wedge P[X \neq X'] \leq \varepsilon \right\}.$$

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- This quantity can be characterized using hypothesis testing (H&Nagaoka'04). We tighten this and show

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Consider an i.i.d. source $\rho_{X^n B^n} = \rho_{XB}^{\otimes n}$ and $0 < \varepsilon < 1$. Then,

$$m^{\varepsilon}(X^n|B^n) = nH(X|B) - \sqrt{nV(X|B)}\Phi^{-1}(\varepsilon) \pm O(\log n).$$

Example of Second Order Asymptotics

- Consider transmission of $|0\rangle$, $|1\rangle$ through a Pauli channel to B (phase and bit flip independent) with environment E. This yields the states

$$\rho_{XB} = \frac{1}{2}\sum |x\rangle\langle x| \otimes \big((1-p)|x\rangle\langle x| + p|1-x\rangle\langle 1-x|\big),$$
$$\rho_{XE} = \frac{1}{2}\sum |x\rangle\langle x| \otimes |\phi^x\rangle\langle\phi^x|, \qquad |\phi^x\rangle = \sqrt{p}|0\rangle + (-1)^x\sqrt{1-p}|1\rangle.$$

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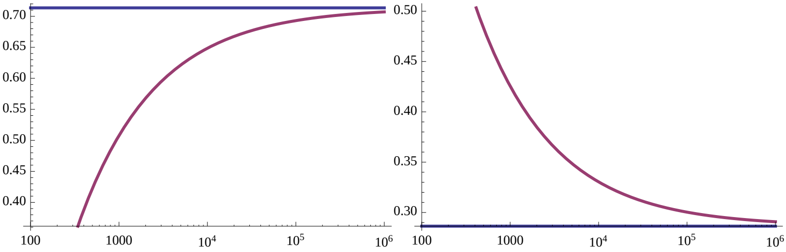
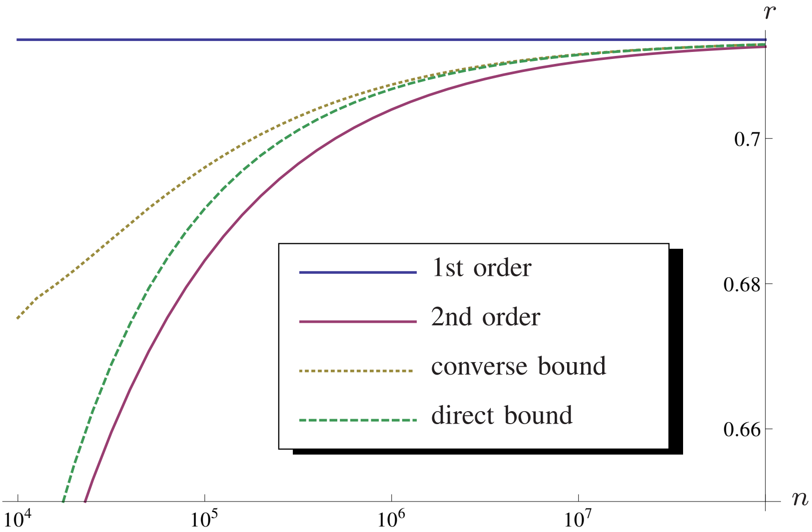


- Plot of first and second order asymptotic approximation of $\frac{1}{n}\ell^{\varepsilon}(X|E)$ and $\frac{1}{n}m^{\varepsilon}(X|B)$ for $p = 0.05$ and $\varepsilon = 10^{-6}$.

Example of Finite Block Length Bounds



Different Layers of Approximation

Class	Role	Quantities
Class 1	Optimal performance of protocol. Calculation is very difficult.	$m^{\varepsilon}(X\|B)_{\rho}$ $\ell^{\varepsilon}(X\|B)_{\rho}$,
Class 2	One-shot bound for general source. SDP tractable for small systems.	$H_h^{\varepsilon}(A\|B)_{\rho}$, $H_{\min}^{\varepsilon}(A\|B)_{\rho}$
Class 3	Quantum information spectrum.	$D_s^{\varepsilon}(\rho\\|\sigma)$
Class 4	Classical information spectrum. Approximately possible for i.i.d.	$D_s^{\varepsilon}(P_{0,\rho,\sigma}\\|P_{1,\rho,\sigma})$
Class 5	Second order asymptotics. Calculation is easy for large n.	$nH(X\|B)+$ $\sqrt{n}\, s(X\|B)\Phi^{-1}(\varepsilon)$

Classes	Difference	Method
$1 \to 2$	$O(\log n)$	Random coding and monotonicity.
$2 \to 4$	$O(\log n)$	Relations between entropies.
$4 \to 5$	$O(1)$	Berry-Essèen Theorem.

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Conclusion / Differences & Overlap

- The methods employed in the two papers are conceptually different.
- The approach employed by Li is more direct, leads to tighter bounds for finite n and better coefficients for the logarithmic term.
- The approach of T&H is more general.

Result	T&H	Li
2nd order asymptotics for hypothesis testing	✓	✓
Finite n bounds for hypothesis testing	✓	✓
2nd order asymptotics of smooth min-entropy	✓	
Application to data compression and randomness extraction with quantum side information	✓	
Hierarchy of information quantities, linking operational quantities, one-shot entropies and asymptotic analysis of quantum tasks	✓	

Open Questions

- There is a difference of $2 \log n$ between the current upper and lower bounds on $D_h^{\varepsilon}(\rho^{\otimes n} \| \sigma^{\otimes n})$. Is this fundamental, i.e. do there exist ρ and σ for which these bounds are tight? Or can this be further improved? (Classically, the upper and lower bounds only differ in the constant.)

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