Majorana Fermions and Topological Quantum Information Processing

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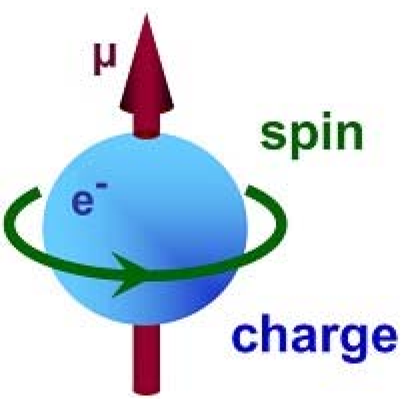
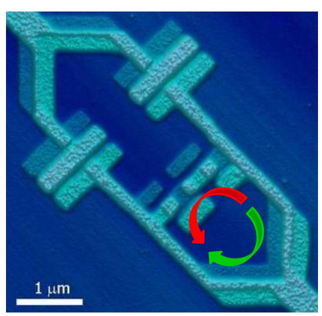
Quantum Systems

Conventional Quantum Systems

- Local degrees of freedom
- E.g., spins, photons, ions, superconducting devices, …
- Merits: arbitrary unitary operations, distant entanglement, …
- Challenges: vulnerable to various imperfections & decoherences



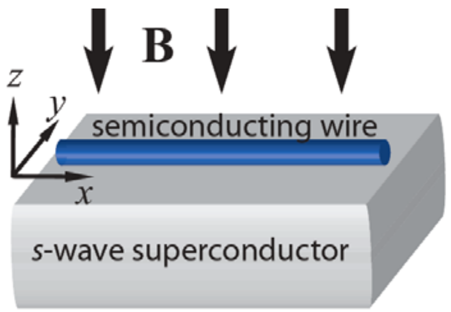


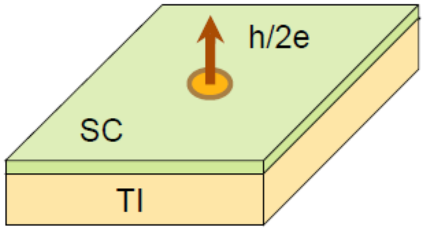
Topological Quantum Systems

- Global degrees of freedom
- E.g., Fractional Quantum Hall Effect, Topological Insulators, Majoranas, …
- Merits: robust against local perturbation/decoherence.
- Challenges: limited unitary operations, quantum network …

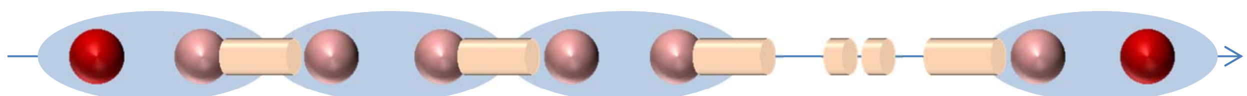
Outline

- Majoranas in 1D
  - Kitaev Model & Semiconductor Wires
  - Experimental signatures of Majoranas
  - Duality (Spin v.s. Particle-hole)
- Hybrid platforms
  - Majoranas in 2D (Fu & Kane)
  - Majoranas qubits & SC qubits





MAJORANAS IN 1D

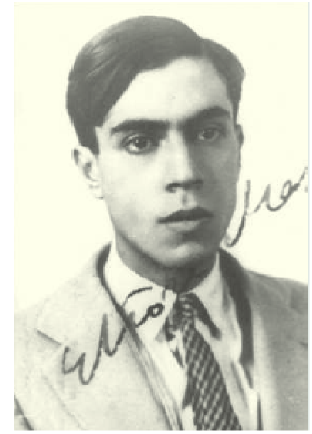


Majorana Bound States

- Majoranas: $\gamma = \gamma^{\dagger}$
  - E.g., "half of a Dirac Fermion"

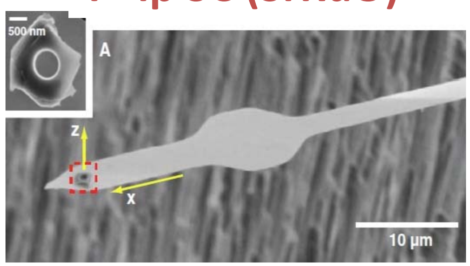
$$\gamma_{2j-1} = \frac{c_j + c_j^{\dagger}}{2} \quad \text{and} \quad \gamma_{2j} = \frac{c_j - c_j^{\dagger}}{2i}$$

- Search for Majorana:



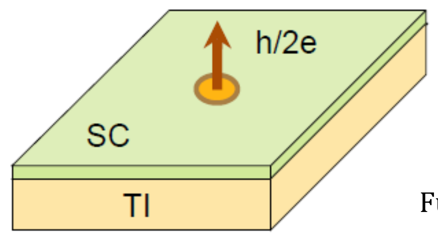
Ettore Majorana (1937)

P+ip SC (SrRuO)



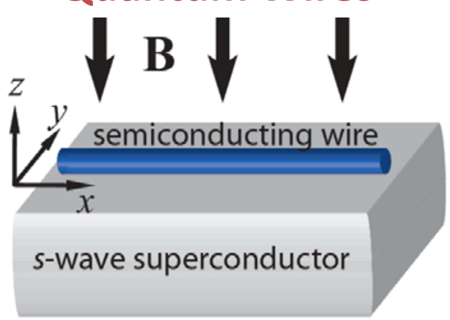
Read & Green, PRB (2000)
Jang, et al., Science (2011)

Topological Insulators



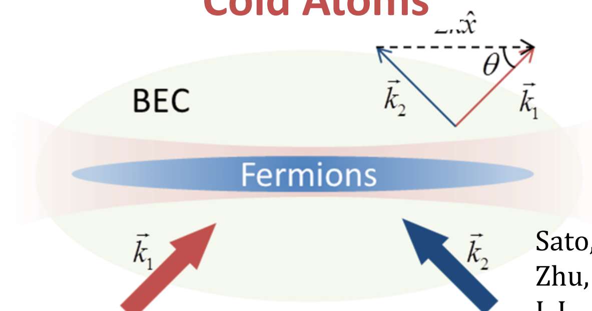
Fu & Kane. PRL (2008),

Quantum Wires



Lutchyn, et al., PRL (2010)
Oreg, et al., PRL (2010)
L.J., Pekker, et al., PRL (2011)

Cold Atoms



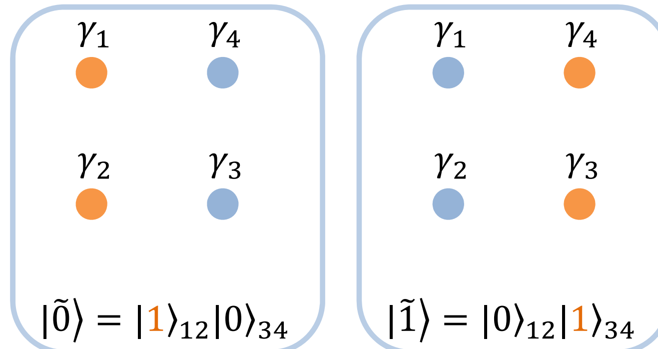
Sato, et al., PRL (2009)
Zhu, et al., PRL (2010)
L J, , Kitagawa, et al., PRL (2011)

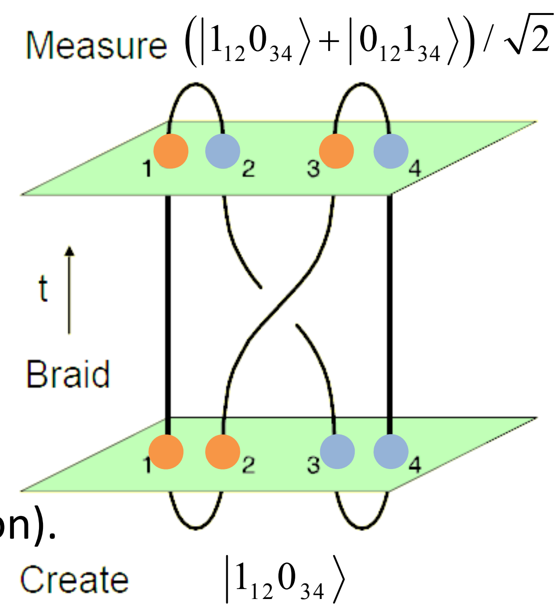
Topological Qubit

- Four Majoranas encode 1 topological qubit
  - Subspace with odd Dirac fermion $\{|1\rangle_{12}|0\rangle_{34}, |0\rangle_{12}|1\rangle_{34}\}$.
- Braiding of Majoranas
  - Non-abelian anyons

$$|\psi_{final}\rangle = U_{AB}|\psi_{init}\rangle$$

$$\text{with } U_{AB} = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$$





- Comments:
  - 2N+2 Majoranas encode N topological qubits
  - Braiding Majoranas using quasi-1D T-junctions
  - Braiding Majoranas is not universal (for computation).

Nayak, et al., RMP 80, 1083 (2008). J. Alicea, et al., Nat. Phys. 7, 412-417 (2010).

Kitaev Quantum Wire

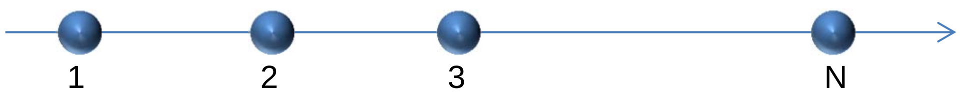
-- Create "*half* of a Dirac Fermion"

- Chain of (**spinless**) Dirac fermions

$$H = -\mu \sum_{j=1}^{N} c_j^\dagger c_j - \frac{1}{2} \sum_{j=1}^{N-1} \left(t c_j^\dagger c_{j+1} + \Delta c_j c_{j+1} + h.c. \right)$$

- Introduce Majoranas:

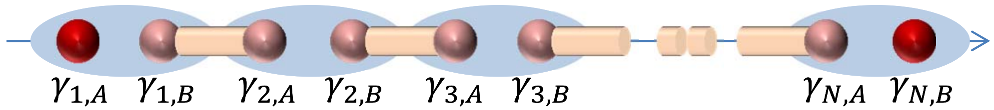
$\gamma_{j,A} = \frac{c_j^\dagger + c_j}{2}$ and $\gamma_{j,B} = \frac{c_j^\dagger - c_j}{2i}$



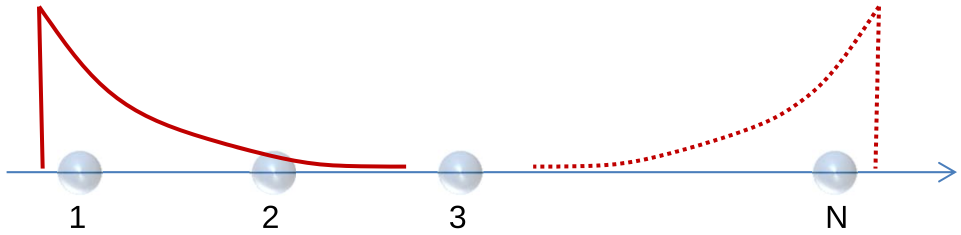
- Consider $\mu = 0$ and $t = \Delta$,

$$H = -it \sum_{j=1}^{N-1} \gamma_{j,B} \gamma_{j+1,A}$$

Two localized Majoranas at the end of the quantum wire!



- For $|\mu| < t$, bound Majoranas exist! With exponential tail.



Kitaev, arXiv: cond-mat/0010440 (2001).

Topological Perspective for Kitaev Quantum Wire

$$H = -\mu \sum_{j=1}^{N} c_j^\dagger c_j - \frac{1}{2} \sum_{j=1}^{N-1} \left(t c_j^\dagger c_{j+1} + \Delta c_j c_{j+1} + h.c. \right)$$

Key challenges:
1. Spinless fermions
2. P-wave pairing

Rewrite in BdG form with $C_k^\dagger = \left(c_k^\dagger, c_{-k} \right)$

$$H = \frac{1}{2} \sum_k C_k^\dagger H_k C_k$$

$$H_k = \begin{pmatrix} \varepsilon_k & \tilde{\Delta}_k^* \\ \tilde{\Delta}_k & -\varepsilon_k \end{pmatrix} = h_{k,x}\sigma_x + h_{k,y}\sigma_y + h_{k,z}\sigma_z$$

with

$$h_{k,z} = \varepsilon_k = -t\cos k - \mu$$

$$h_{k,x} + i h_{k,y} = \tilde{\Delta}_k = -i|\Delta| e^{-i\phi} \sin k$$

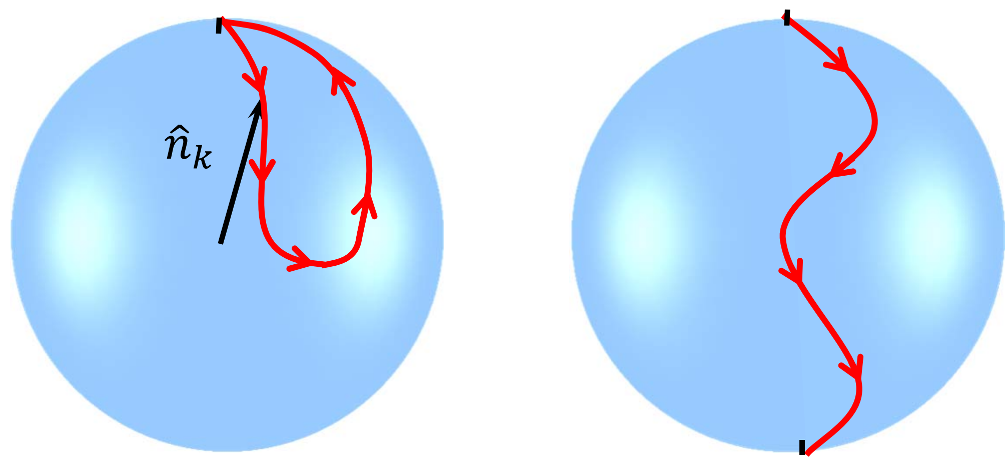
Define unit vector: $\hat{n}_k \equiv \vec{h}_k / |\vec{h}_k|$

$$\hat{n}_0 = \pm \hat{z} \quad \& \quad \hat{n}_\pi = \pm \hat{z}$$

The Z_2 topological invariance

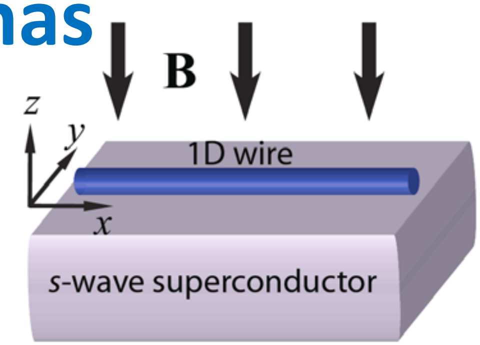
$$\nu = \hat{n}_0 \cdot \hat{n}_\pi = \mathrm{sgn}\left(\mu^2 - t^2 \right) = \pm 1$$

$\hat{n}_0 = \hat{n}_\pi$ $\qquad$ $\hat{n}_0 = -\hat{n}_\pi$



$\nu = 1$ (trivial) $\quad |\mu| > |t|$ $\qquad$ **$\nu = -1$ (topological)** $\quad |\mu| < |t|$

Spin-1/2 Fermions to Create Majoranas



- Fermions with two internal states $a_p = \left(a_{p,\uparrow}, a_{p,\downarrow}\right)$

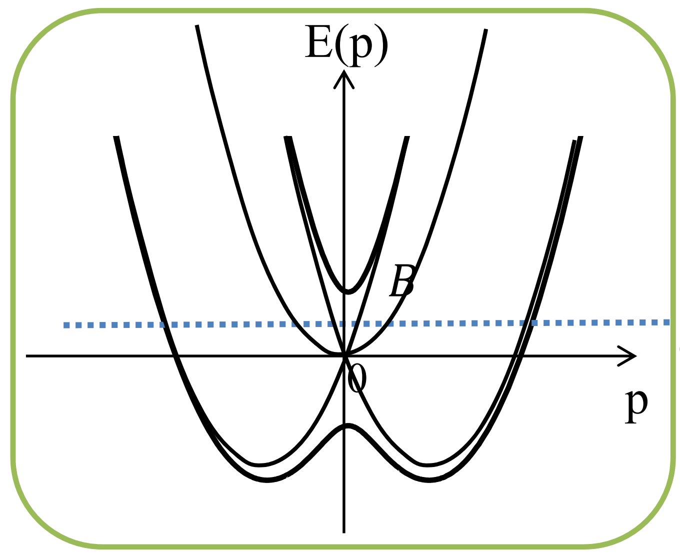
Effective energy scale: $\sqrt{\mu^2 + \Delta^2}$

$$H = \sum_p a_p^\dagger \left(\frac{p^2}{2m} - \mu + up\sigma_z \right) a_p + \left(B a_{p,\uparrow}^\dagger a_{p,\downarrow} + \Delta a_{p,\uparrow}^\dagger a_{-p,\downarrow}^\dagger + \text{H.c.} \right)$$

Kinetic energy

Spin-orbital coupling: $up\sigma_z$

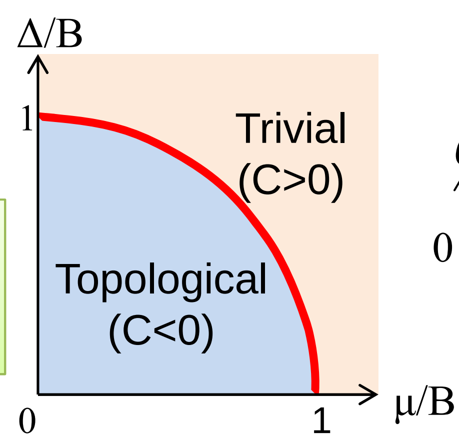
Magnetic field: B

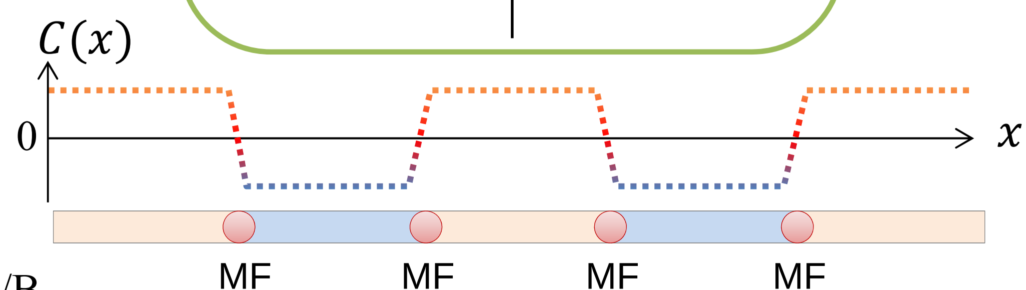


Effectively Spinless

Define quantity:
$\boldsymbol{C \equiv \Delta^2 + \mu^2 - B^2}$

MFs appear at the boundary between two phases

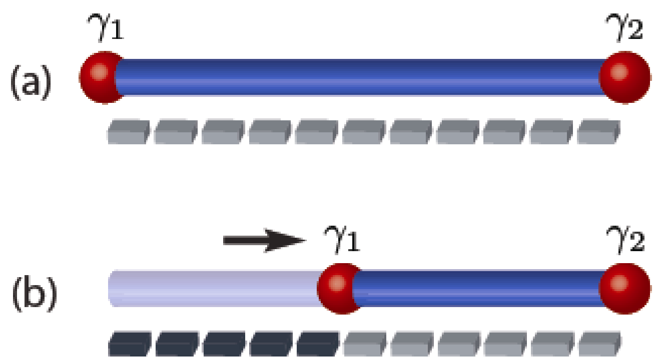




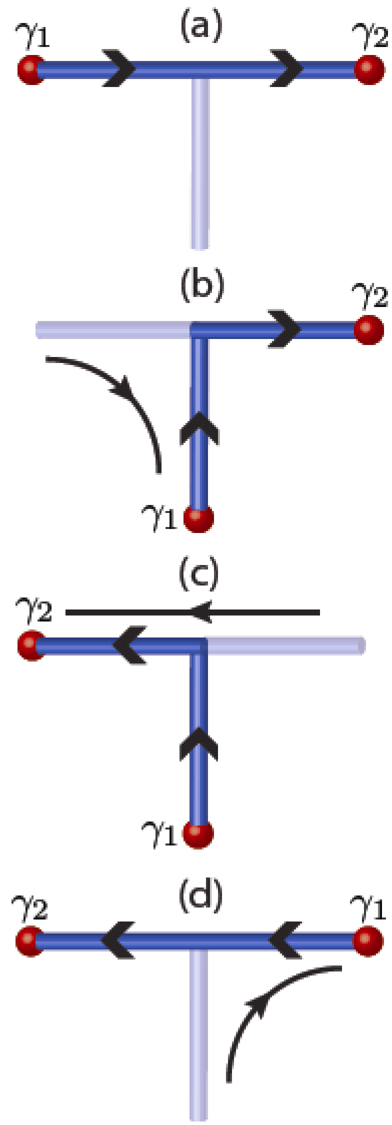
Lutchyn, Sau, Das Sarma, PRL 105, 077001 (2010); Oreg, Refael, von Oppen, PRL, 105, 177002 (2010)
L. J., Kitagawa, Alicea, et al., PRL, 106, 220402 (2011); L. J., Pekker, Alicea, et al., PRL, 106, 236401 (2011)

Braiding of Majoranas in Quasi-1D Systems

1. Move Majoranas along the wire

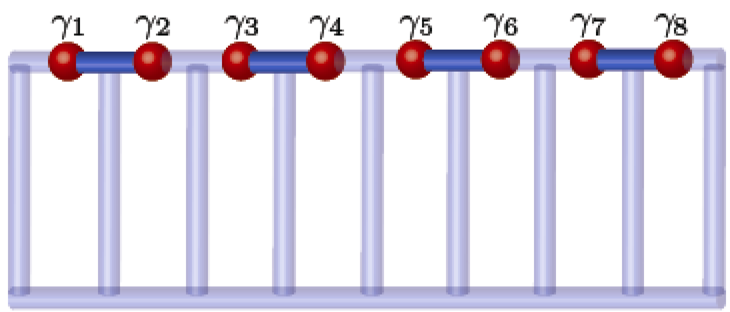


2. Exchange Majoranas via a T-junction



$$U_{12} = \exp\left(\pi \gamma_2 \gamma_1 / 4\right)$$

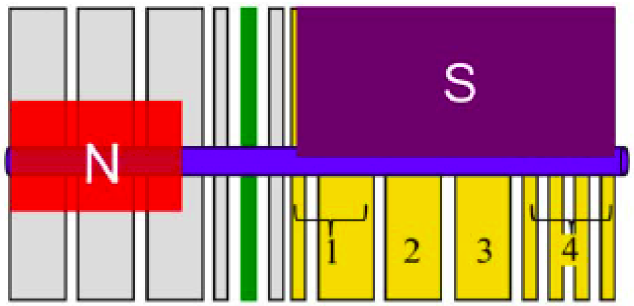
3. Wire networks for efficient exchange of many Majoranas

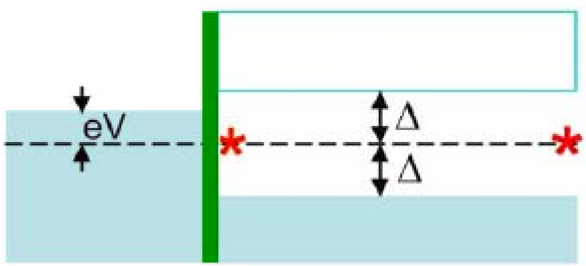


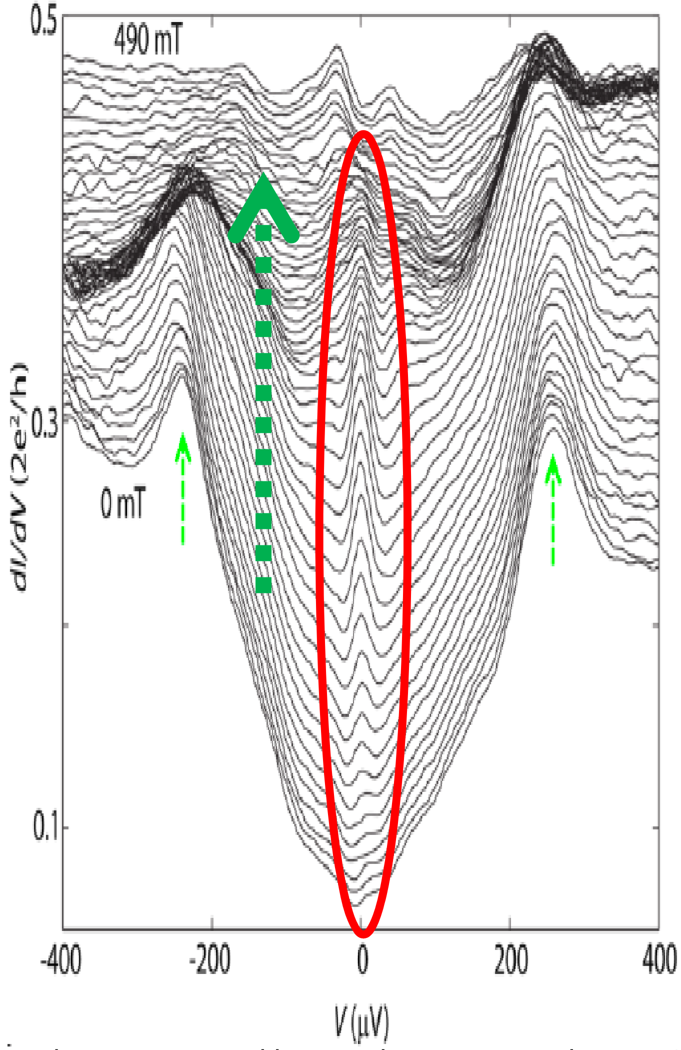
Alicea, Oreg, Refael, von Oppen, Fisher, *Nat. Phys.* ***7, 412-417 (2011).***

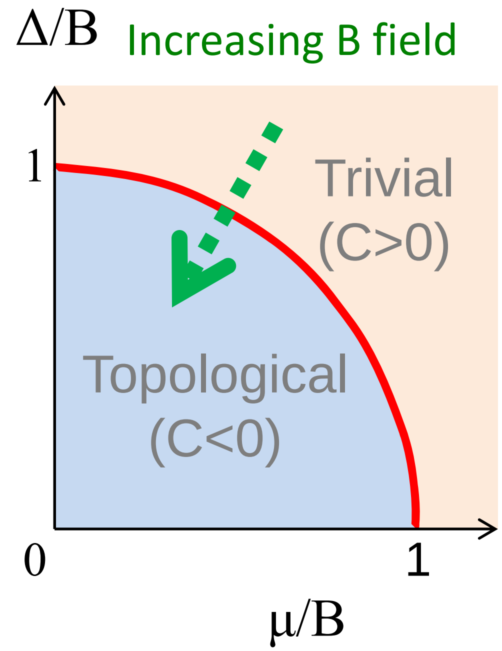
Experimental Signatures – *Zero Bias Peaks*

Majoranas $\gamma = \gamma^\dagger$
implies $E = -E = 0$.









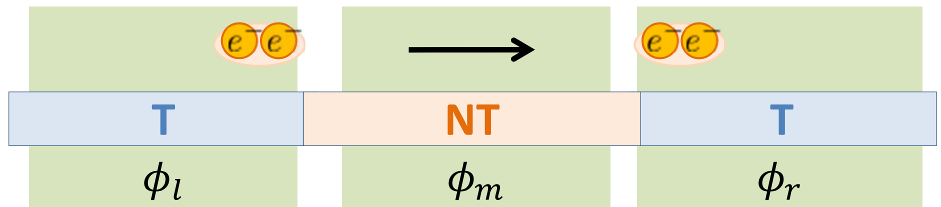
1. V. Mourik, K. Zuo, S. M. Frolov, S. R. Plissard, E. P. A. M. Bakkers and L. P. Kouwenhoven, *Science* ***336,*** *1003-1007 (2012).*
2. M. T. Deng, C. L. Yu, G. Y. Huang, M. Larsson, P. Caroff and H. Q. Xu, *Nano Lett.* ***12****, 6141-6419 (2012).*
3. A. Das, Y. Ronen, Y. Most, Y. Oreg, M. Heiblum and H. Shtrikman, *Nat. Phys.* ***8****, 887-895 (2012).*

Experimental Signatures – *Fractional Josephson Effects*

$$H = \sum_p a_p^\dagger \left(\varepsilon_p - \mu + up\sigma_z\right) a_p + \left(B a^\dagger_{p,\uparrow} a_{p,\downarrow} + \Delta e^{i\phi} a^\dagger_{p,\uparrow} a^\dagger_{-p,\downarrow} + \mathrm{H.c.}\right)$$

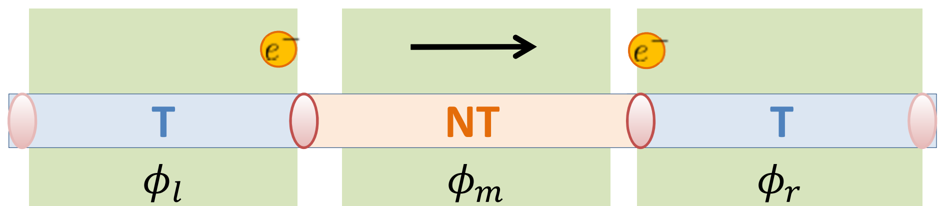
- **Conventional** Josephson effect

$$I_r = I_J \sin(\phi_l - \phi_r)$$



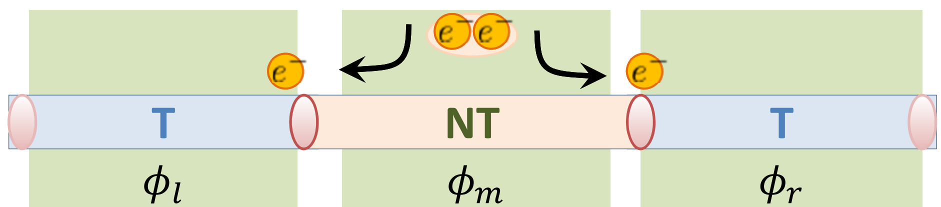
- **Majorana** Josephson effect

$$I_r = I_M \sin\left(\frac{\phi_l - \phi_r}{2}\right)$$



- **Zipper Majorana** Josephson effect

$$I_r = I_Z \sin\left(\frac{\phi_l + \phi_r}{2} - \phi_m\right)$$



A. Kitaev, Physics-Uspekhi 44, 131 (2001); Kwon, Sengupta, Yakovenko EPJB 37, 349 (2003);
Fu, Kane, PRB 79, 161408 (2009); Lutchyn, Sau, Das Sarma PRL 105, 077001 (2010); Akhmerov et al, PRL 106, 057001 (2011);
L. J., Pekker, Alicea, Refael, Oreg, von Oppen, PRL, 106, 236401 (2011)

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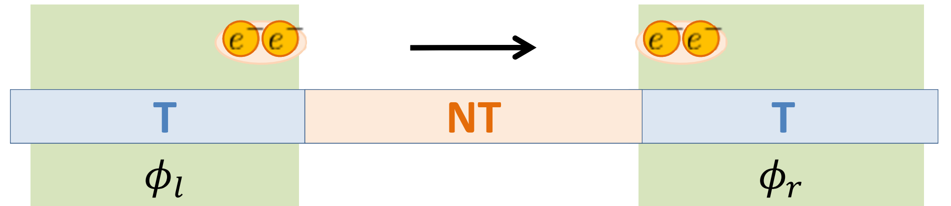

- **Majorana** Josephson effect

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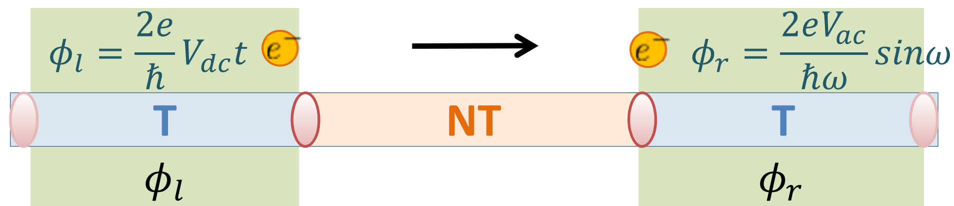
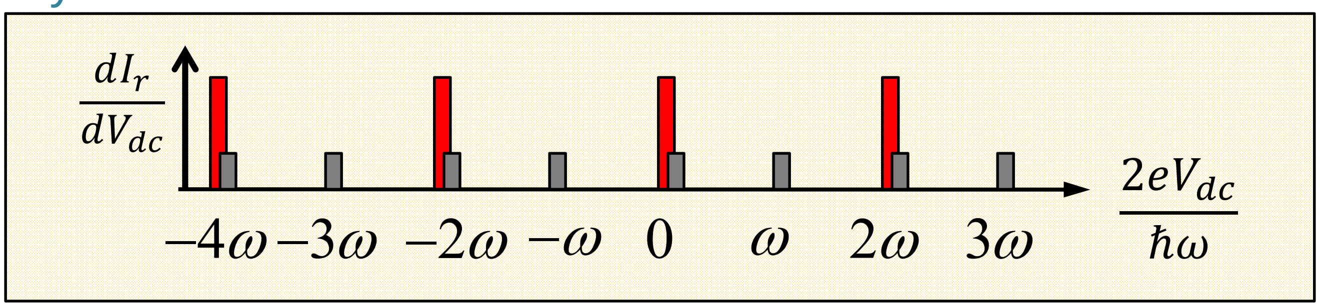


- DC voltage on left-leg.
- AC voltage on right leg.
- Current on right leg.
- Even steps only!

$$I_r = I_M \sum_{n=-\infty}^{\infty} \left[J_n\left(\frac{eV_{ac}}{\hbar\omega}\right) \right] \cdot \sin\left(\frac{1}{2} \frac{2e}{\hbar} V_{dc} t - n\omega t + \phi_0 \right)$$



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L. J., Pekker, Alicea, Refael, Oreg, von Oppen, PRL, 106, 236401 (2011)

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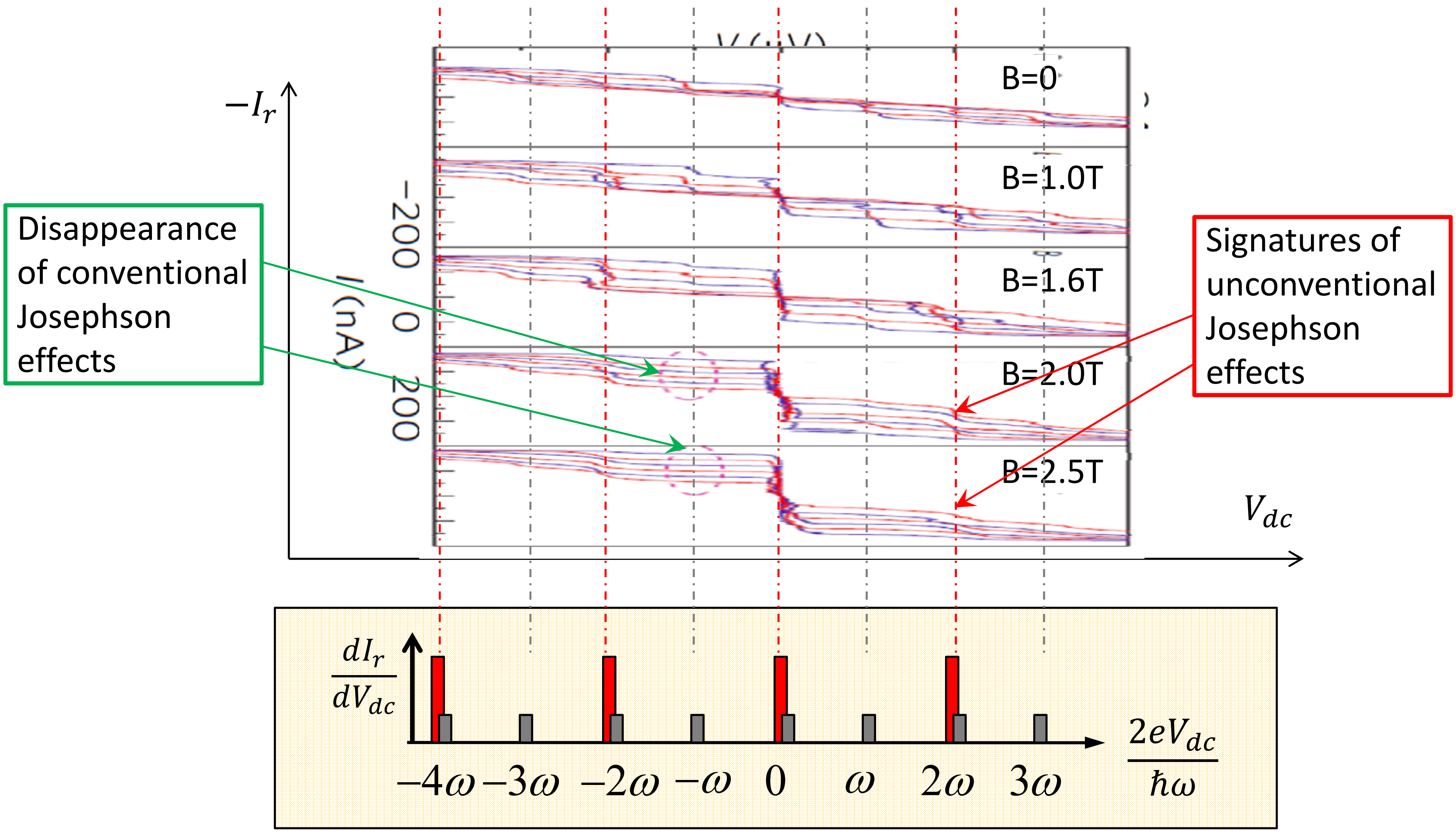


L. P. Rokhinson, X. Liu and J. K. Furdyna, *Nat Phys* ***8 (11), 795-799 (2012).***

Duality in 1D Quantum Wire

$$H = \sum_p a_p^\dagger \left(up\sigma_z + \varepsilon_p - \mu\right) a_p + \left(B a_{p,\uparrow}^\dagger a_{p,\downarrow} + \Delta a_{p,\uparrow}^\dagger a_{-p,\downarrow}^\dagger + \mathrm{H.c.}\right)$$

Use Nambu spinor basis
$\Psi^{\mathrm{T}} = \left(a_\uparrow, a_\downarrow, a_\downarrow^\dagger, -a_\uparrow^\dagger\right)$

$$H = u\, p\, \sigma^z \tau^z + \left(\varepsilon_p - \mu\right)\tau^z + B\sigma^x + \Delta\tau^x$$

Generalize with
ϕ, θ, and B_z.

$$H = up\,\sigma^z\tau^z + \left(\varepsilon_p - \mu\right)\tau^z + \Delta\left(\tau^x \cos\phi - \tau^y \sin\phi\right) + B_z\sigma^z + B_\perp\left(\sigma^x \cos\theta - \sigma^y \sin\theta\right)$$

Duality in 1D Quantum Wires

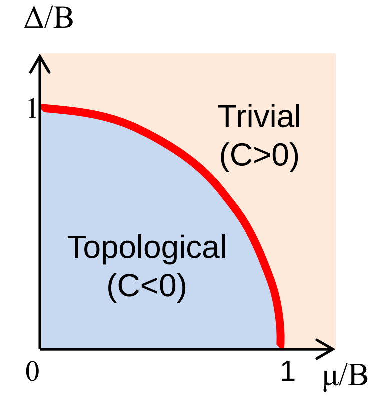
$$H = vp\sigma^z\tau^z + \left(\varepsilon_p - \mu\right)\tau^z + \Delta\left(\tau^x \cos\phi - \tau^y \sin\phi\right) + B_z\sigma^z + B_\perp\left(\sigma^x \cos\theta - \sigma^y \sin\theta\right)$$

Physical Parameter	Dual Parameter
τ	σ
Δ	$B_\perp$
ϕ	θ
$\varepsilon_p - \mu$	B_z
Charge current $I_Q \propto \frac{\partial H}{\partial \phi}$	Spin current $I_S \propto \frac{\partial H}{\partial \theta}$
Majorana Josephson Effect	Majorana Spintronics Effect

L. J., Pekker, Alicea, Refael, Oreg, Brataas, von Oppen, arXiv: 1206.1581.
L. J., Pekker, Alicea, Refael, Oreg, von Oppen, Phys. Rev. Lett. 107, 236401 (2011).

Duality in 1D Quantum Wires

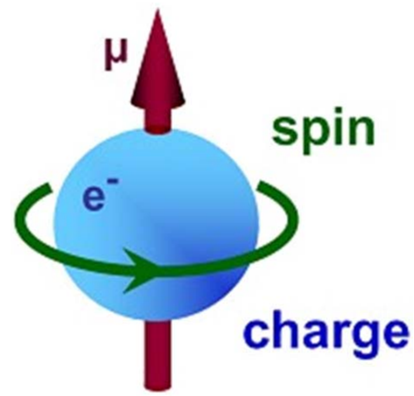
$$H_p = vp\sigma^z\tau^z + \left(\frac{p^2}{2m} - \mu\right)\tau^z + \Delta\left(\tau^x \cos\phi - \tau^y \sin\phi\right) + B_\perp\left(\sigma^x \cos\theta - \sigma^y \sin\theta\right)$$



	 $\{\boldsymbol{\phi_l}, \theta_l\}$ $\{0,0\}$ $\{\boldsymbol{\phi_r}, \theta_r\}$	Probe in Quantum Wires
Charge current $j_r^Q = \frac{2e}{\hbar}\frac{\partial\langle H\rangle}{\partial\phi_r}$	**Fractional Josephson Effects** ➢ j_r^Q is 4π periodic in ϕ_l, ϕ_r	Shapiro-step measurement
	Fractional Magneto-Josephson ➢ j_r^Q is 4π periodic in θ_l, θ_r	Rotate the magnetic angle $\theta(t)$
Spin current $j_r^S = \frac{\partial\langle H\rangle}{\partial\theta_r}$	**Fractional Phase-Driven Spin Current** ➢ j_r^S is 4π periodic in ϕ_l, ϕ_r	*Ultrasensitive spin torque*
	Fractional Spin-Josephson ➢ j_r^S is 4π periodic in θ_l, θ_r	*Use shift in FMR in nano-particle*

L. J., Pekker, Alicea, Refael, Oreg, Brataas, von Oppen, arXiv: 1206.1581 & In preparation

TOPOLOGICAL & CONVENTIONAL QUANTUM SYSTEMS

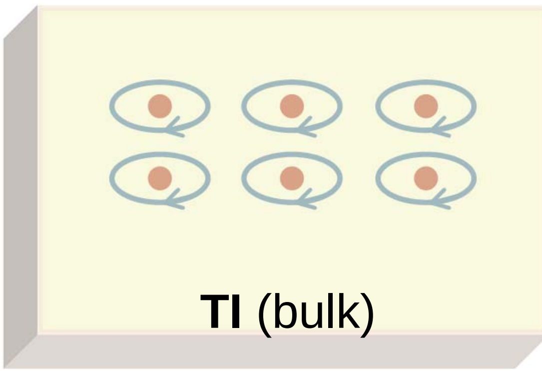


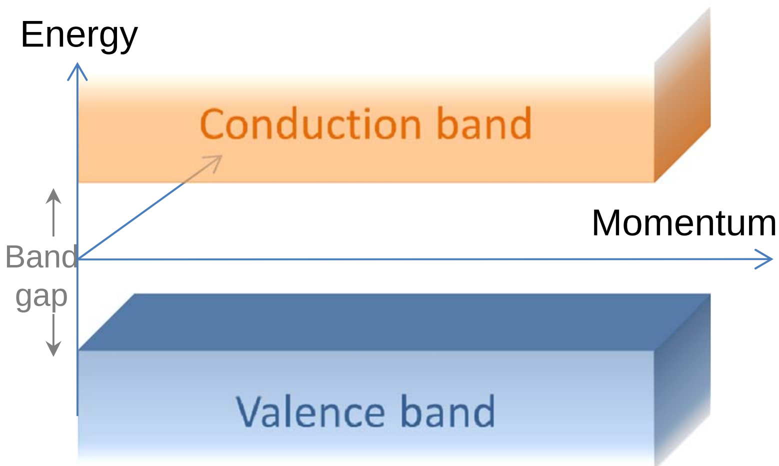
Hybrid Platforms
– between topological and conventional systems

- Goal
  1. Coherent coupling
  2. Switch on/off coupling without fine-tuning (to restore topological protection)
- Idea
  1. Use conventional system to control the evolution/braiding of topological system

Another approach to create Majoranas? – Topological Insulator

Interior: gapped insulator

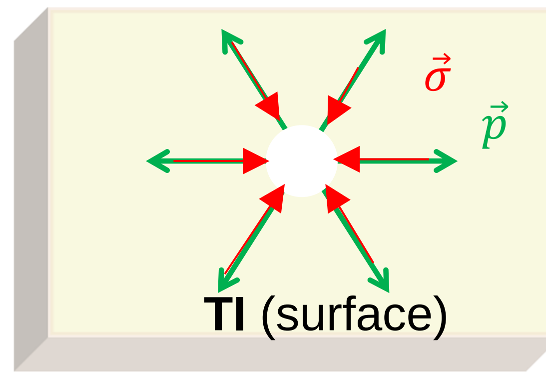


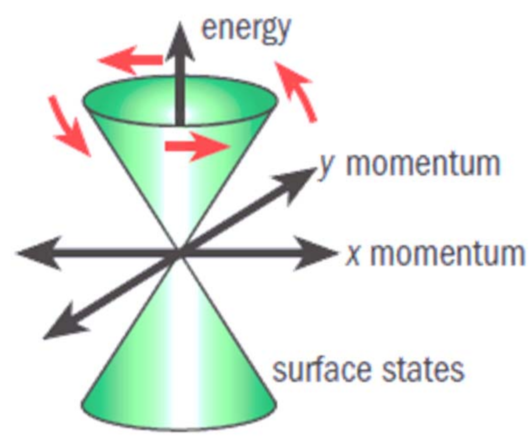


Hasan and Kane, RMP 82, 3045 (2010);
Qi and Zhang, RMP 83, 1057 (2011).

Surface: spin-locked conductor

$$H_{TI} = \psi^\dagger(v\vec{\sigma}\cdot\vec{p} - \mu)\psi$$



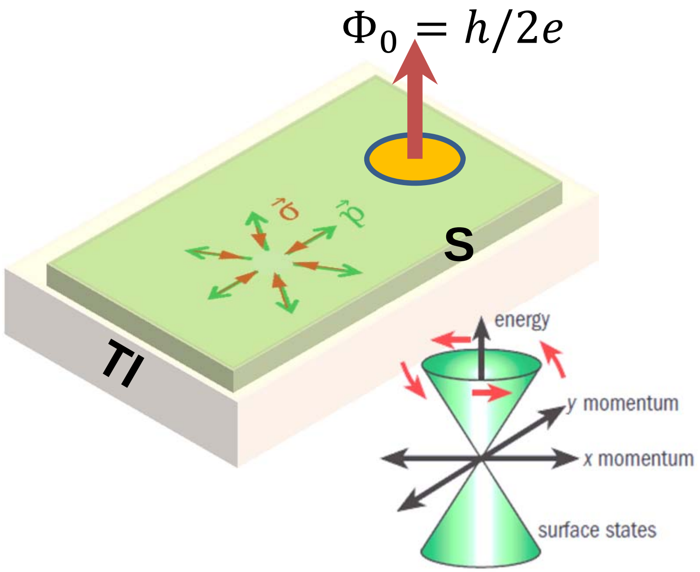


Brüne, Liu, Novik, et al., PRL 106, 126803 (2011).

Another approach to create Majoranas? – Topological Insulator

- 3D Topological Insulator
  - Interior: gapped insulator
  - Surface: spin-locked conductor

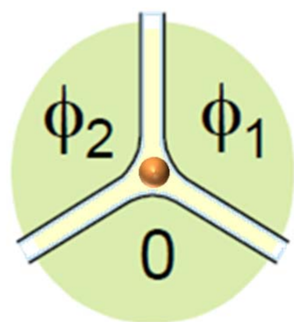
$$H_{TI} = \psi^{\dagger}(v\vec{\sigma} \cdot \vec{p} - \mu)\psi$$



- S-wave superconductor

$$H_S = \Delta\psi_{\uparrow}^{\dagger}\psi_{\downarrow}^{\dagger} + h.c.$$

- Similar to p+ip superconductor
  - Support Majoranas at vortices (e.g., Tri-Junction of SC islands)

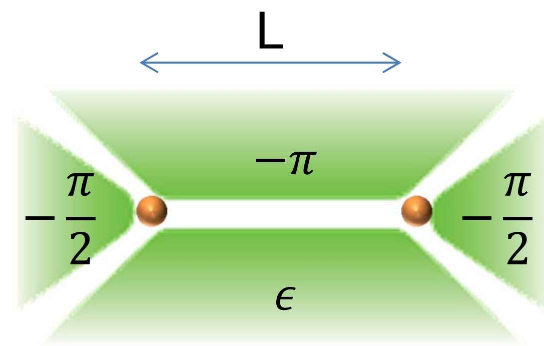


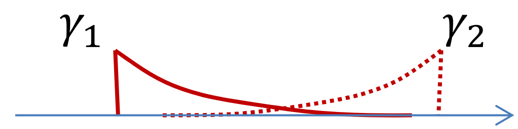
Fu & Kane, PRL 100, 096407 (2008), Read & Green, PRB 61, 10267 (2000).

Two Majoranas with Coupling

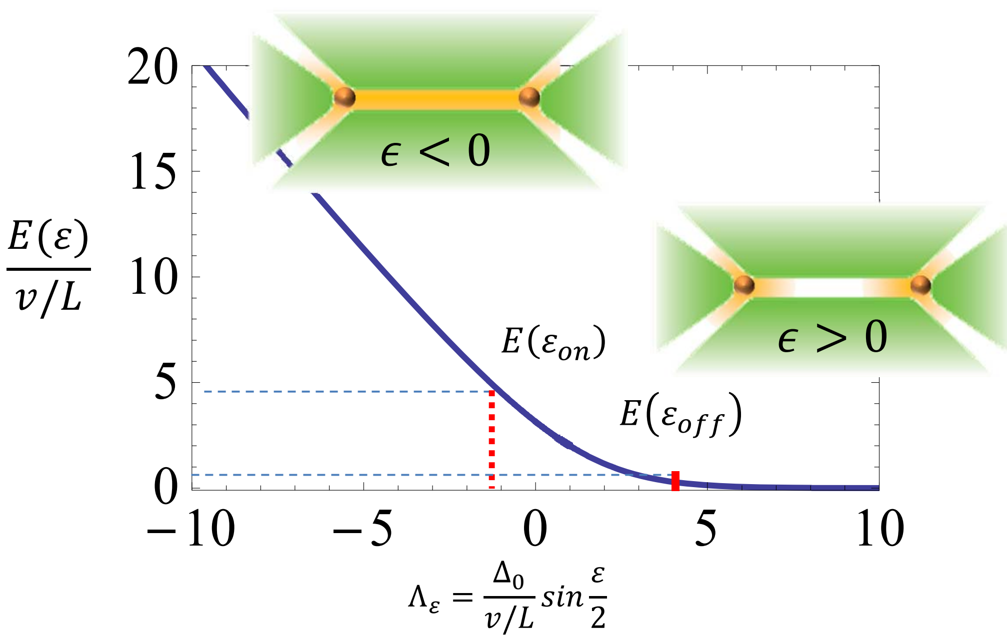
- Using two <u>tri-junctions</u> connected by a <u>quantum wire</u>
  - Majorana wavefunction controlled by ε
  - Interact along quantum wire

$$H_{12}^{MBS} = iE(\varepsilon)\gamma_1\gamma_2 \cong E(\varepsilon)Z_{topo}$$
with $Z_{topo} \cong n_{1,2} = 0,1.$





$$H_{wire} = -iv\tau^x\partial_x + \delta_\phi\tau^z$$



Fu and Kane, PRL 100, 096407 (2008)

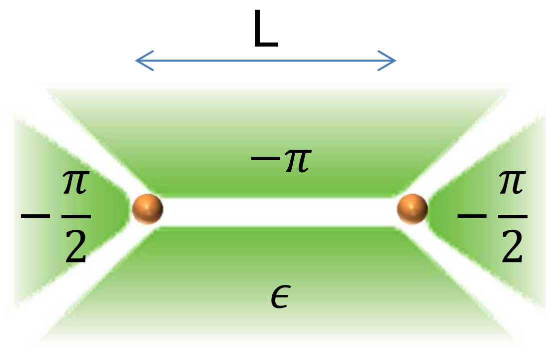
Superposition of Evolutions

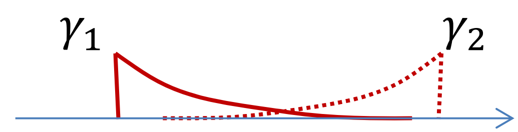
- Interaction between Two Majoranas
  - Overlap along quantum wire

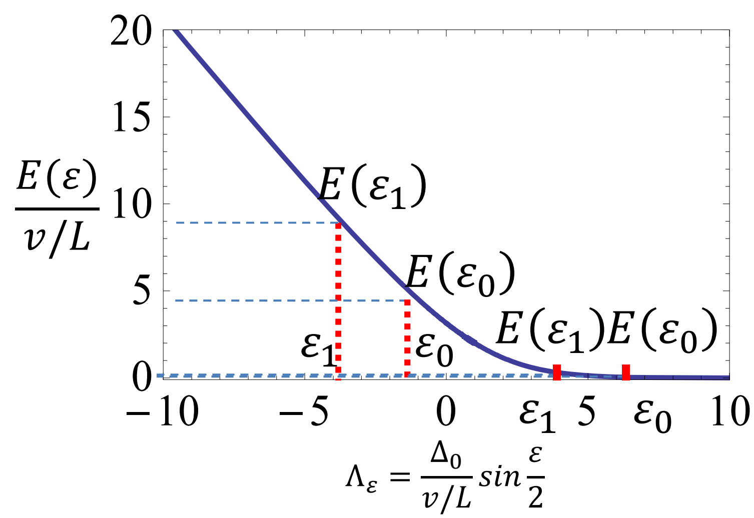
$$H_{12}^{MBS} = iE(\varepsilon)\gamma_1\gamma_2 \cong E(\varepsilon)Z_{topo}$$

- Observation
  - $\hat{\varepsilon} \rightarrow |\varepsilon_0\rangle + |\varepsilon_1\rangle$ induces superposition of evolutions (i.e., Ctrl-Phase evolution)
  - Highly non-linear (good for switch on/off)

How to achieve $|\varepsilon_0\rangle + |\varepsilon_1\rangle$?







Switch on Interaction

Switch off Interaction

L.J., C. L. Kane and J. Preskill, PRL 106, 130504 (2011).

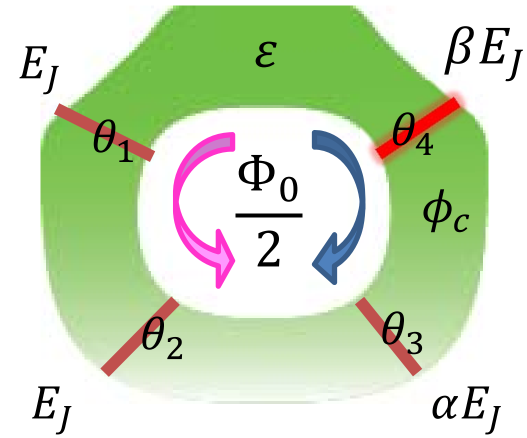
Flux Qubit -- to achieve $|\varepsilon_0\rangle + |\varepsilon_1\rangle$

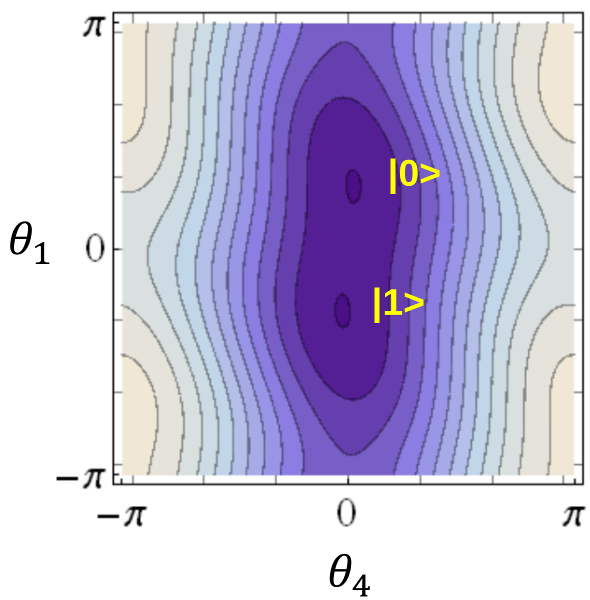
- Series of Josephson Junctions
  - Josephson (potential) energy

$$U = -\sum_i E_{J,i} \cos\theta_i$$

with phase constraint $\sum_i \theta_i \equiv \pm\pi$

and two potential minimum $(1/2 < \alpha < 1, \beta \gg 1)$





- SC Flux Qubit
  - CW/CCW current
  - Superposition of two values of ε

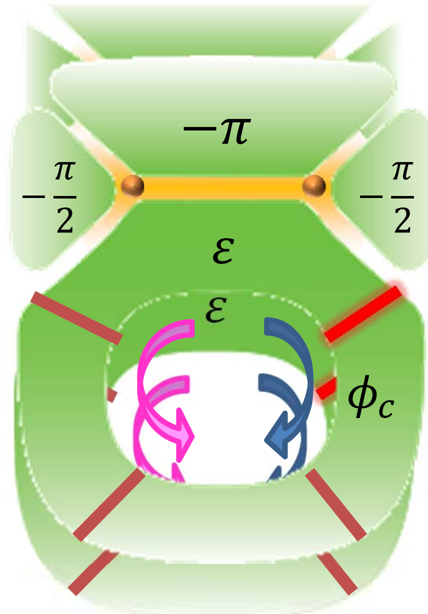
$$\varepsilon = \phi_c + \Delta\varepsilon \cdot Z_{flux}$$

$$\Delta\varepsilon \approx \frac{1}{\beta}\sqrt{1 - \frac{1}{4\alpha^2}} \text{ for } \beta \gg 1.$$

Hybrid System

- Topological Quantum Wire
  & Flux Qubit
- ε coherently controls the coupling between Majoranas:

$$H_{topo} \Rightarrow \begin{cases} E(\varepsilon_0) Z_{topo} & \text{for } \varepsilon = \varepsilon_0 \text{ with } |0\rangle_{flux} \\ E(\varepsilon_1) Z_{topo} & \text{for } \varepsilon = \varepsilon_1 \text{ with } |1\rangle_{flux} \end{cases}$$



- Coupling for Controlled-Phase Gate

$$H_I = \frac{g}{4} Z_{flux} Z_{topo}$$

with coupling strength

$$g \approx \left(E(\varepsilon_1) - E(\varepsilon_0)\right) + \frac{1}{4}\left(E''(\varepsilon_1) - E''(\varepsilon_0)\right)\delta\varepsilon^2 + \ldots$$

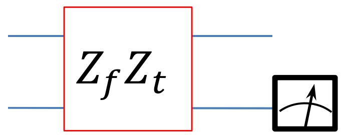
L.J., C. L. Kane and J. Preskill, PRL 106, 130504 (2011).

Transfer Quantum Information

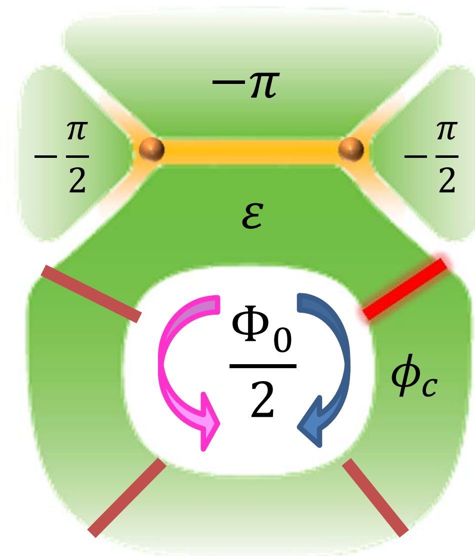
- QND Repetitive measurement

Topo qubit:. $|0\rangle, |1\rangle$

Flux qubit: $|0\rangle + |1\rangle$



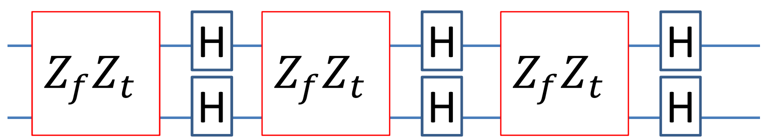
$|0\rangle \pm |1\rangle$



- **SWAP** quantum state between topological qubit to flux qubit

Topo qubit:. $|\psi_1\rangle$

Flux qubit: $|\psi_2\rangle$

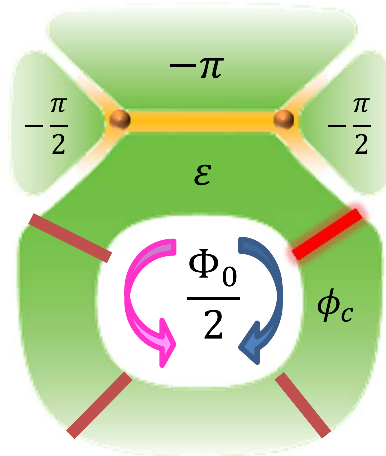


$|\psi_2\rangle$

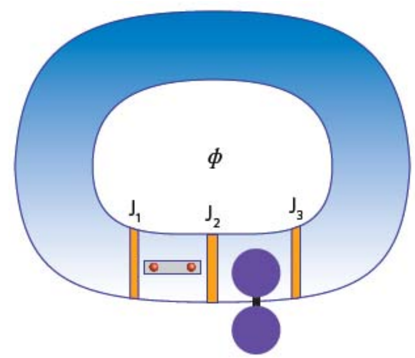
$|\psi_2\rangle$

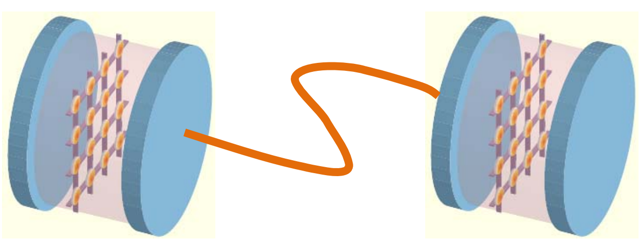
L.J., C. L. Kane and J. Preskill, PRL 106, 130504 (2011).

Other Approaches to Hybrid Platforms



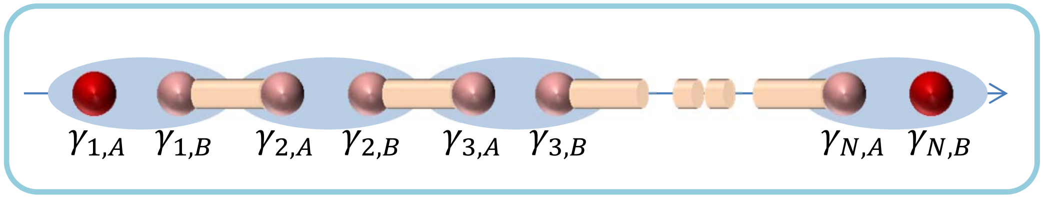
- Hybrid system of topological and flux qubits
  - Measure, probe anyonic statics, connect different topological systems, …
- Various related proposals
  - Semiconductor quantum wire + SC qubit
    - Hassler et al., NJP 12, 125002 (2010)
    - Bonderson, Lutchyn, PRL 106, 130505 (2011)
    - Pekker, et al., arXiv 1301.3161.
  - Topological quantum wire + SC flux/phase qubit
    - Jiang, Kane, Preskill, PRL 106, 130504 (2011)
- Topological quantum networks



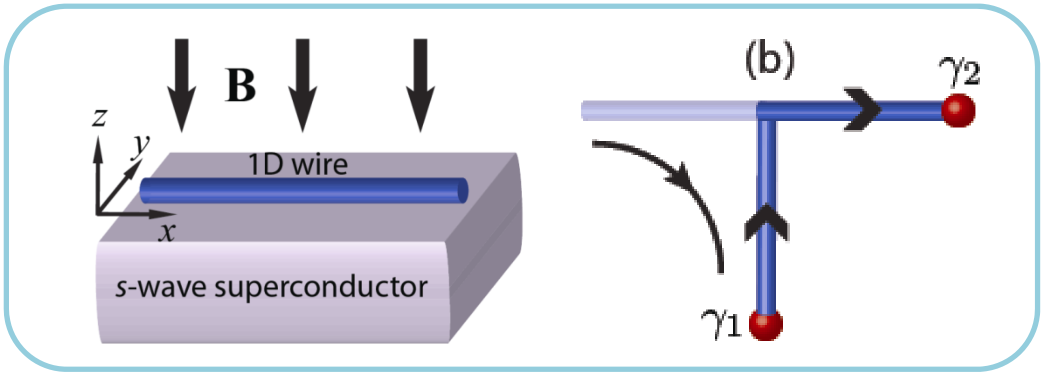


Summary

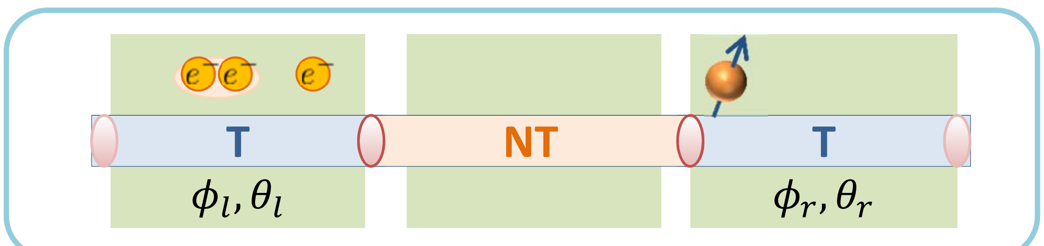
Kitaev Quantum Wire – Majoranas



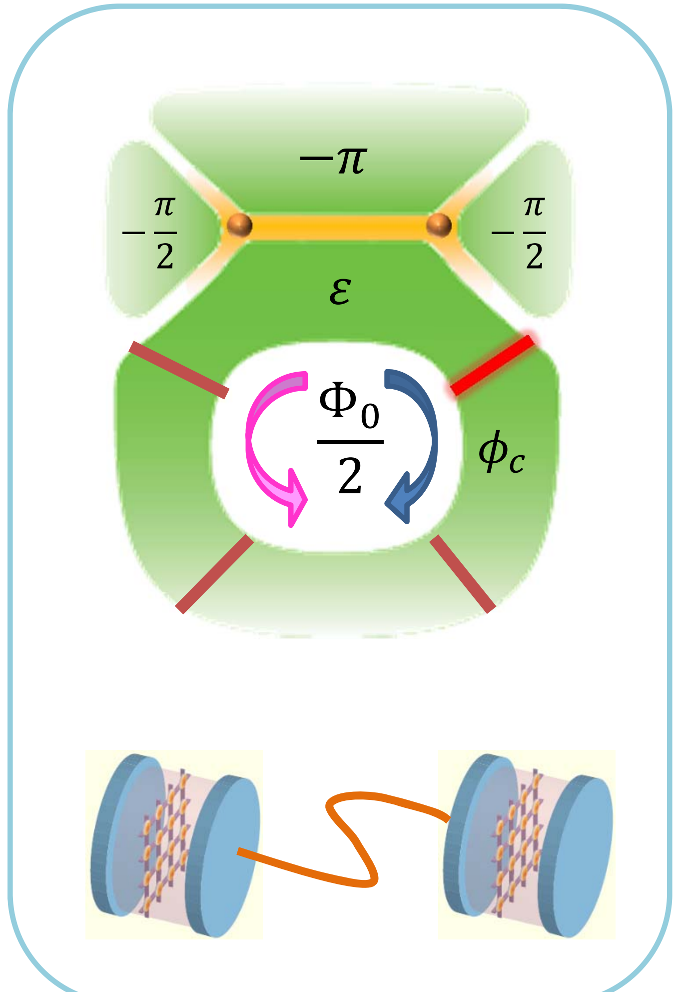
Semiconductor Wire & Braiding



Duality in 1D Wire (Spin & Particle-hole)



Hybrid Platforms between Topological & Conventional Systems



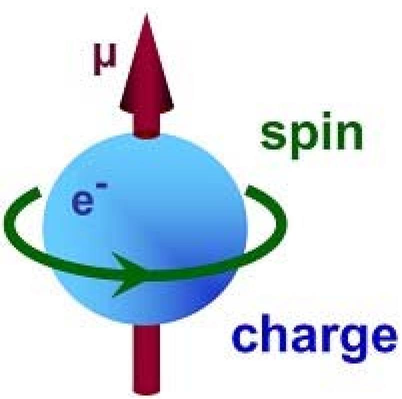
Quantum Systems

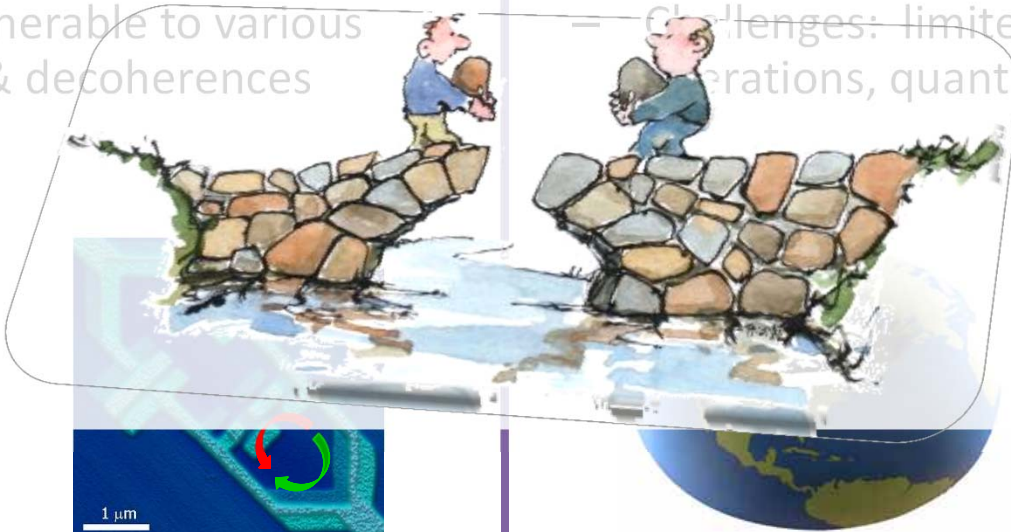
Conventional Quantum Systems

- Local degrees of freedom
- E.g., spins, photons, ions, superconducting devices, …
- Merits: arbitrary unitary operations, distant entanglement, …
- Challenges: vulnerable to various imperfections & decoherences

Topological Quantum Systems

- Global degrees of freedom
- E.g., Fractional Quantum Hall Effect, Topological Insulators, Majoranas, …
- Merits: robust against local perturbation/decoherence.
- Challenges: limited unitary operations, quantum network …





New hybrid systems combine merits from both systems

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