# Local topological order inhibits thermal stability in 2D

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joint work with David Poulin

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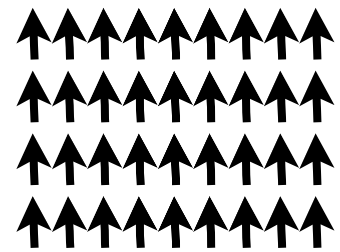
Encoding $|\psi_i\rangle$ —Noise→  → $|\psi_i'\rangle$ —Decoding→ $|\psi_f\rangle$

Code = degenerate groundspace of a local Hamiltonian of spin particles (qudits) on a 2D lattice.

# Self-correcting classical memories

## 2D ferromagnetic Ising model

$$H_{\text{Ising2D}} = -\sum_{\langle i,j \rangle} \sigma_z^i \otimes \sigma_z^j$$



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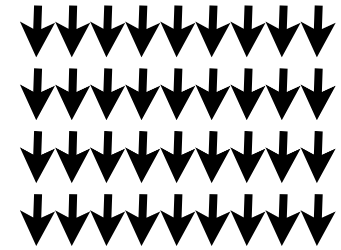

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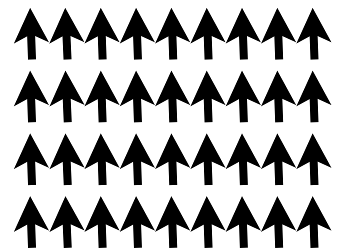
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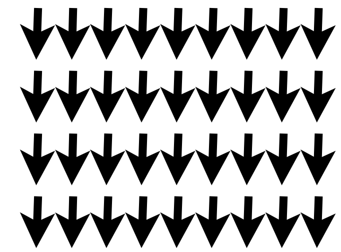
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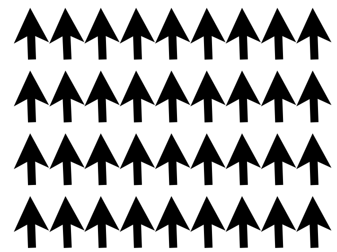
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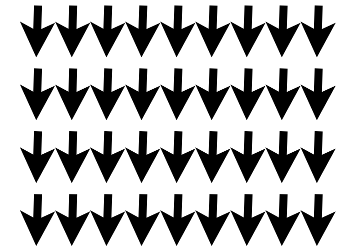
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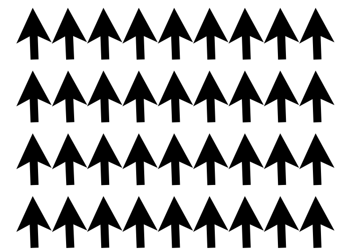

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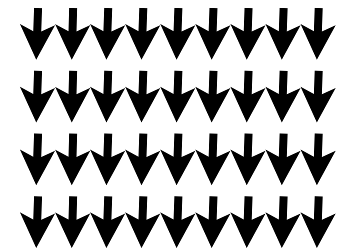

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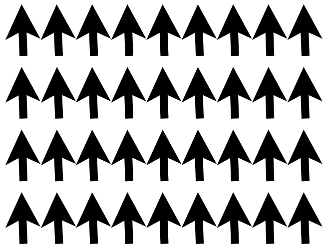

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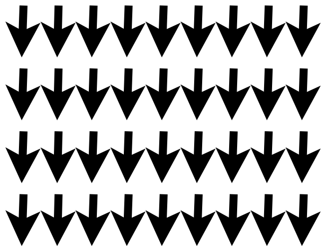

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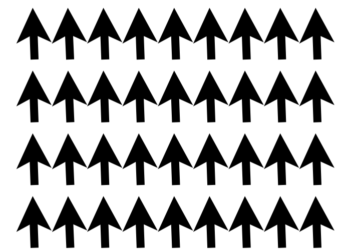
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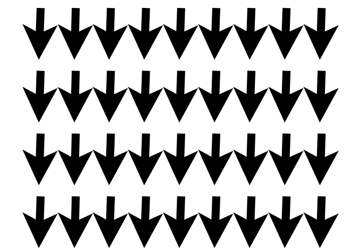
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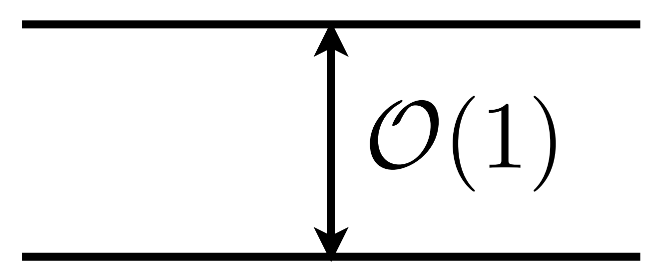

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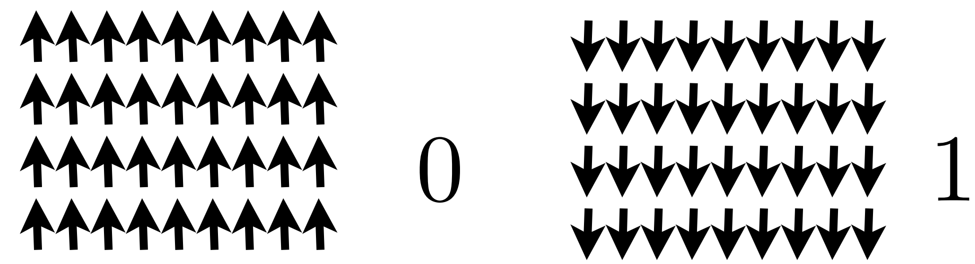


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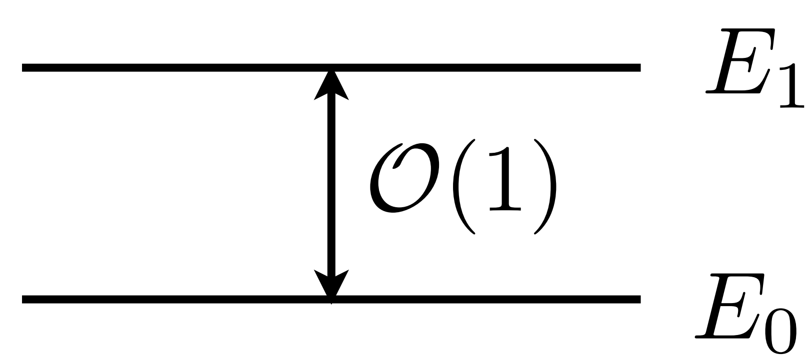


Topologically ordered system !

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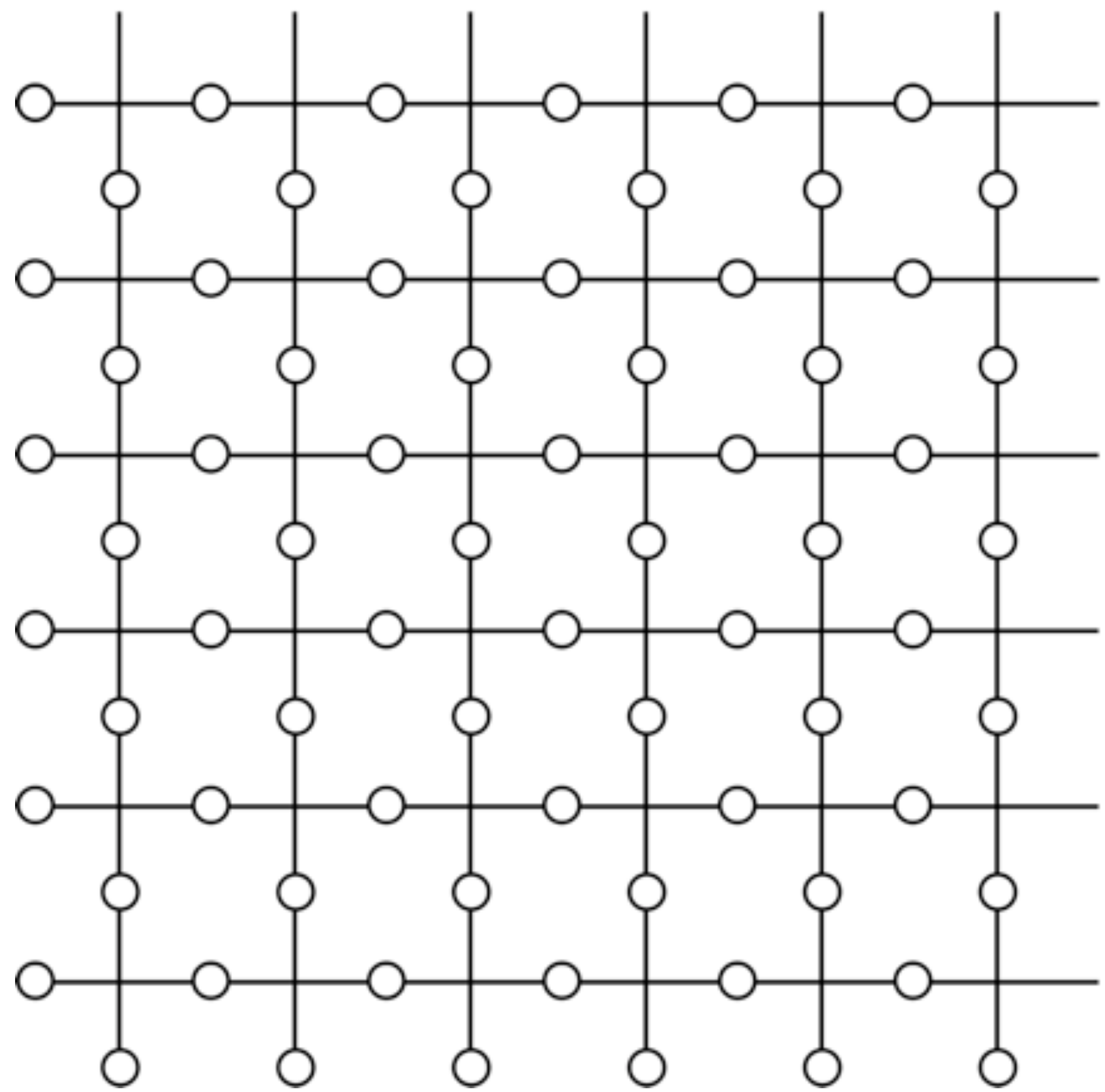

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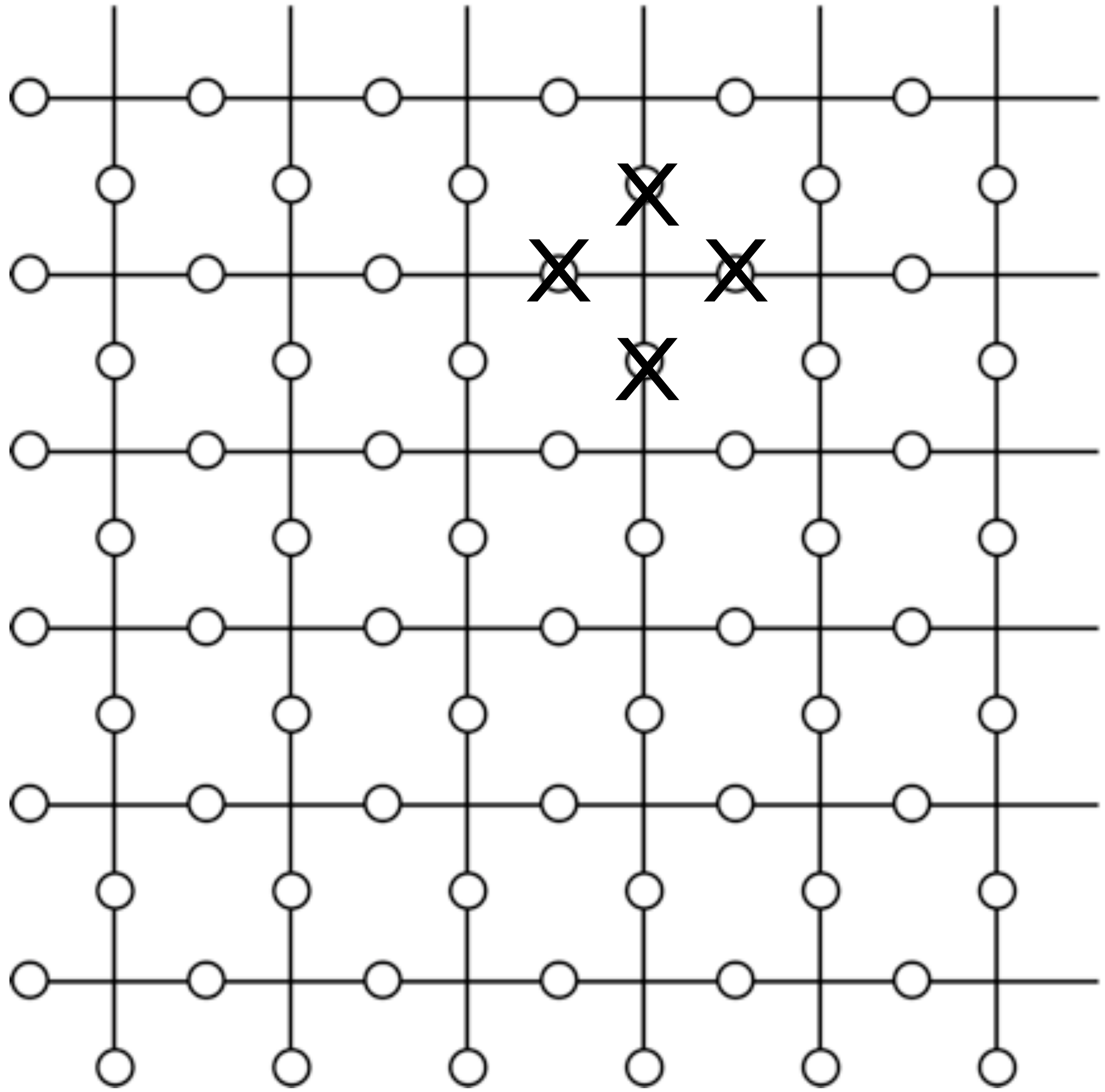

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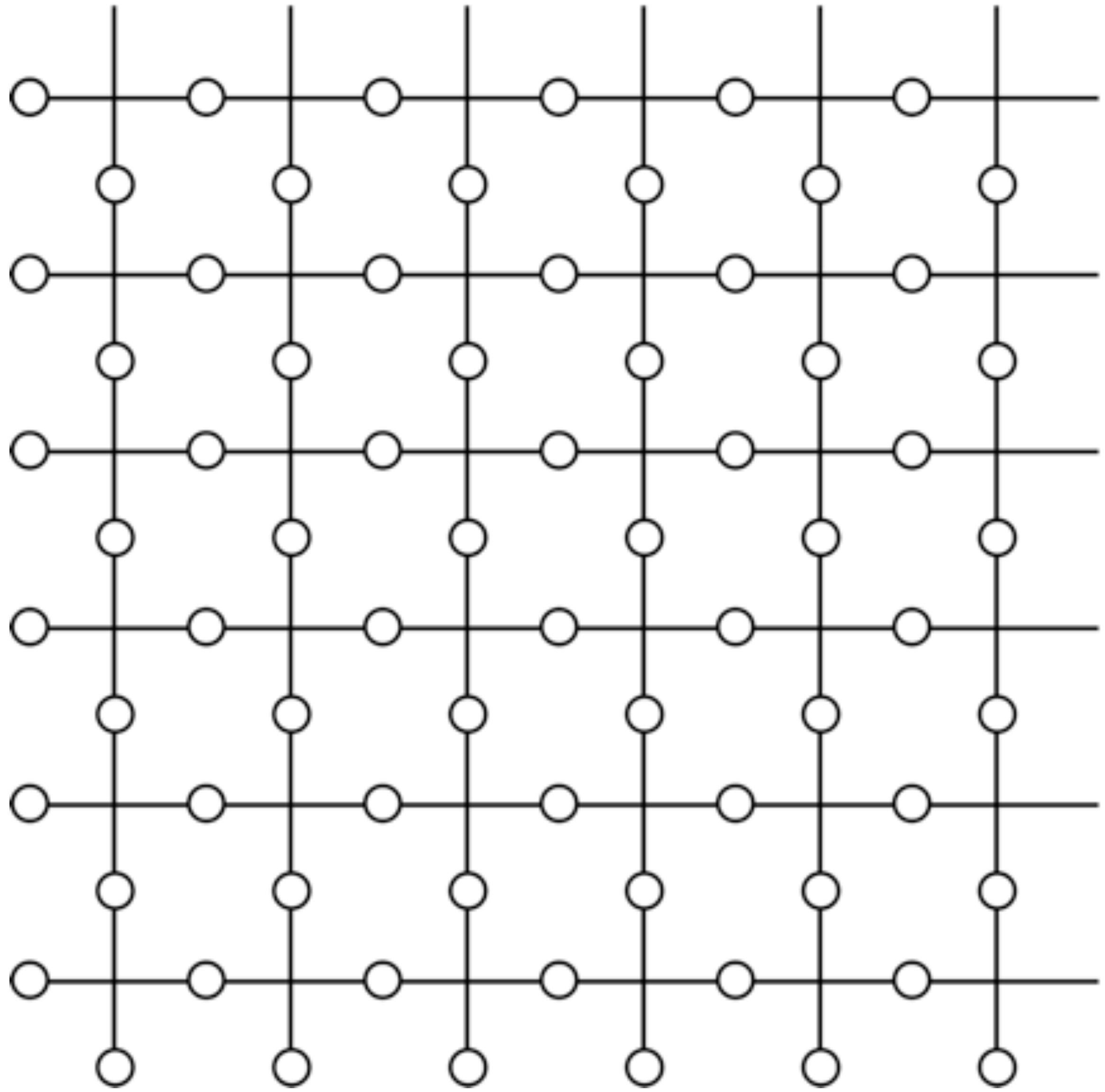

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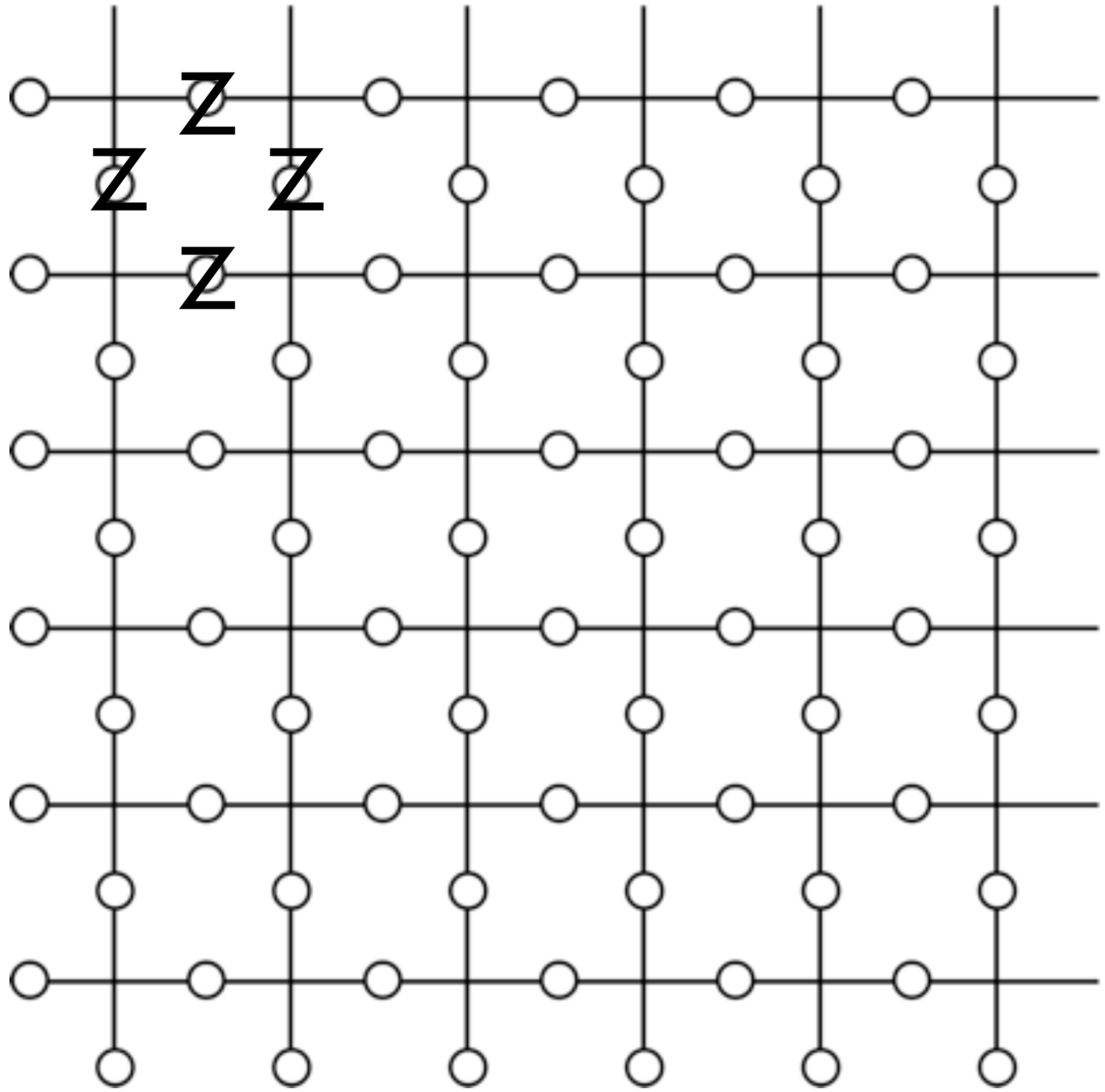

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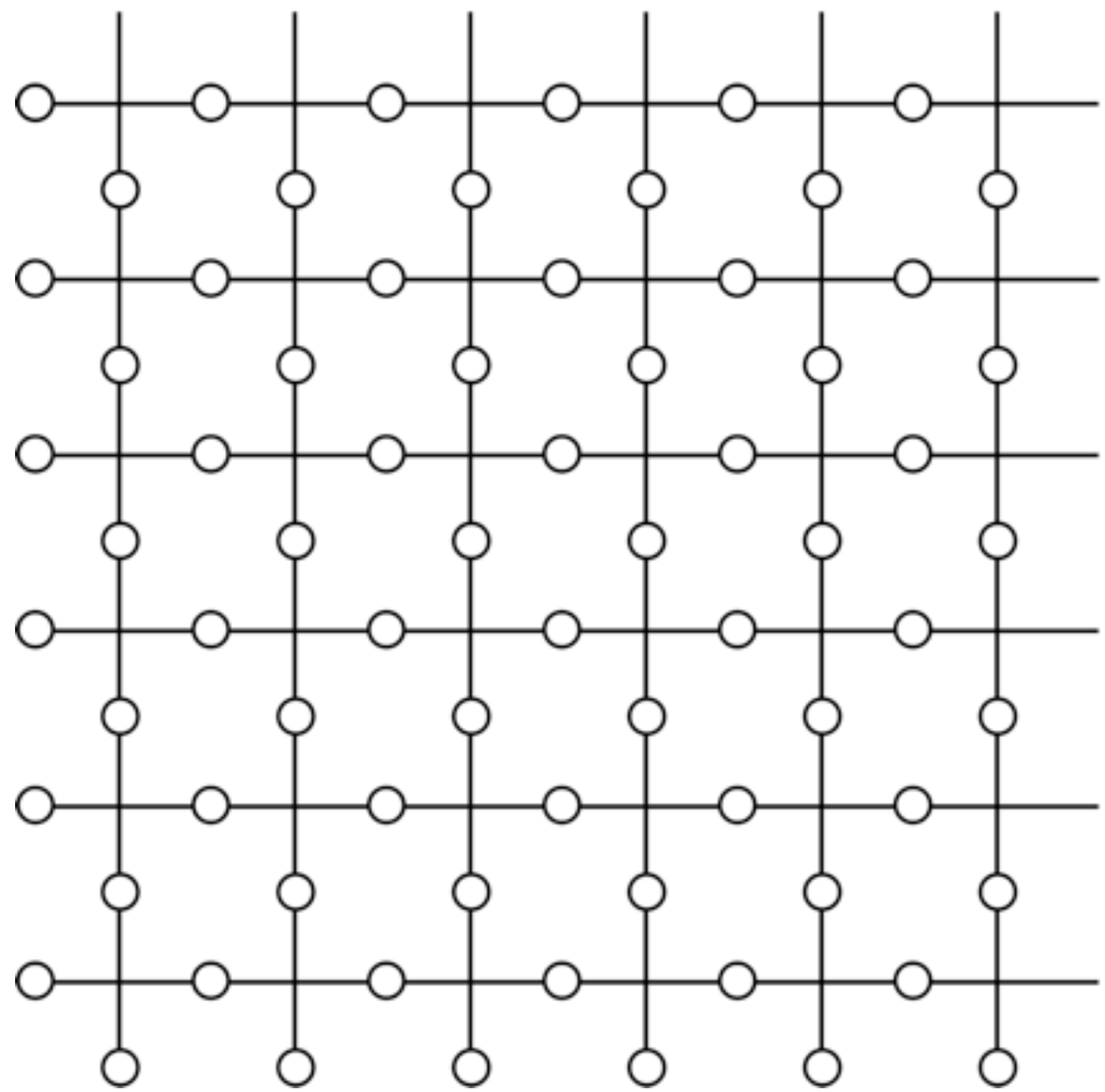

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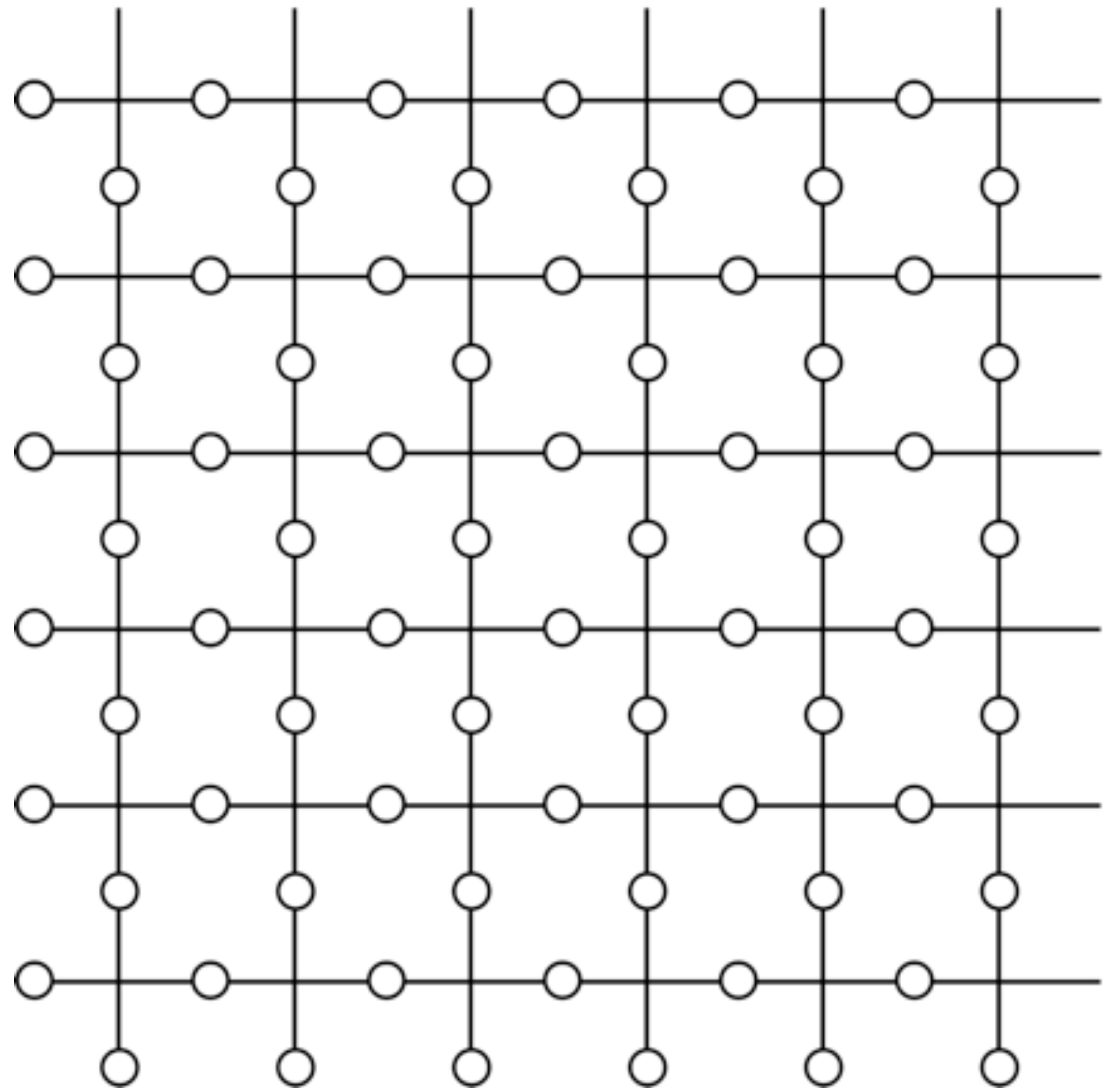

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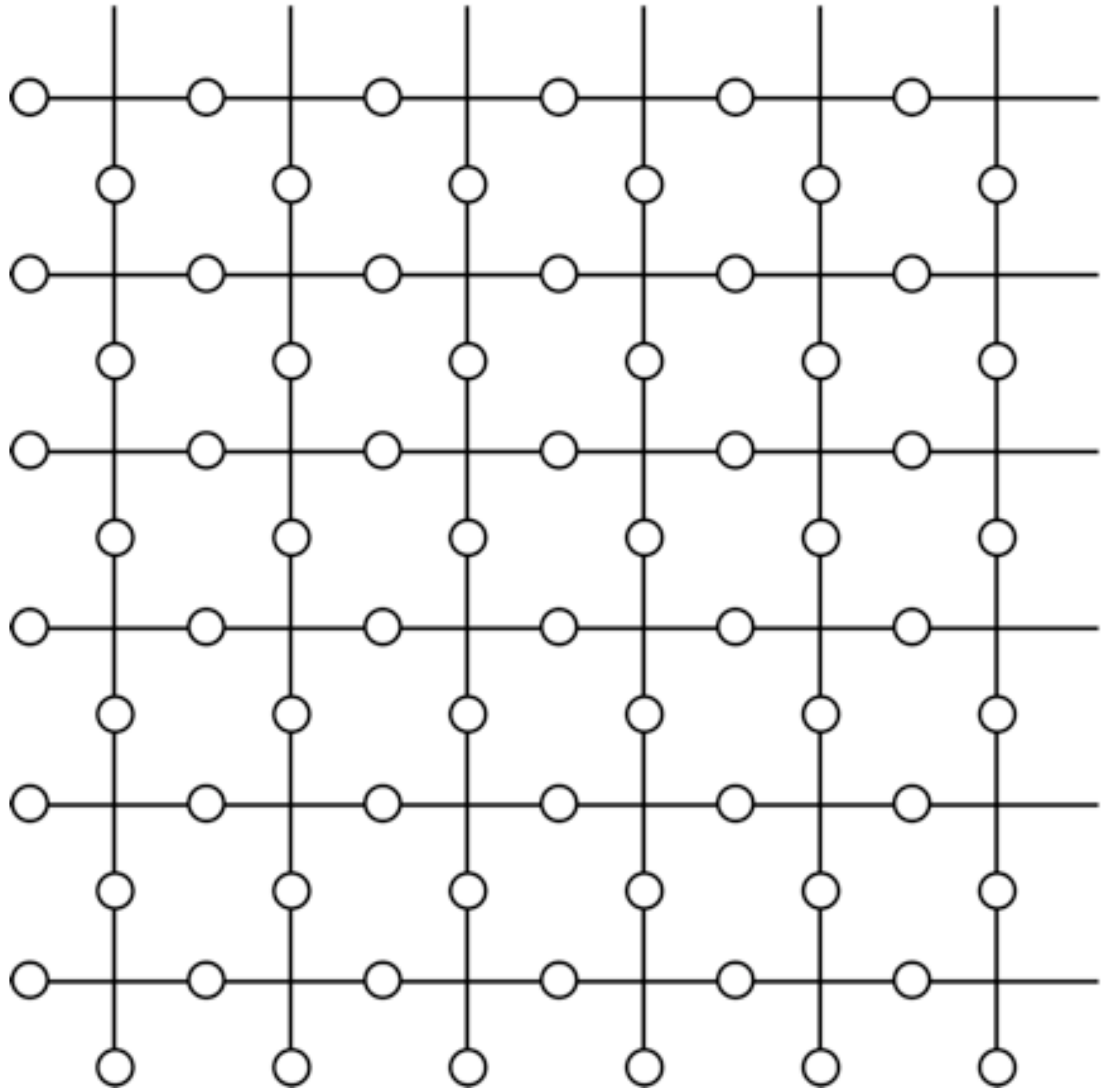

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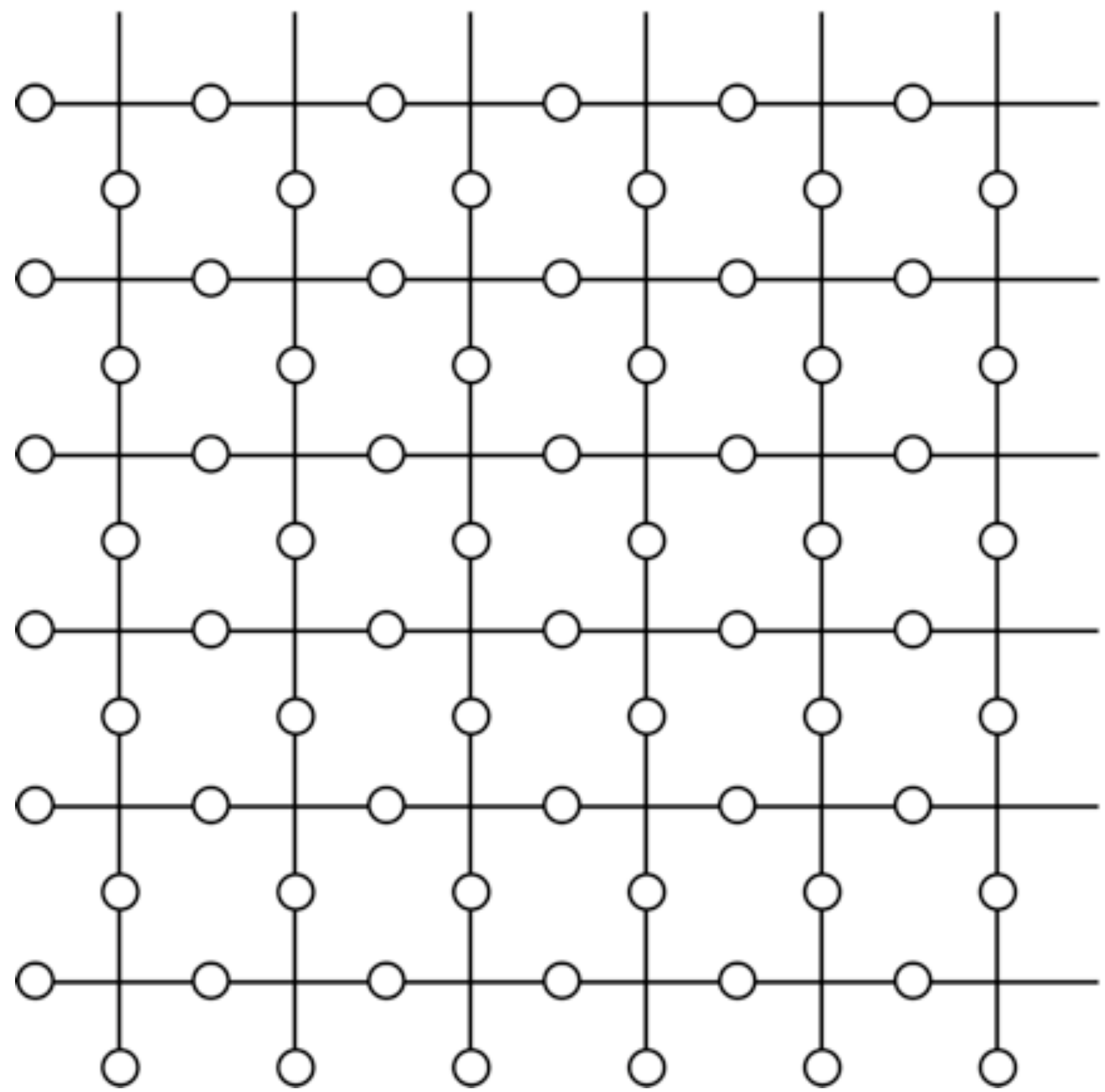

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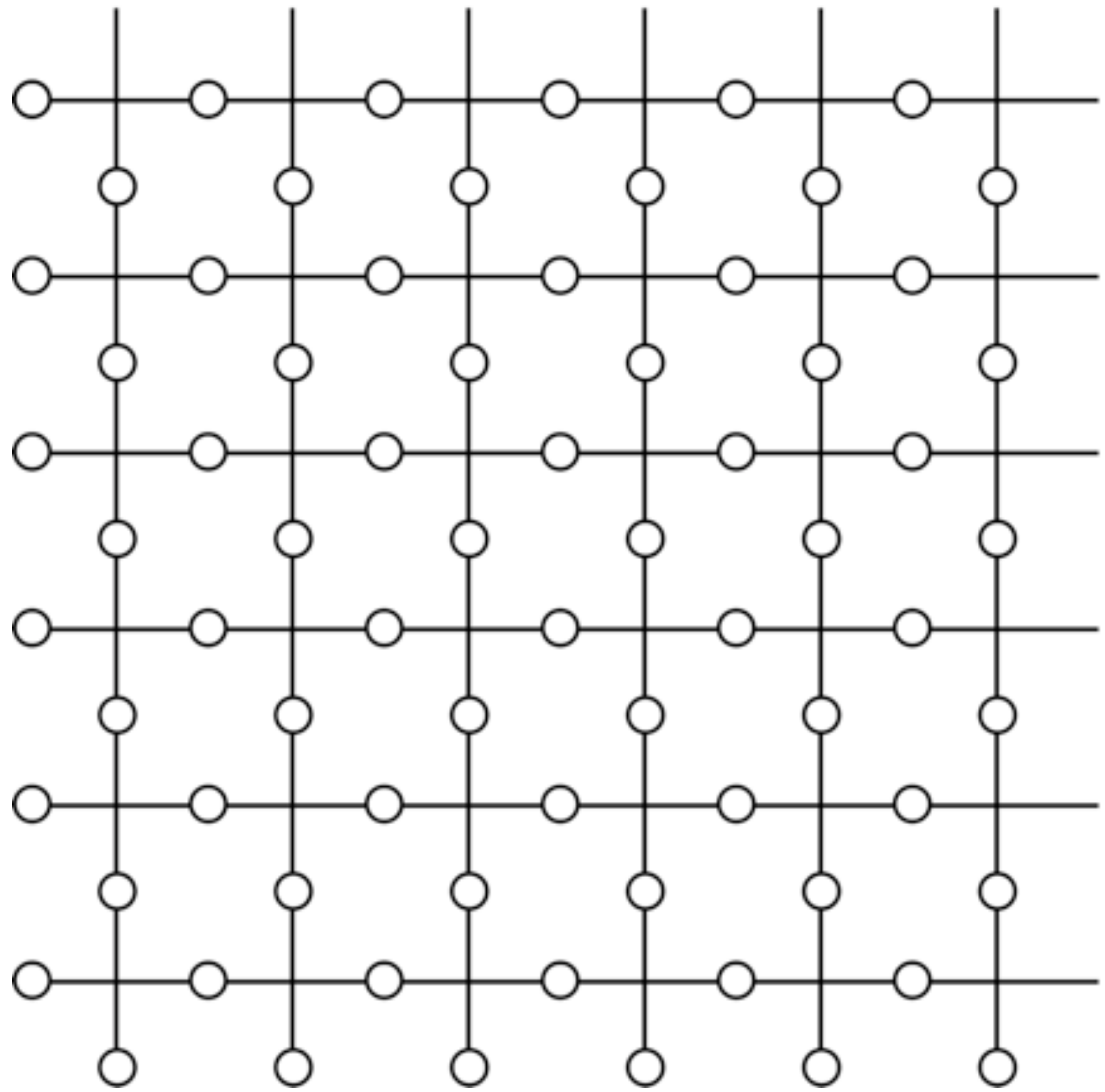

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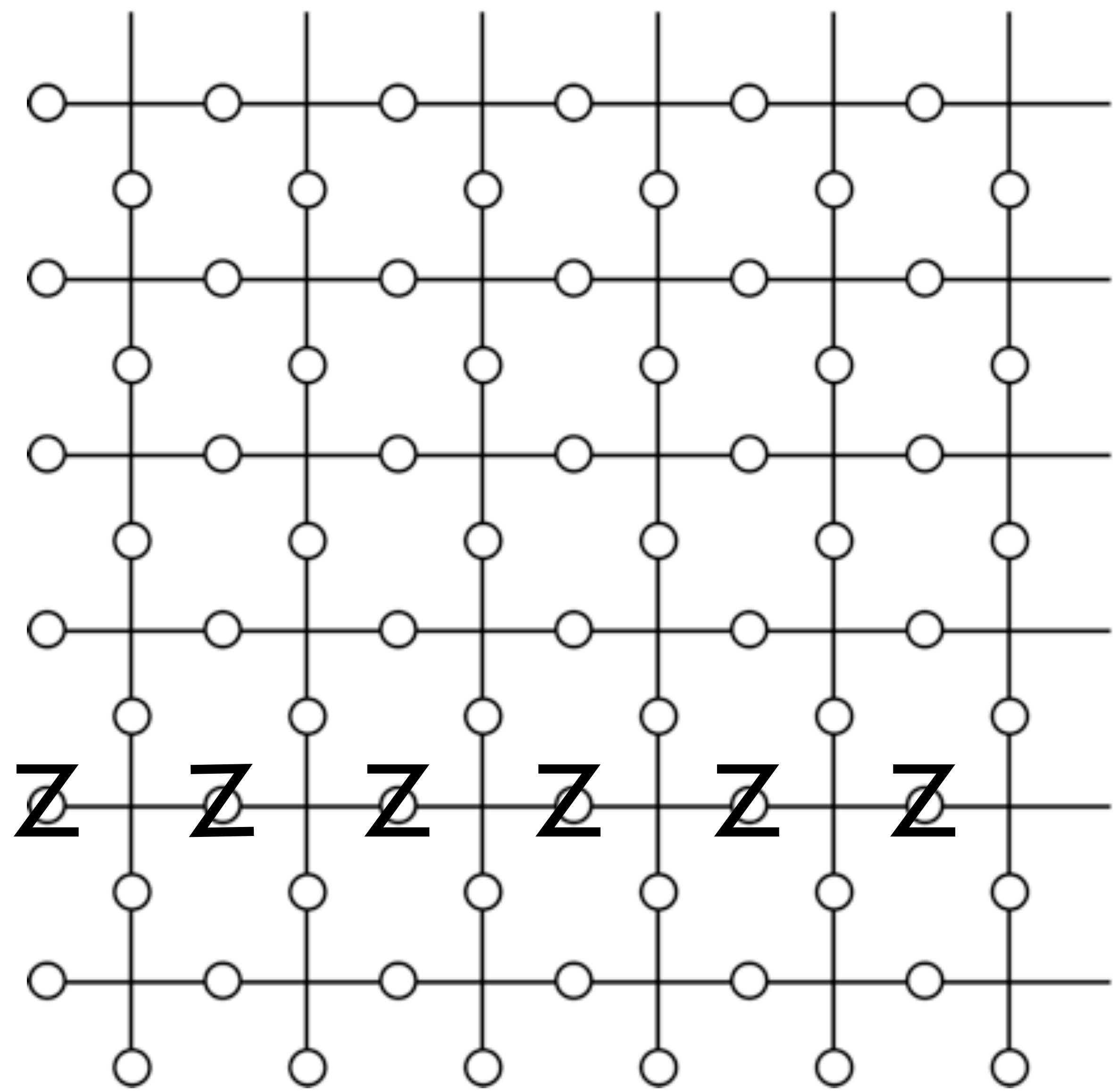

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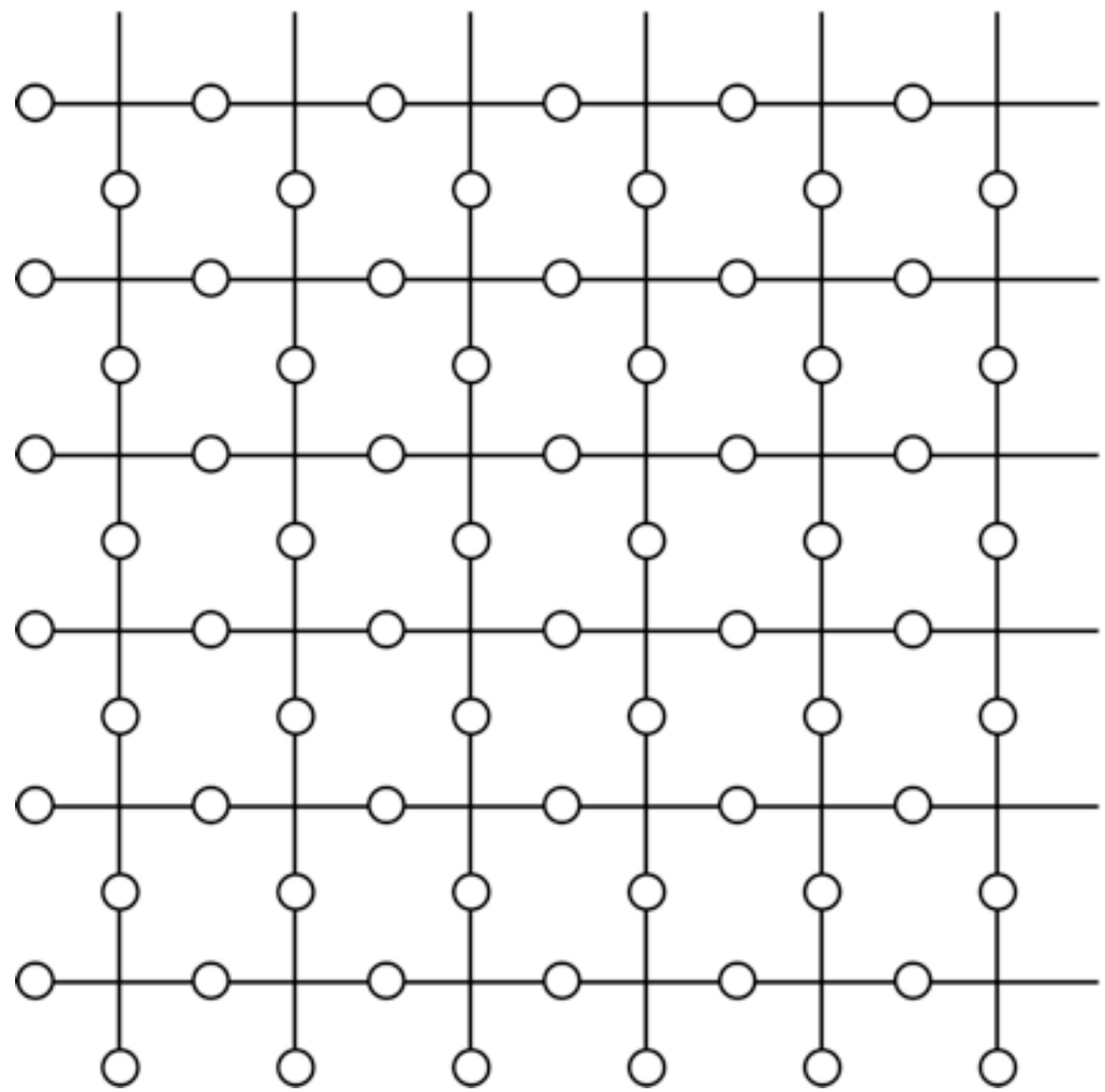

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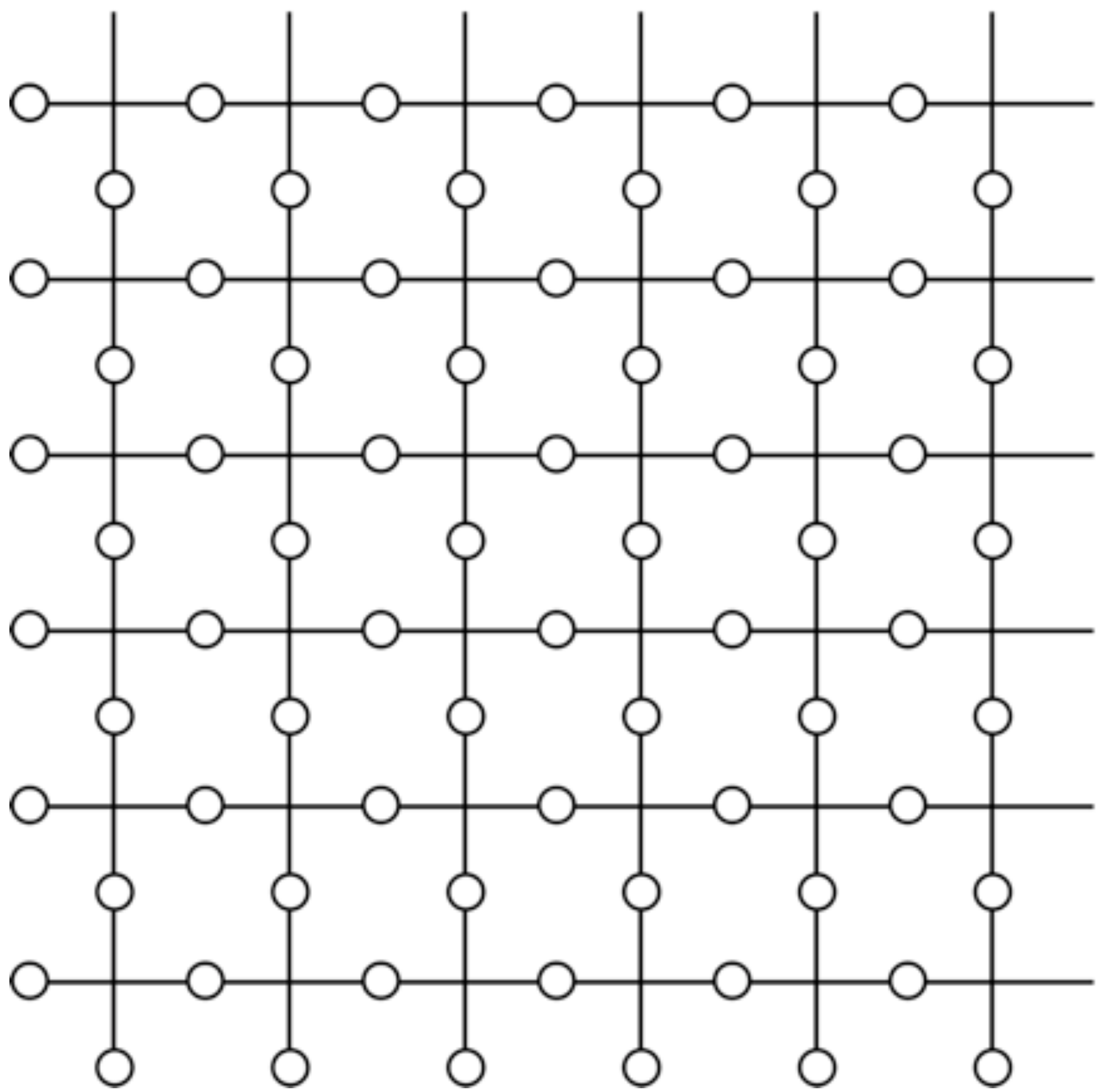

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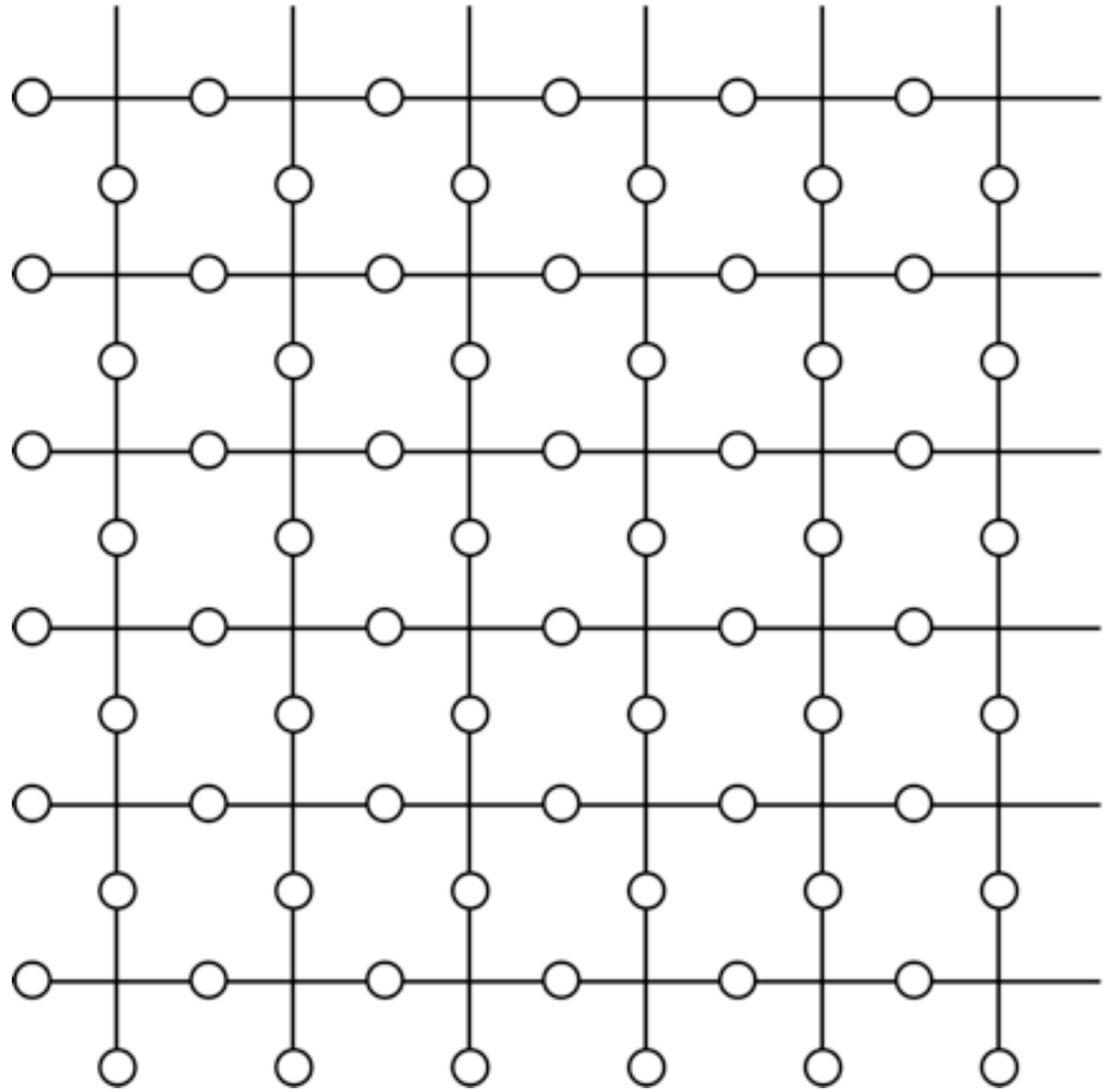

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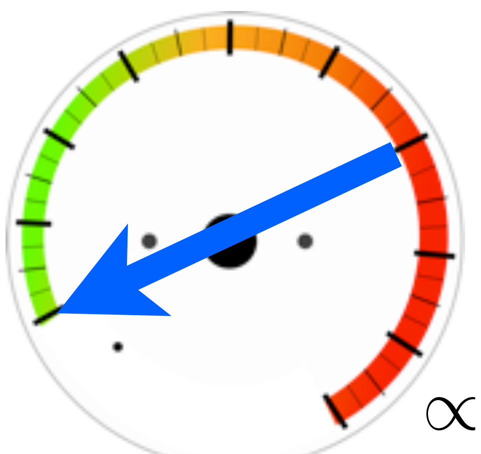

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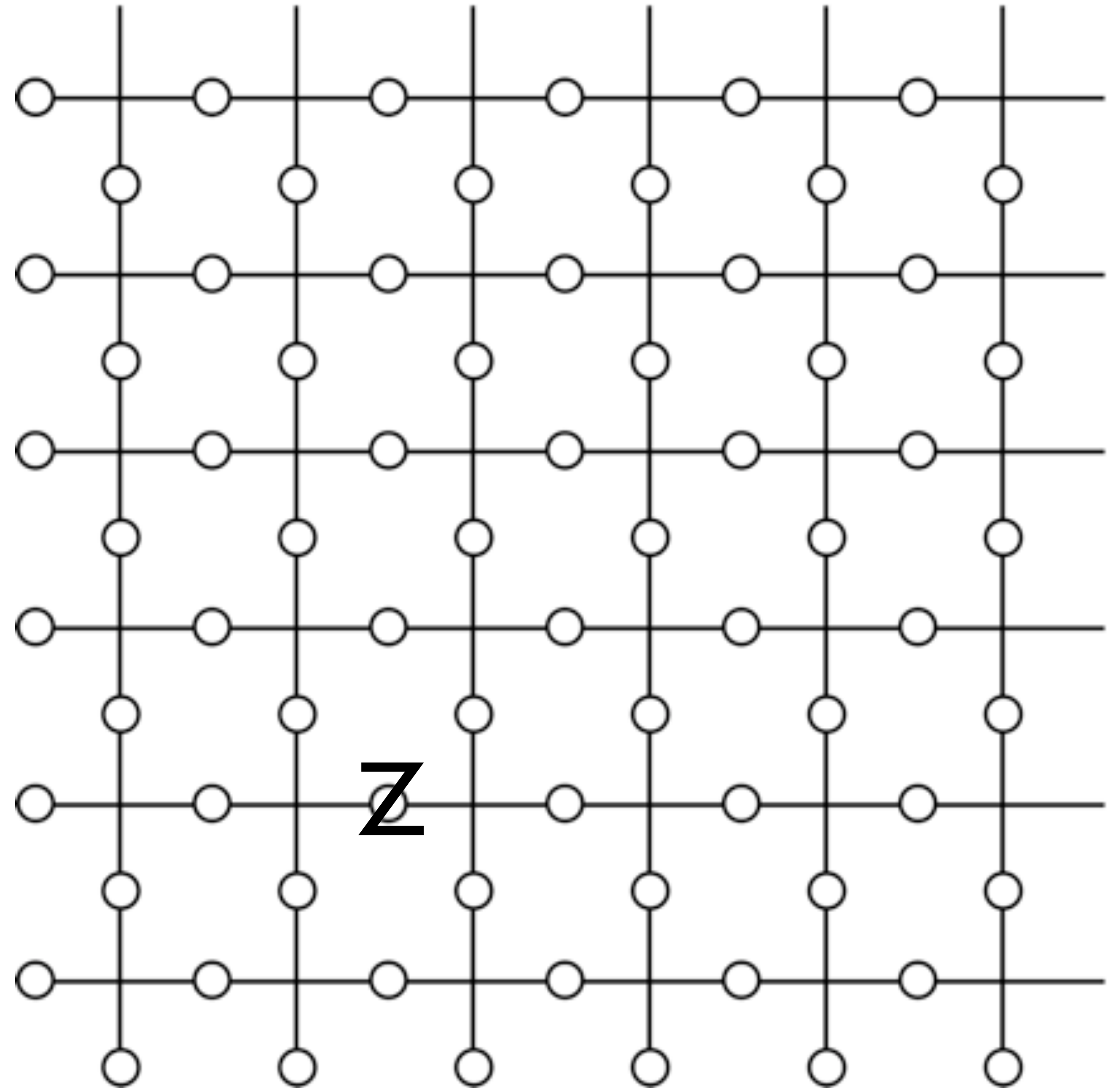


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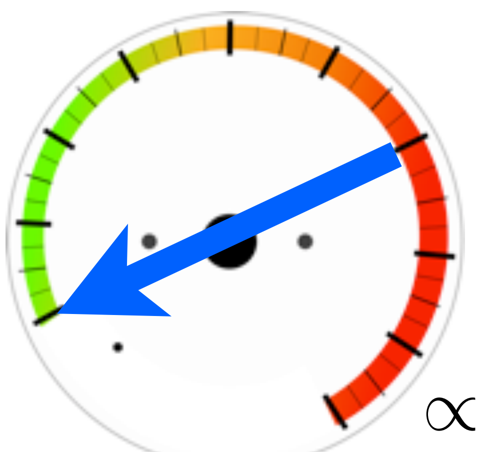

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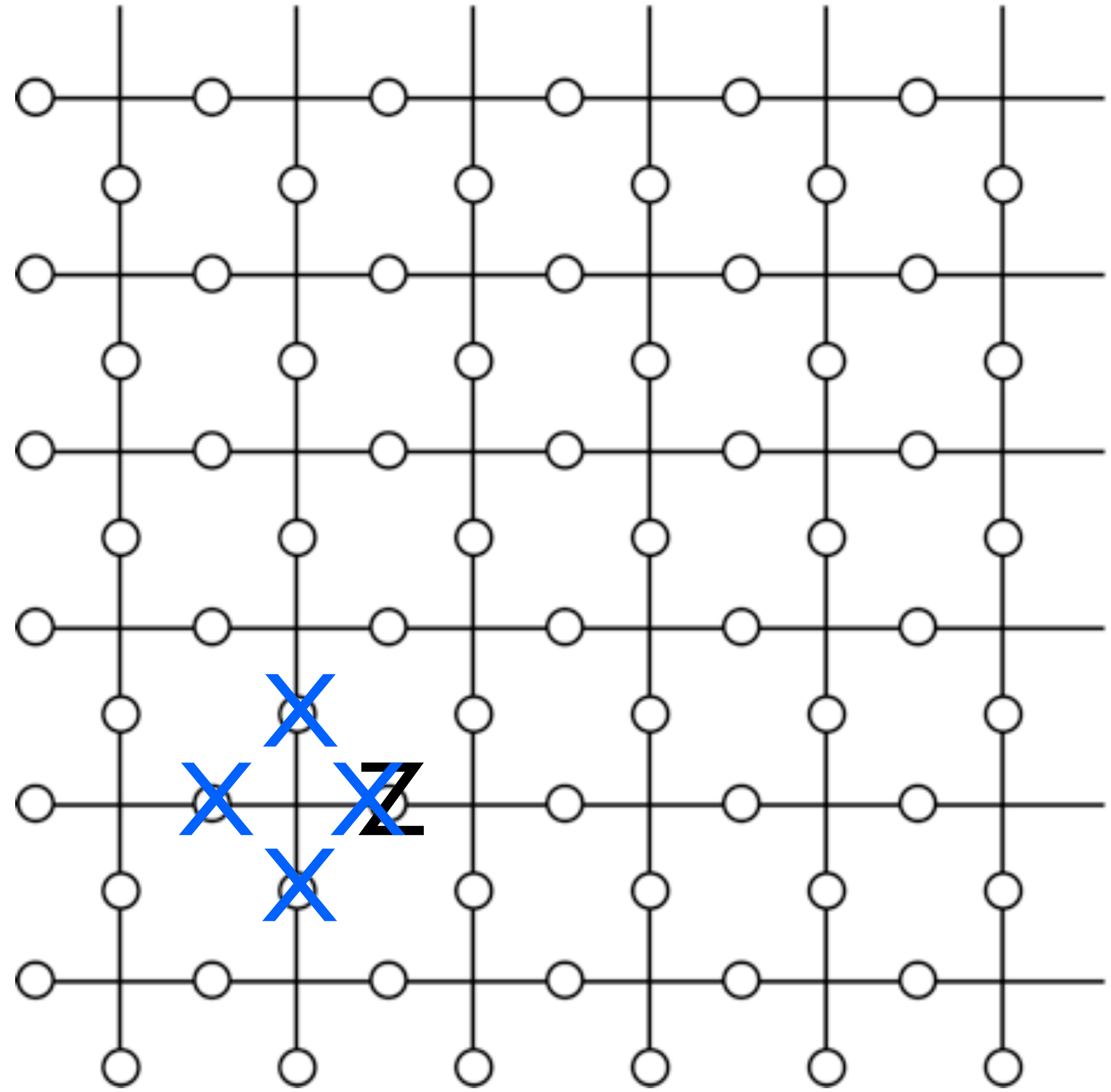


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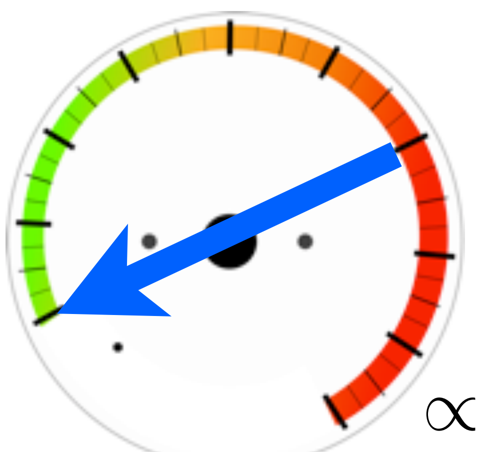

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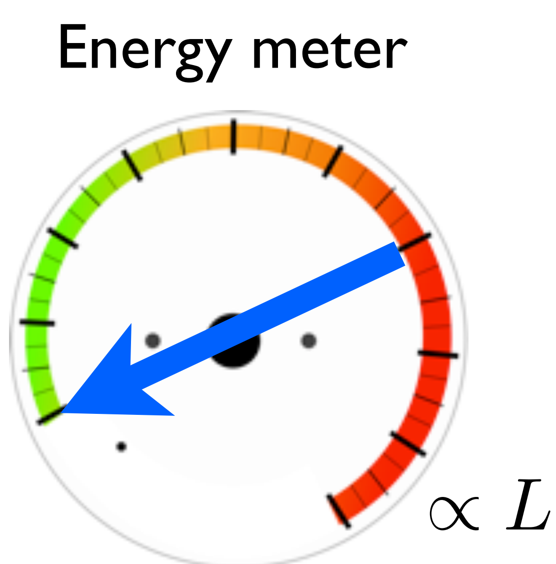


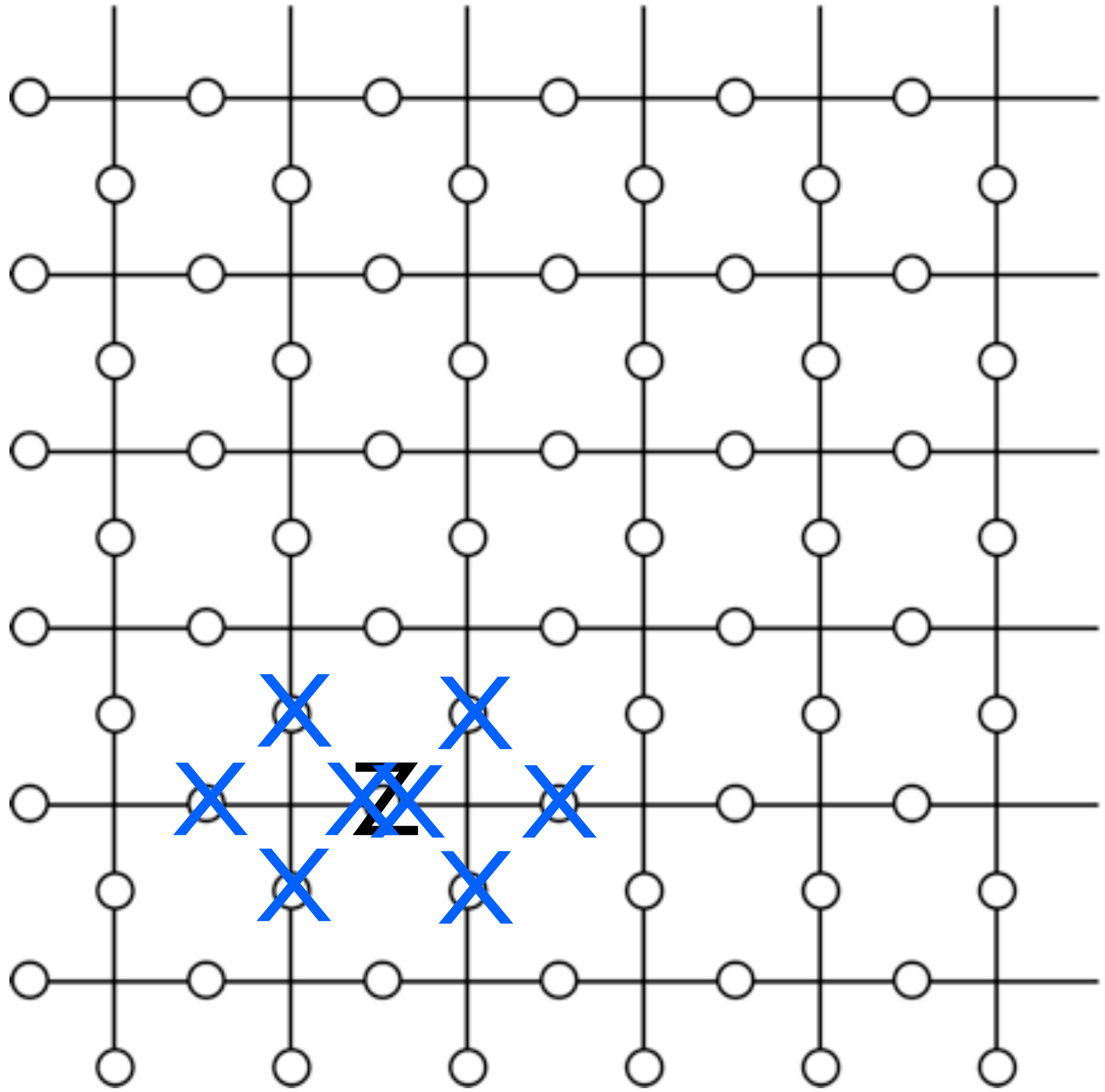

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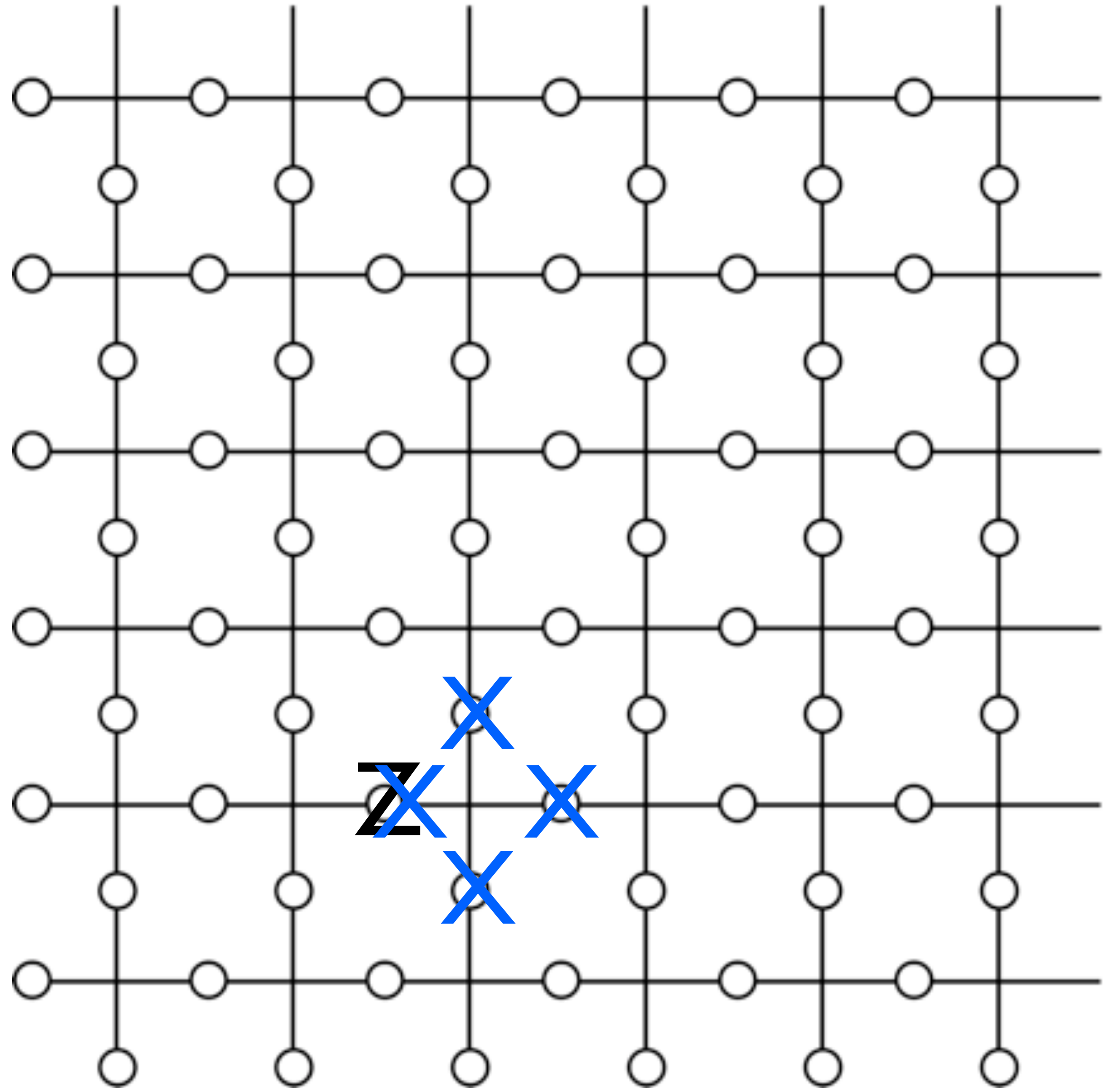


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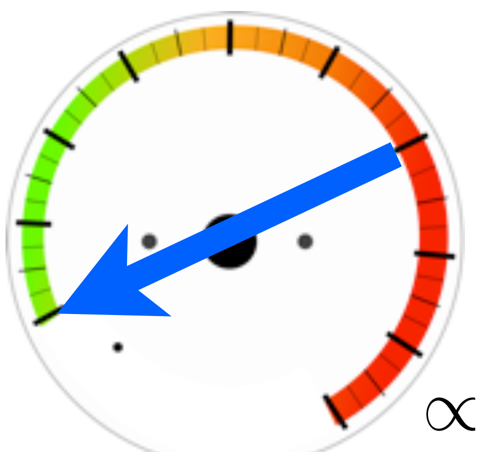

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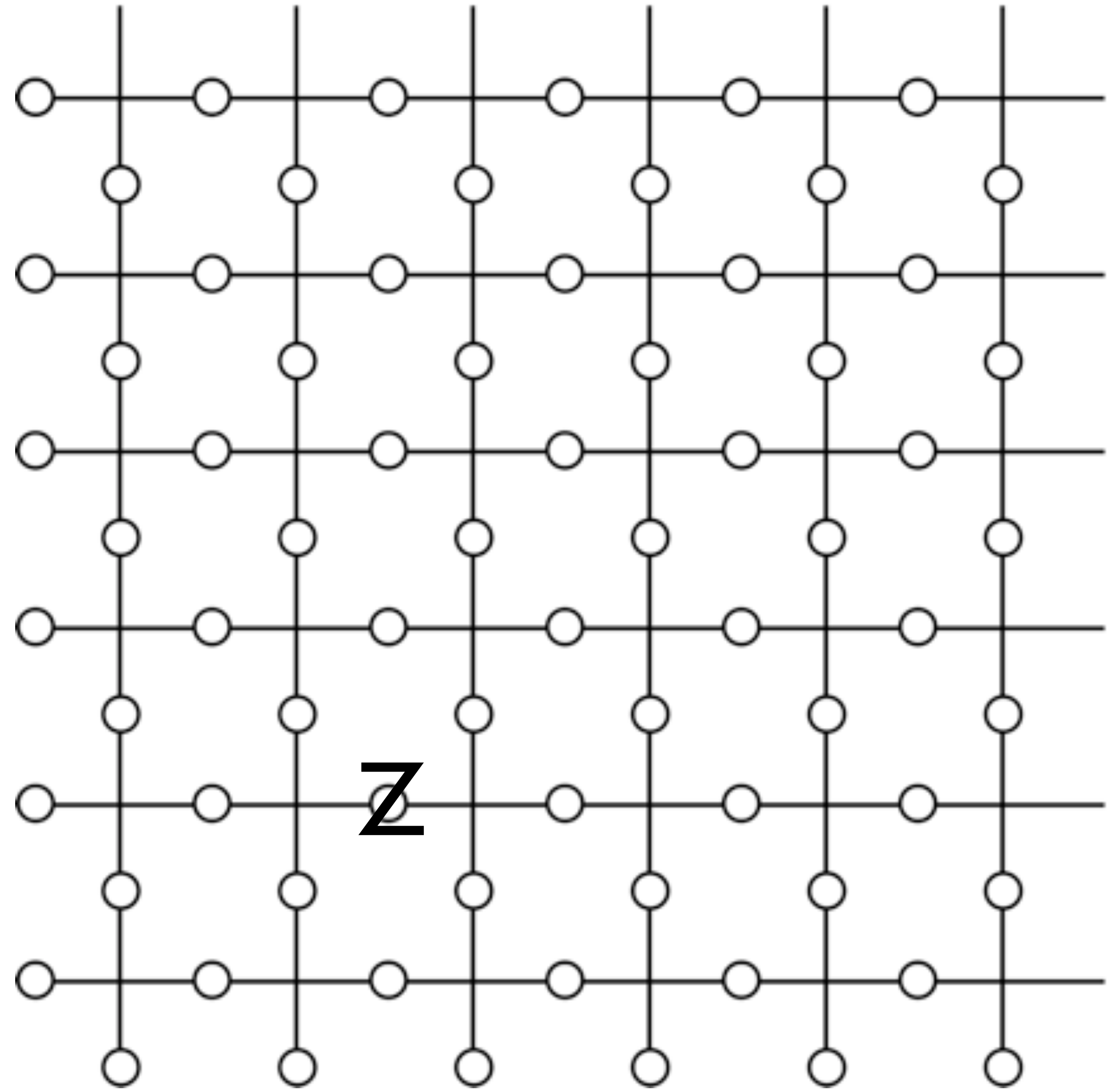


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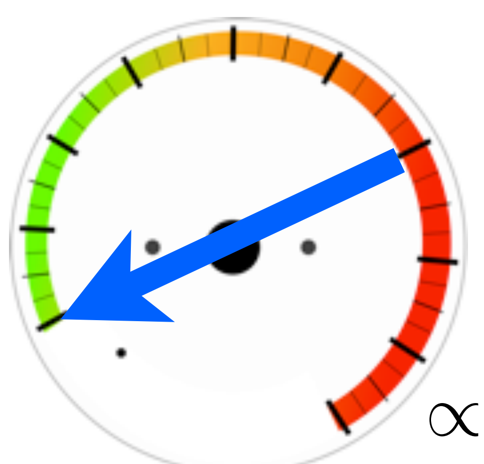

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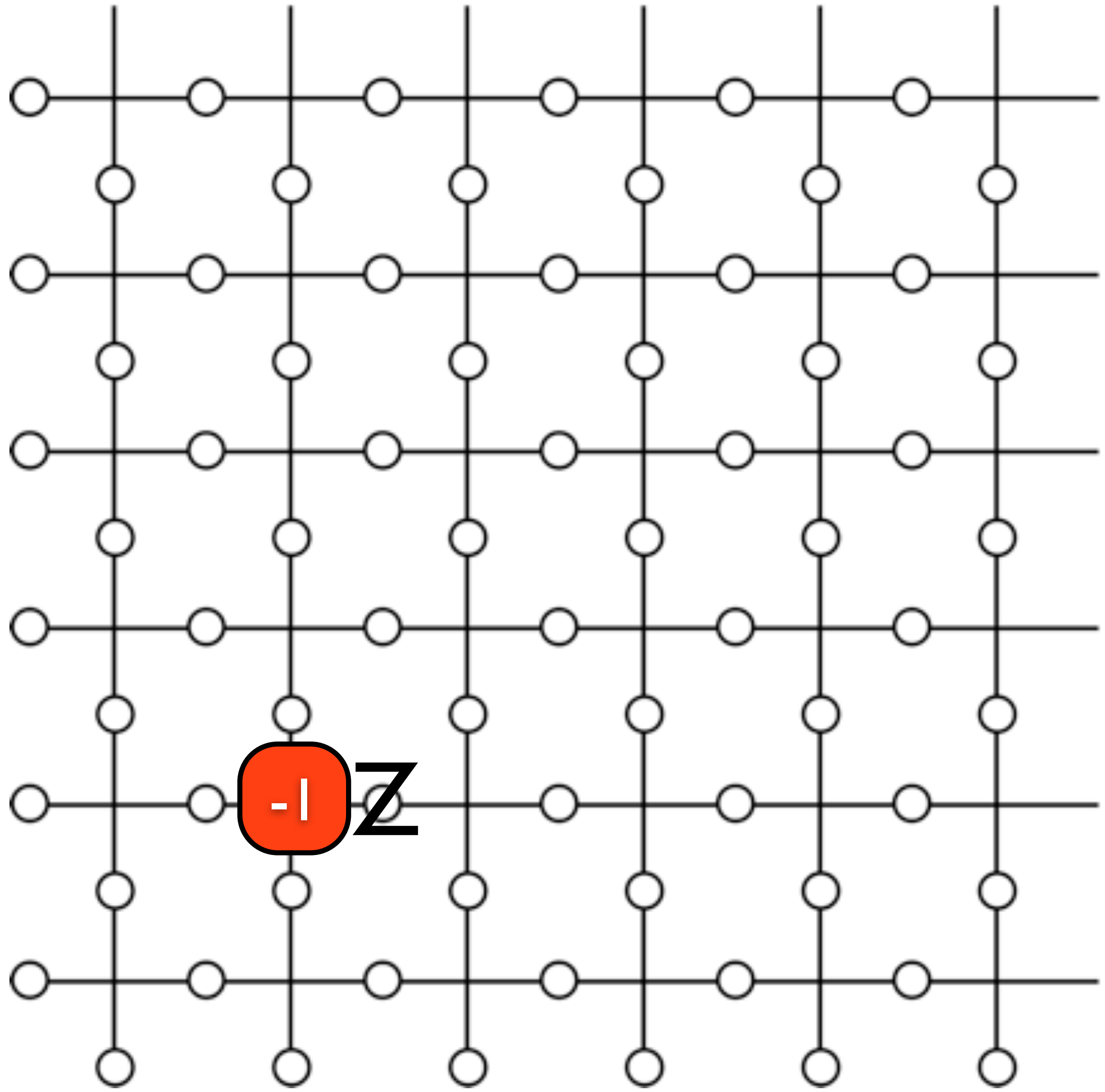


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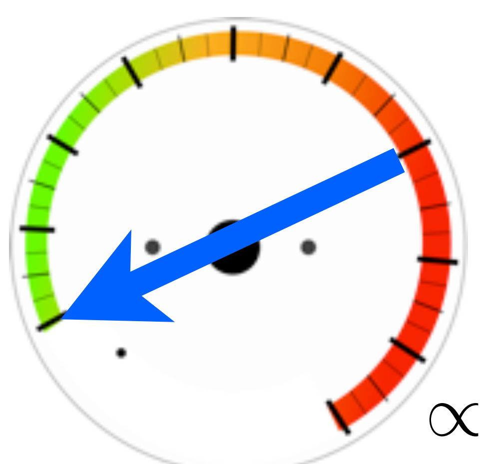

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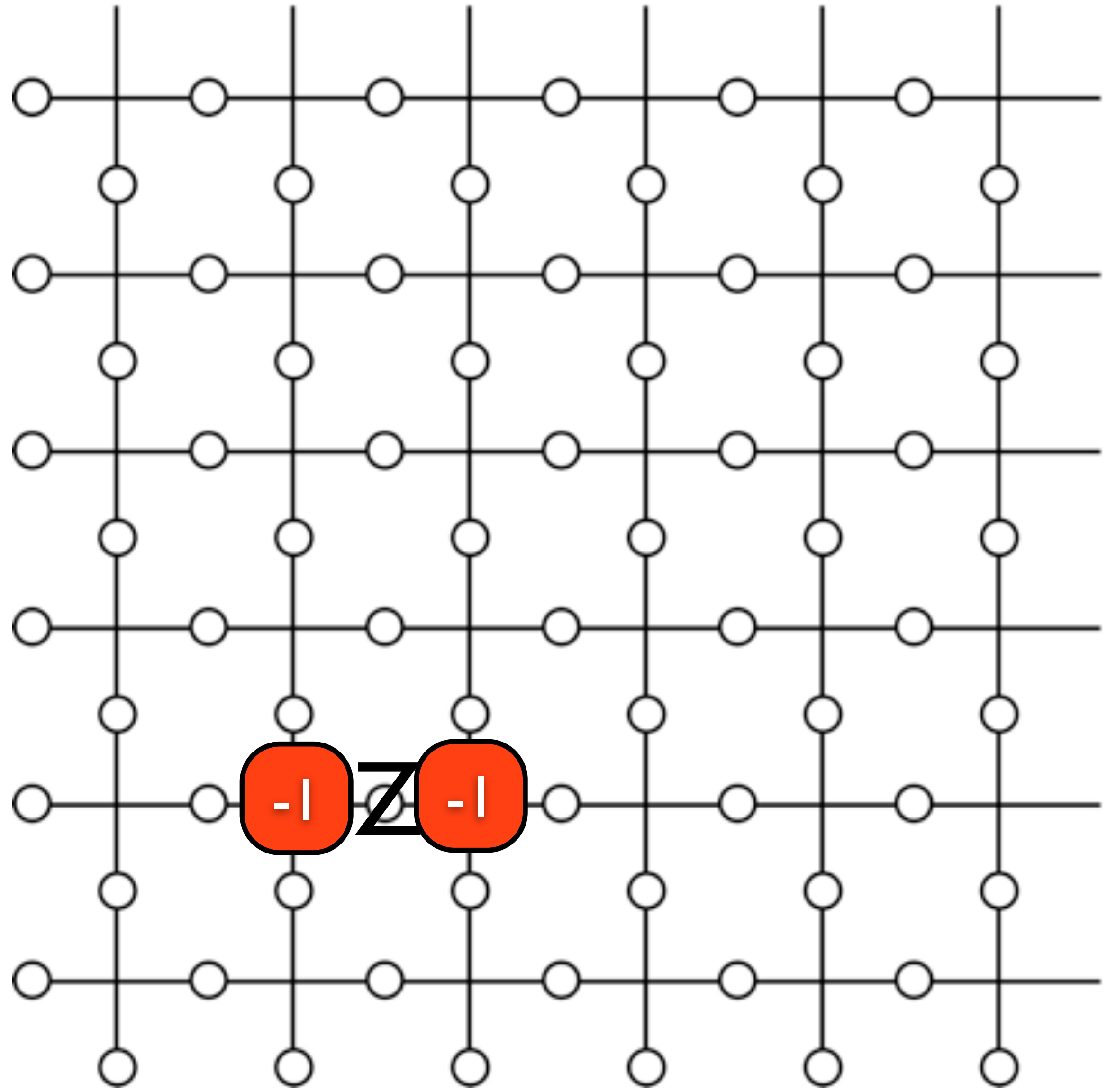


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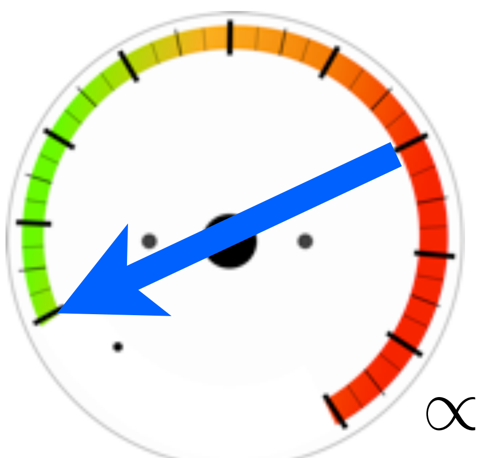

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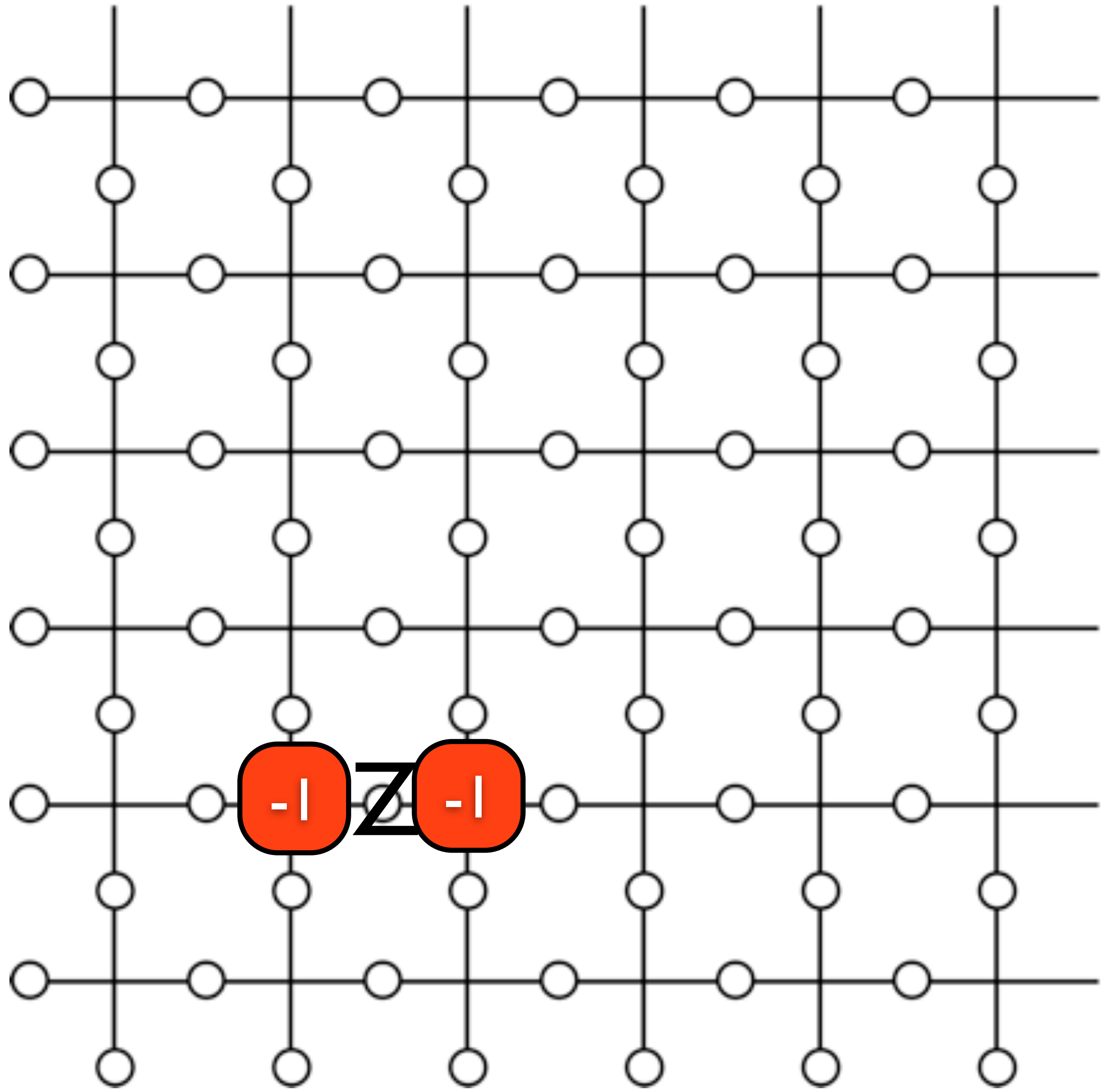


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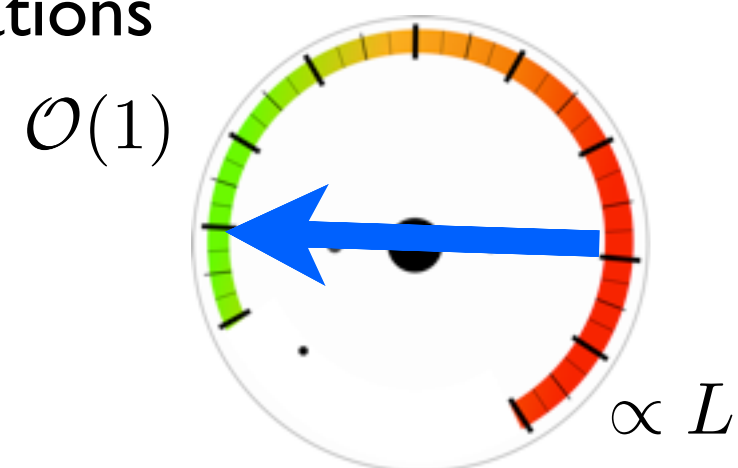

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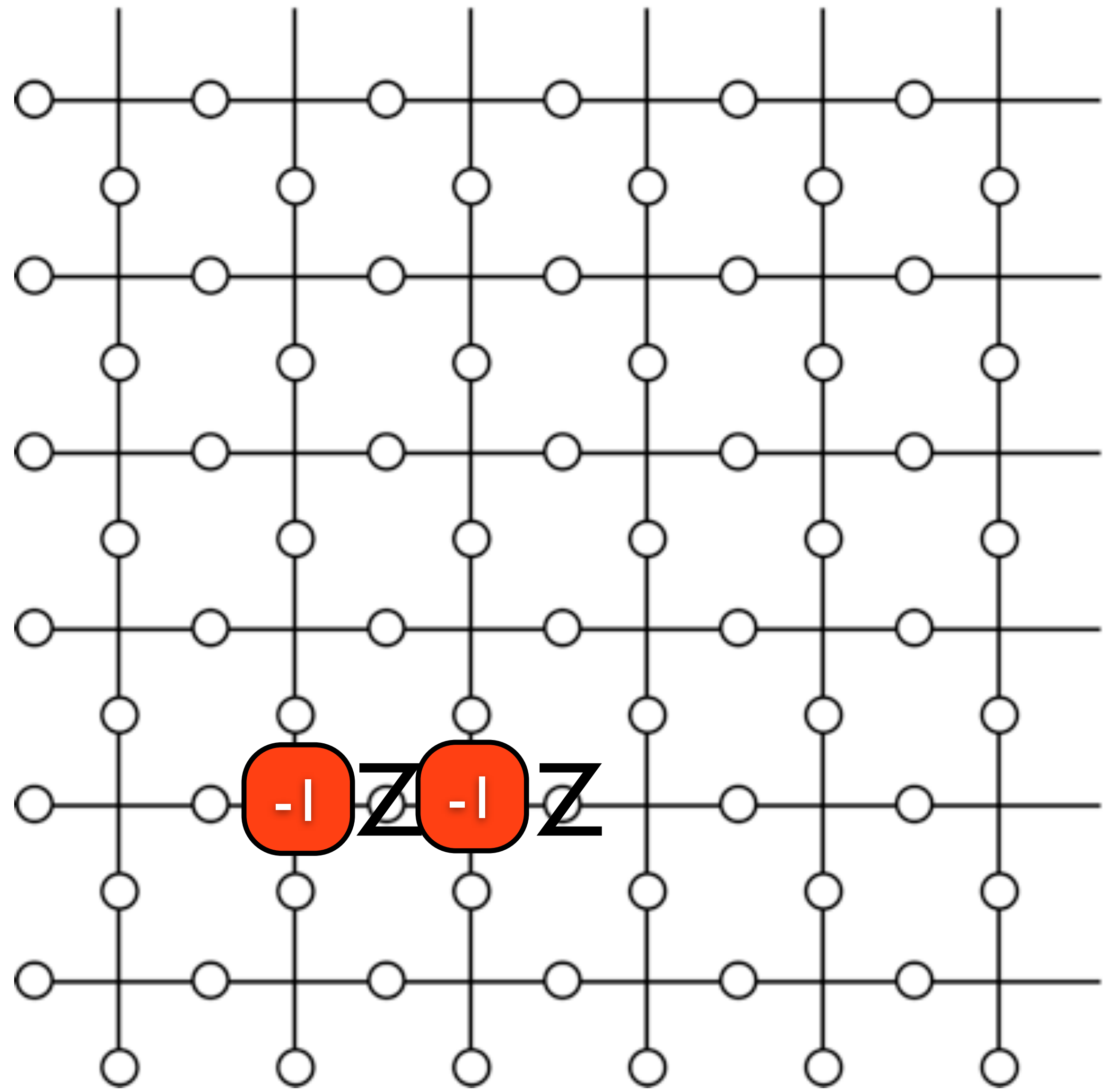


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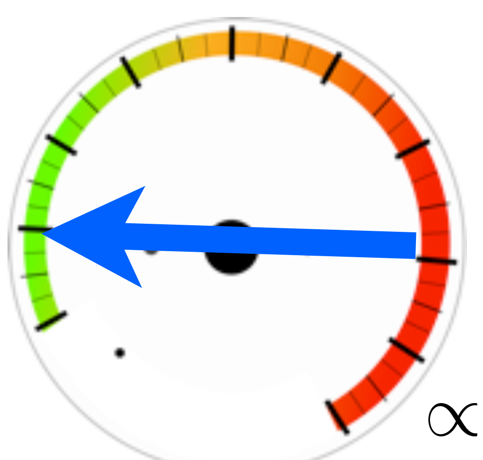

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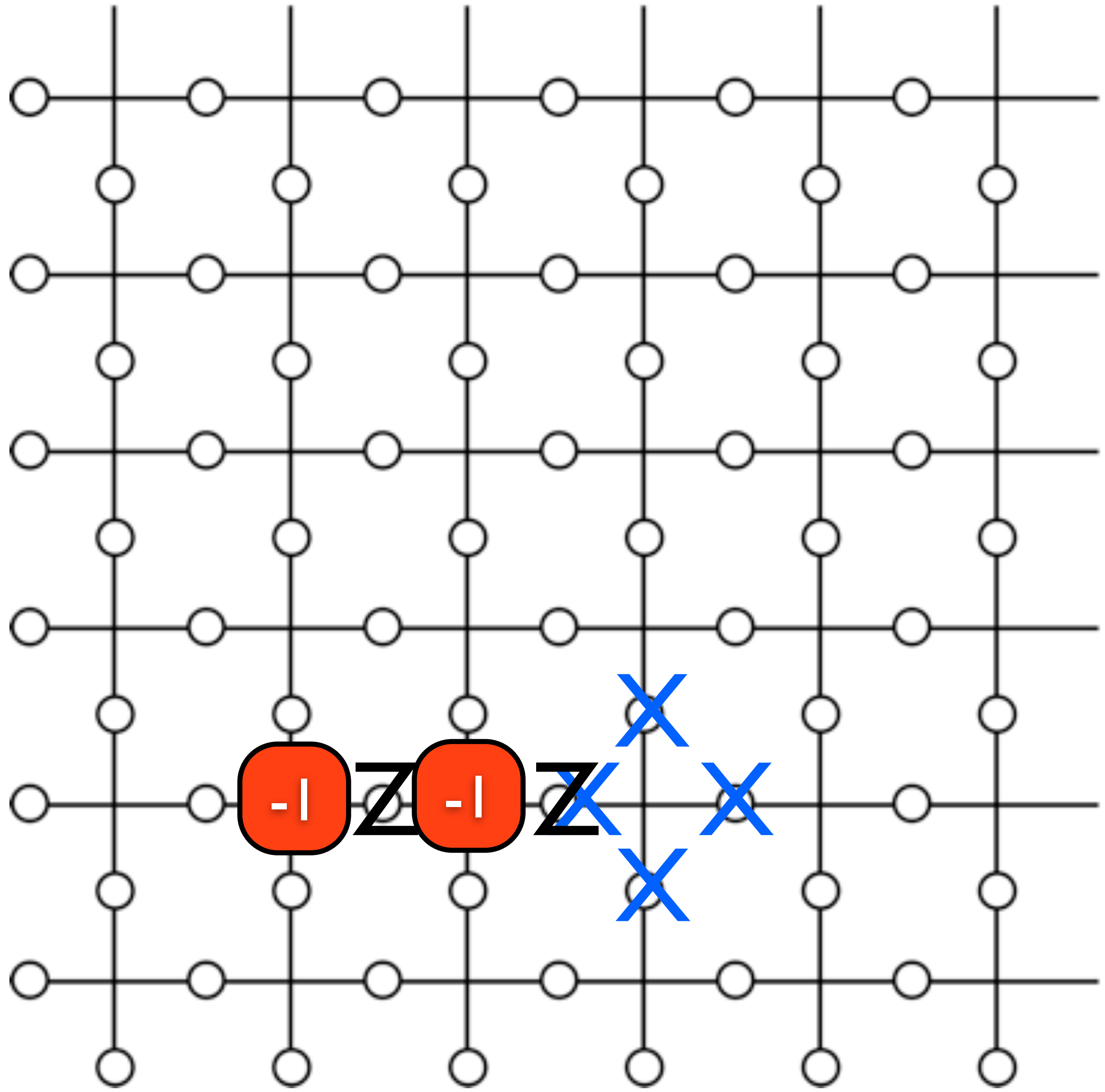


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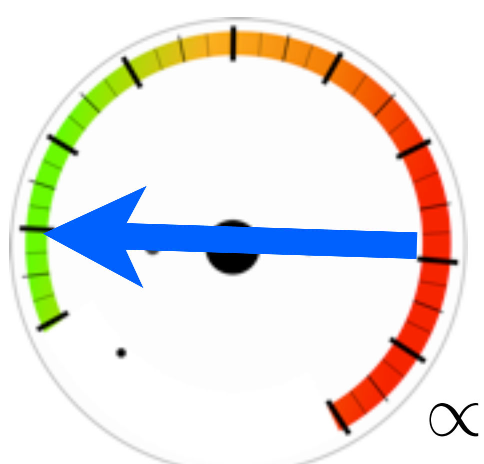

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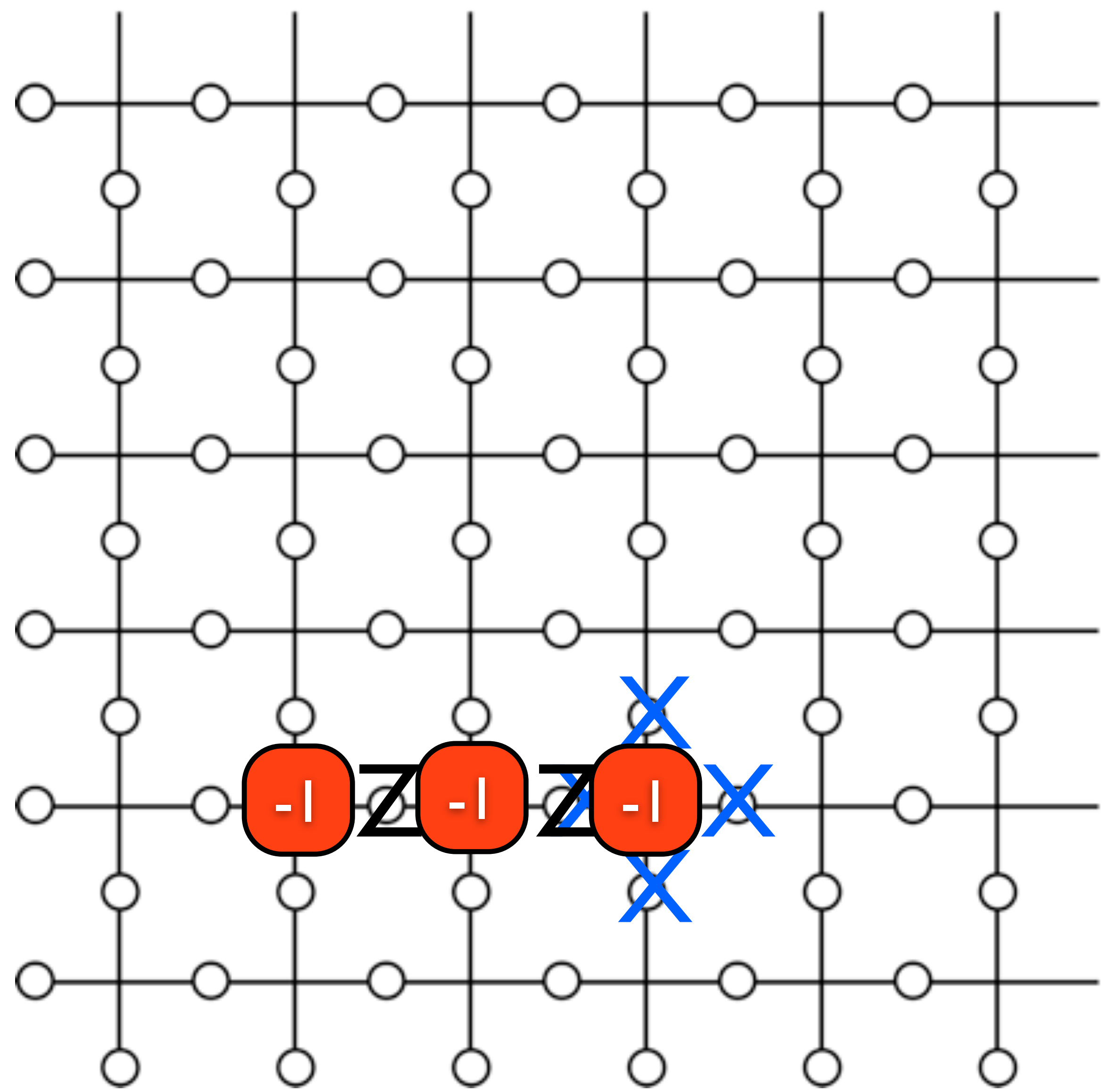


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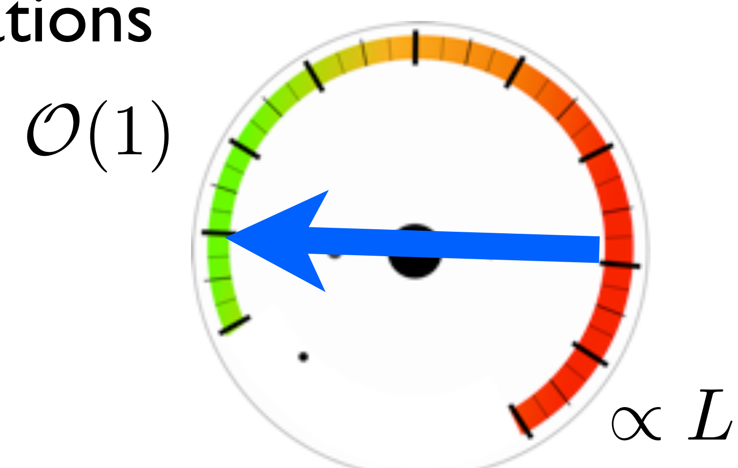

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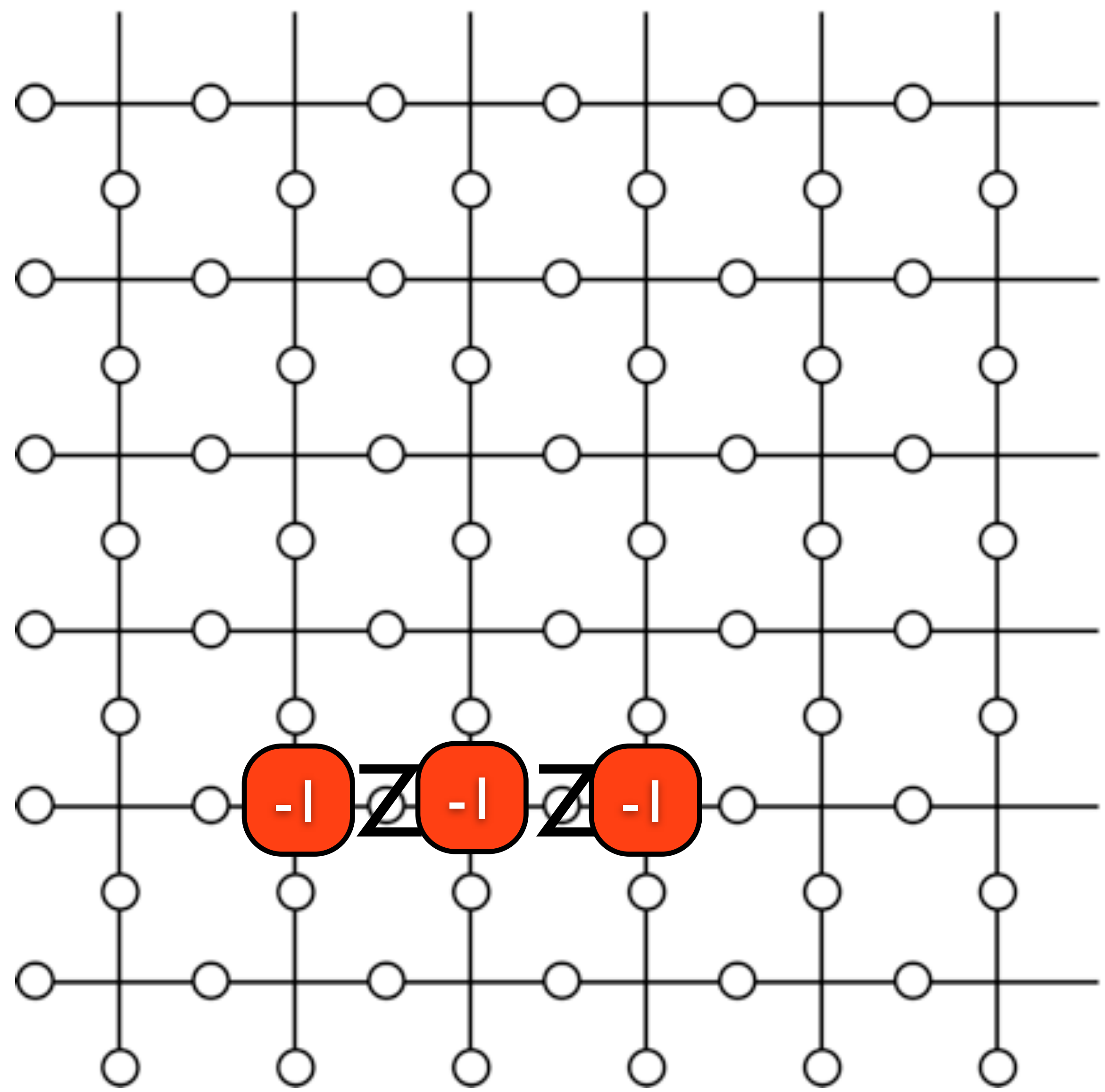


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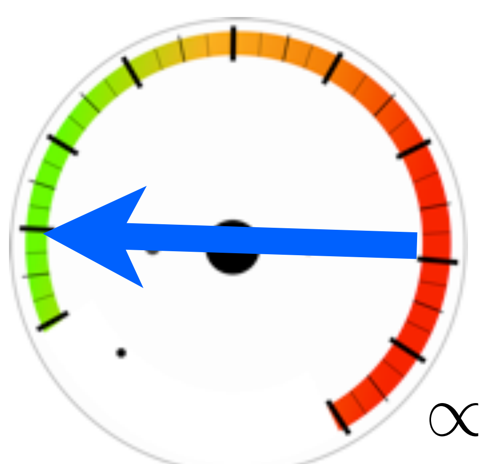

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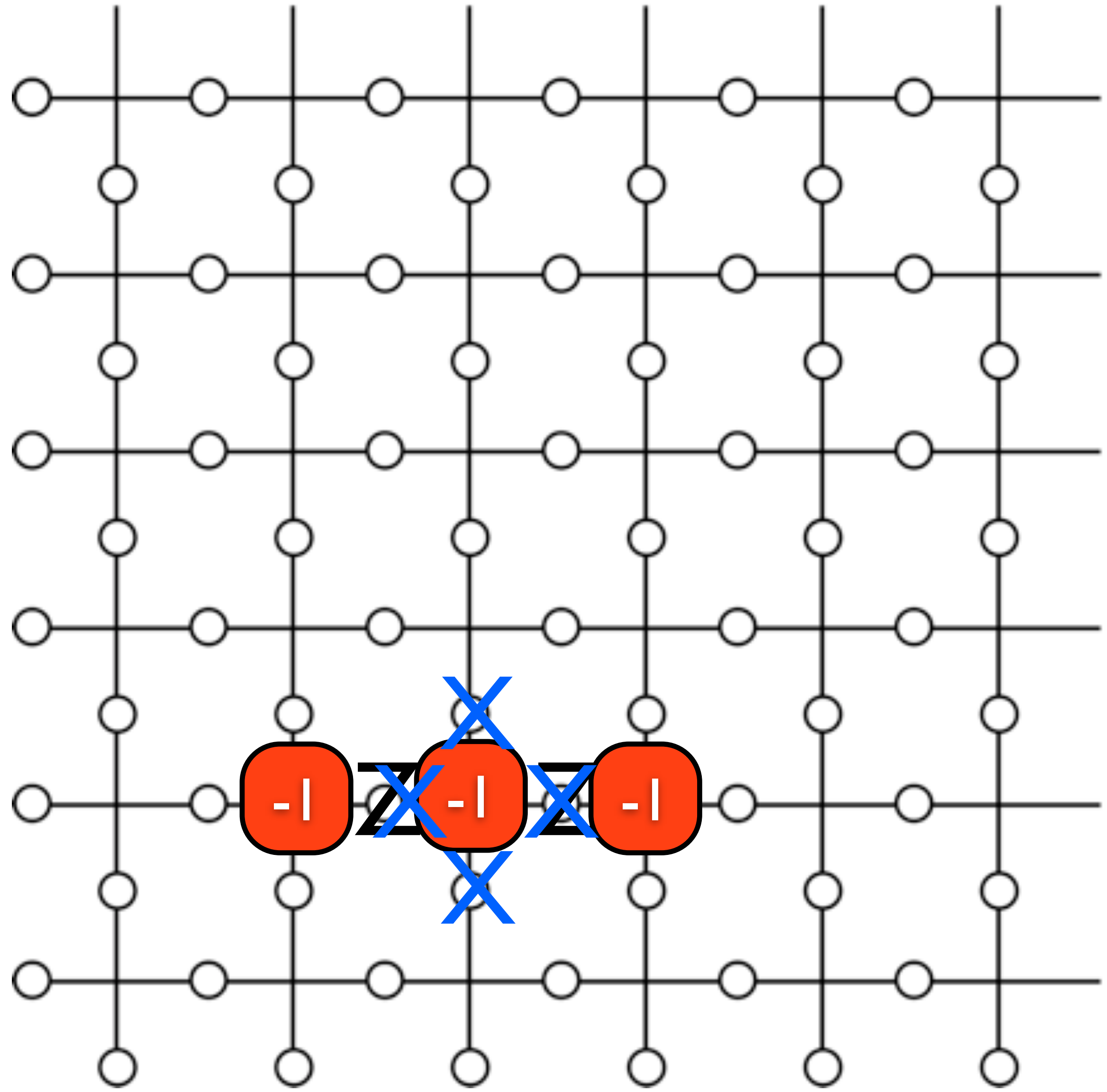


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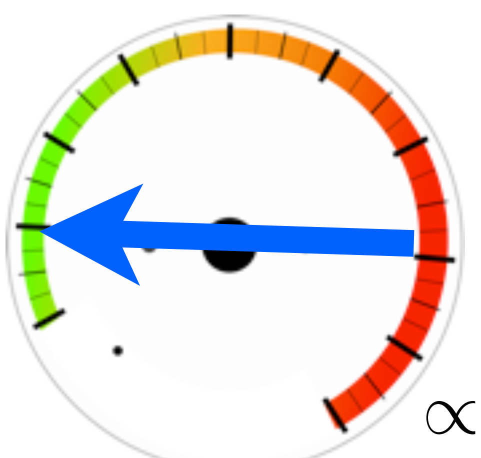

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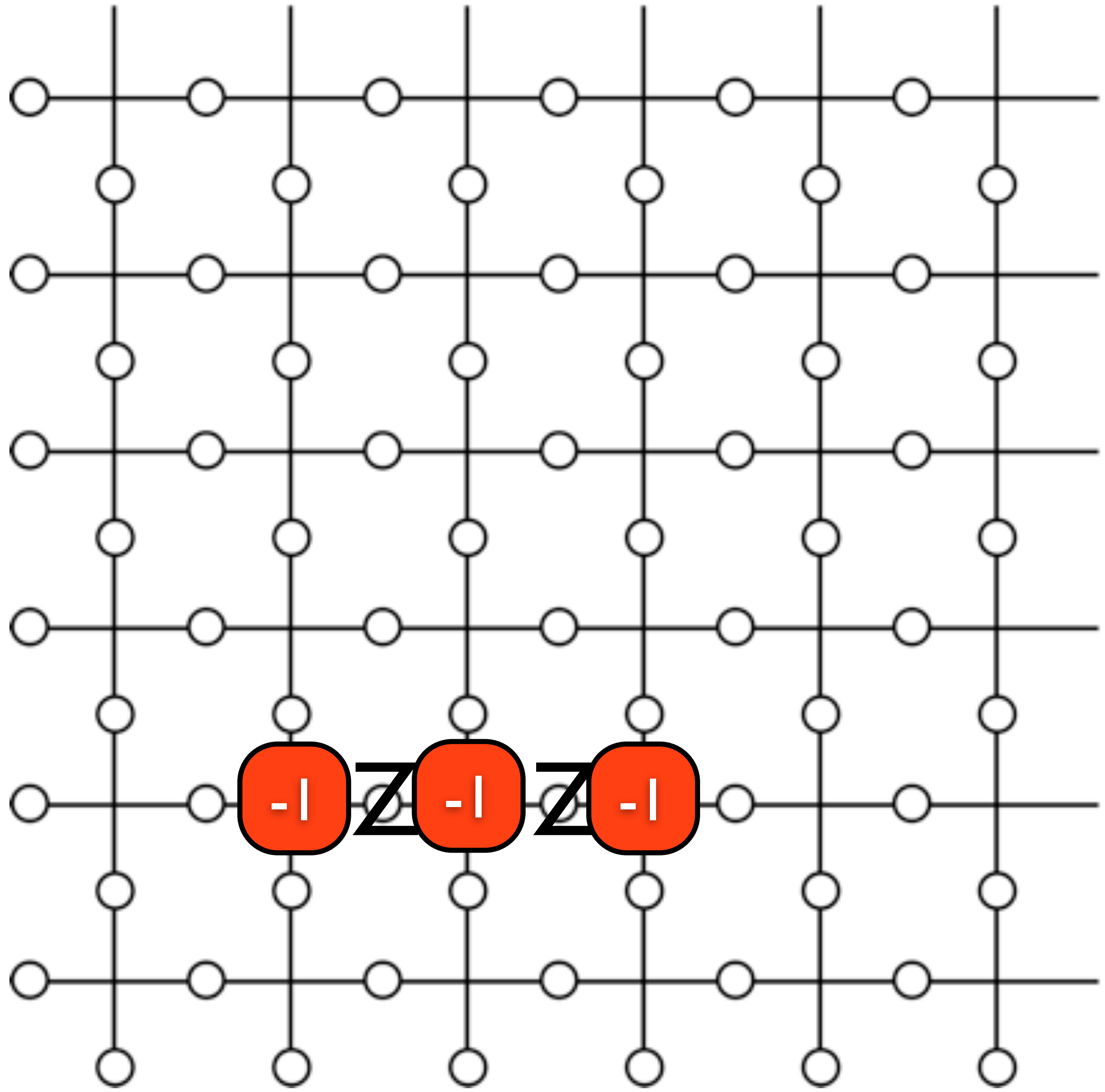


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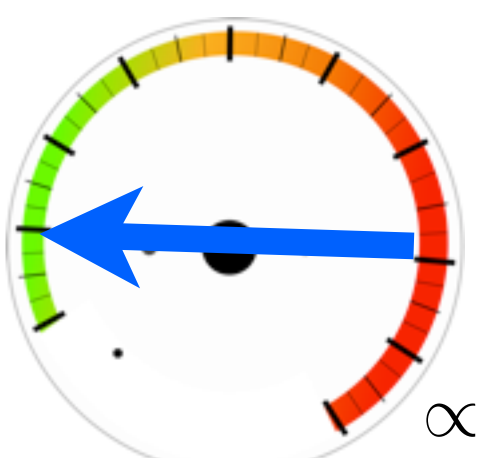

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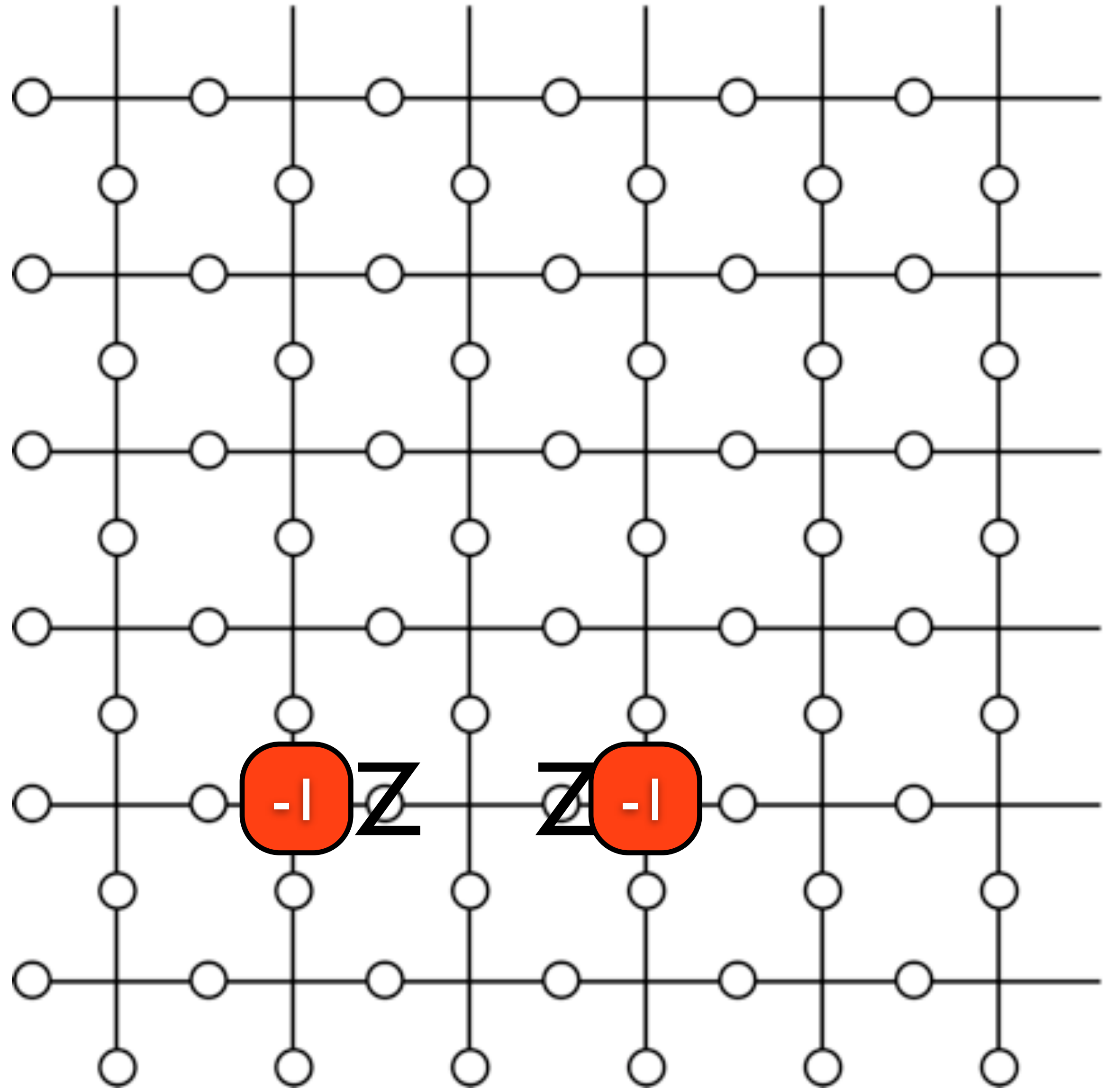


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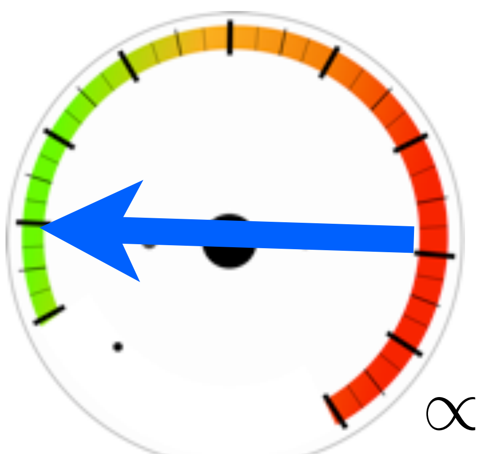

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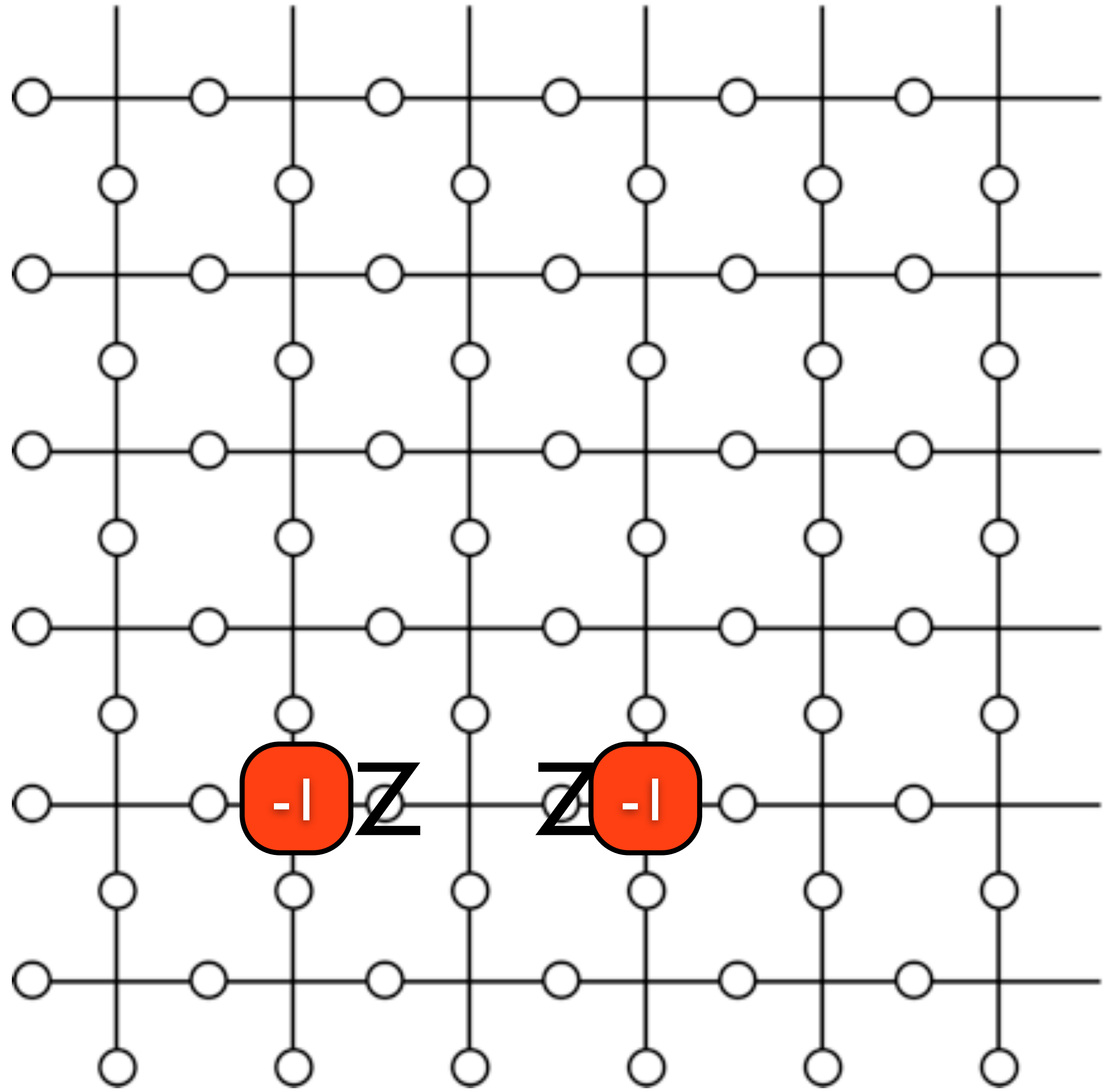


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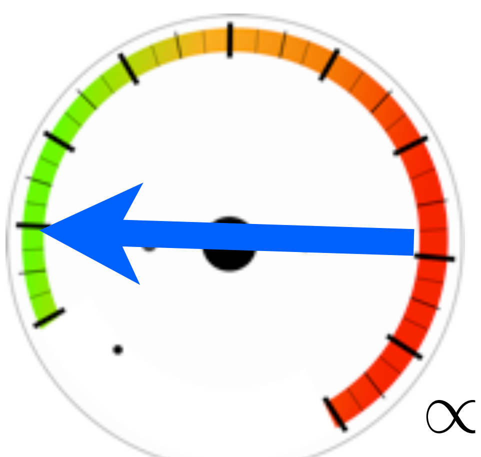

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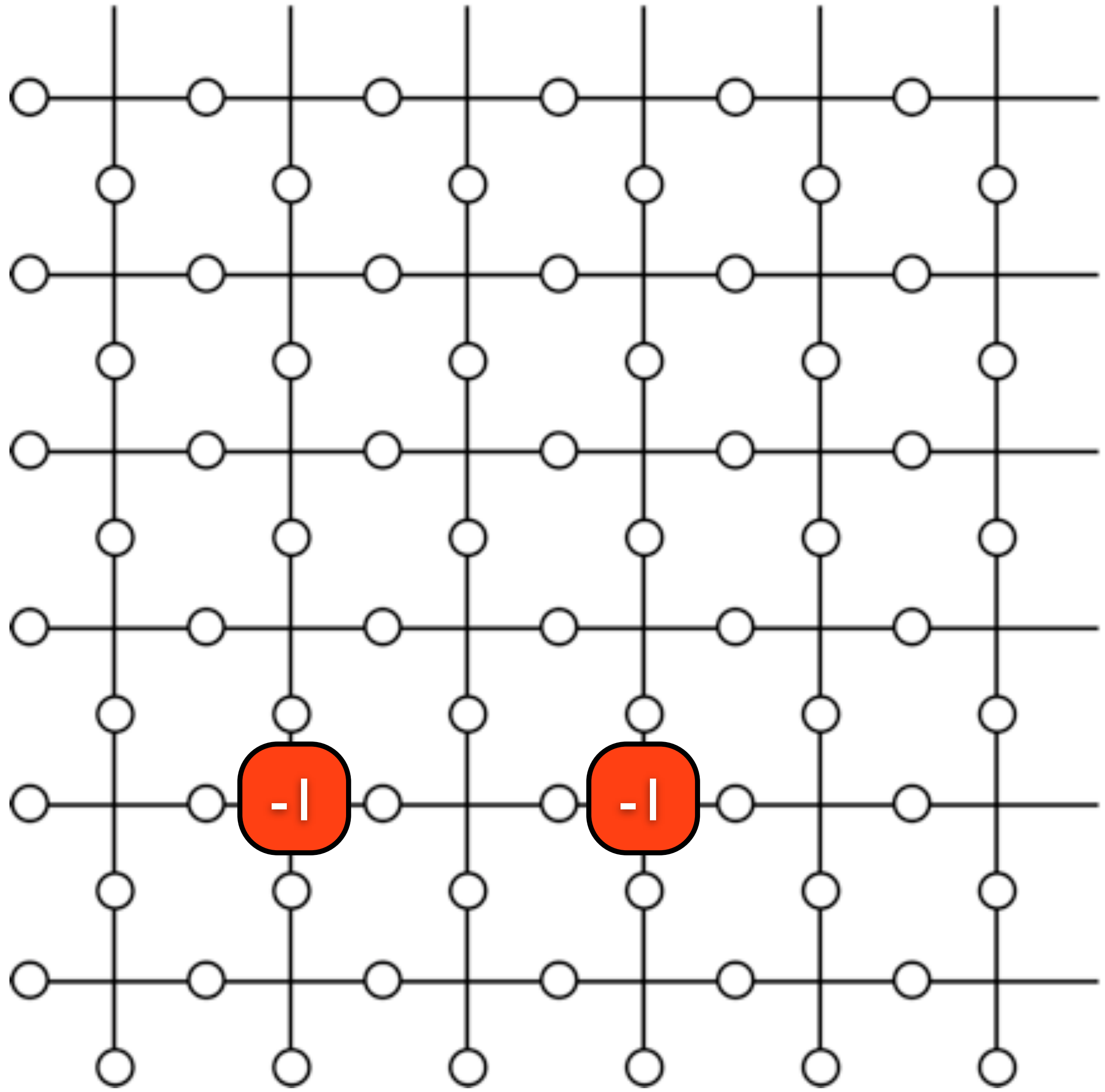


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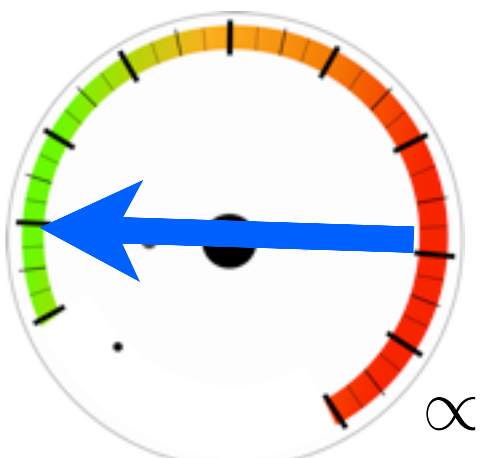

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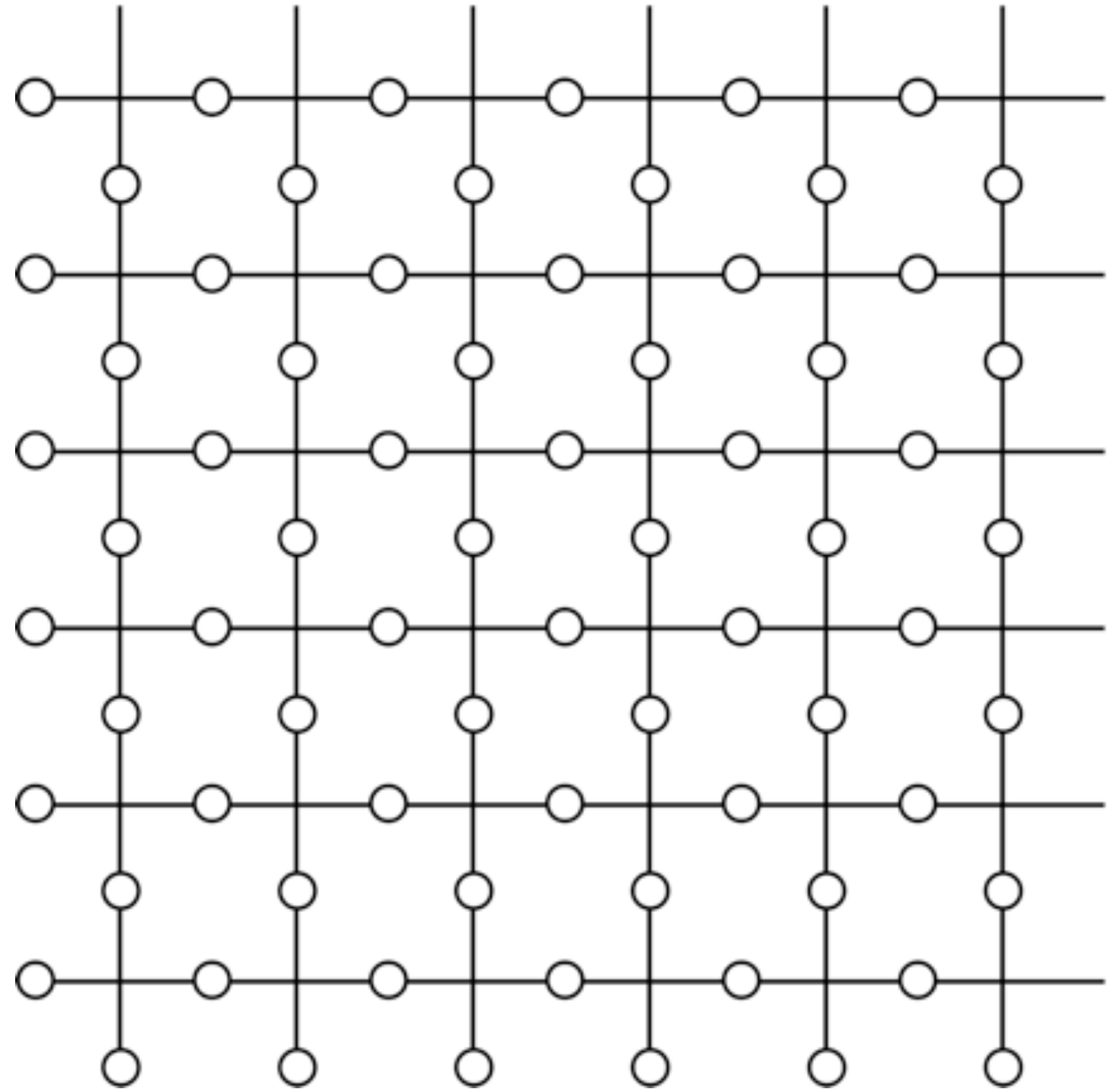

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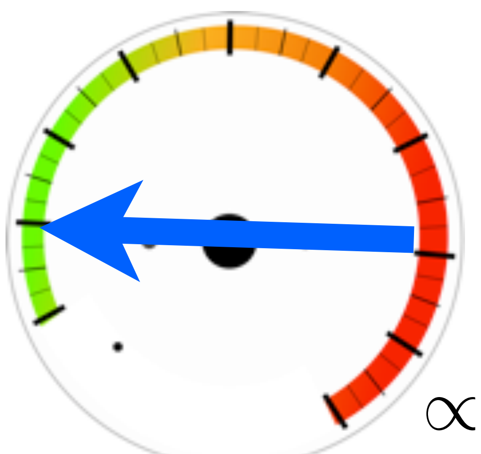

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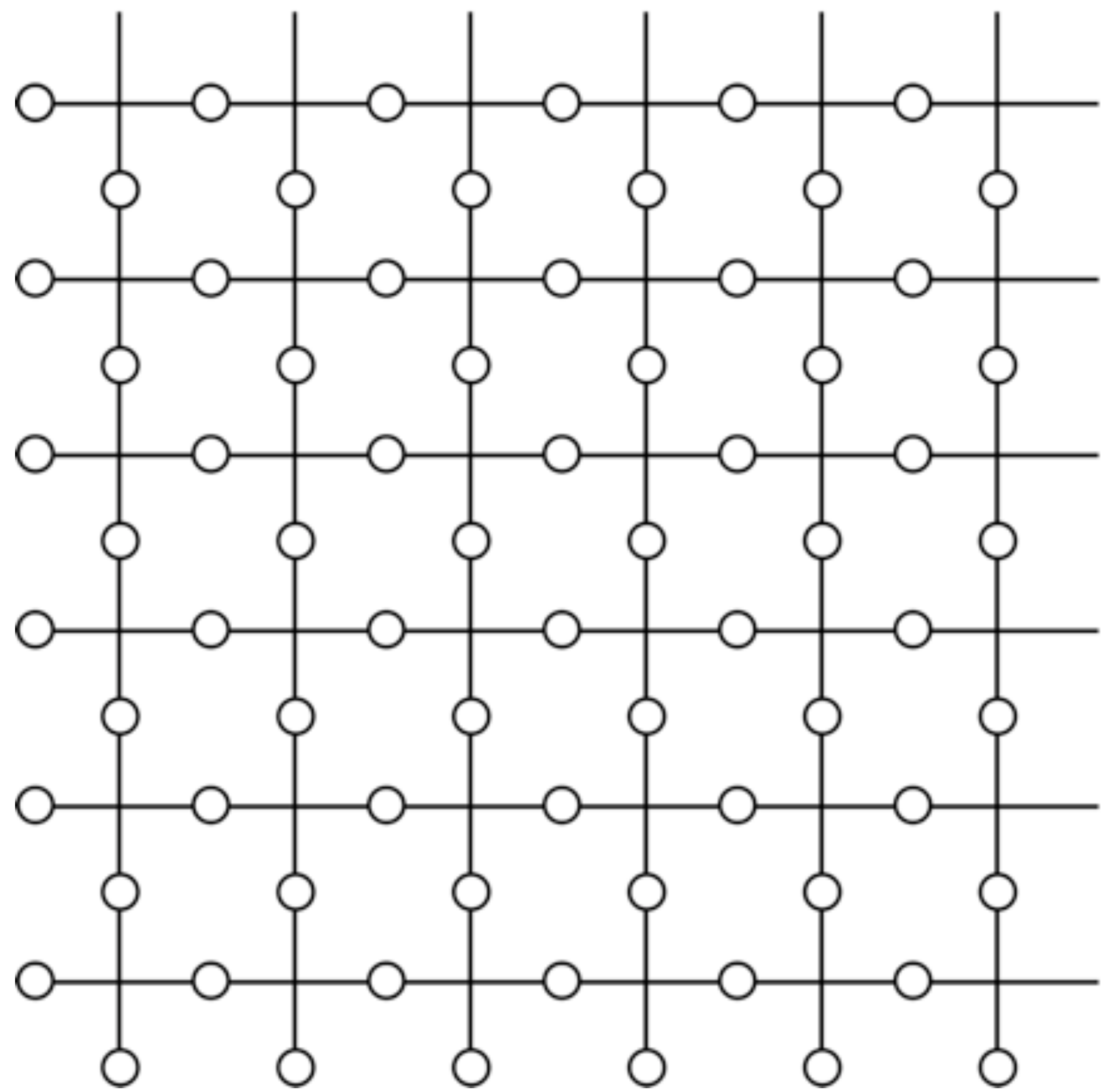

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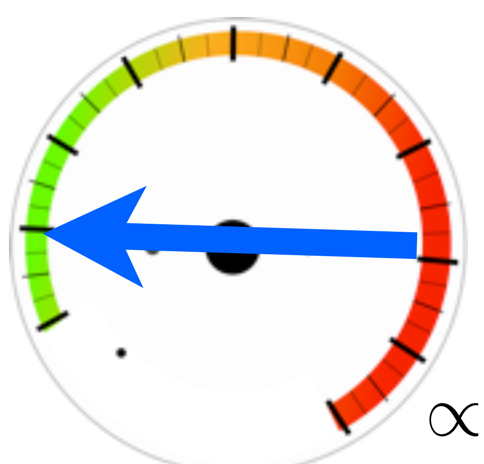

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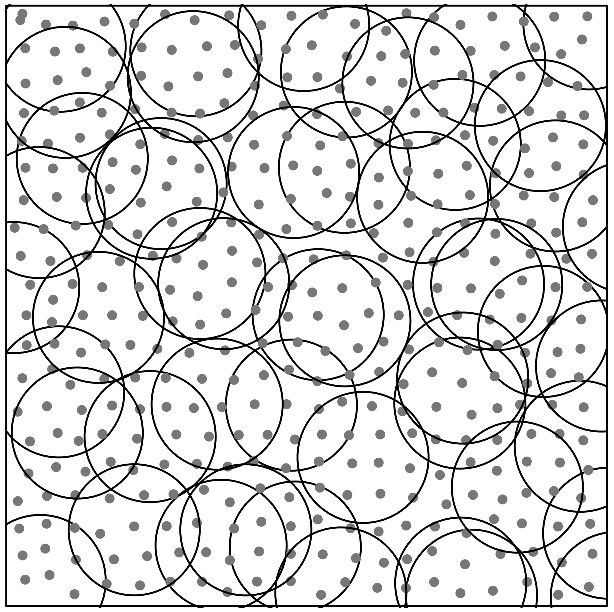

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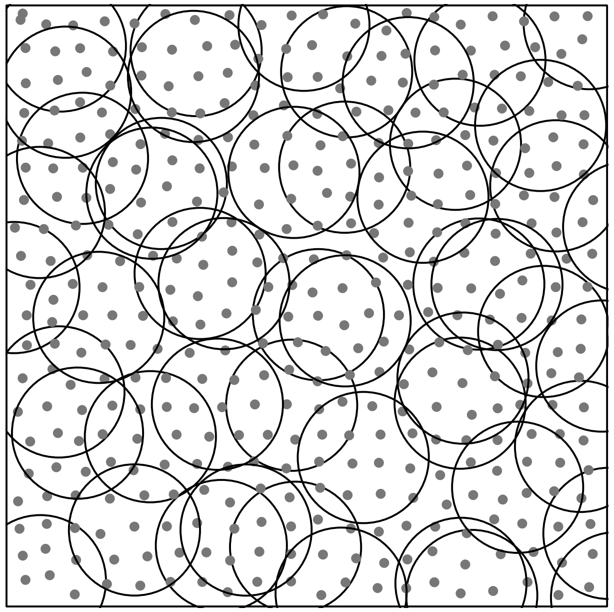

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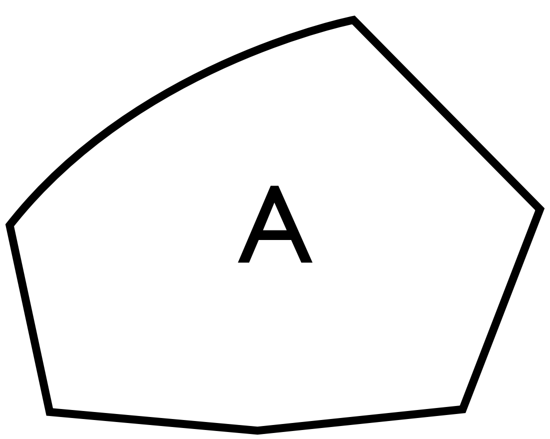

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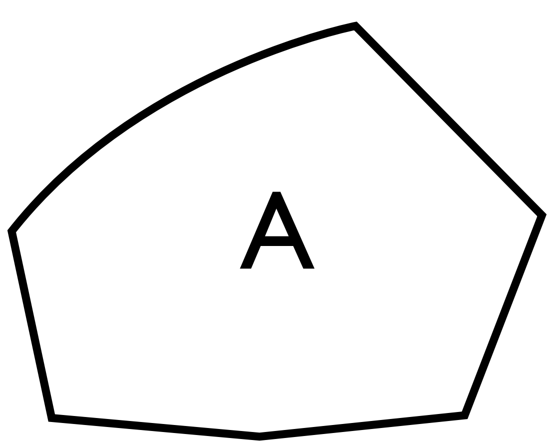

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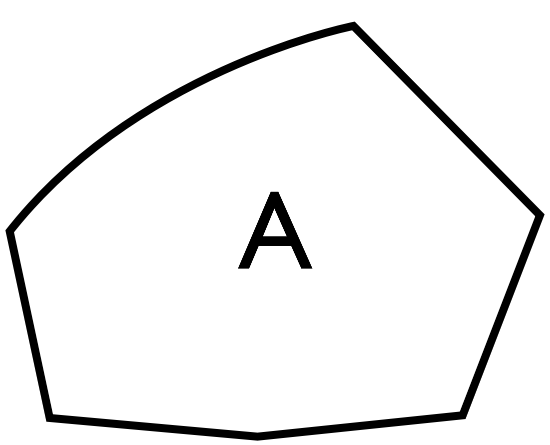

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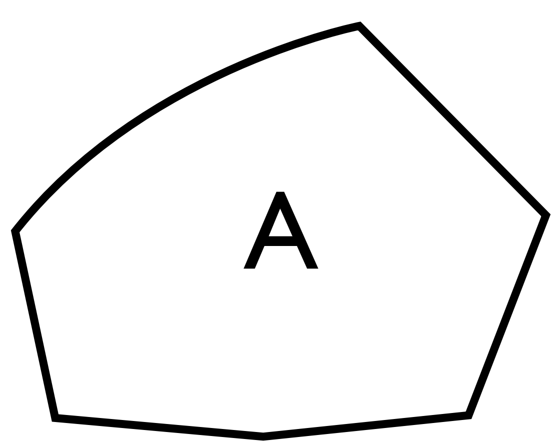

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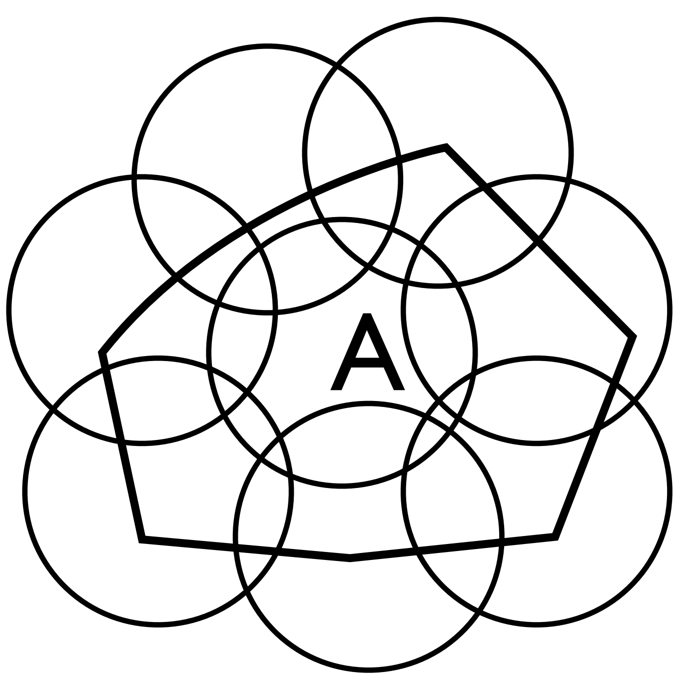

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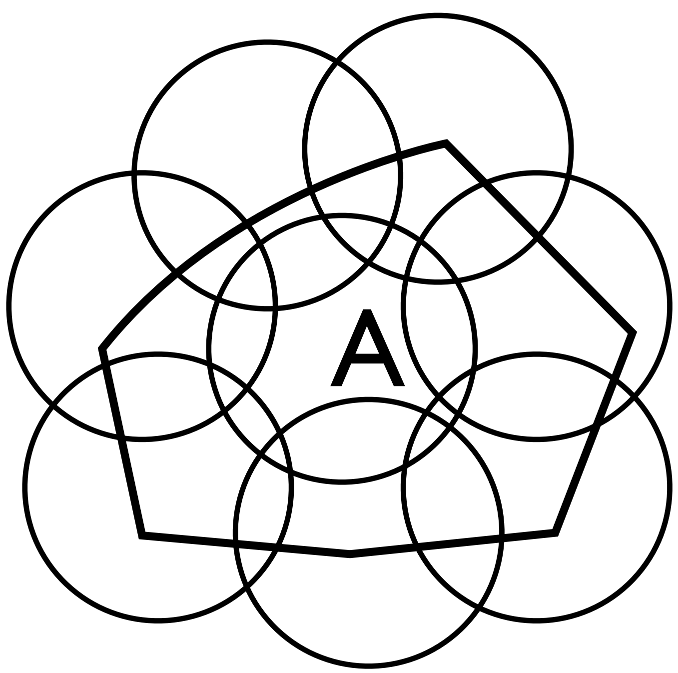

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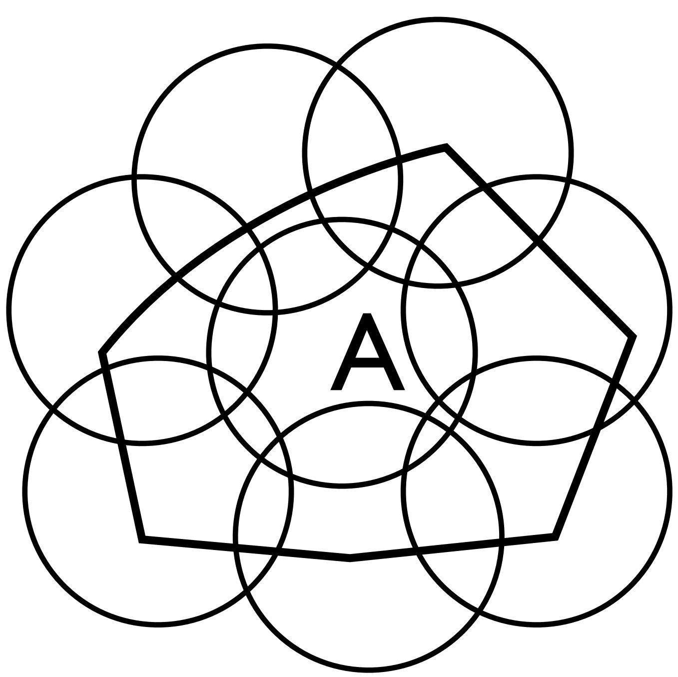

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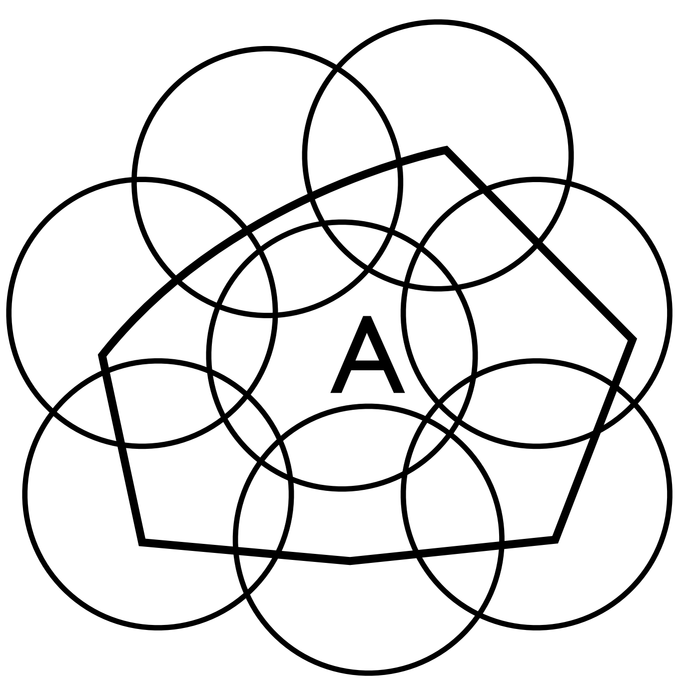


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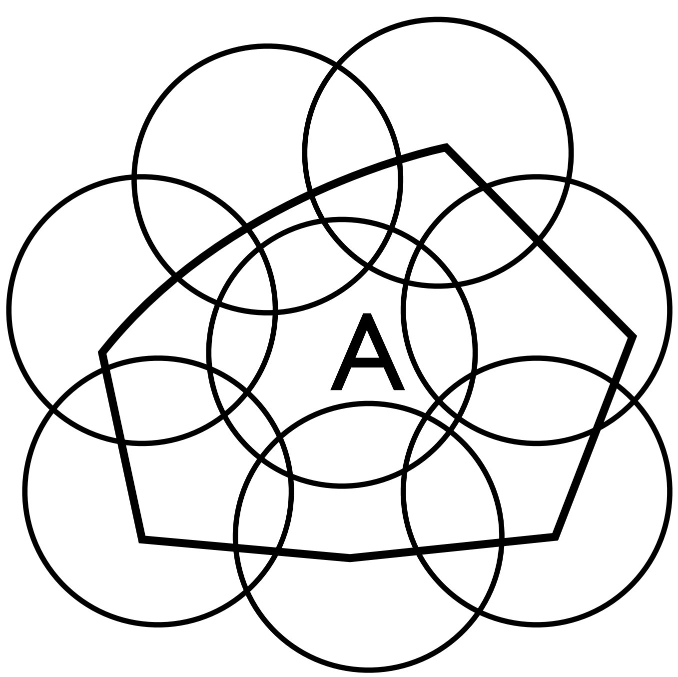


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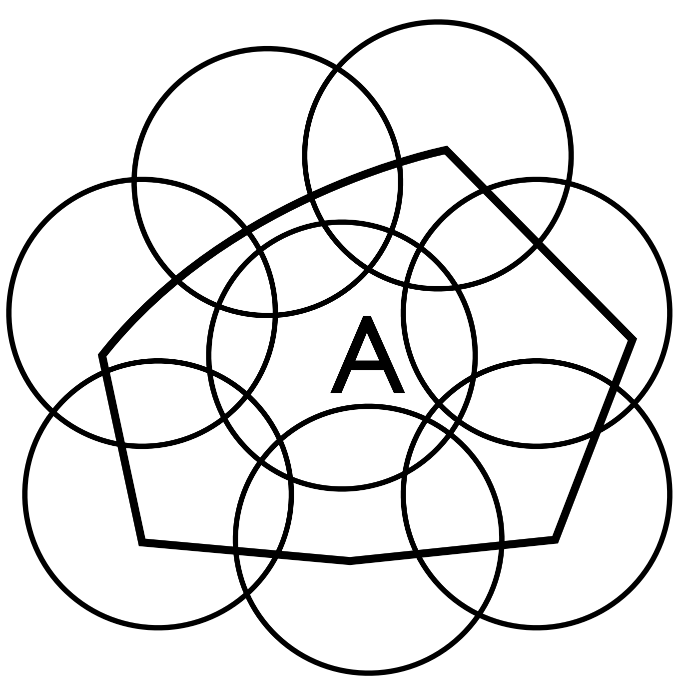


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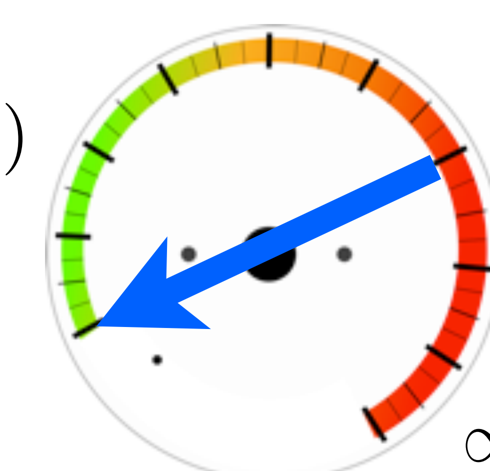

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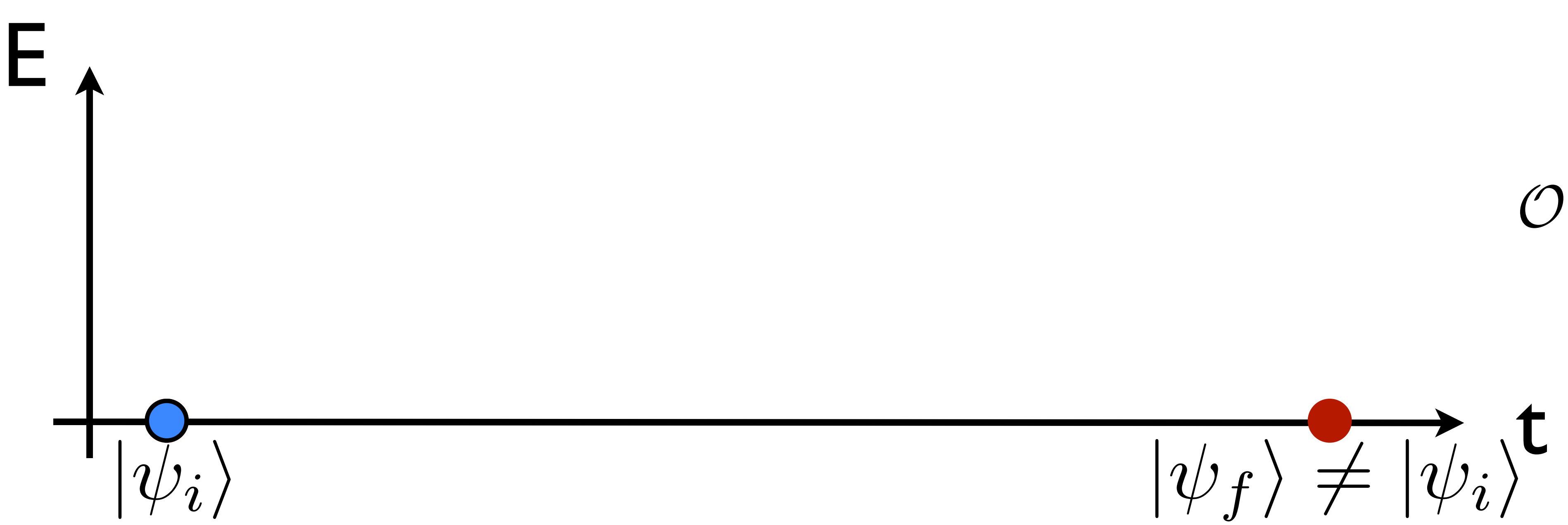


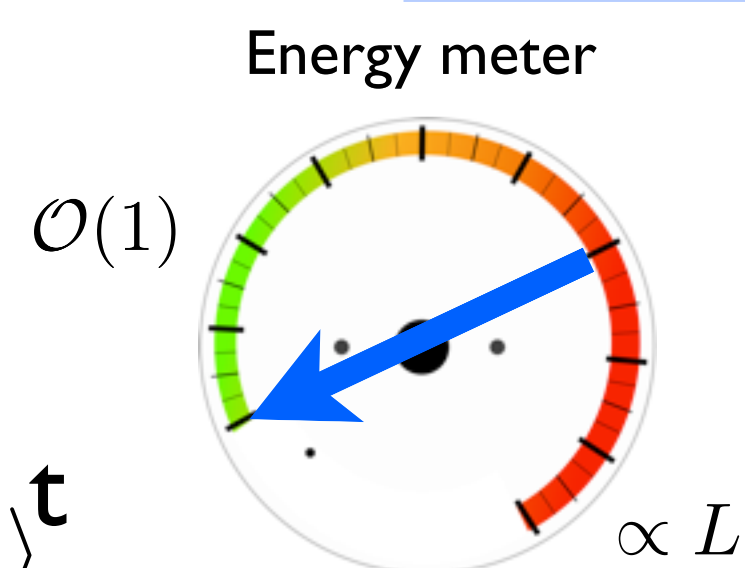

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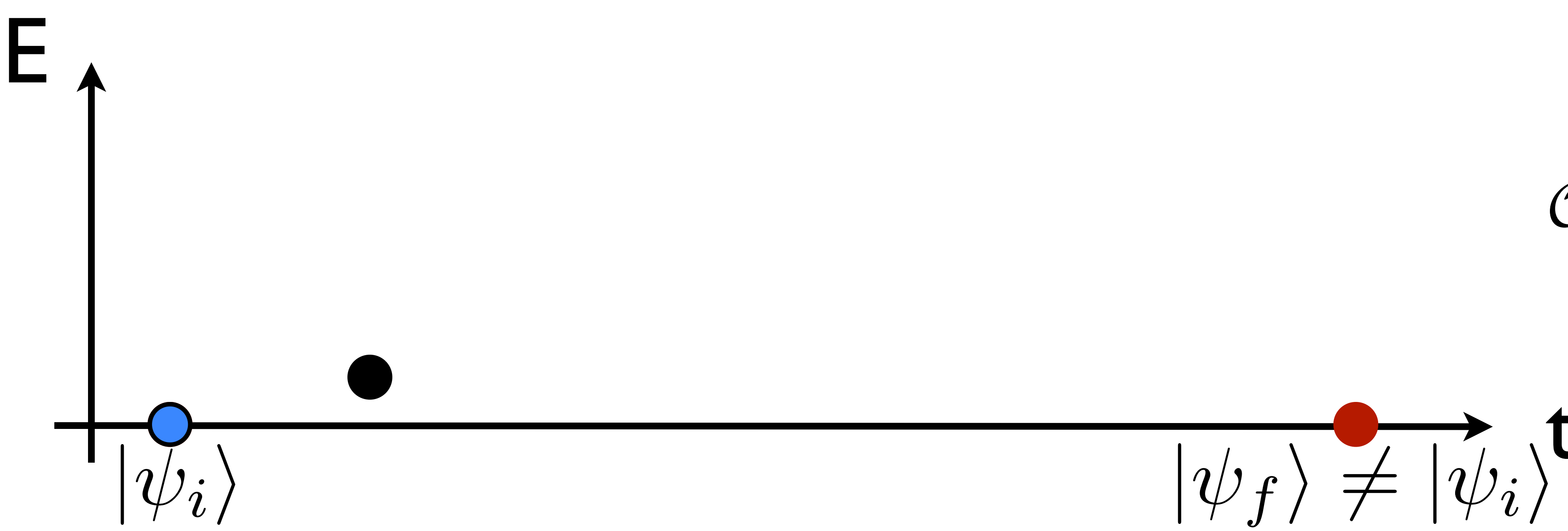


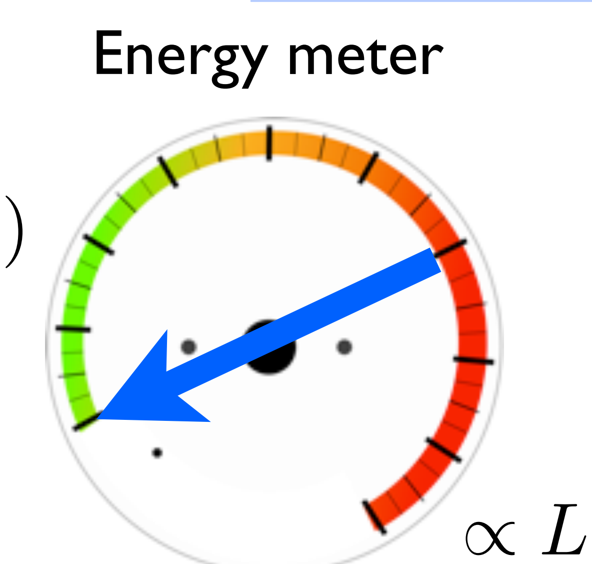

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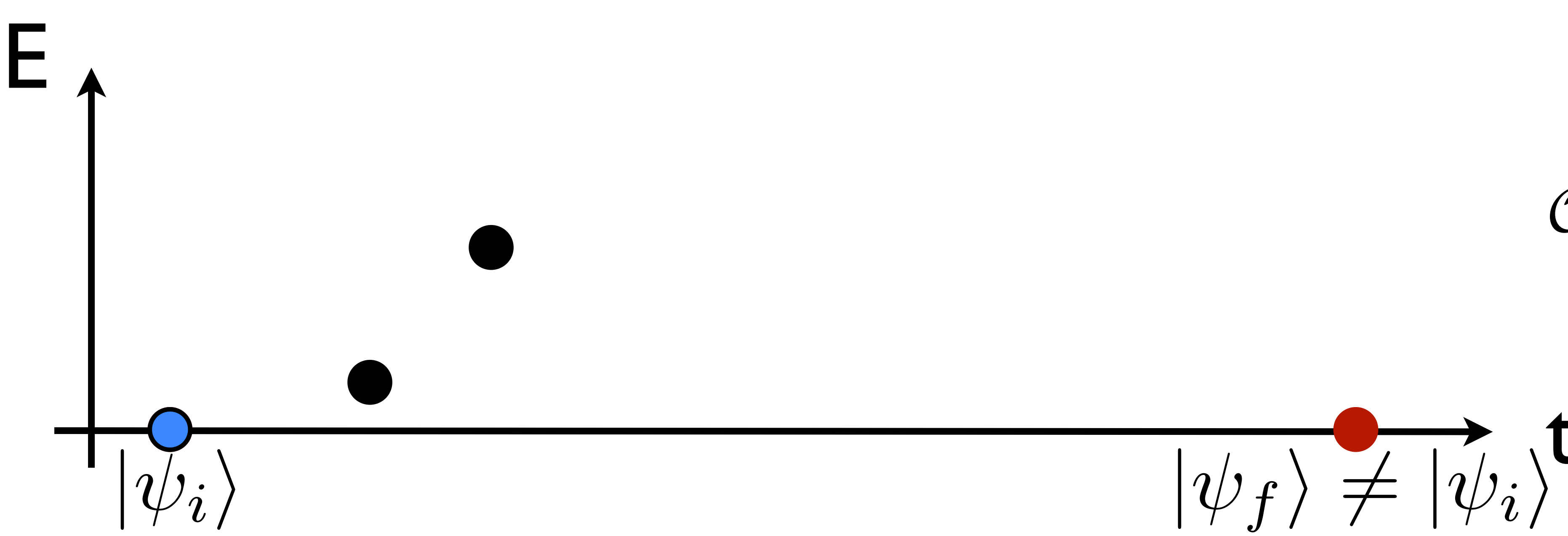


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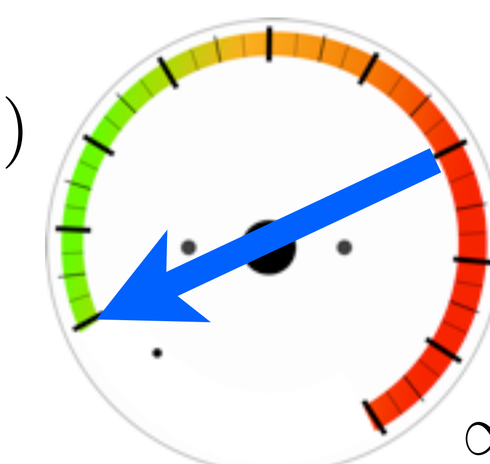




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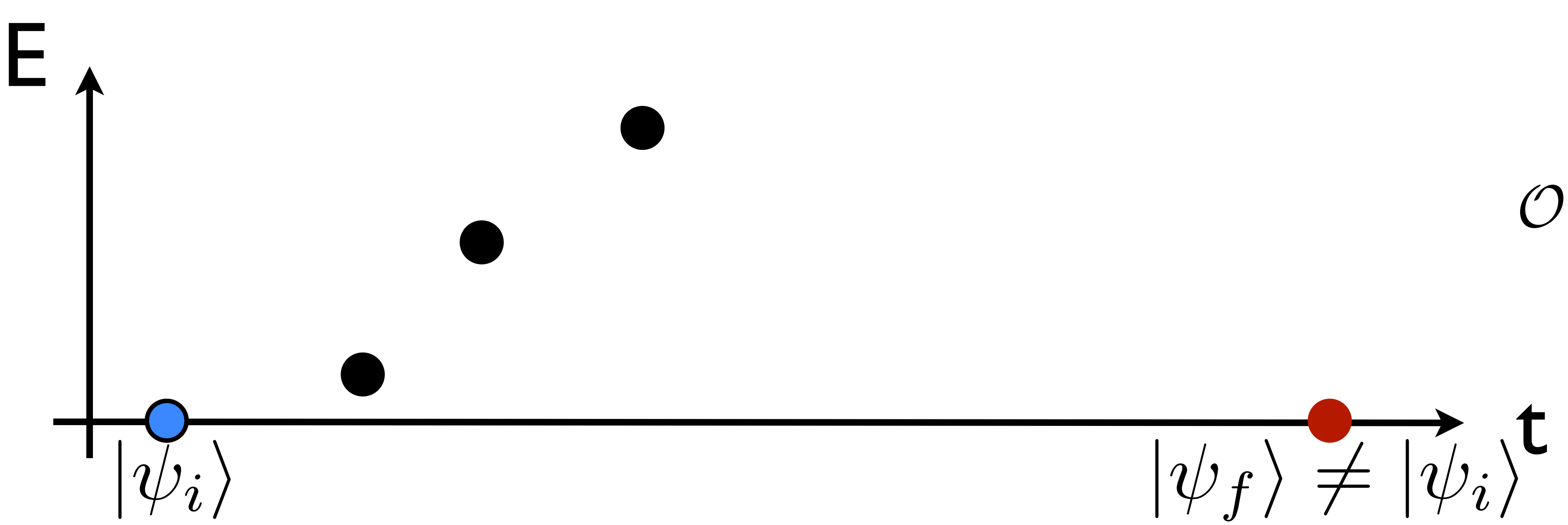


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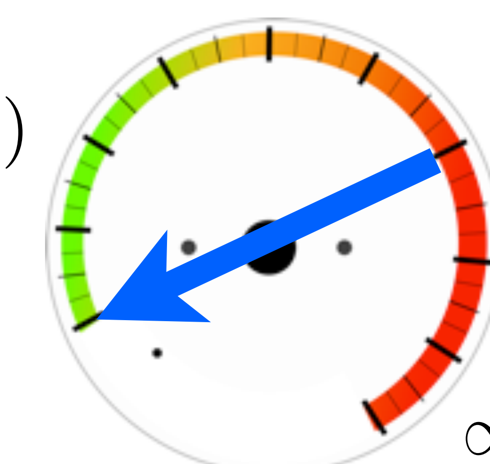

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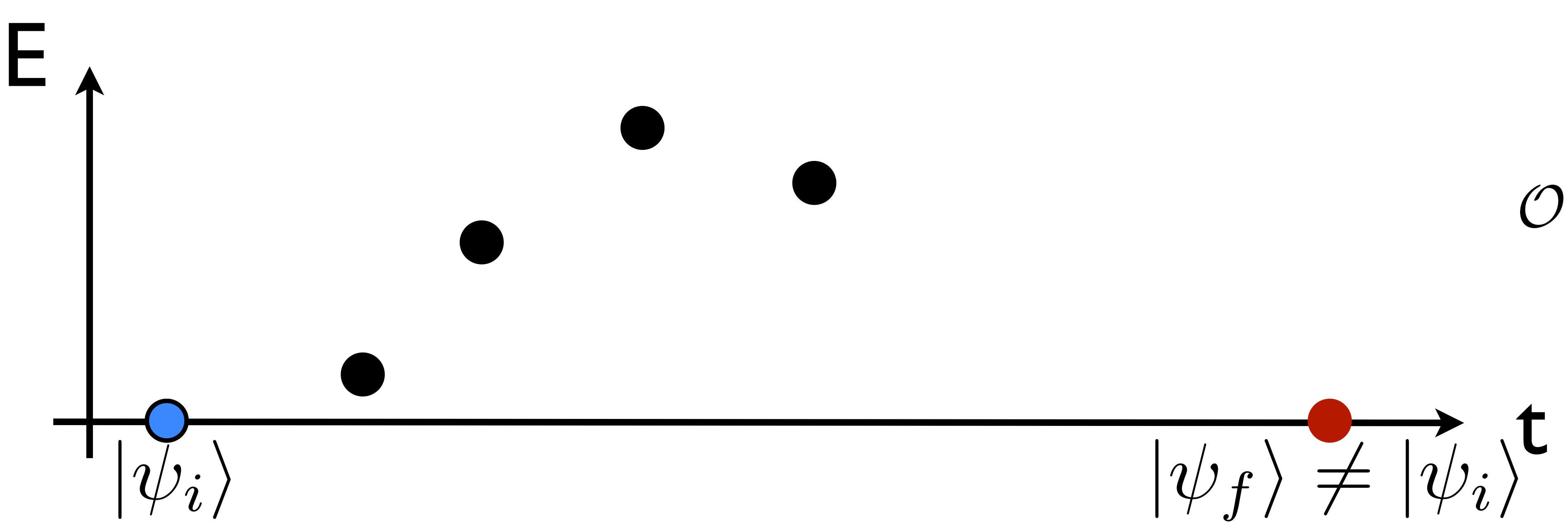


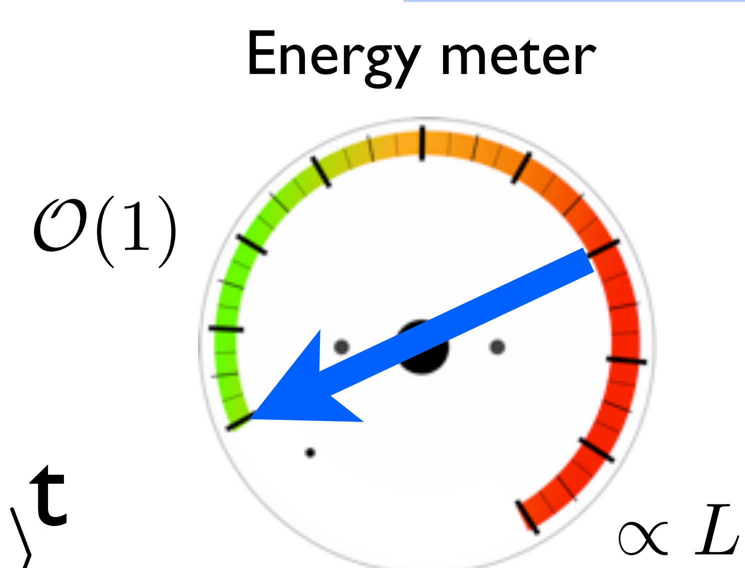

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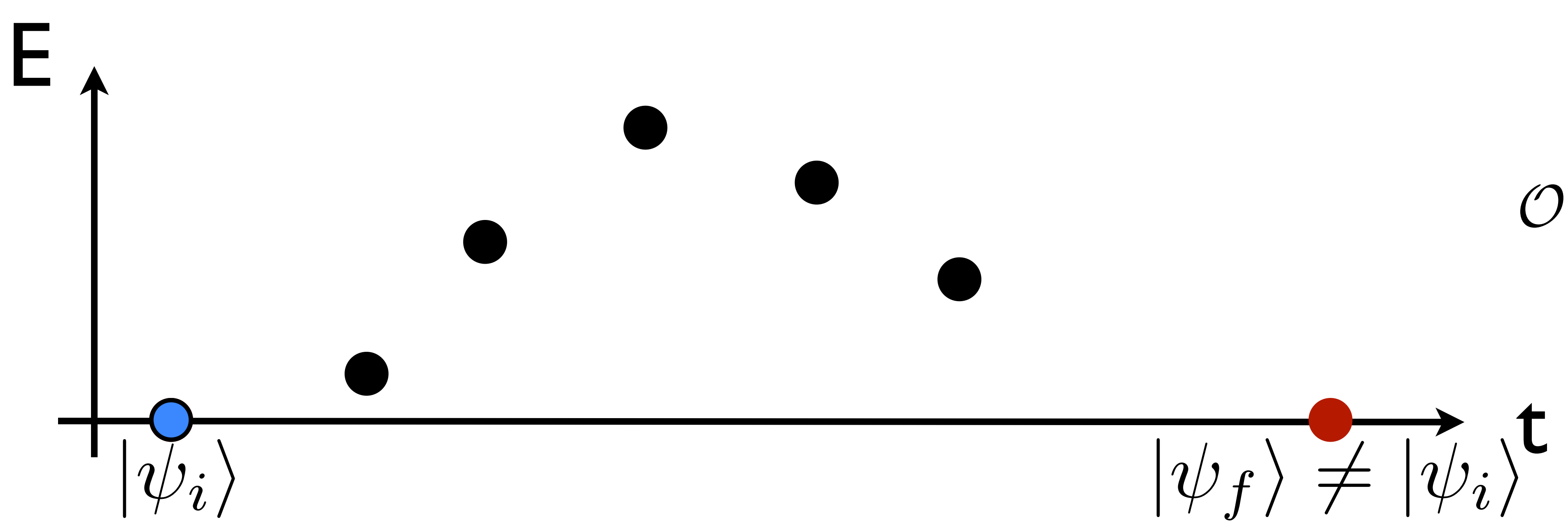


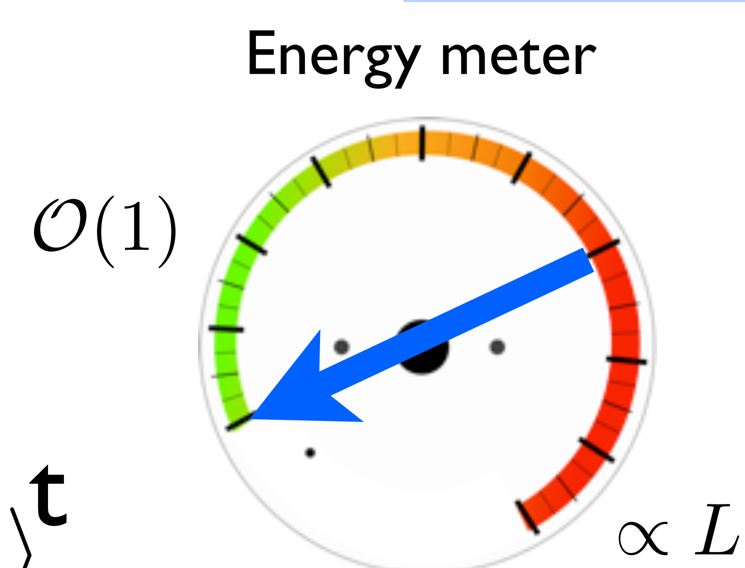

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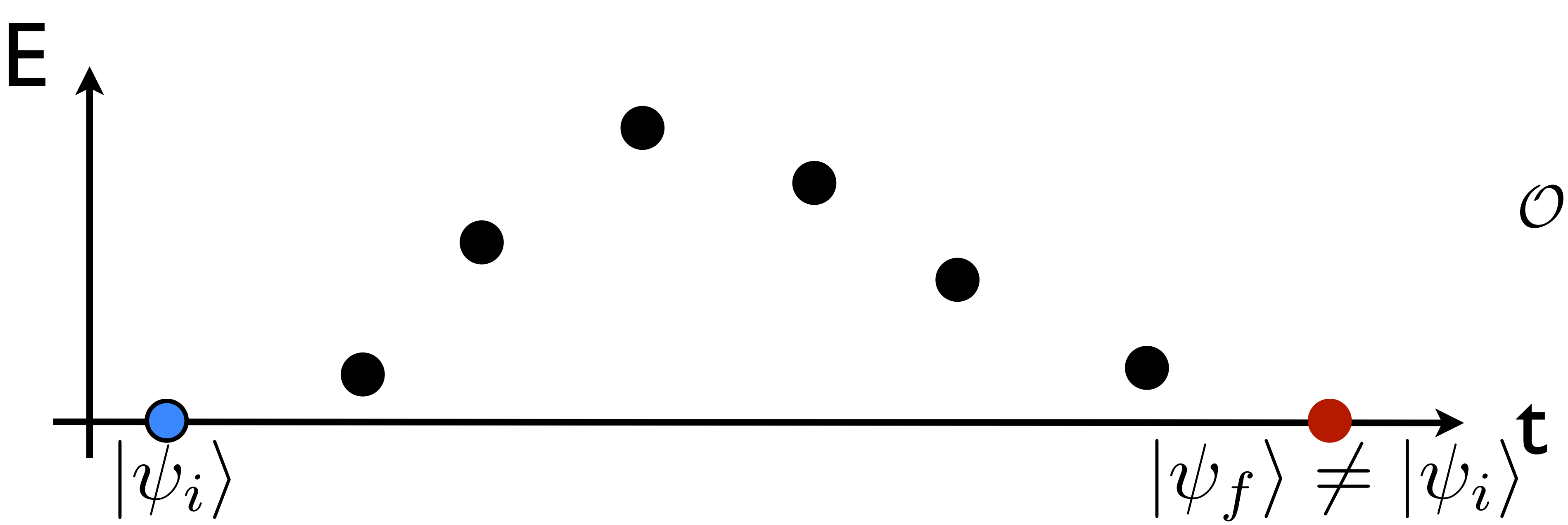


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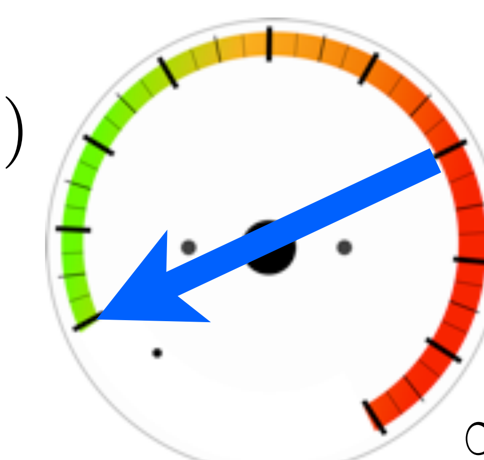

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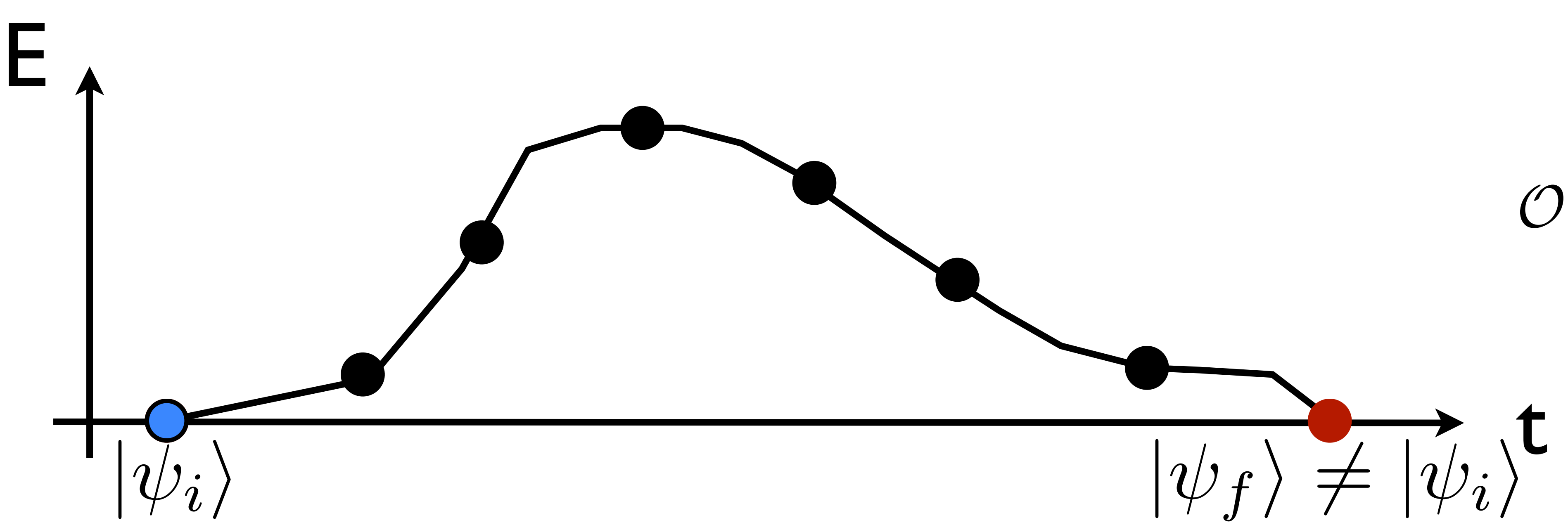


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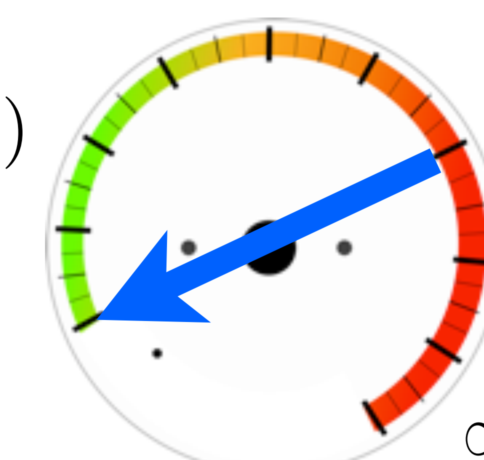



**TQO inhibits thermal stability**

Olivier Landon-Cardinal

Introduction
Thermal stability
Spectral stability

Background
LCPC
Topo order
Self-correction
Known results

Main result
Noise model
No dead-ends
Sketch of proof

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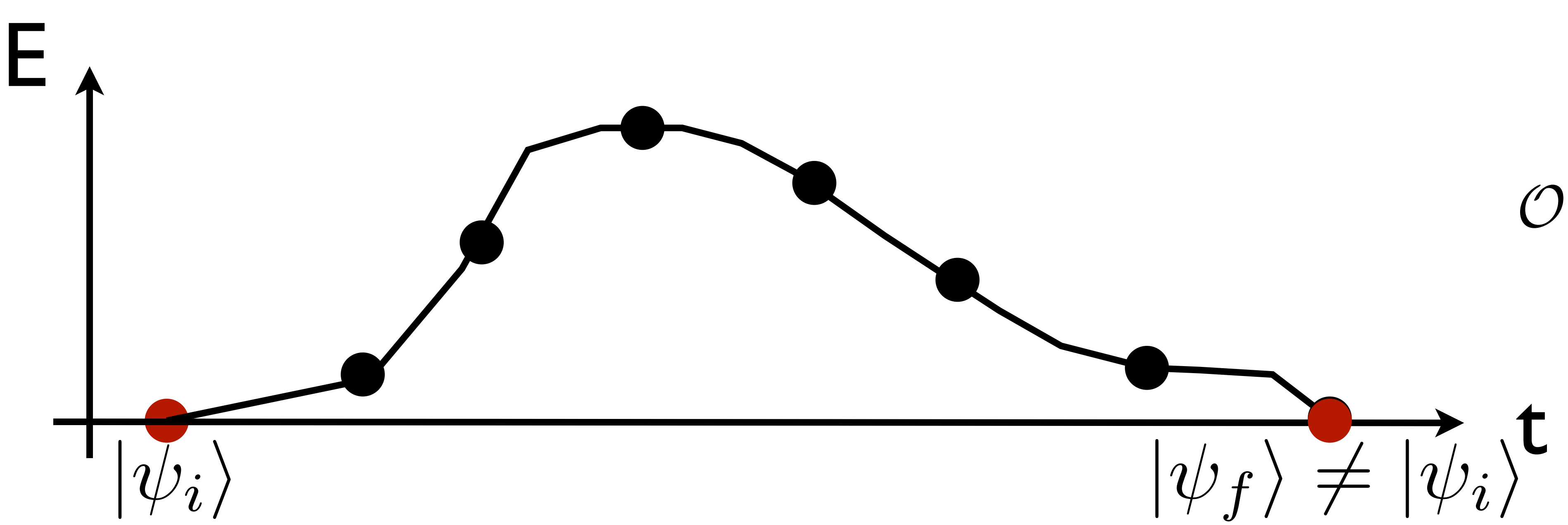


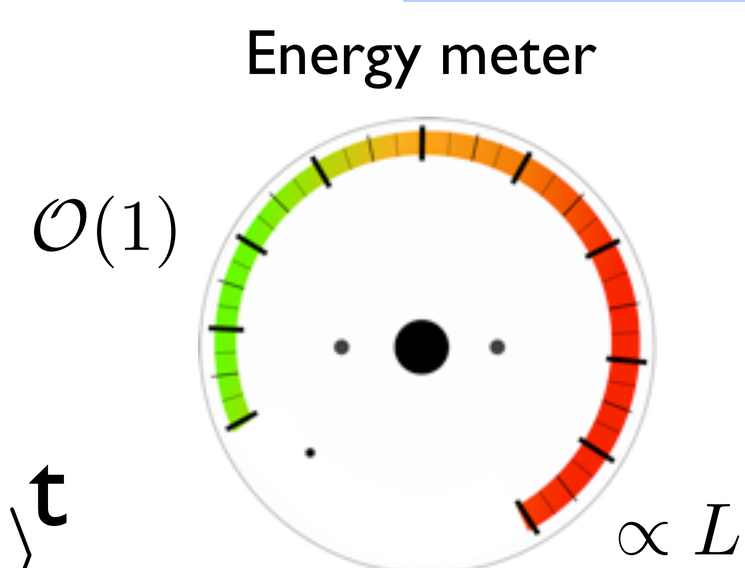

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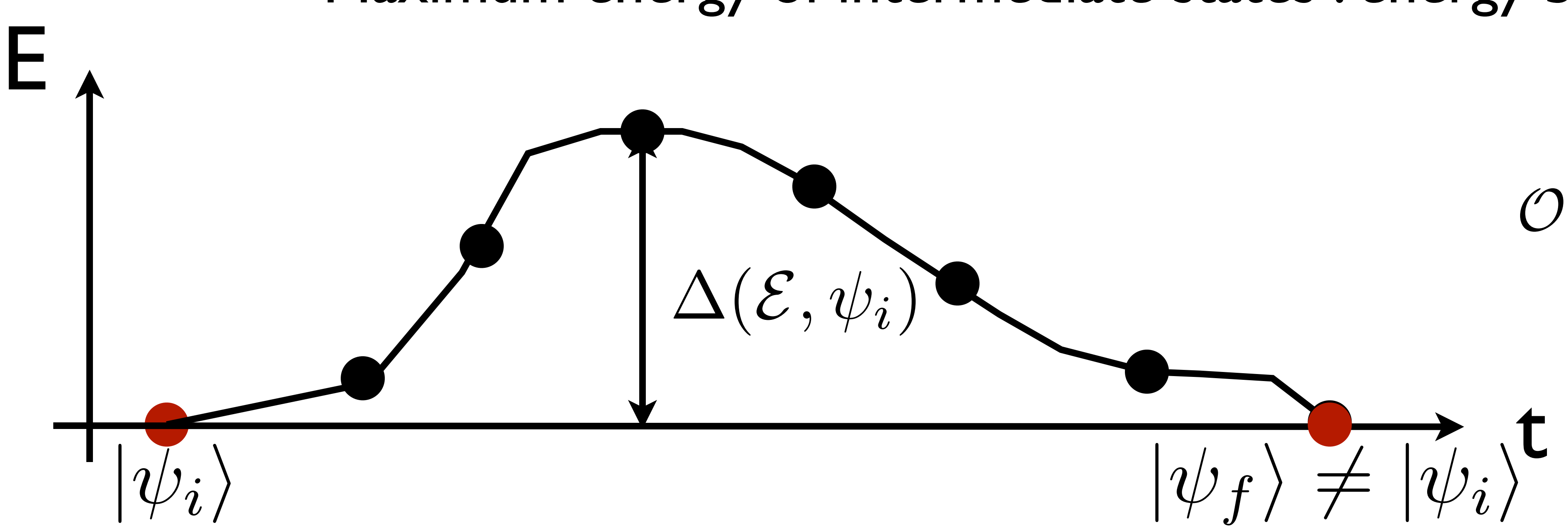


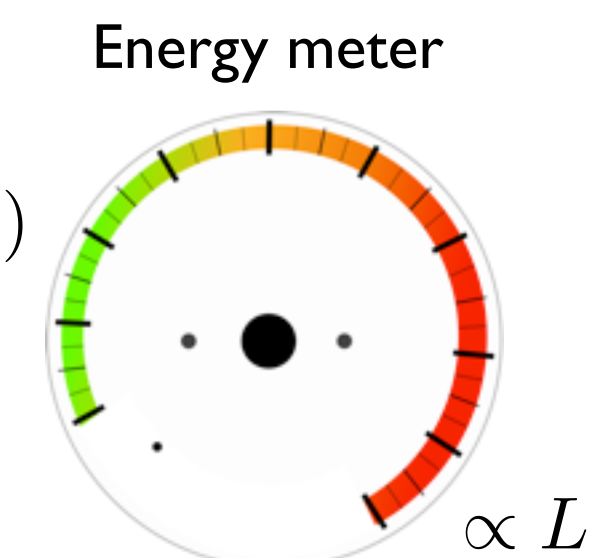

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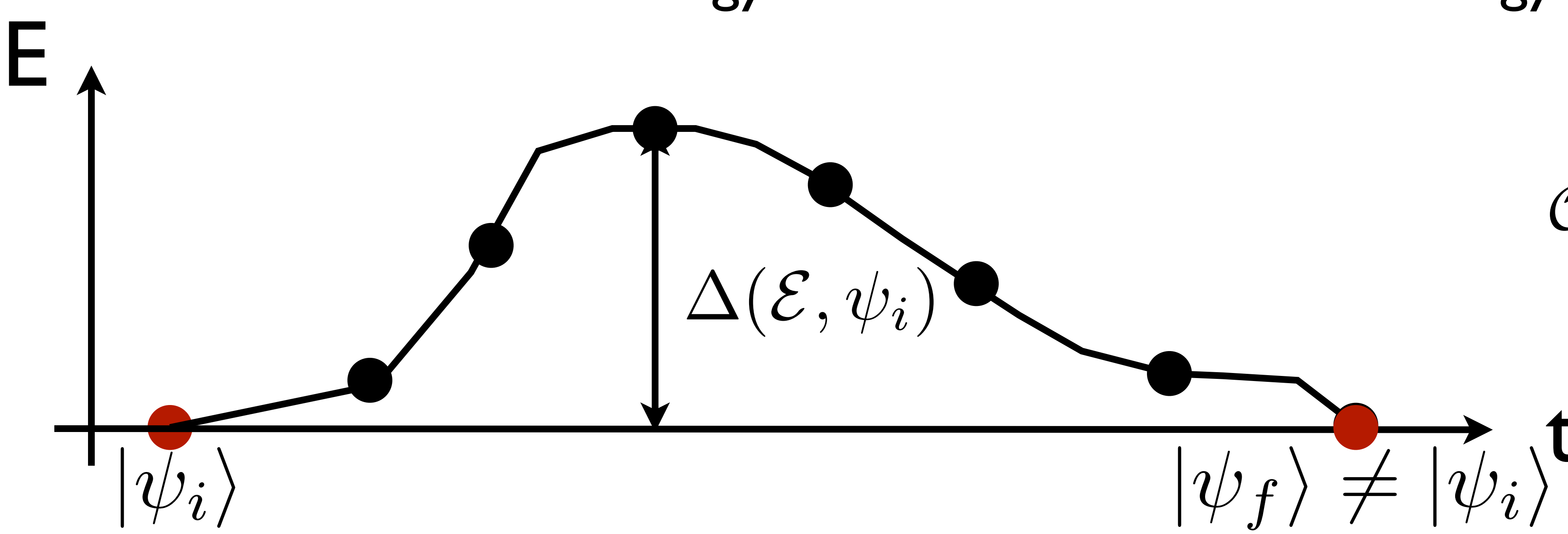


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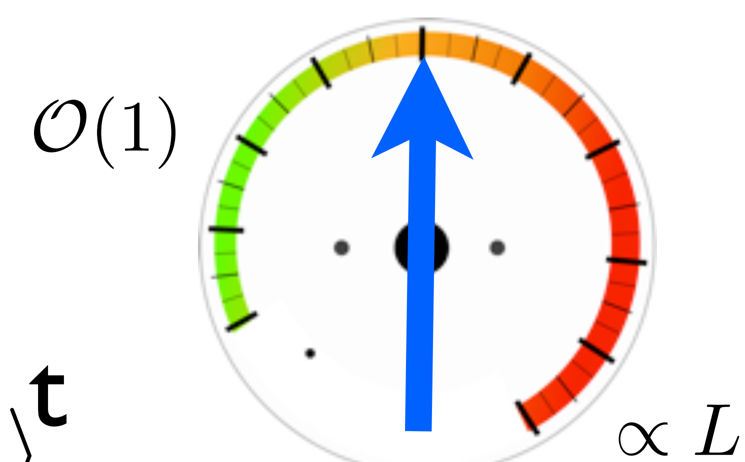
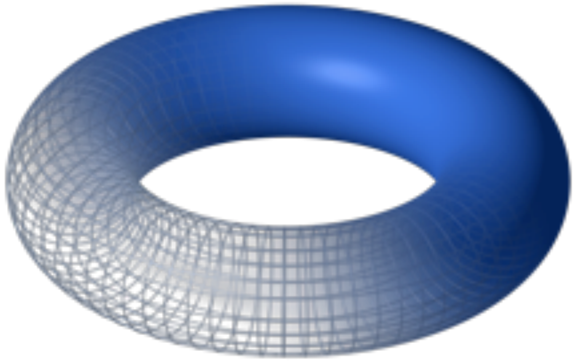


# Known results: 2D stabilizer codes & LCPCs

Instability in Kitaev's toric code



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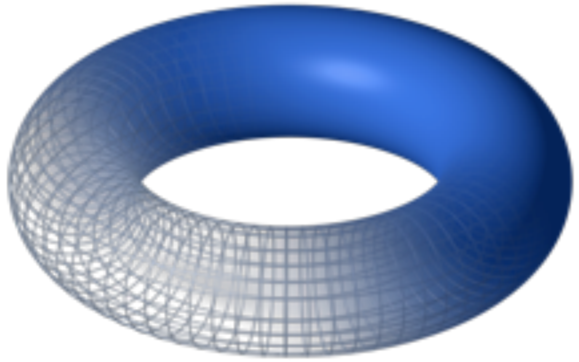

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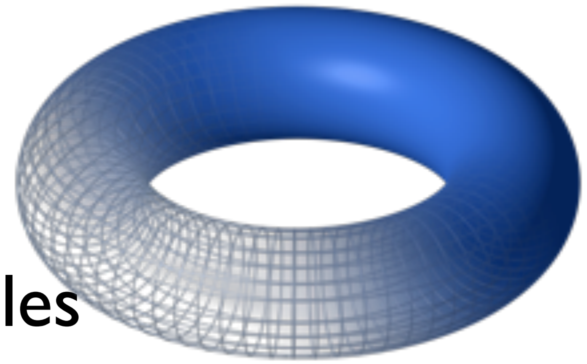

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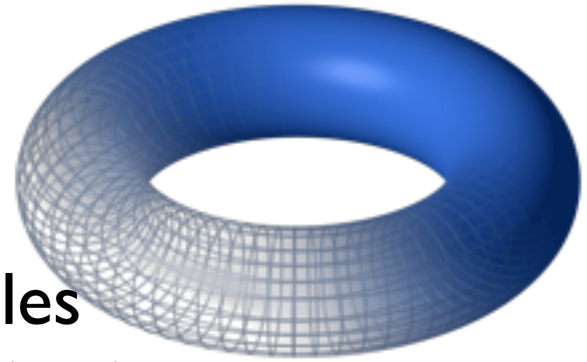

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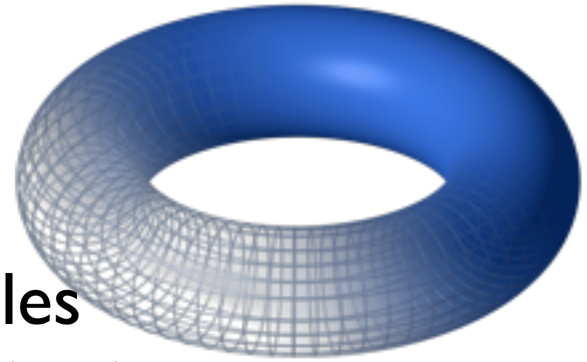

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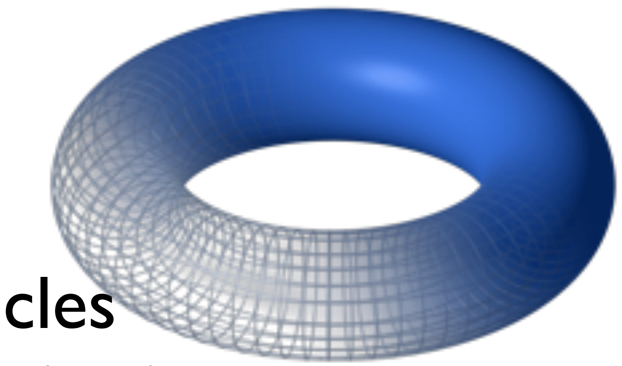

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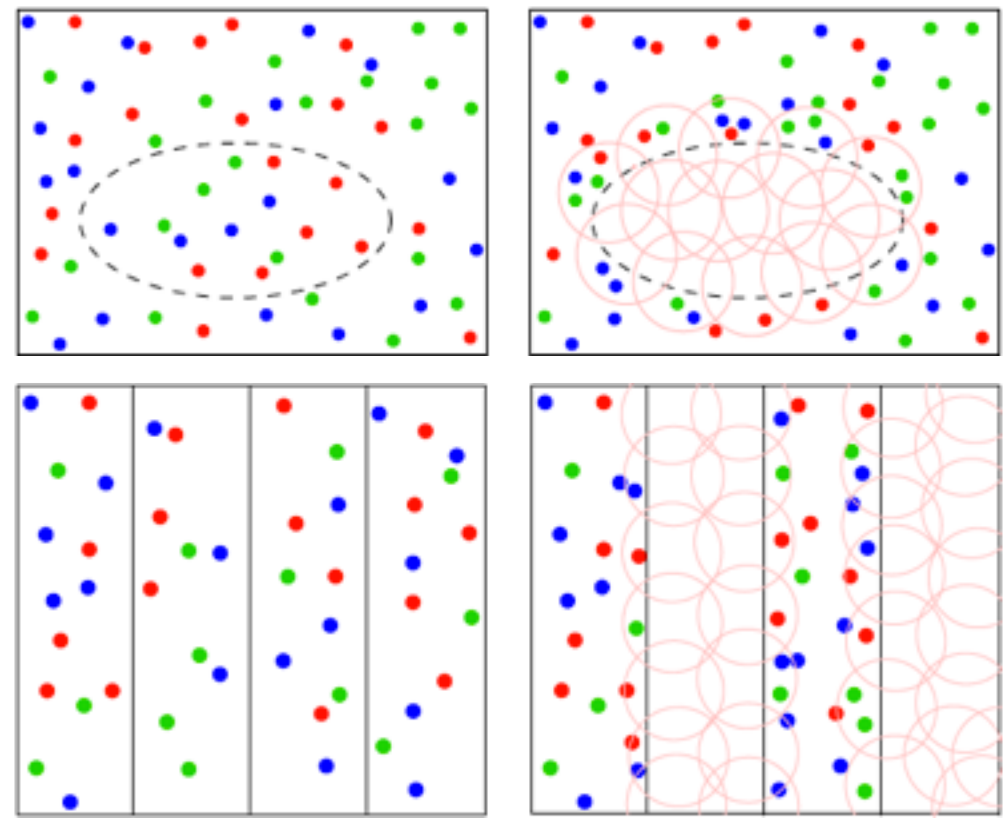

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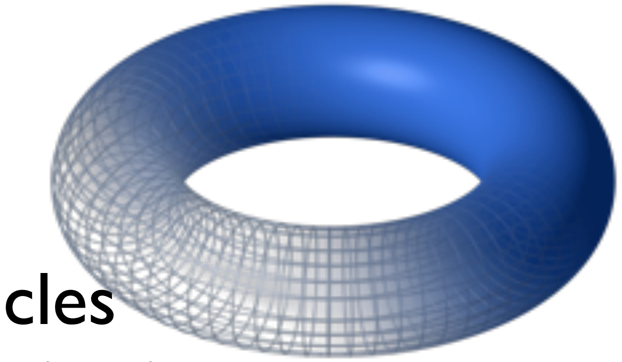

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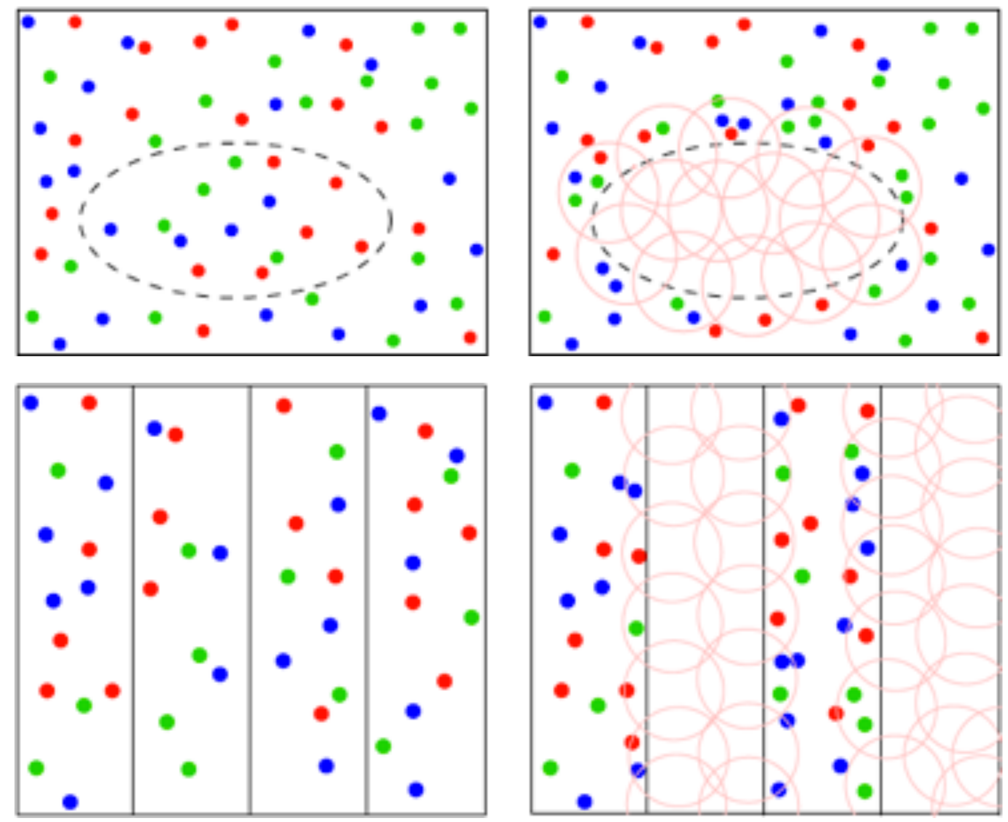

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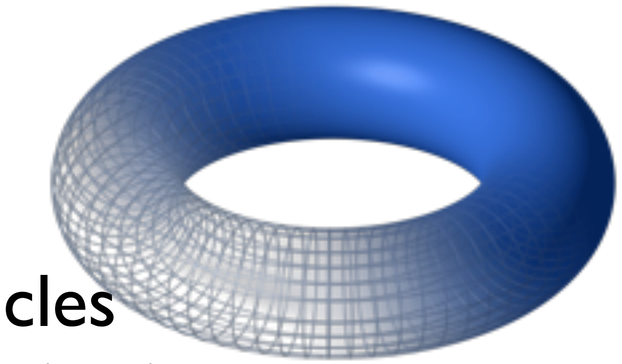

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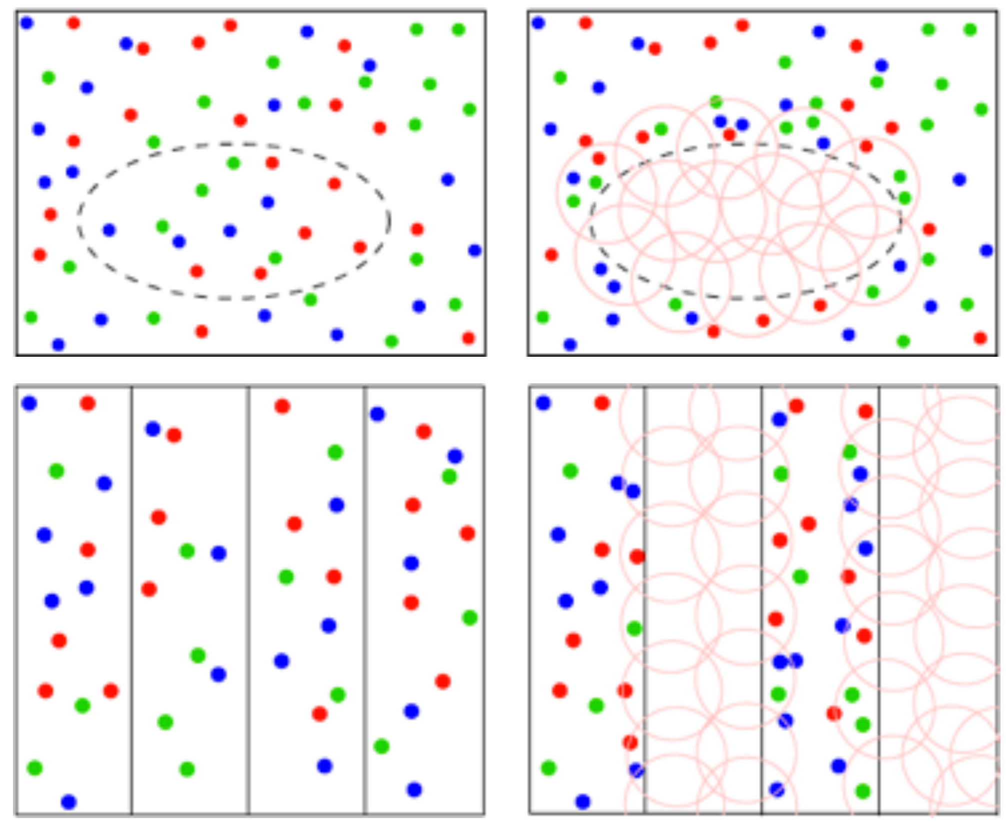

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2D LCPCs : logical operator supported on a 1D strip, but not a tensor product.

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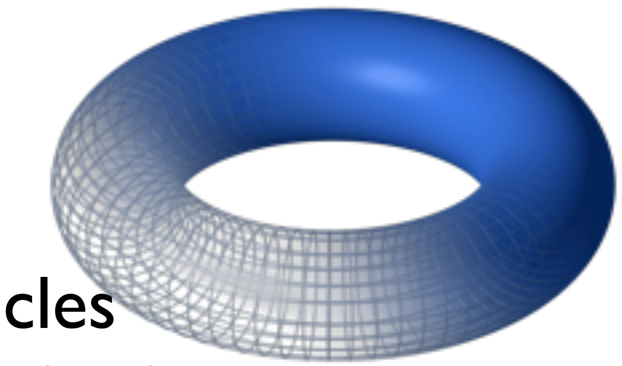

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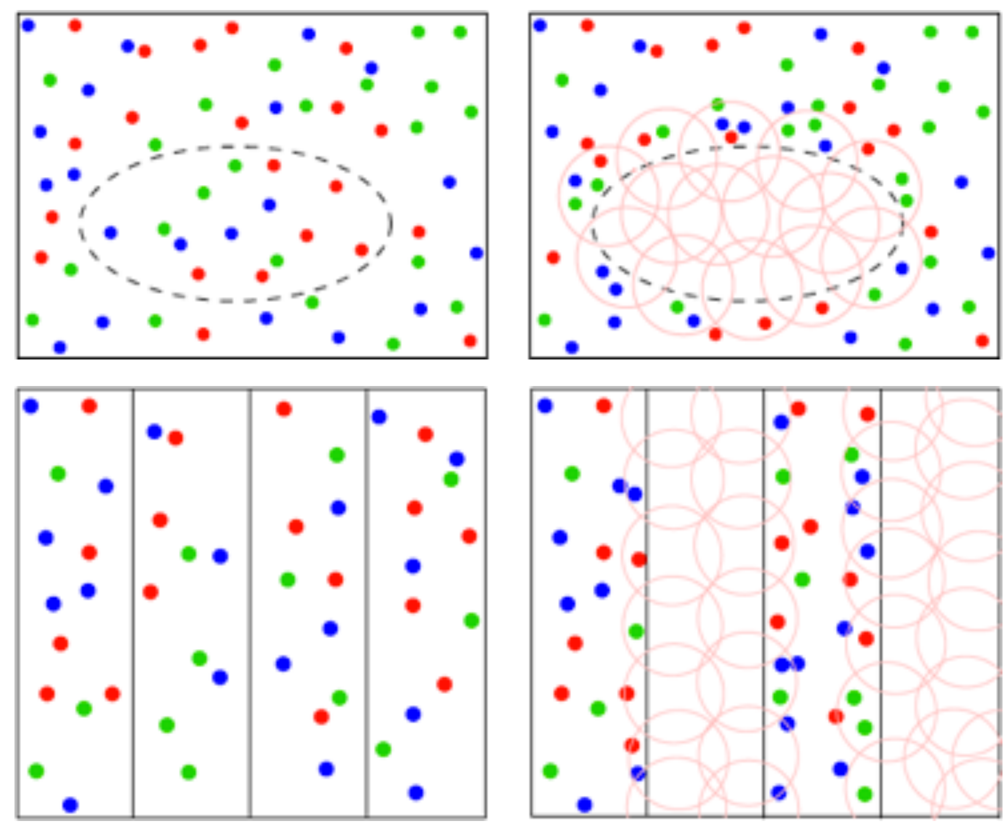


General result for 2D stabilizer codes

➡ cleaning lemma (Bravyi & Terhal '09)

Generalization to 2D LCPCs

➡ disentangling lemma

Bravyi, Poulin & Terhal '10

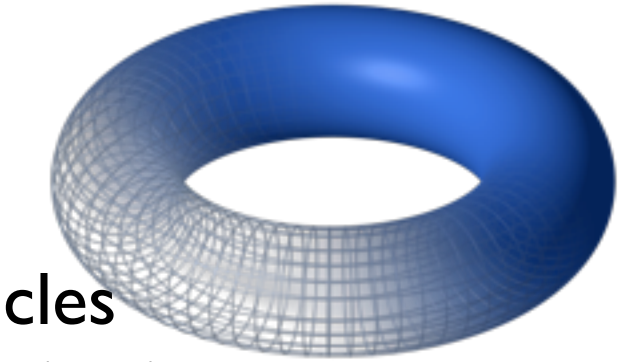
➡ Haah & Preskill '12

2D LCPCs : logical operator supported on a 1D strip, but not a tensor product.

# Known results: 2D stabilizer codes & LCPCs



Instability in Kitaev's toric code



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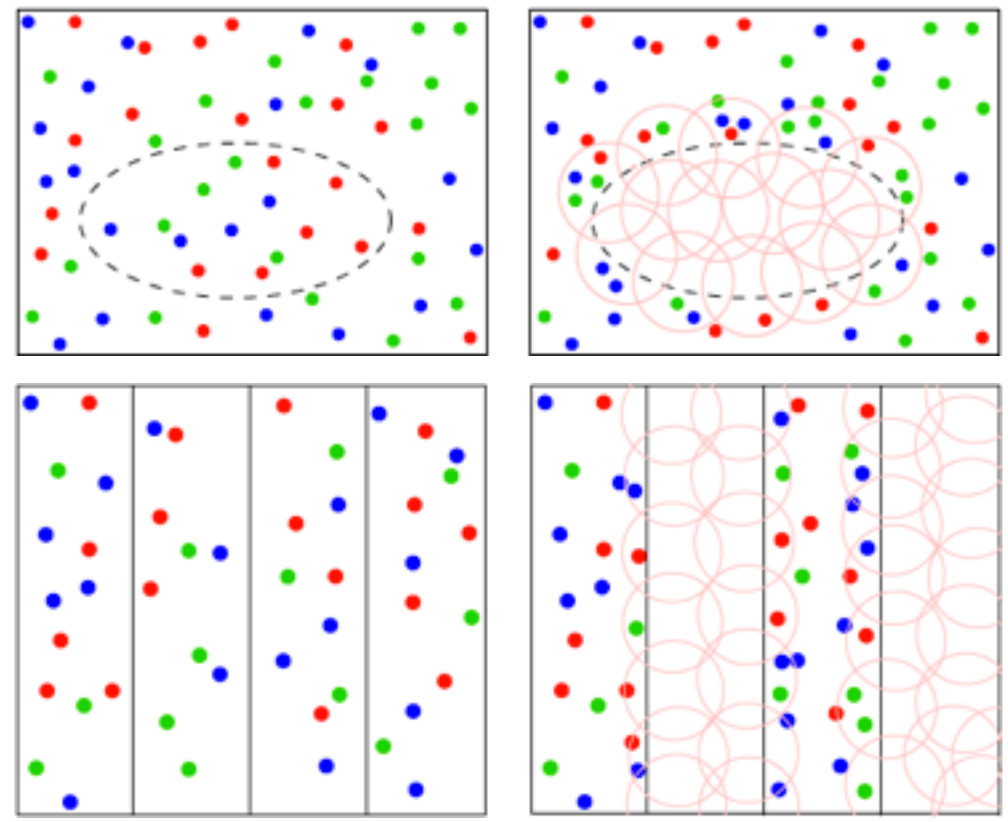

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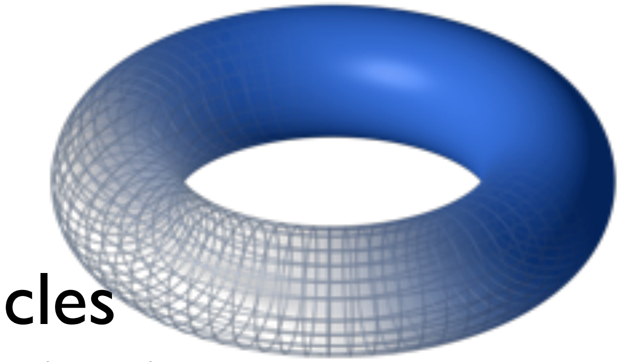

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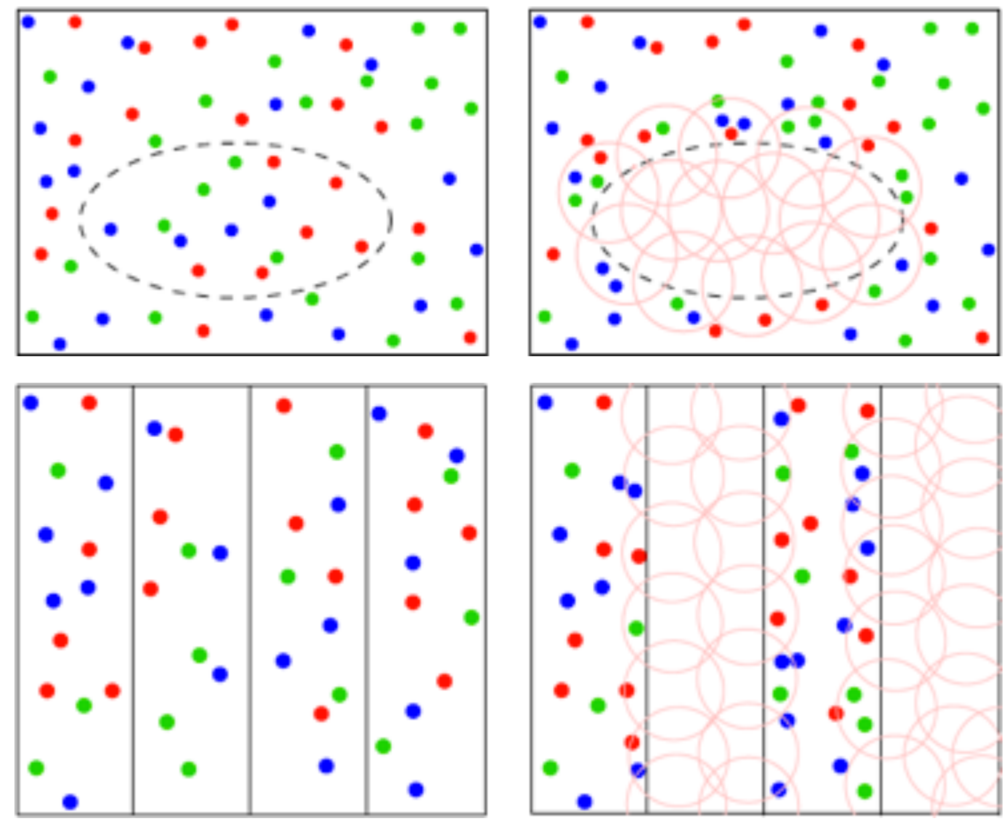

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How to apply it
- through a sequence of local CPTP maps?



Olivier Landon-Cardinal

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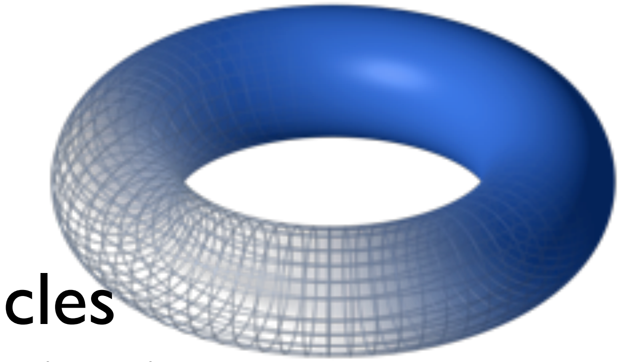

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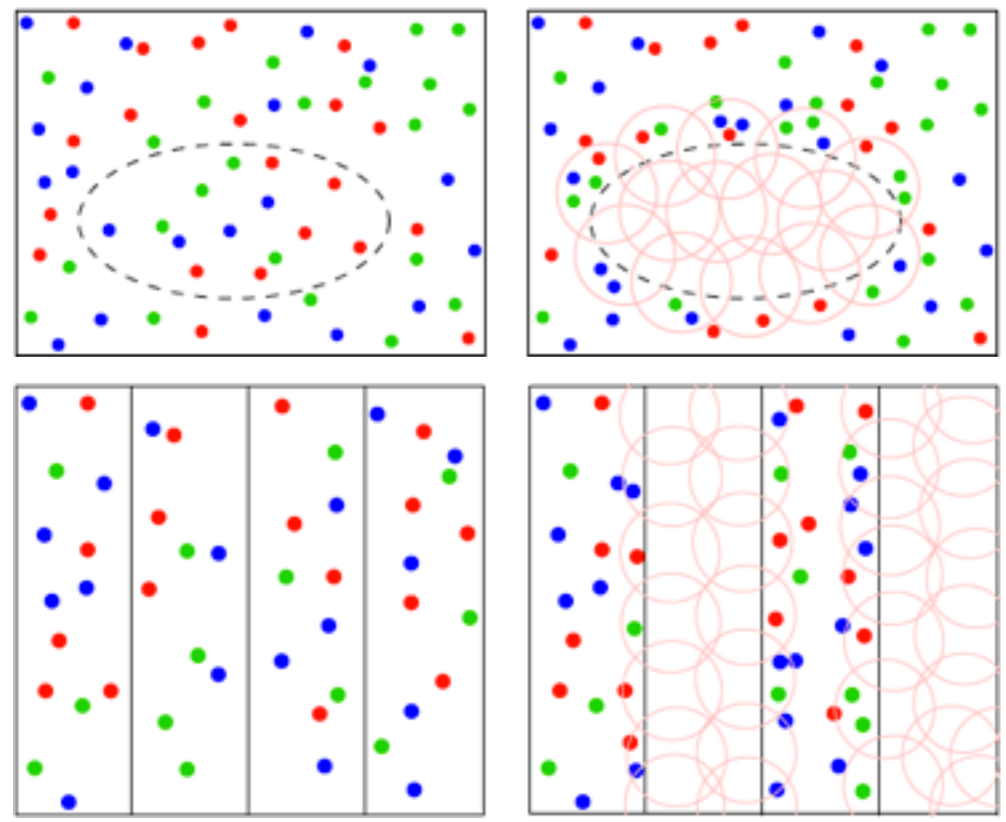

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2D LCPCs : logical operator supported on a 1D strip, but not a tensor product.

How to apply it

- through a sequence of local CPTP maps?
- without creating too much energy?

# Main result

**Main result** (arXiv:1209.5750)
For any 2D *local topologically ordered* LCP code, we exhibit an error model corrupting the information.

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BHM '10

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Local topological order → (BHM '10) → Spectral stability

Local topological order ↓ (this work)

Thermal instability

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BHM '10

Local topological order → Spectral stability

↓ this work

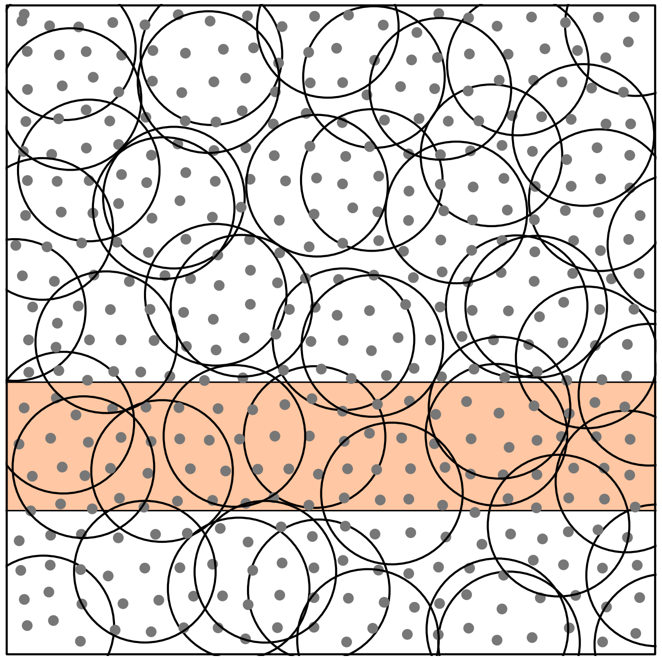
Thermal instability

Tradeoff between spectral and thermal stability.




Olivier Landon-Cardinal

# Sketch of the proof (1): coarse-graining



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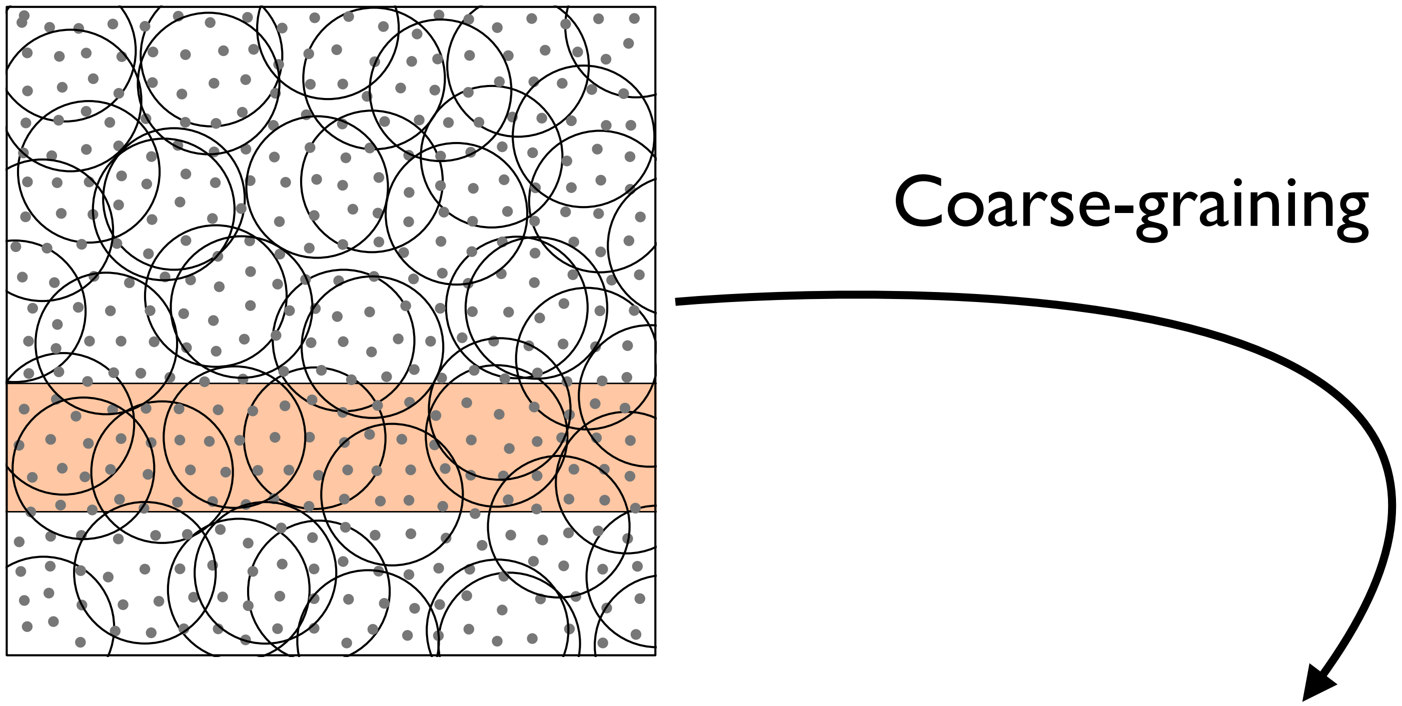

Coarse-graining

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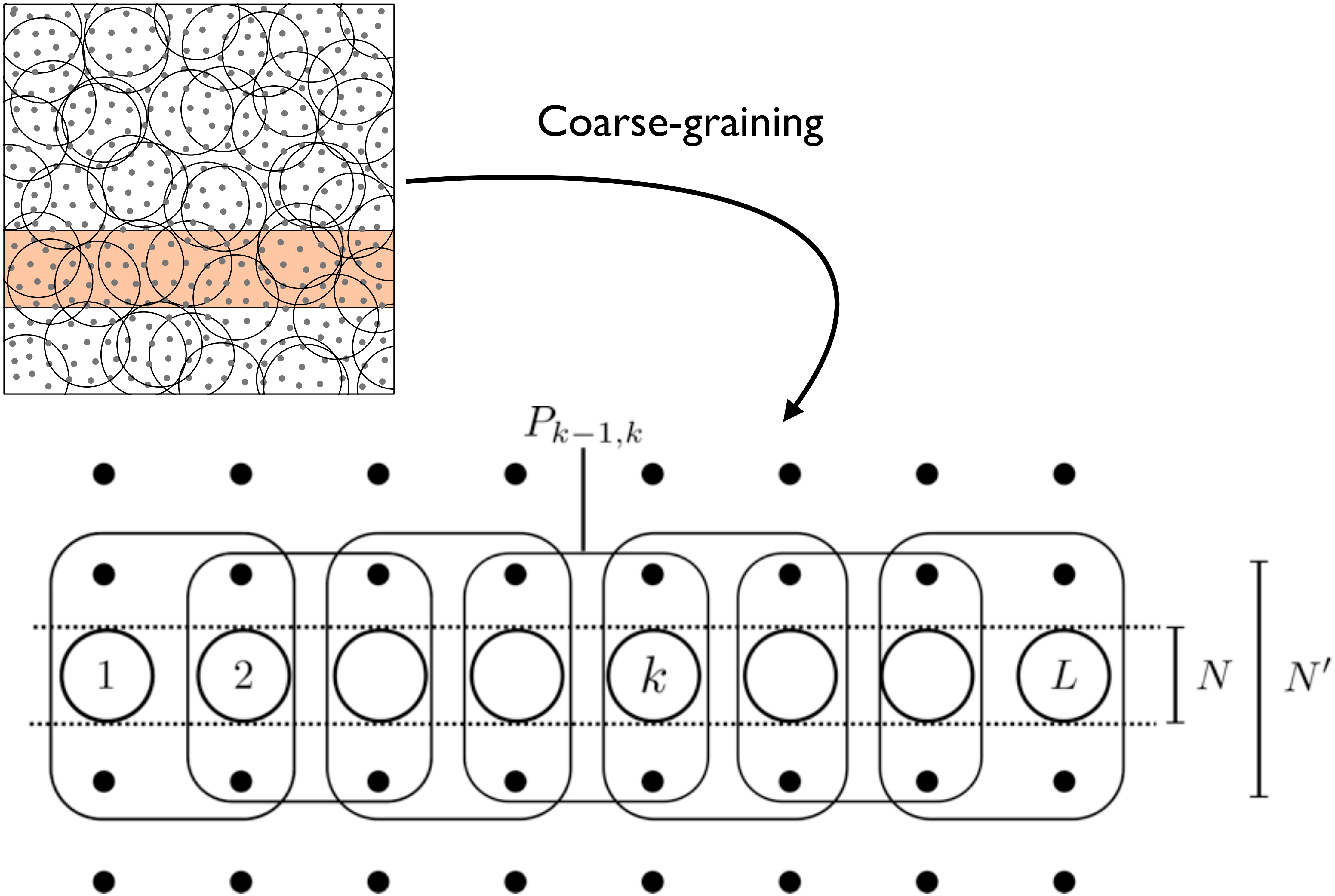

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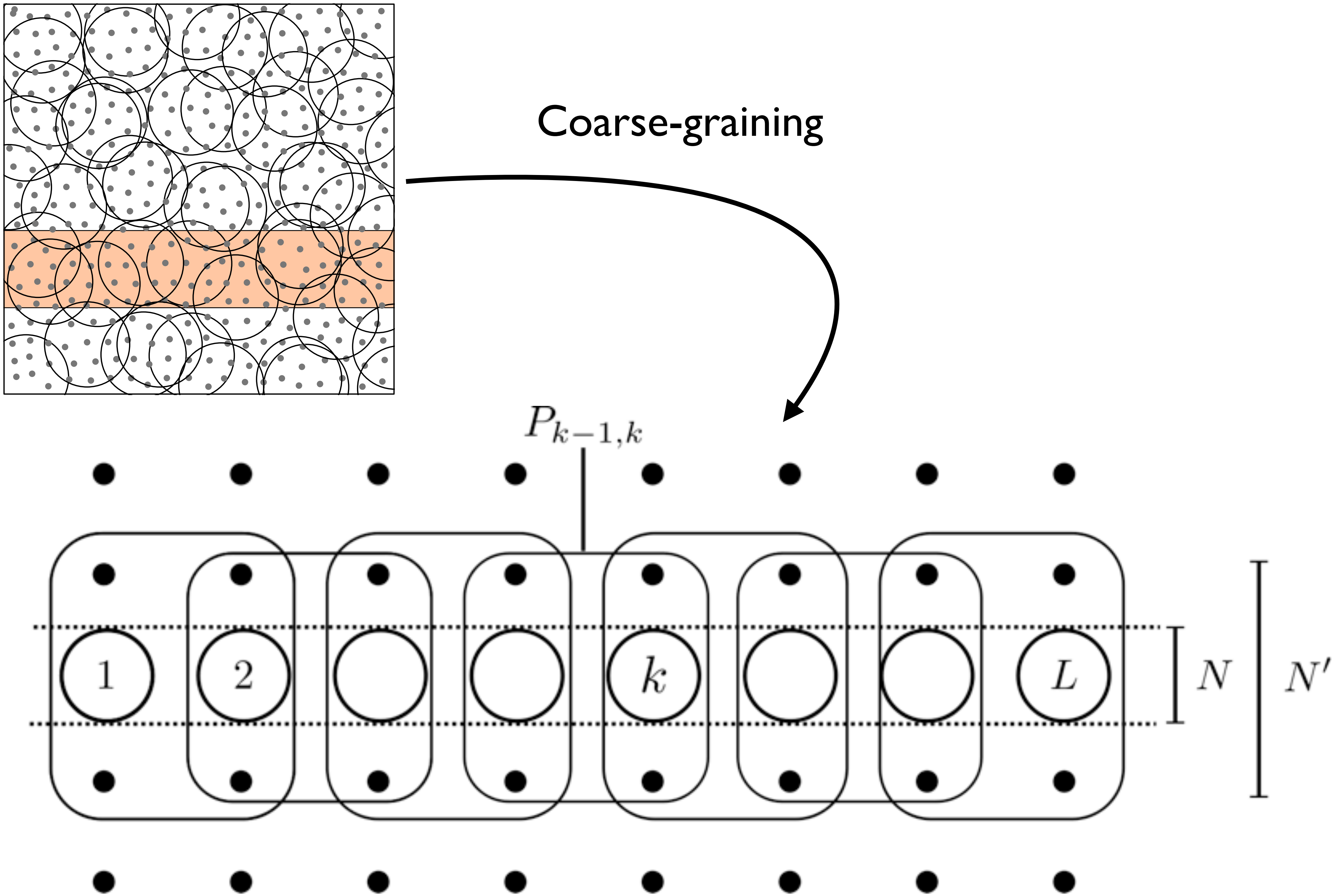
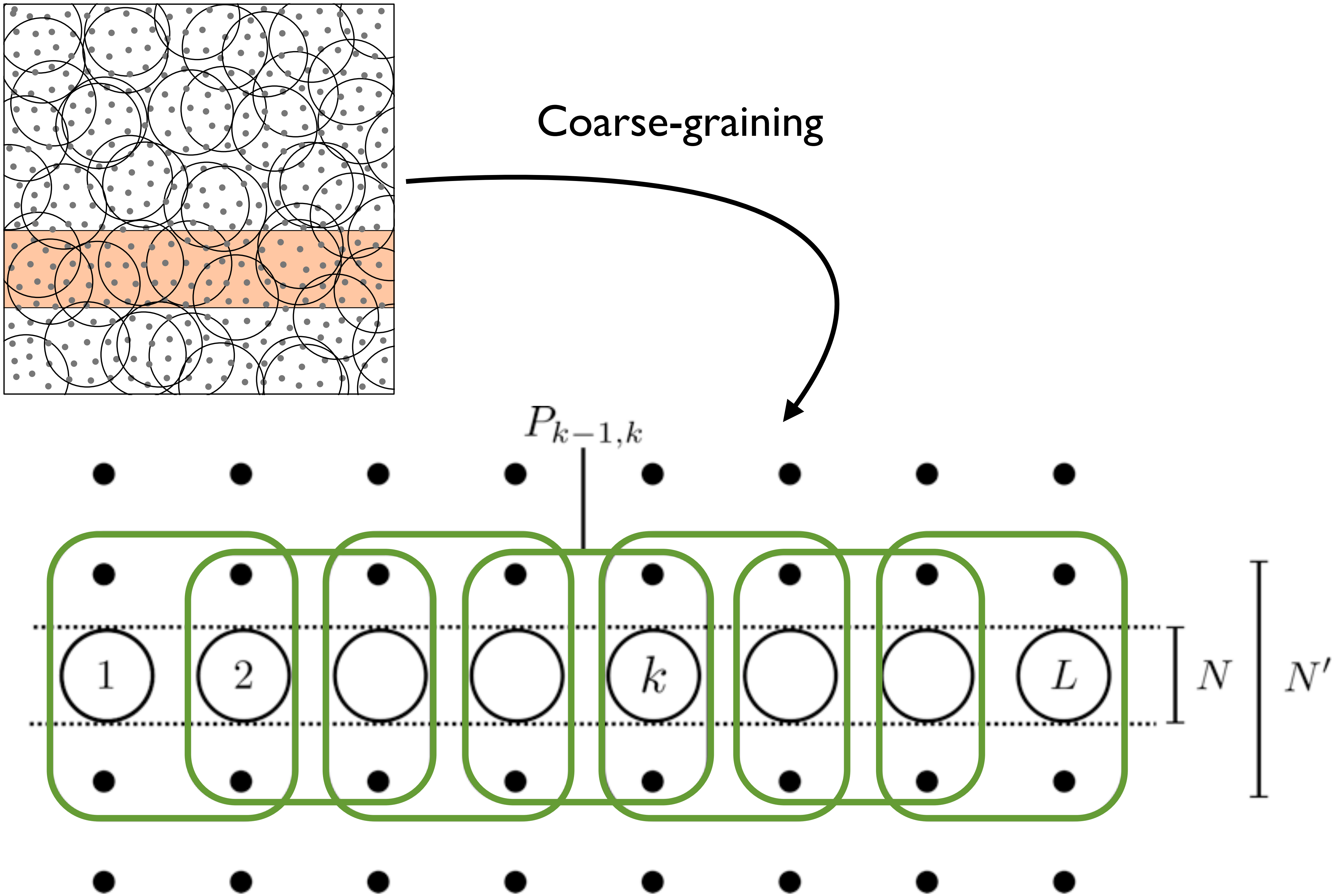



- *Sites* on the strip

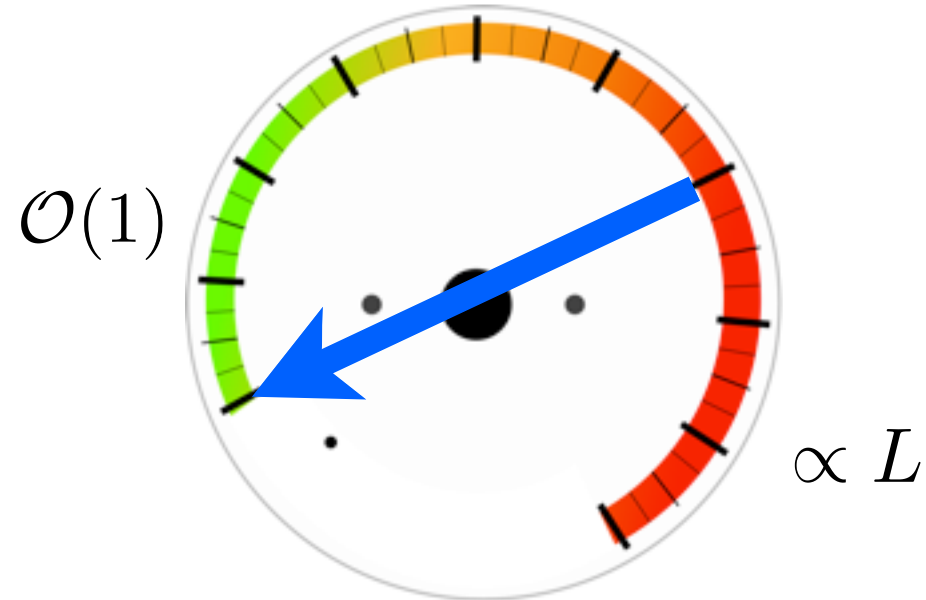
# Sketch of the proof (1): coarse-graining

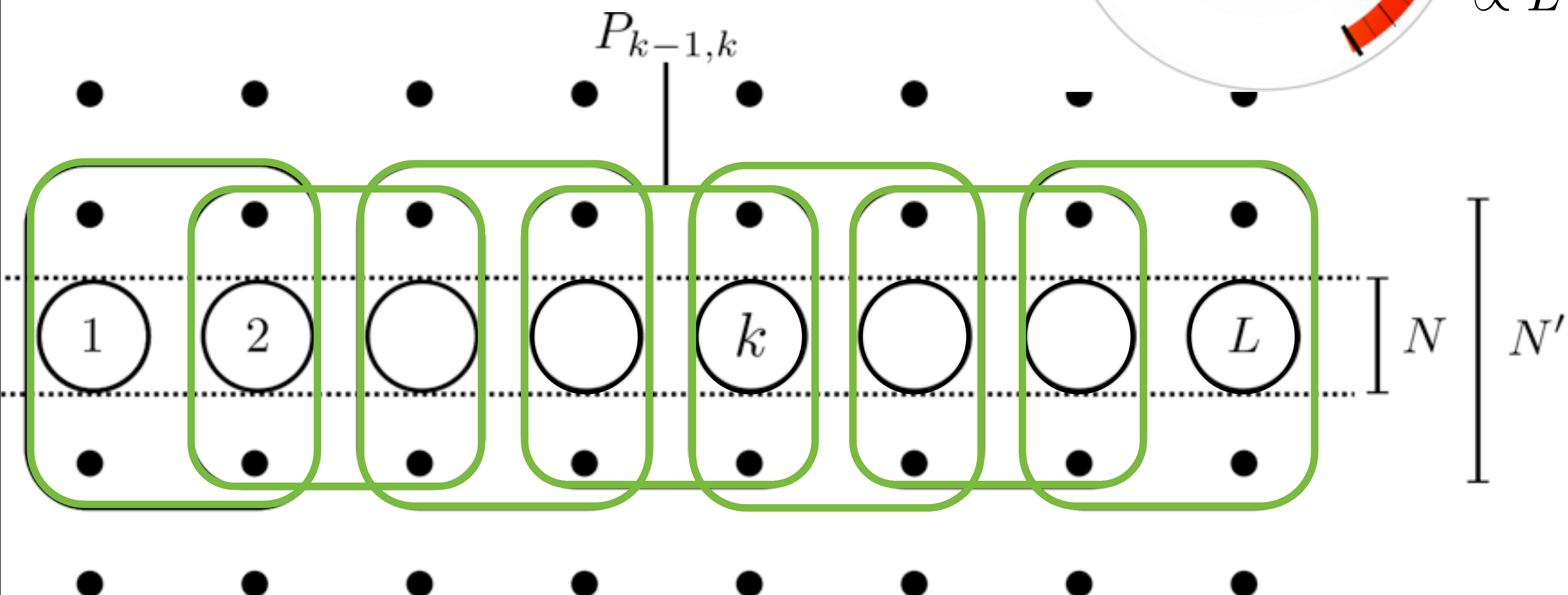



- *Sites* on the strip
- Local constraints

# Sketch of the proof (II): iterative randomization model

## Iterative randomization model

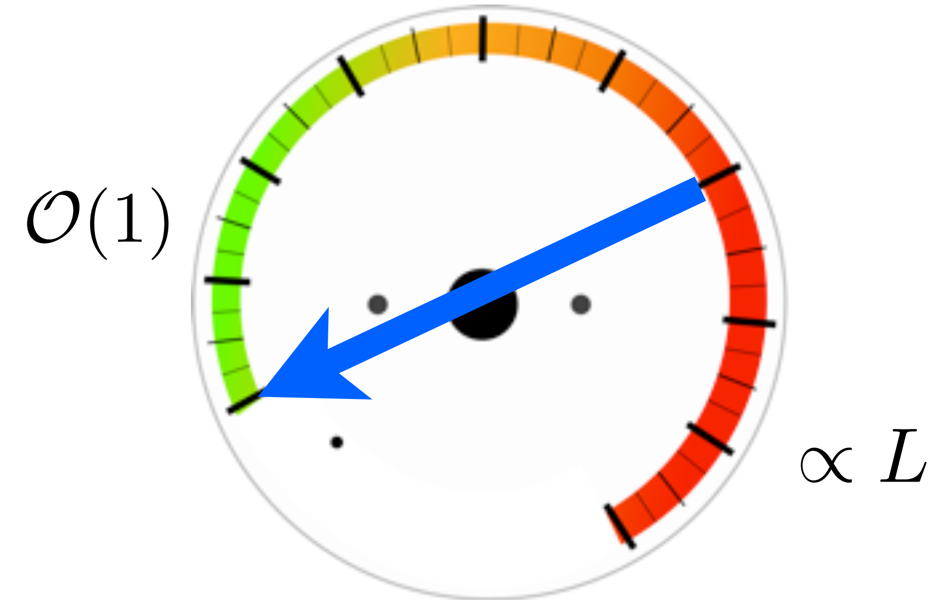


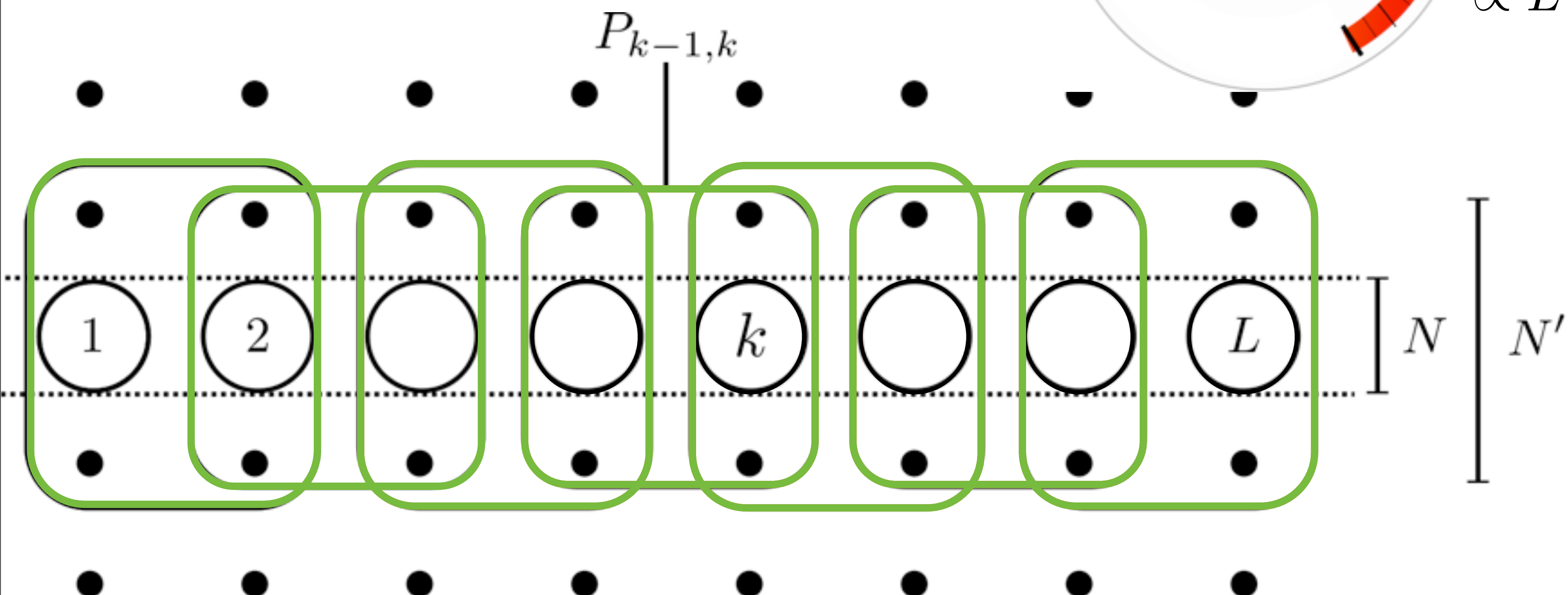



# Sketch of the proof (II): iterative randomization model

Iterative randomization model

For every site k (iteration),



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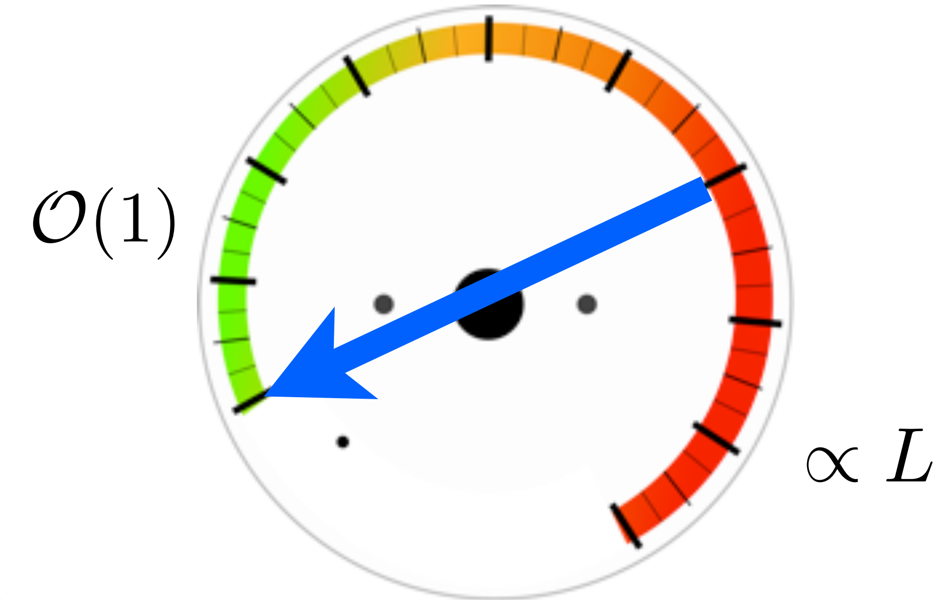


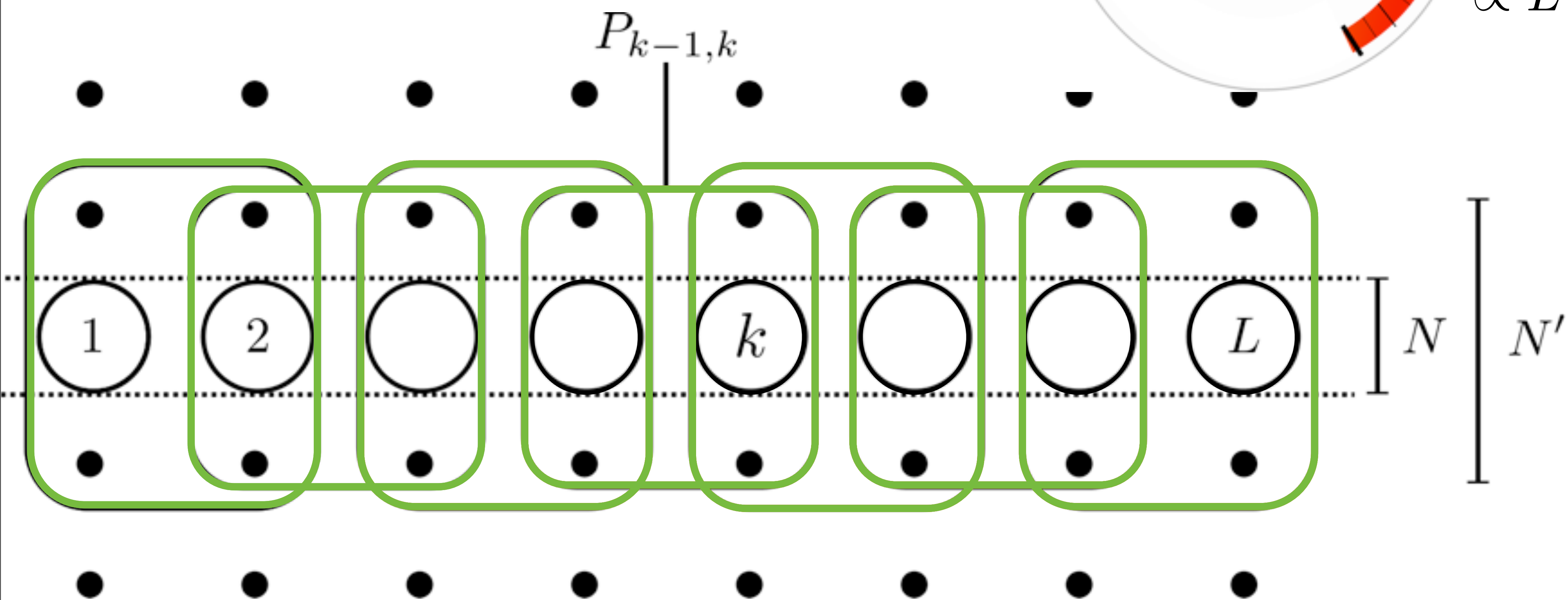

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For every site k (iteration),
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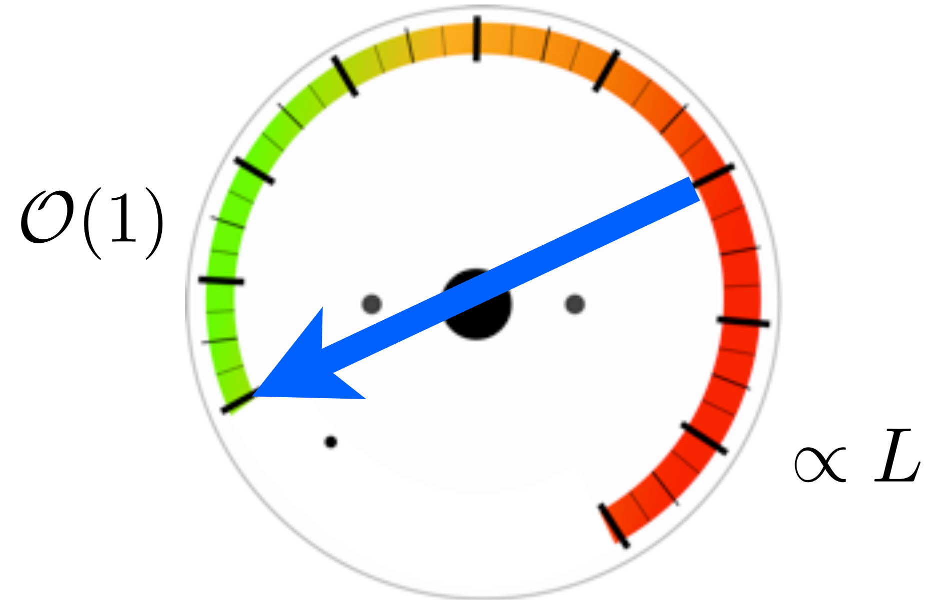


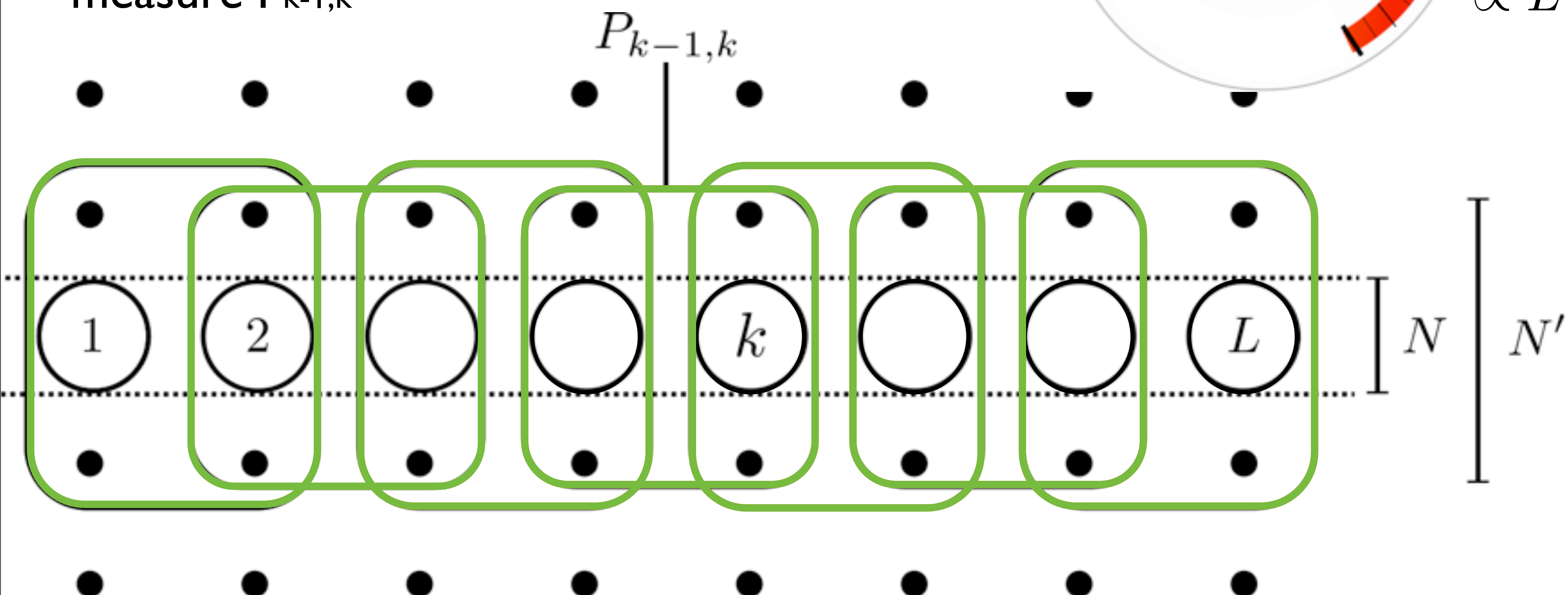

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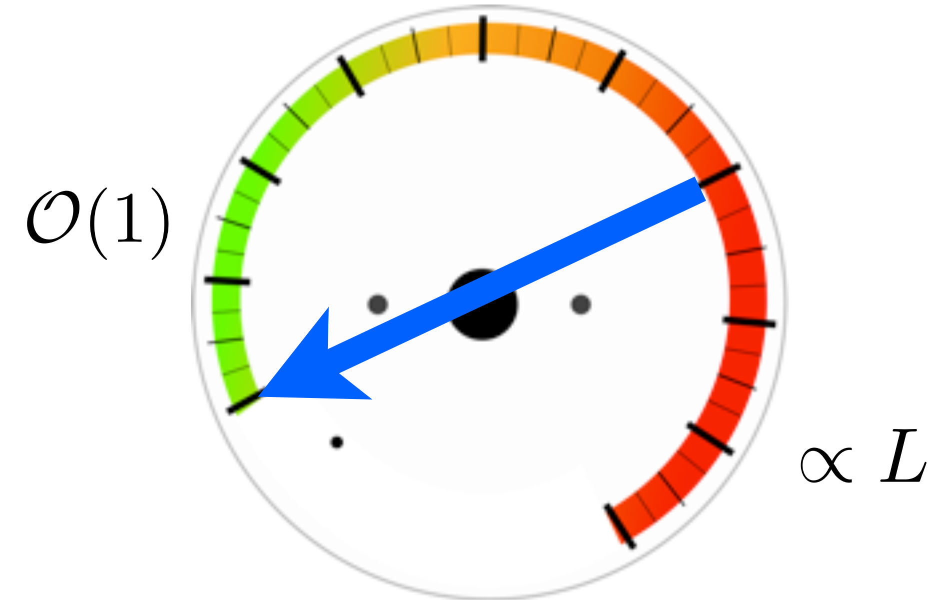


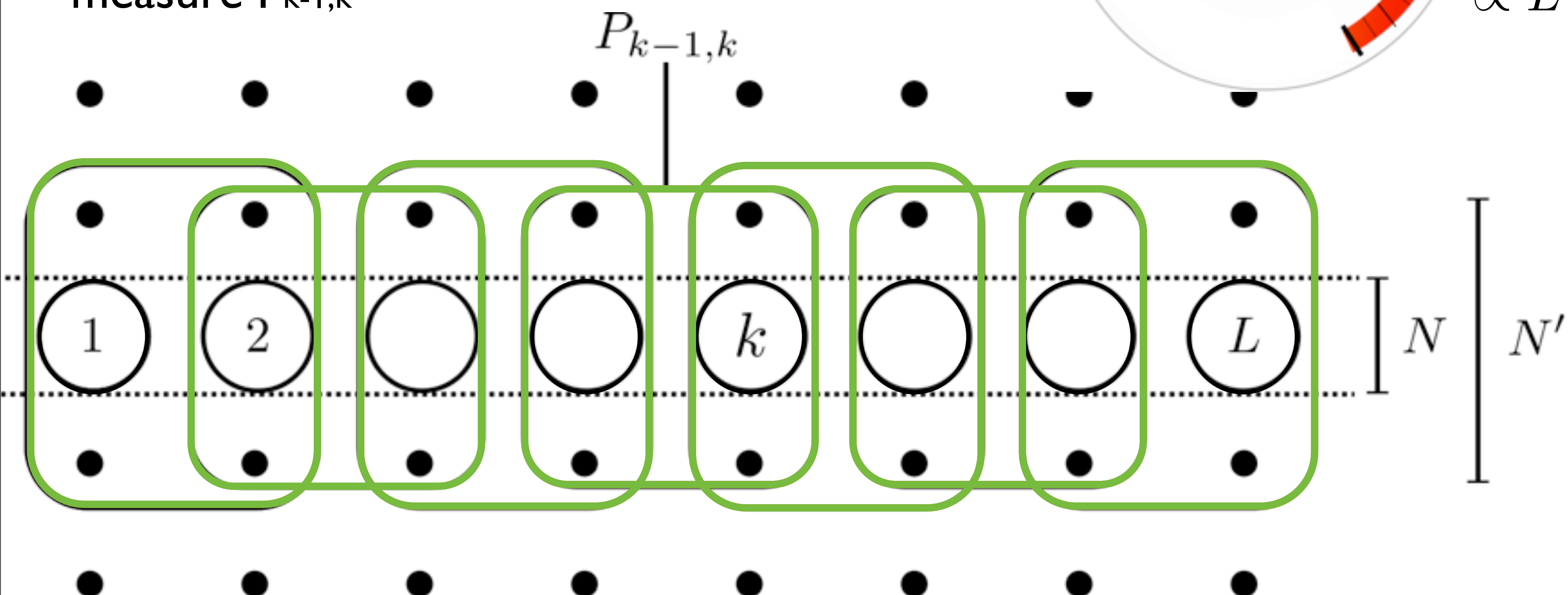

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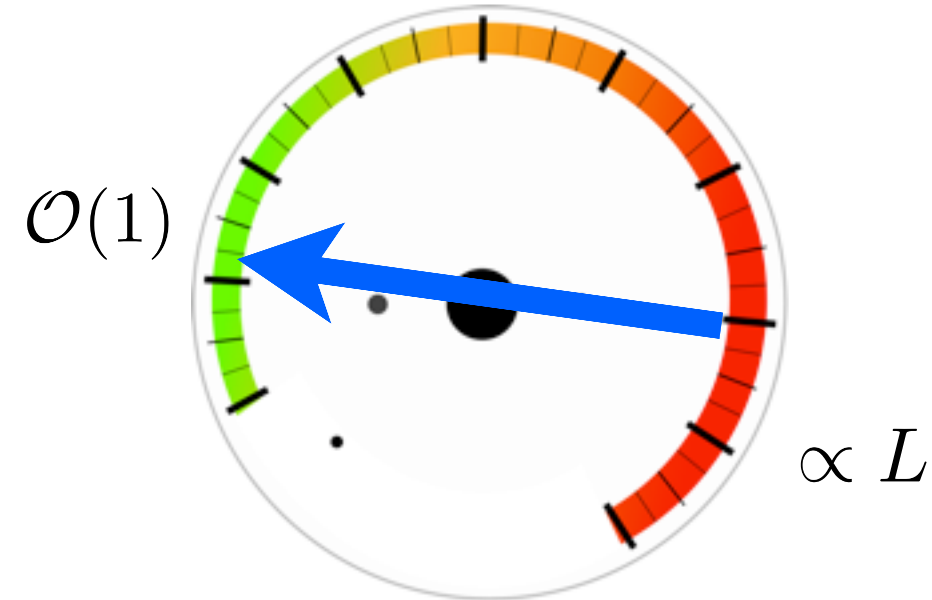


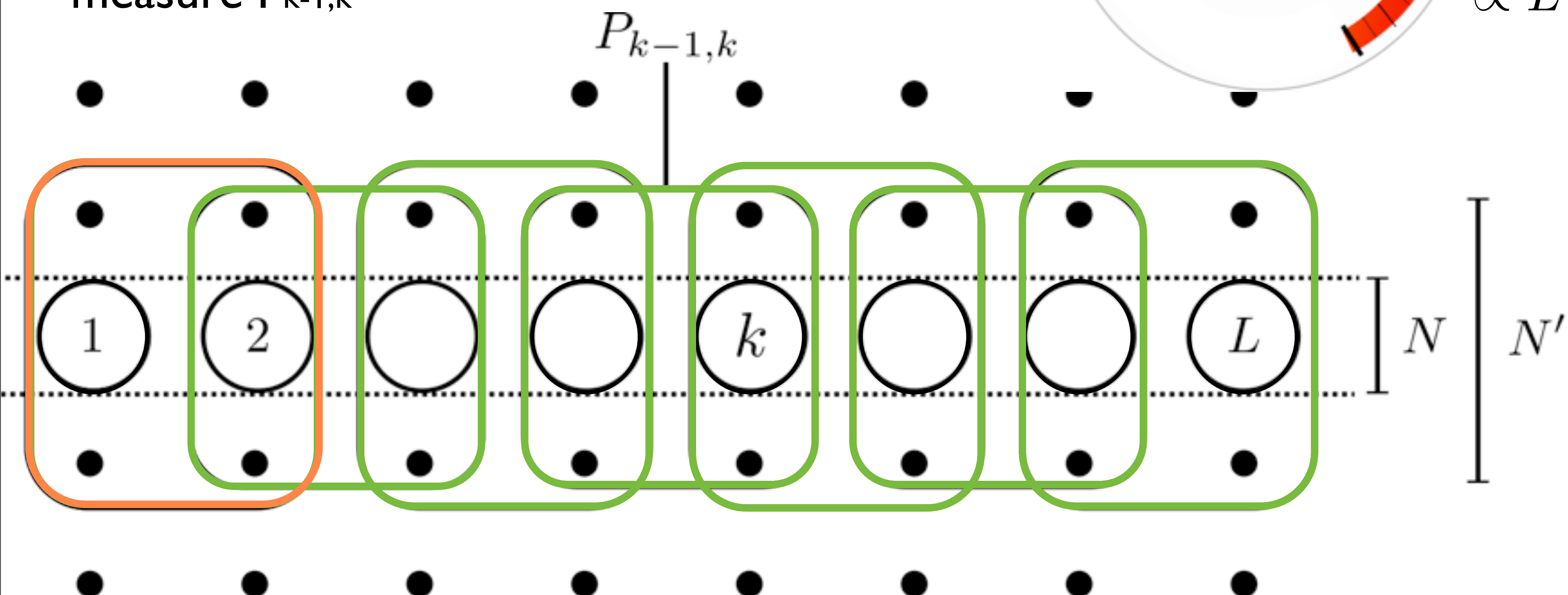

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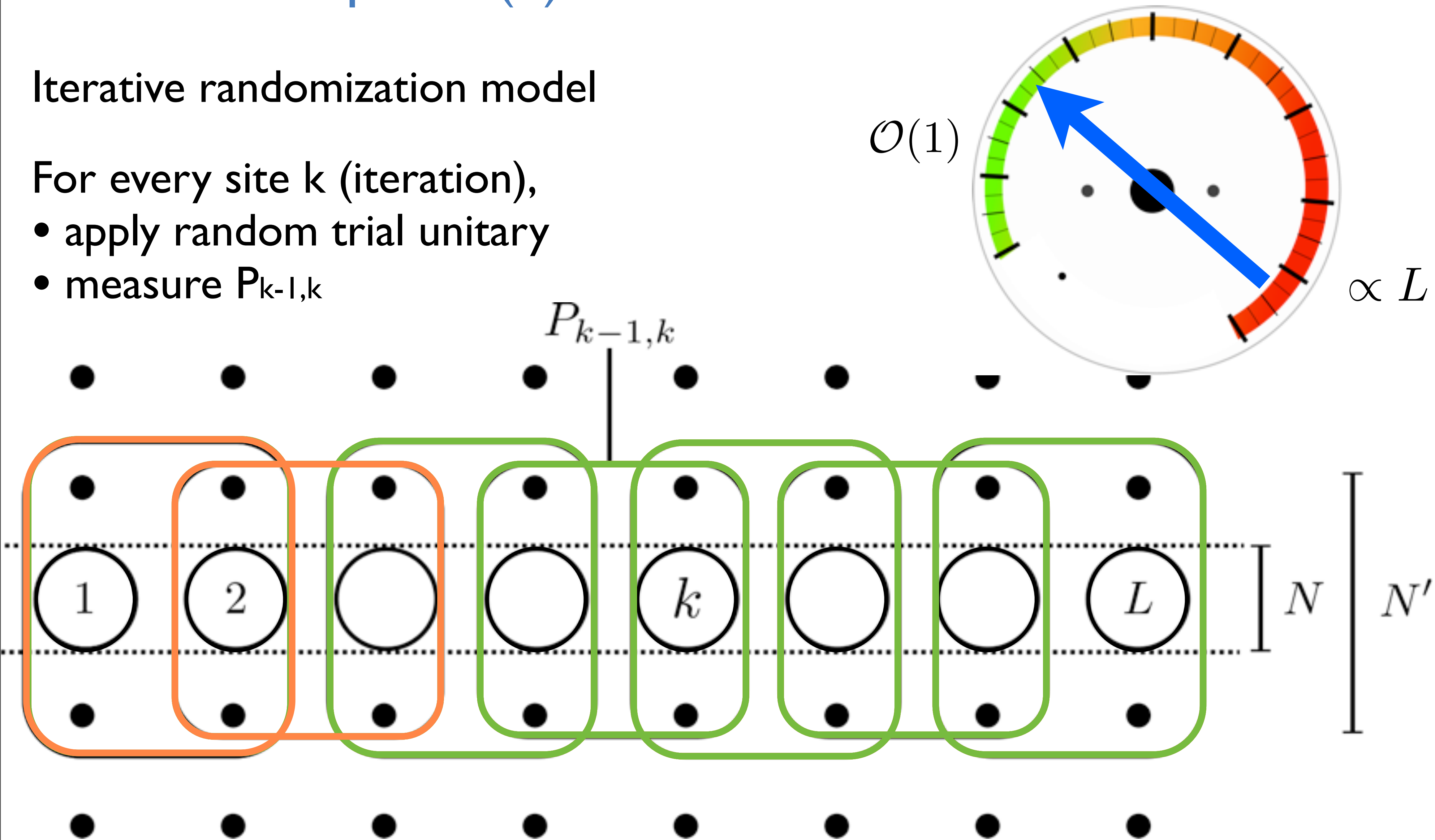

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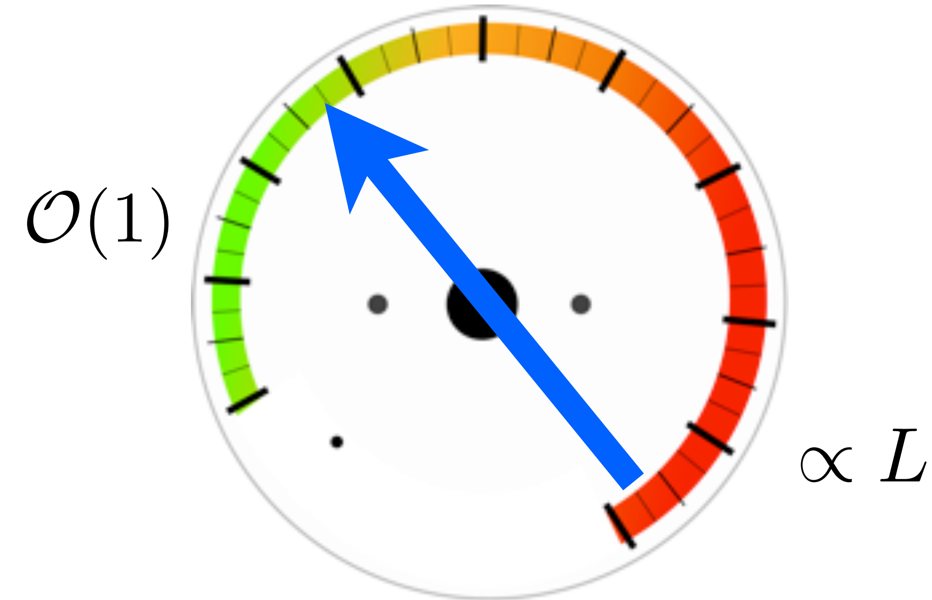


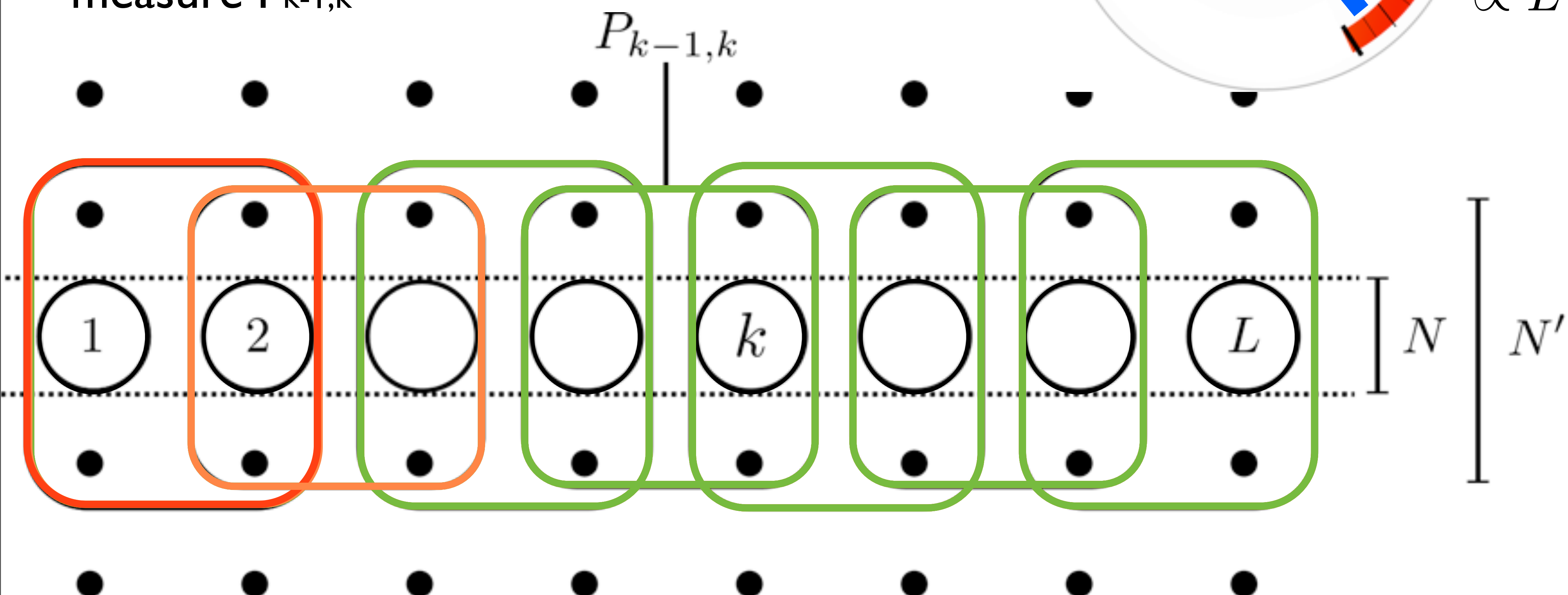

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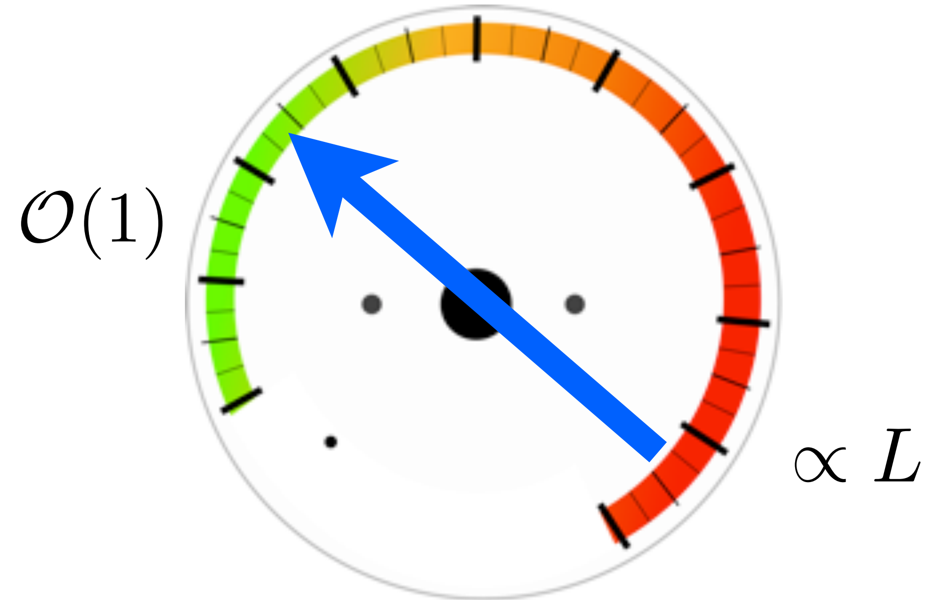


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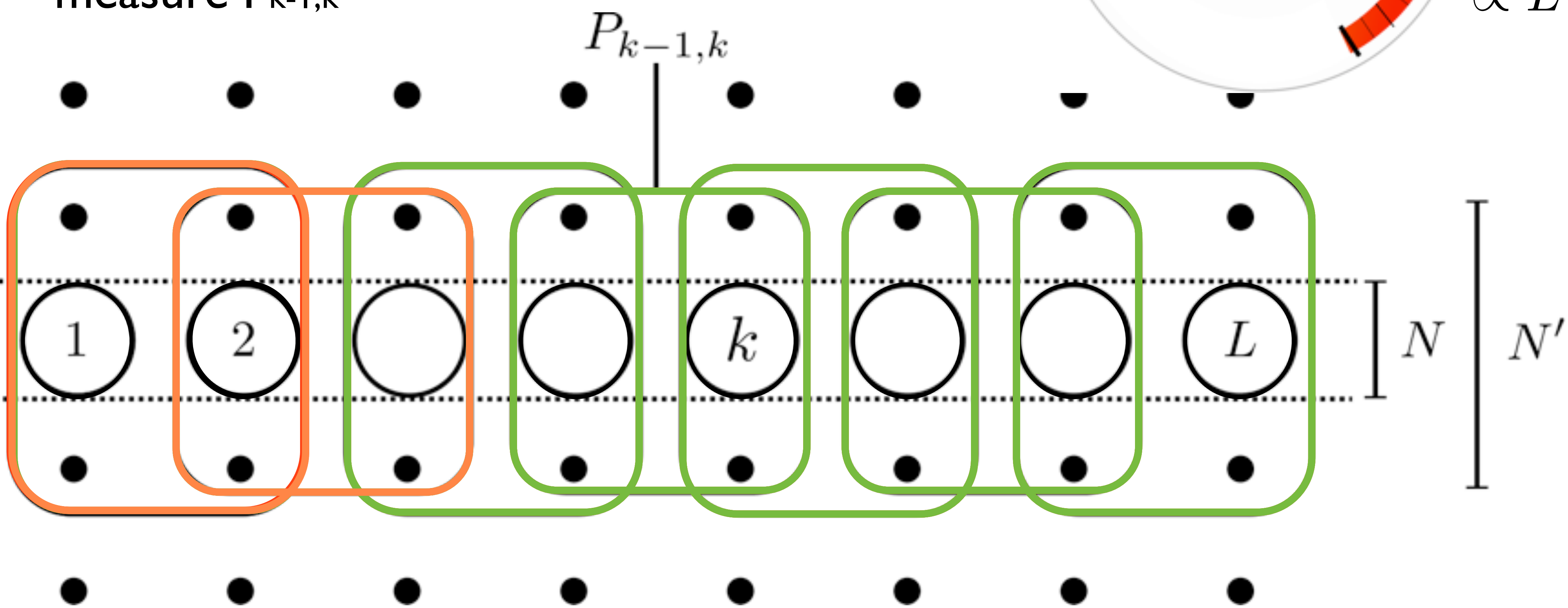

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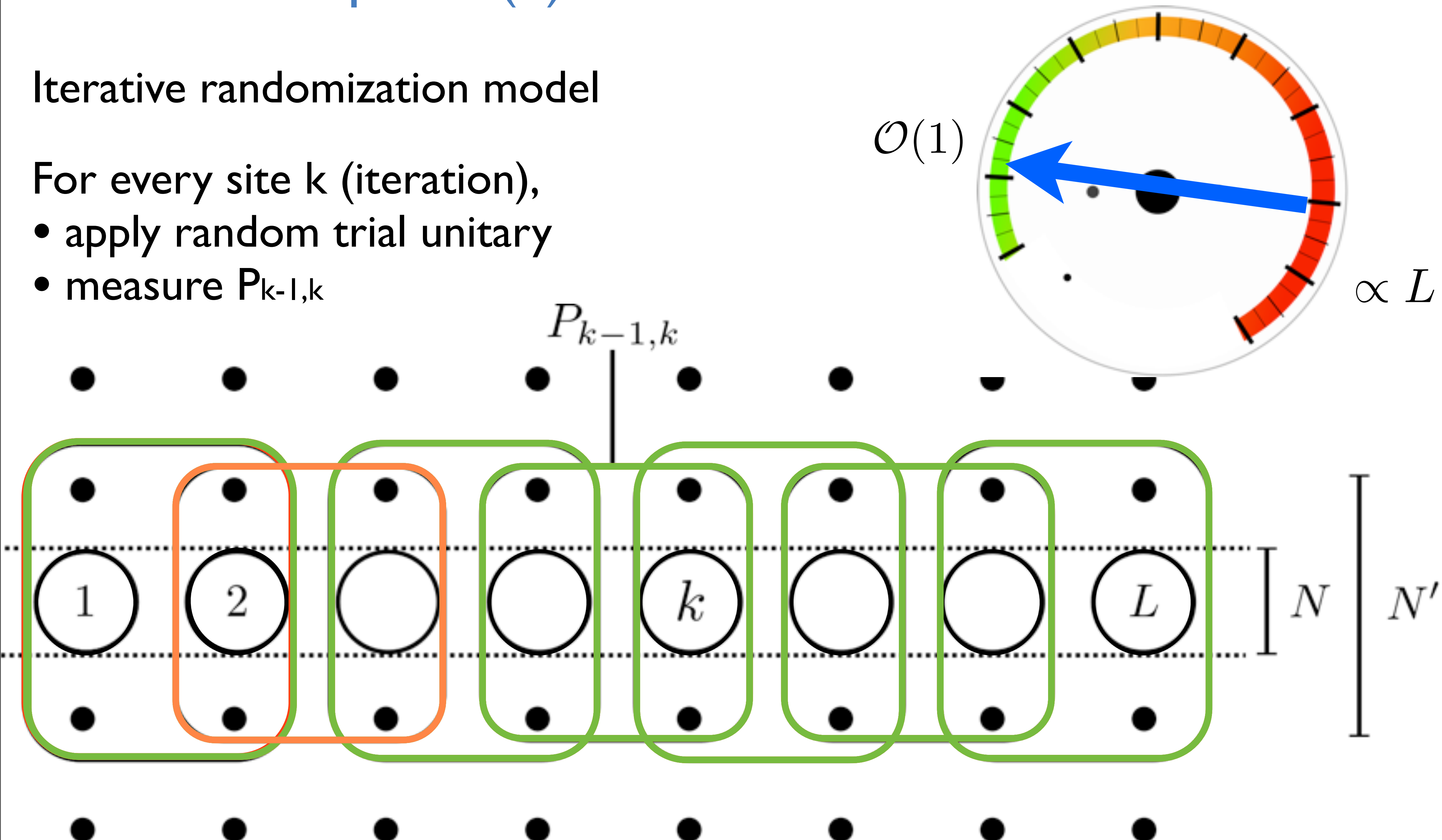

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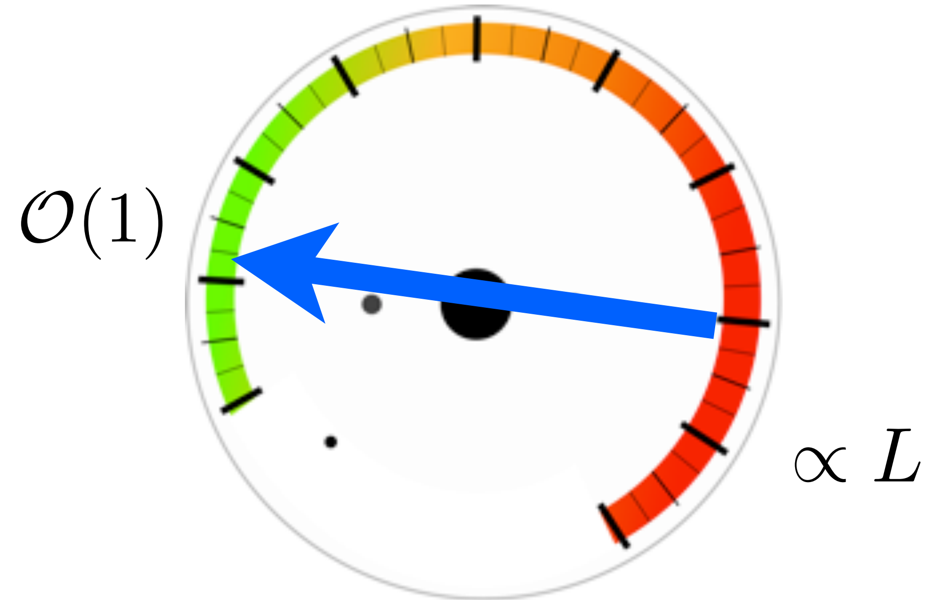


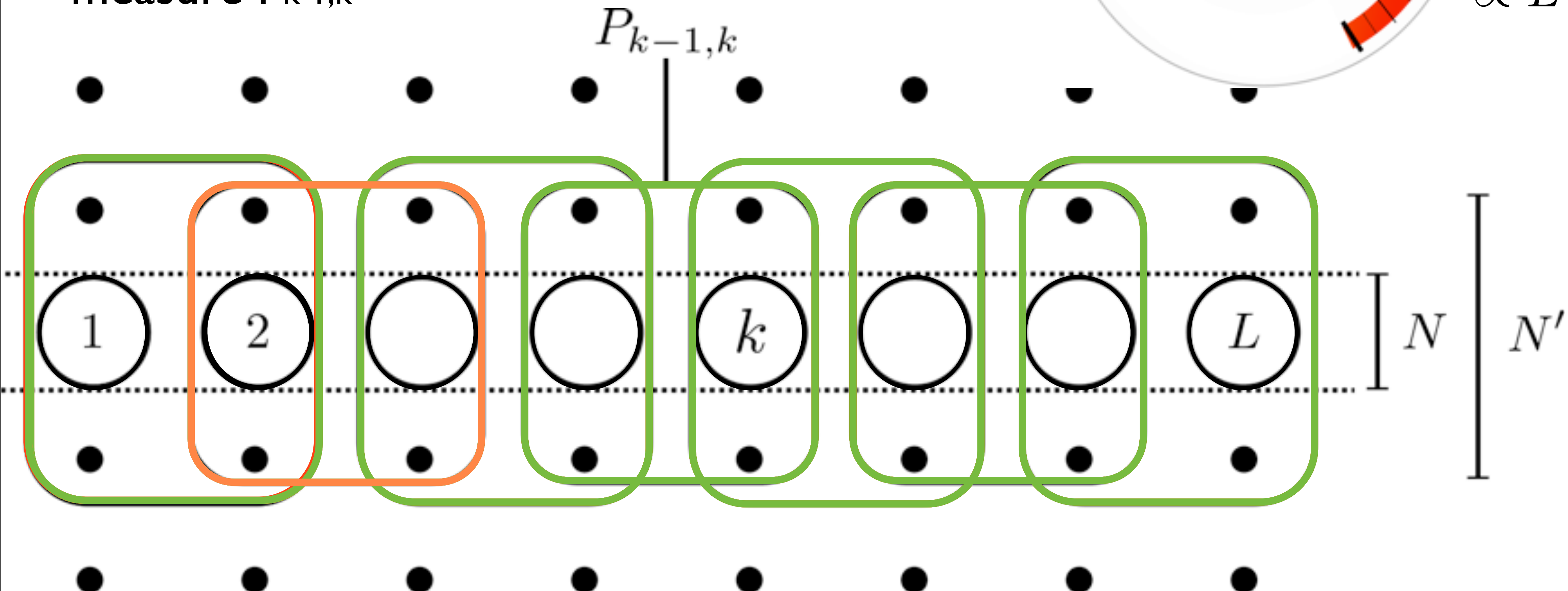

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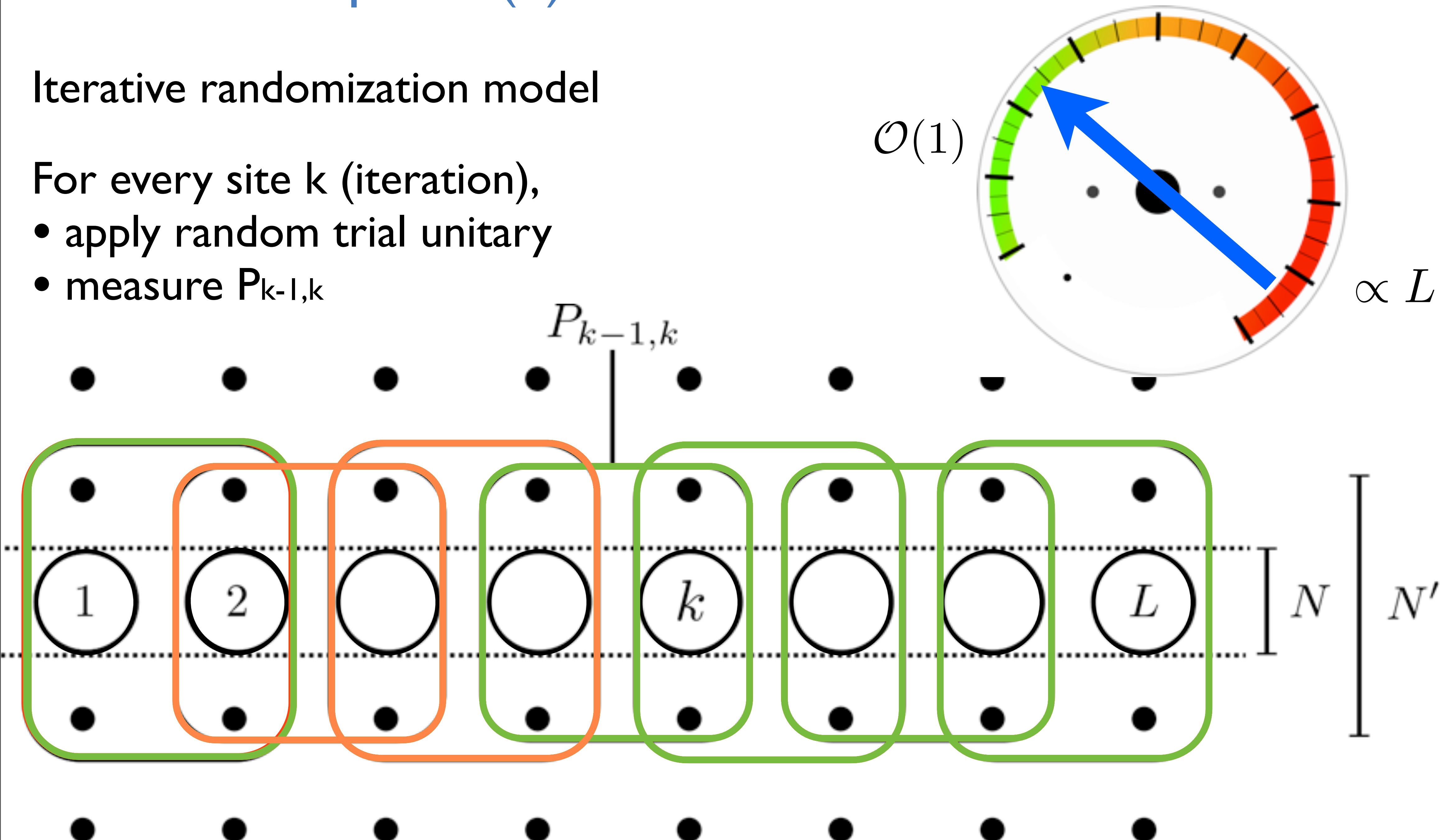

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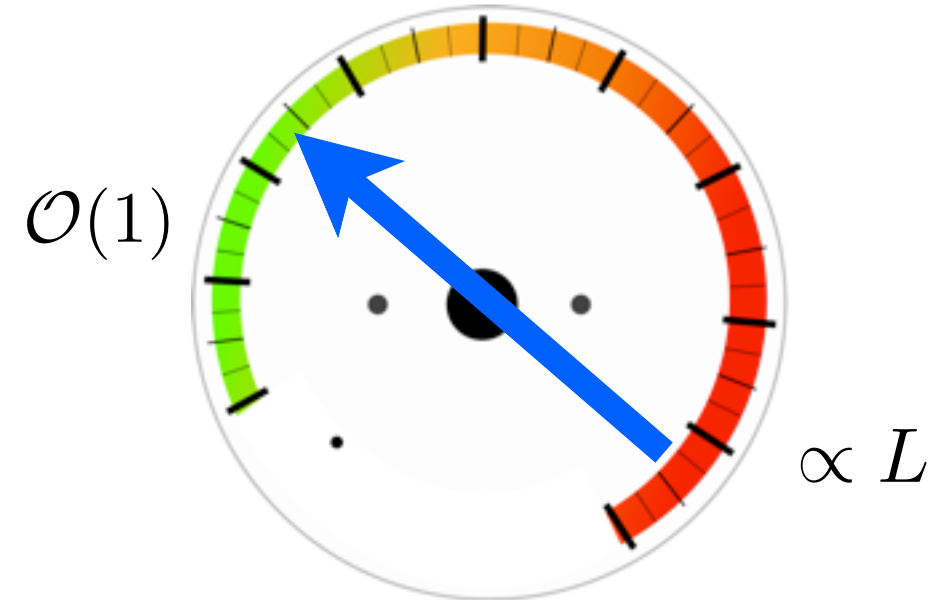


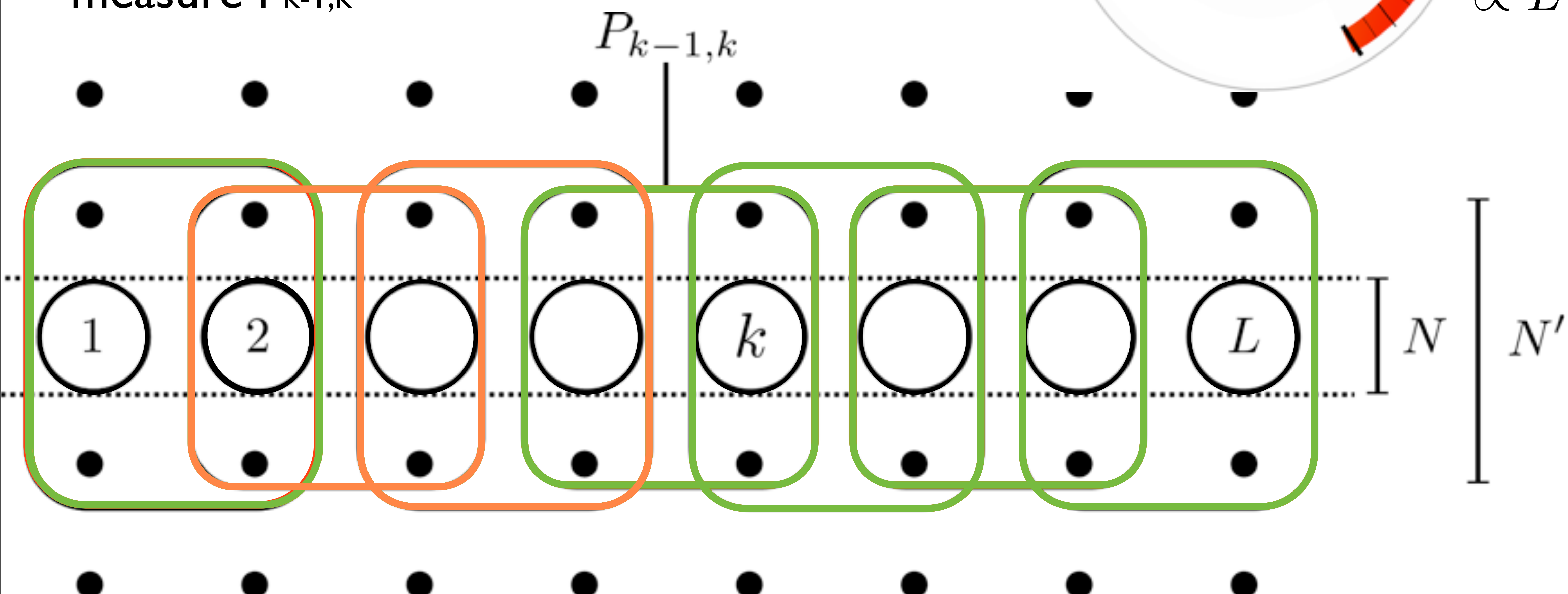


Immediate properties

# Sketch of the proof (II): iterative randomization model

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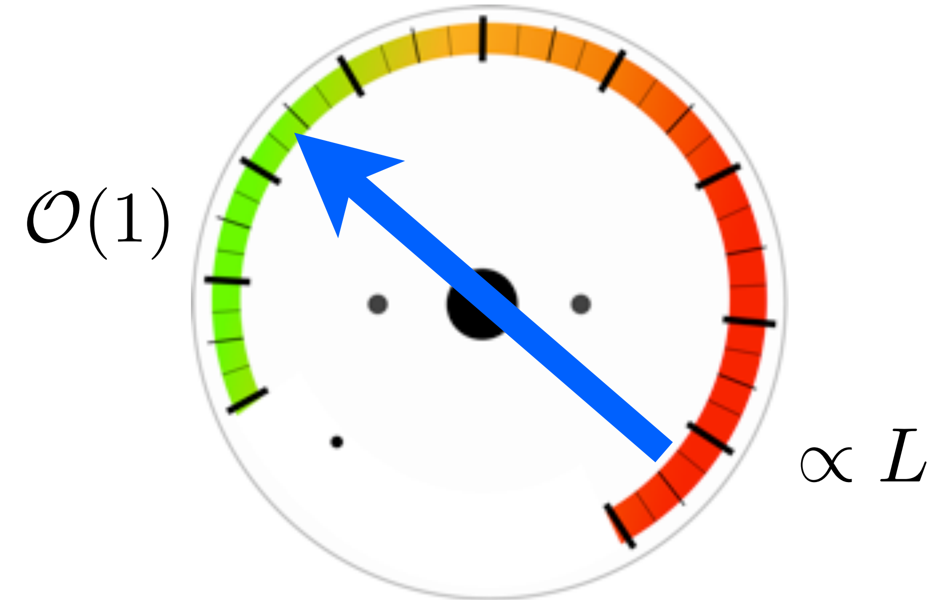
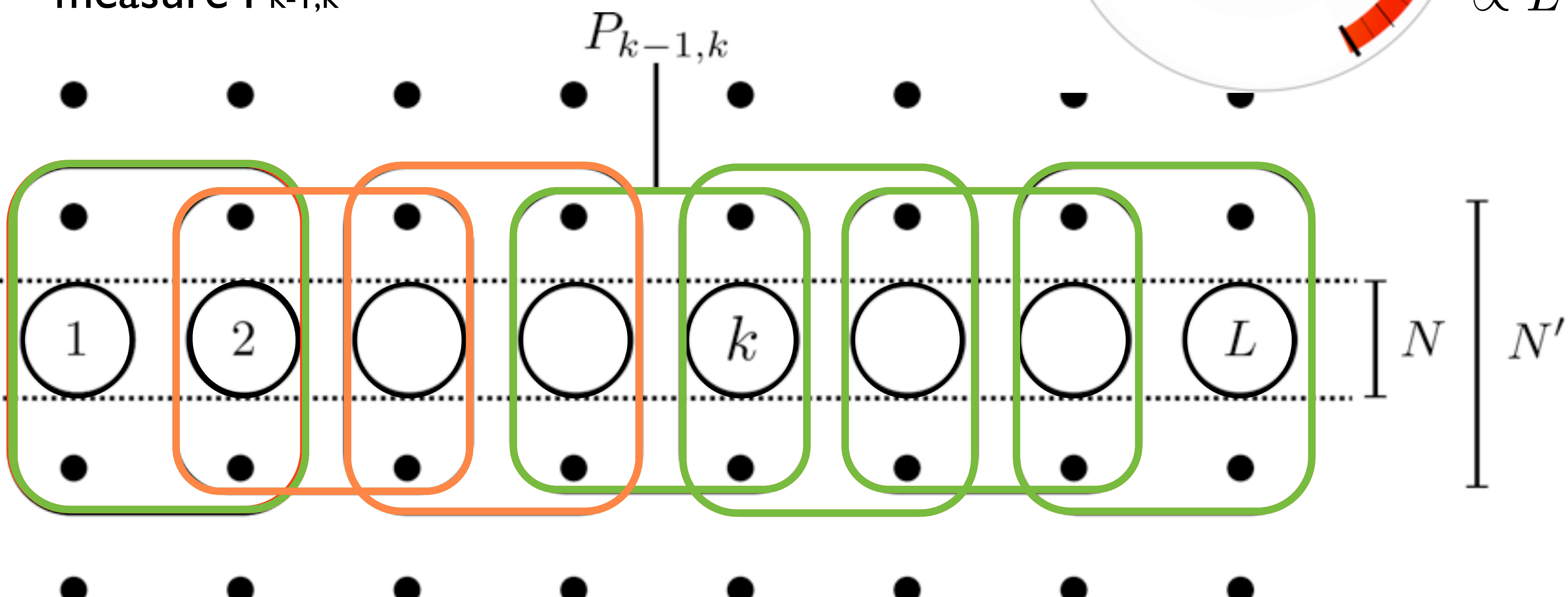
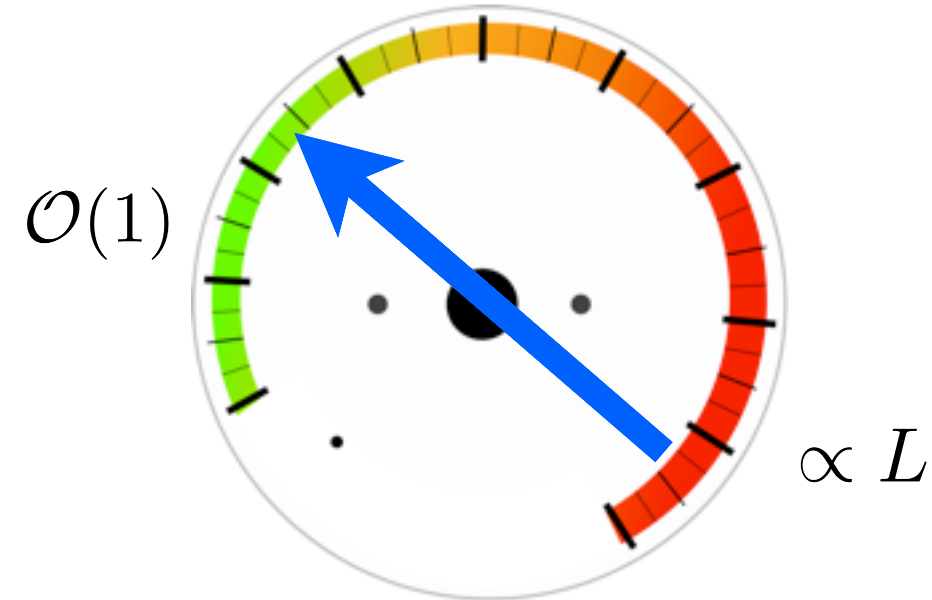



Immediate properties
- at any step, the energy is constant above the gs energy

# Sketch of the proof (II): iterative randomization model

Iterative randomization model

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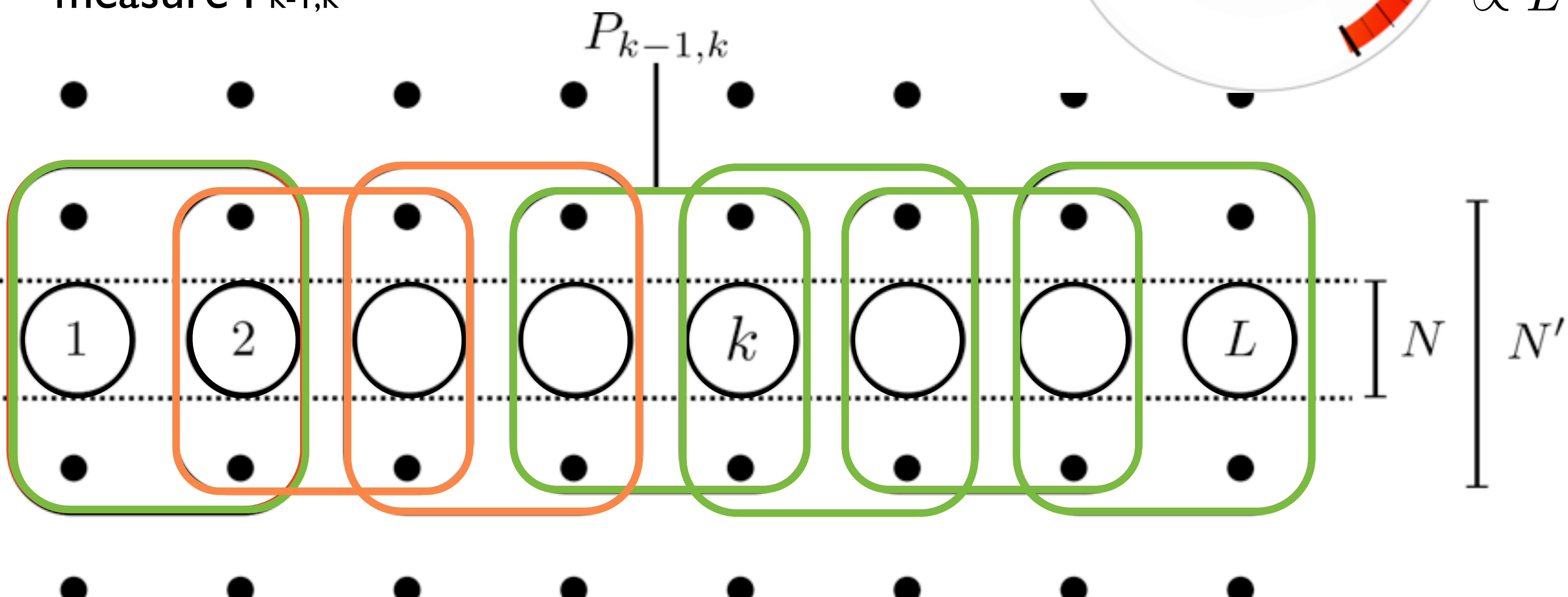
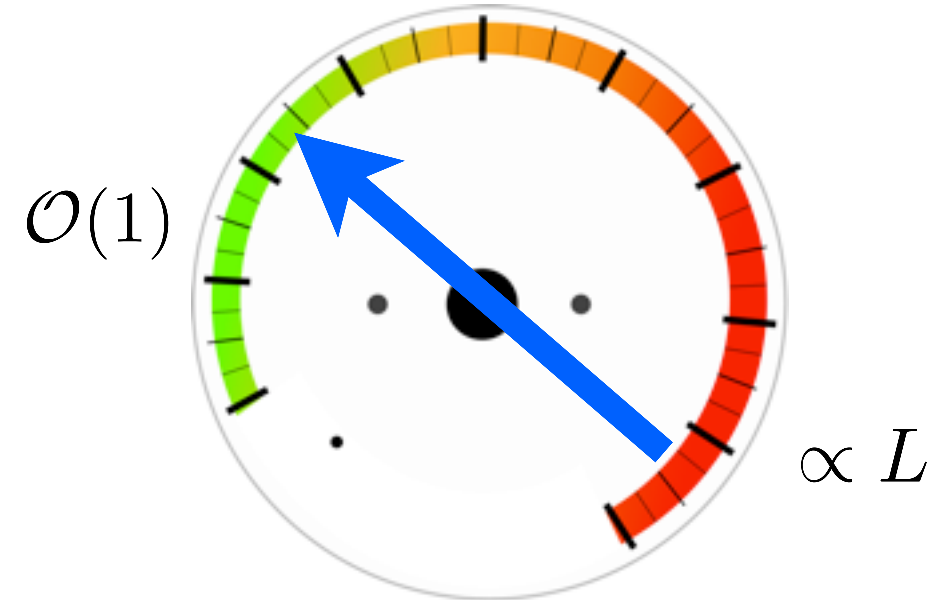



Immediate properties
- at any step, the energy is constant above the gs energy
- no need to backtrack

# Sketch of the proof (II): iterative randomization model




Iterative randomization model

For every site k (iteration),
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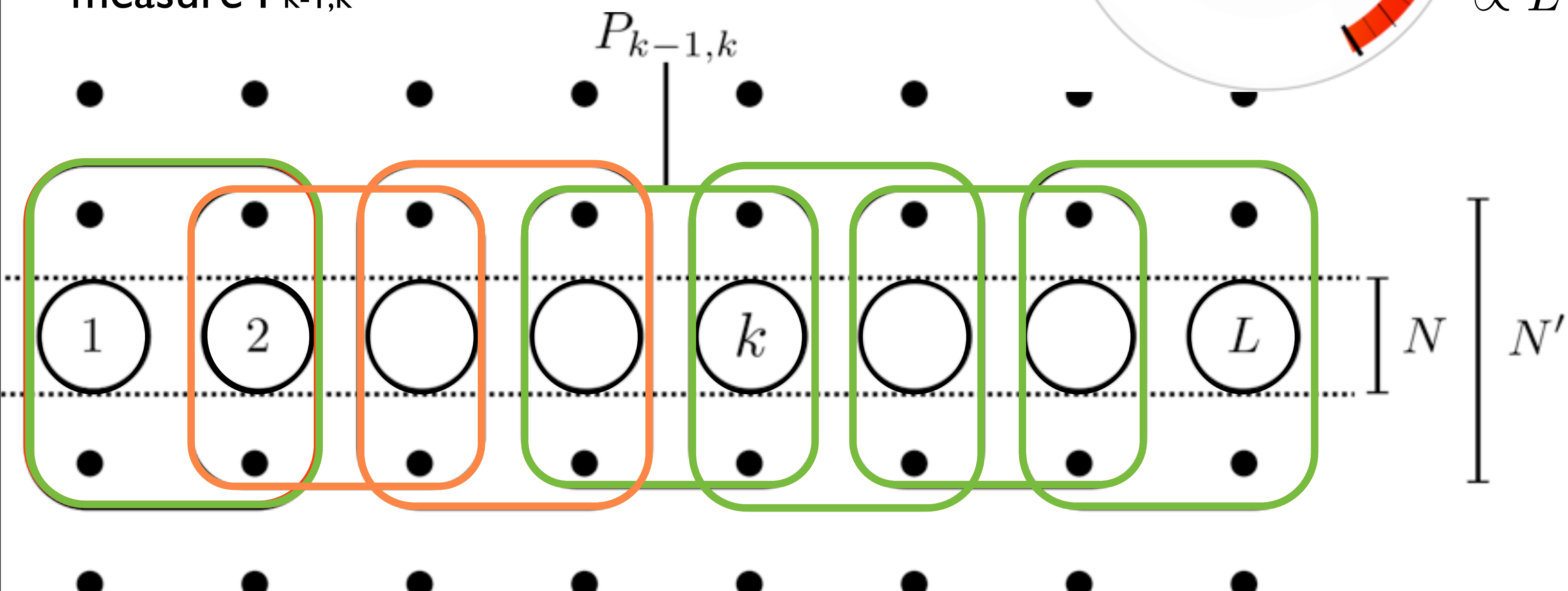


Immediate properties
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To show

# Sketch of the proof (II): iterative randomization model

Iterative randomization model

For every site k (iteration),
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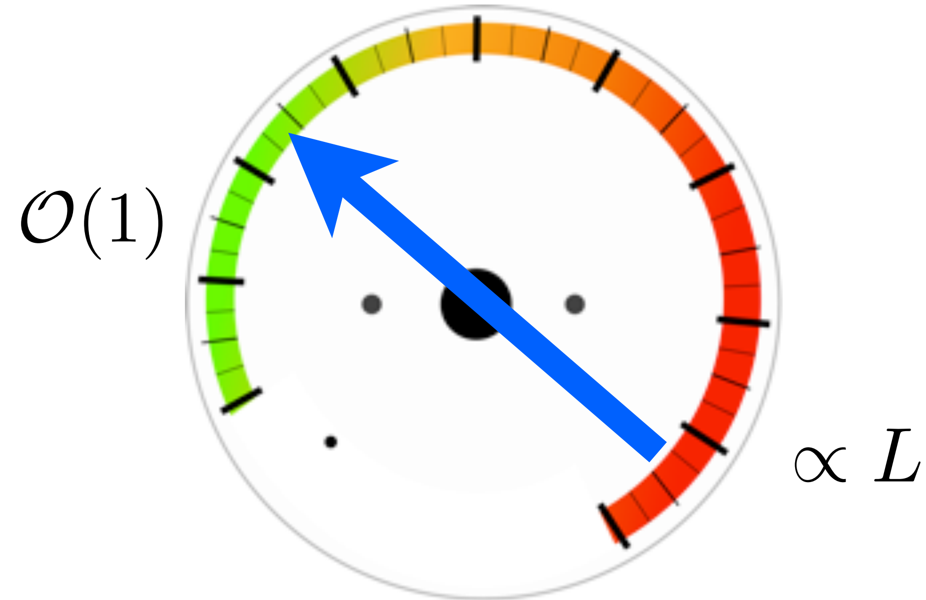


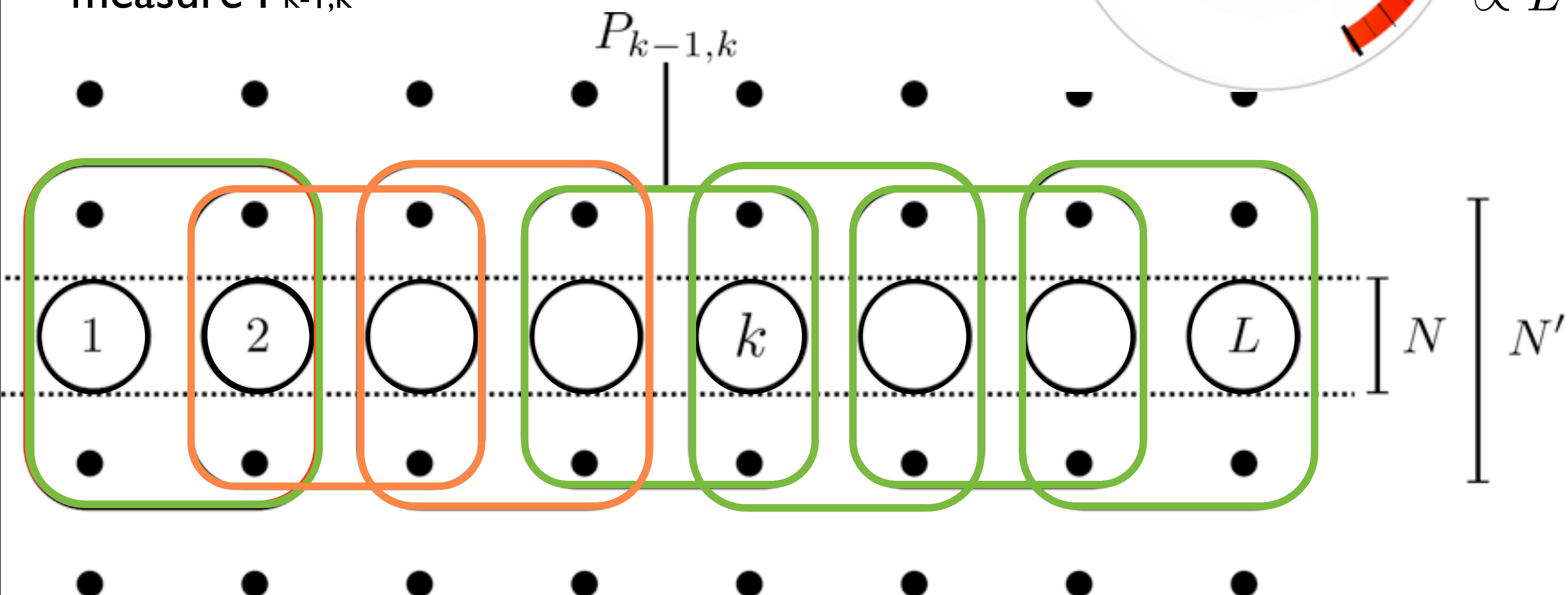


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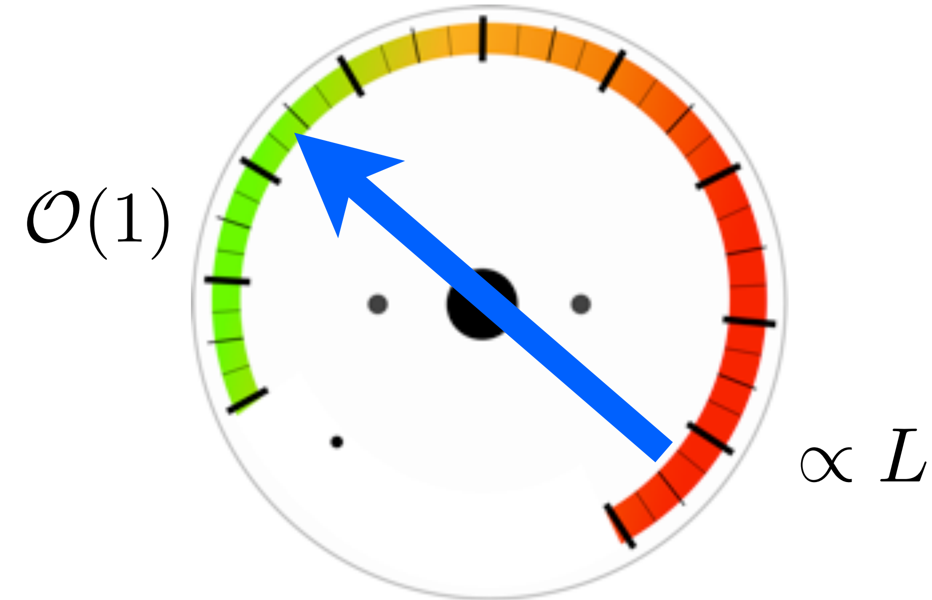
To show
- no dead-end and expected number of trials at each iteration is constant

# Sketch of the proof (II): iterative randomization model

Iterative randomization model




For every site k (iteration),
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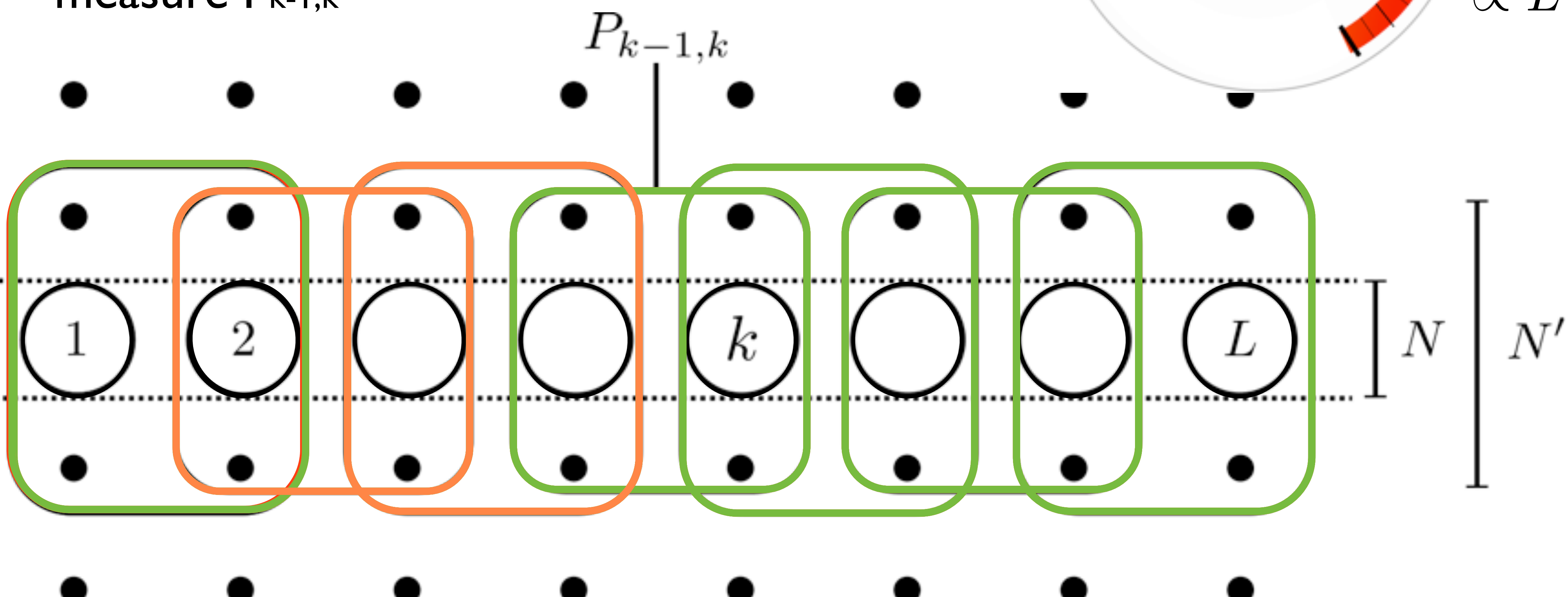


Immediate properties
- at any step, the energy is constant above the gs energy
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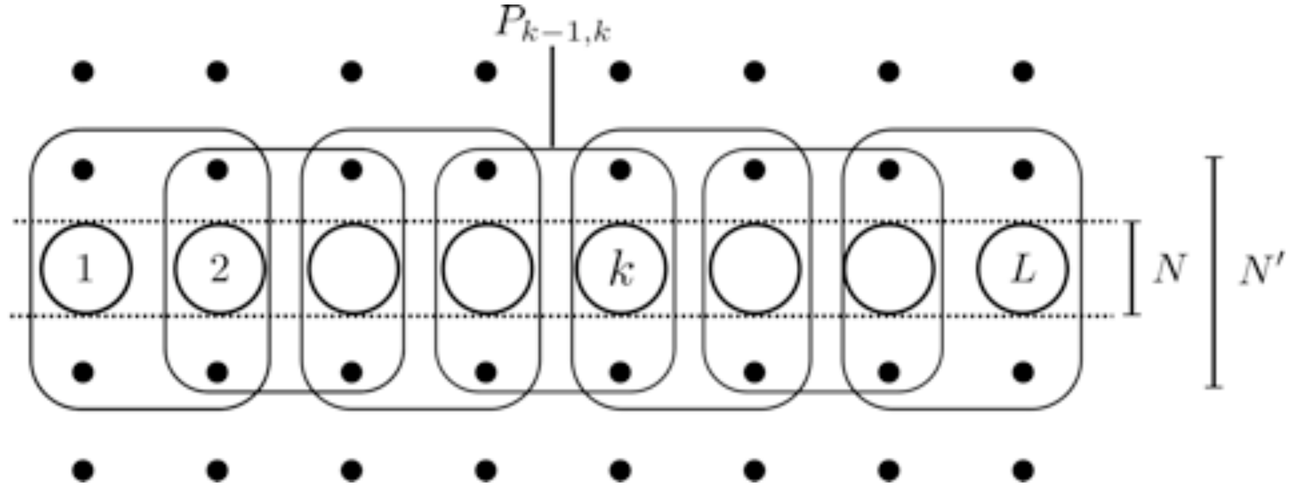
To show
- no dead-end and expected number of trials at each iteration is constant
- non-trivial average effect

# Sketch of the proof (IV): no dead-end

Iterative randomization model



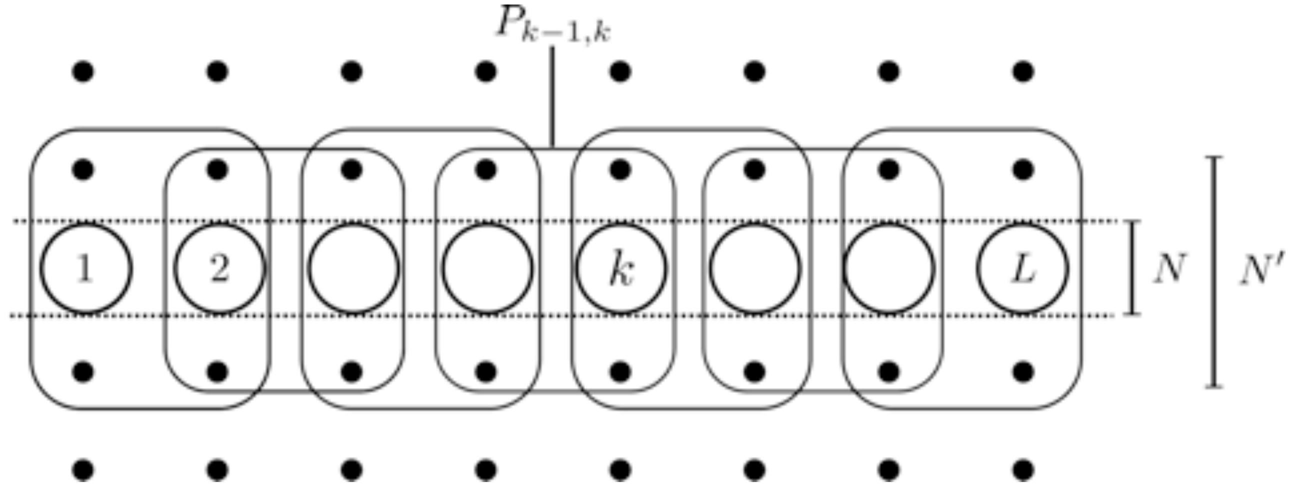

For every site k,
- apply random trial unitary
- measure $P_{k-1,k}$

Dead-end = impossible to find eligible unitary at a given iteration.
State of the strip, yet consistent with previous constraints, can't be extended.

# Sketch of the proof (IV): no dead-end

Iterative randomization model

For every site k,
- apply random trial unitary
- measure $P_{k-1,k}$




Dead-end = impossible to find eligible unitary at a given iteration.
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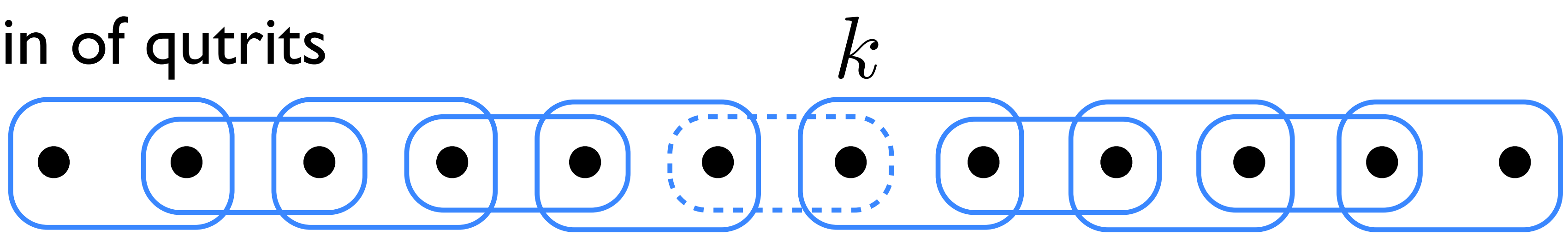
Simple example: chain of qutrits




$$P_{i,i+1} = |00\rangle\langle 00| + |11\rangle\langle 11| + |22\rangle\langle 22|$$

$$P^*_{k-1,k} = |00\rangle\langle 00| + |11\rangle\langle 11|$$

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# Sketch of the proof (IV): no dead-end

Iterative randomization model

For every site k,
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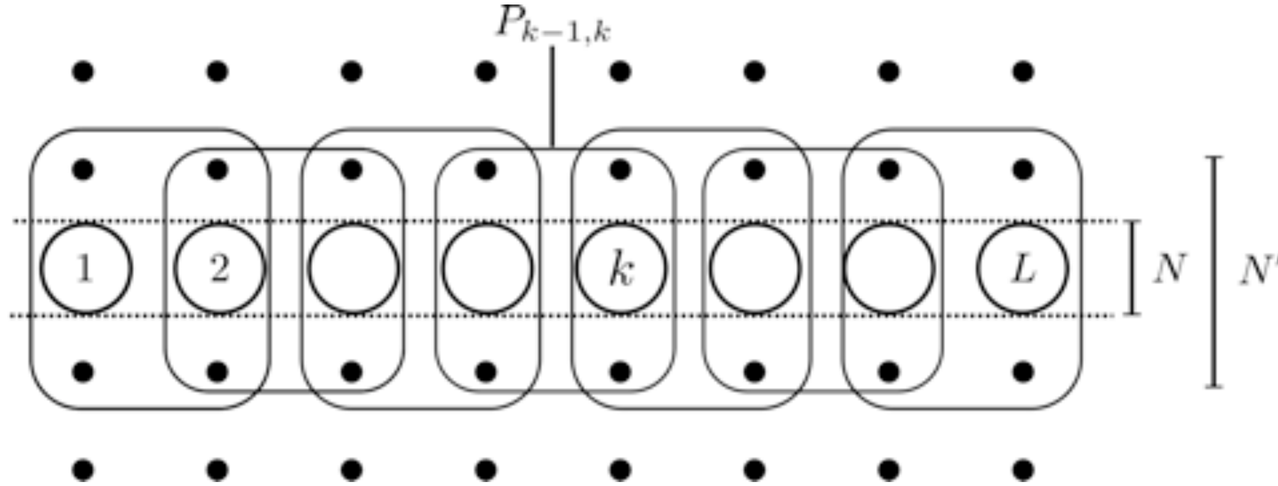


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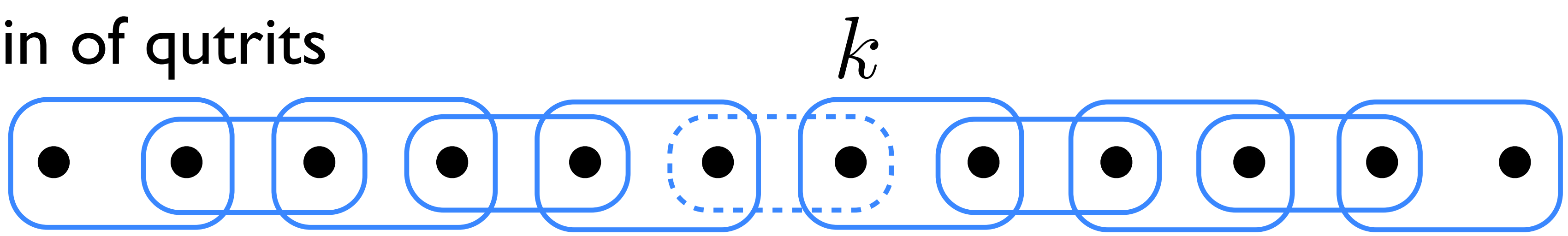


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# Sketch of the proof (IV): no dead-end

Iterative randomization model

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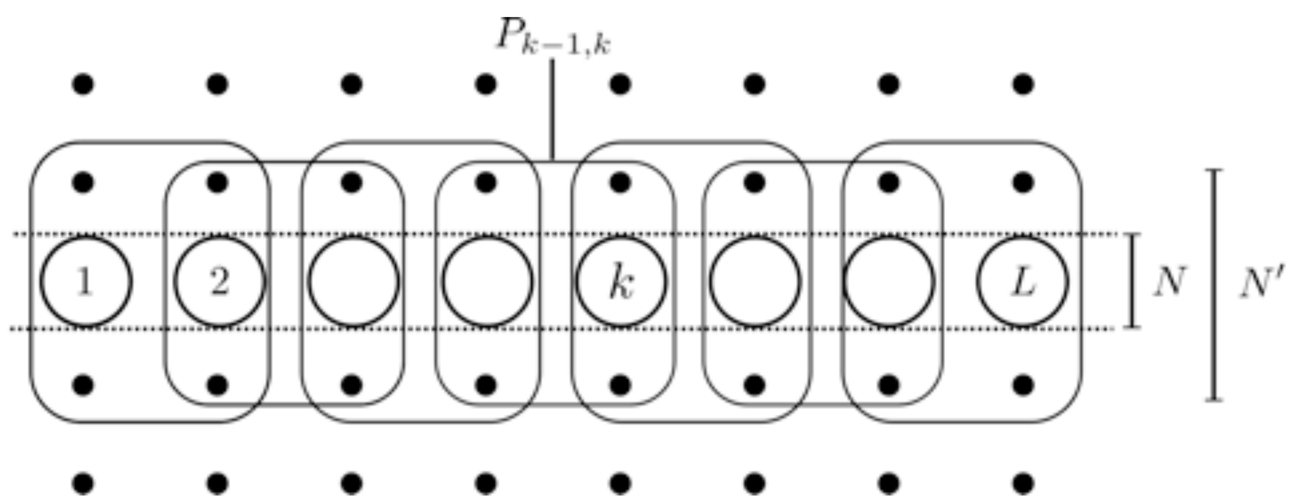


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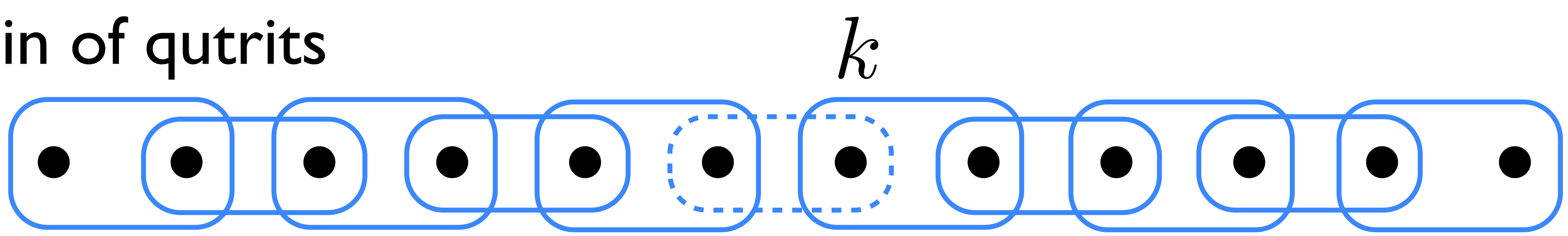


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Dead-end: start preparing all 2 state...

# Sketch of the proof (IV): no dead-end

Iterative randomization model

For every site k,

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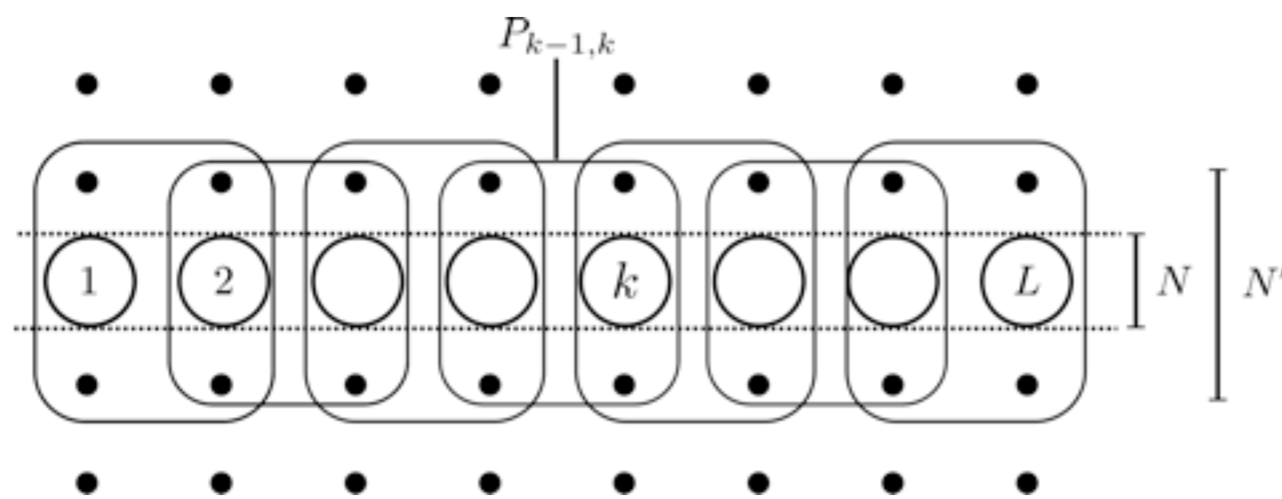
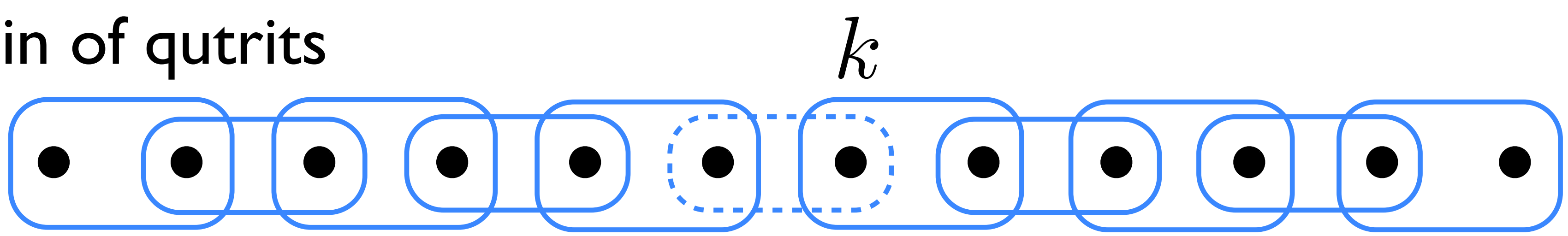



Dead-end = impossible to find eligible unitary at a given iteration.
State of the strip, yet consistent with previous constraints, can't be extended.

Simple example: chain of qutrits

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# Sketch of the proof (IV): no dead-end

Iterative randomization model

For every site k,
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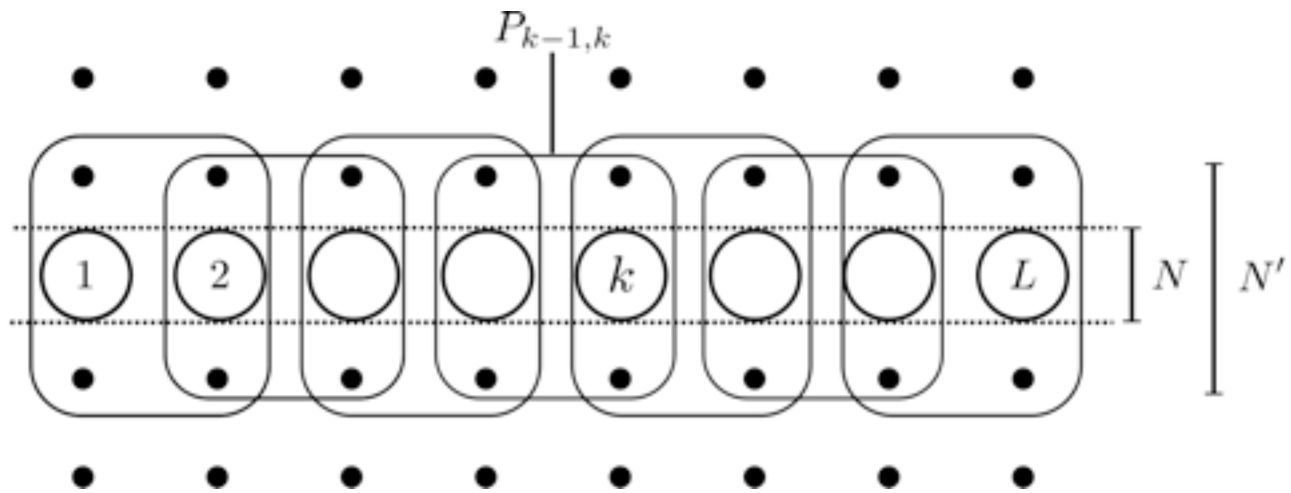


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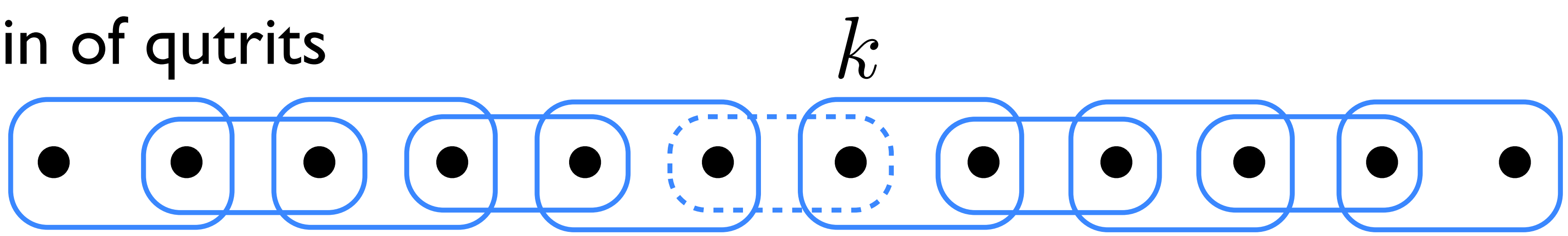


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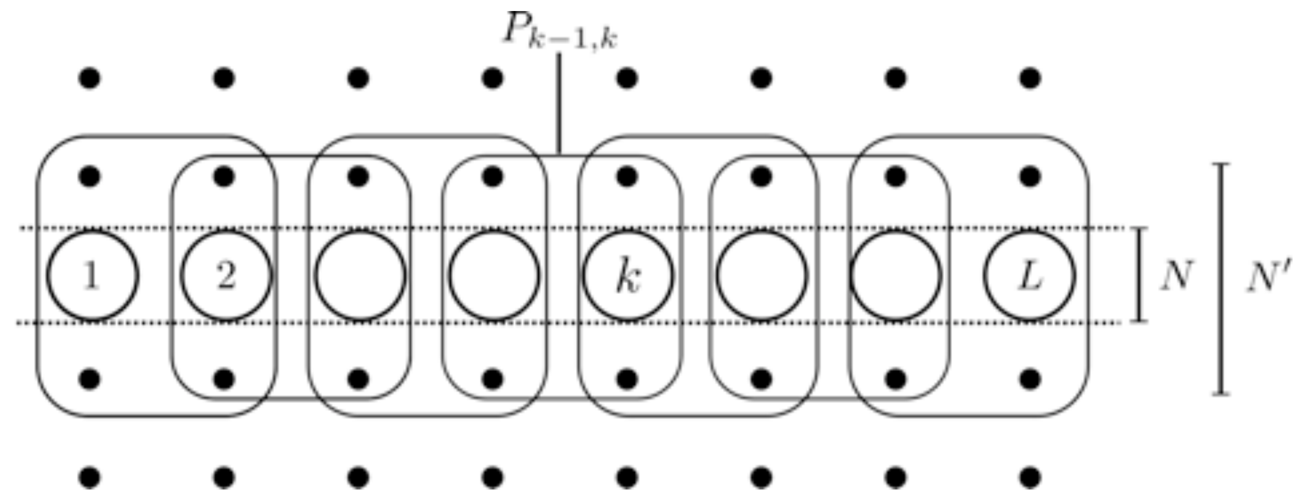


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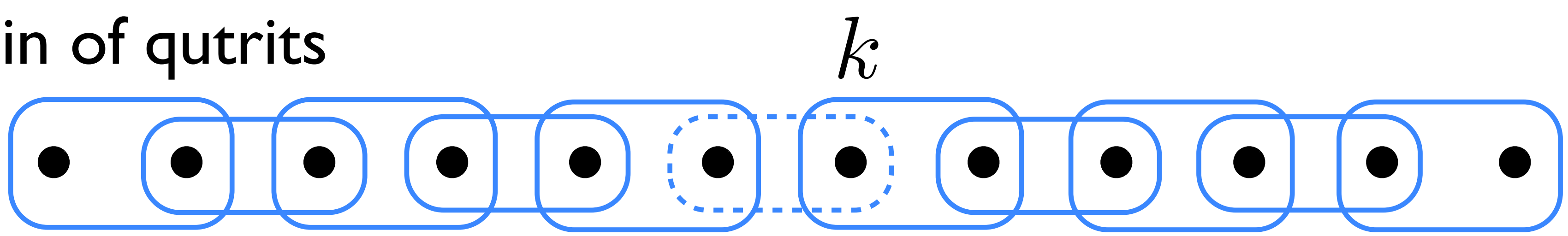


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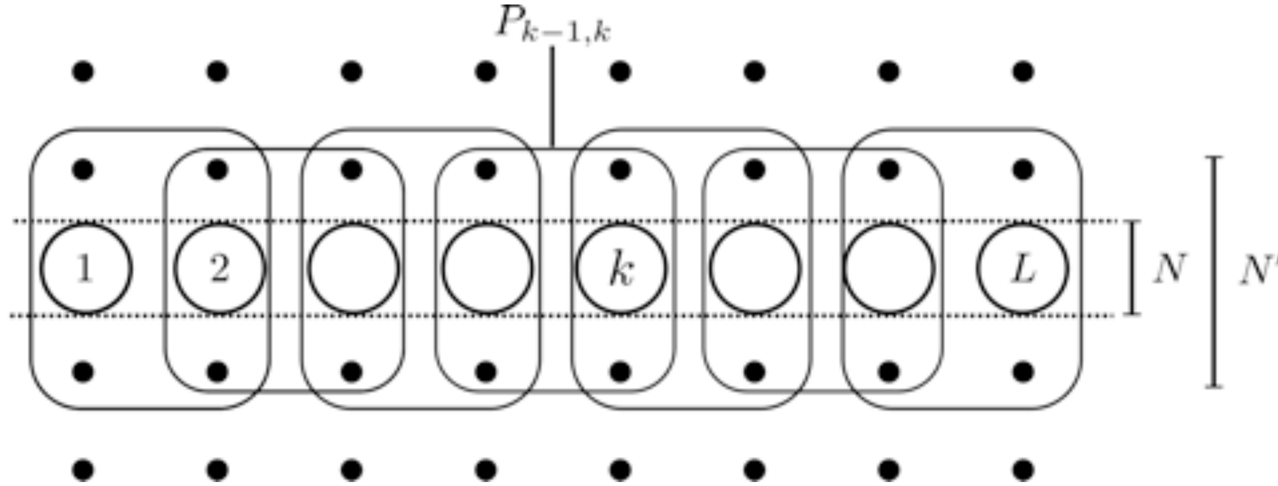


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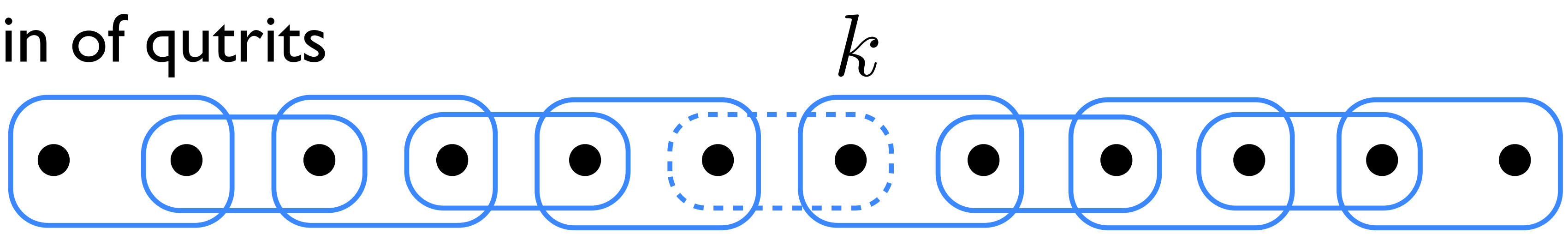


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different kernels

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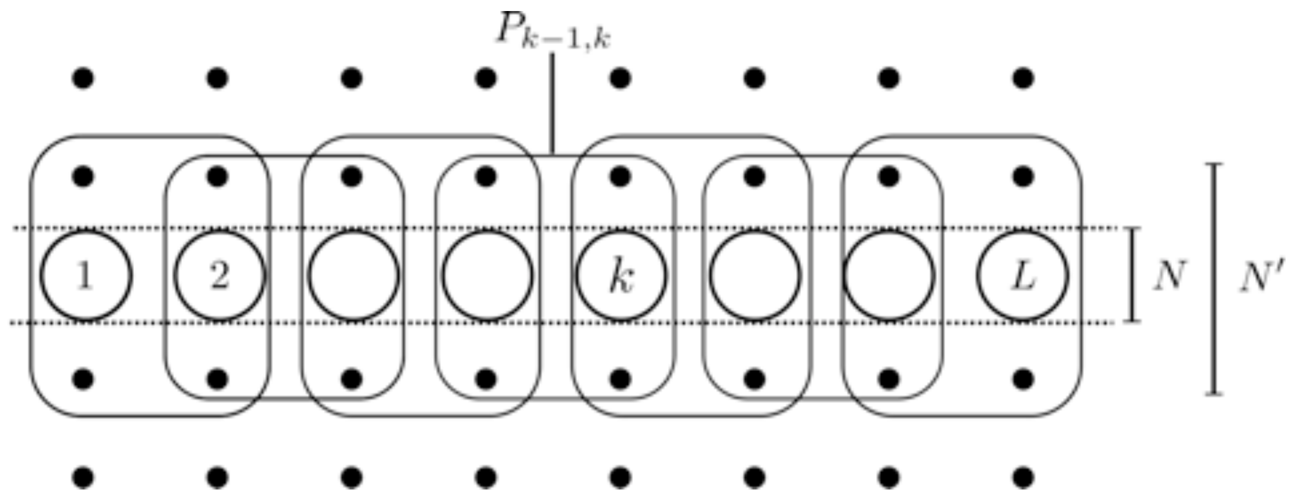

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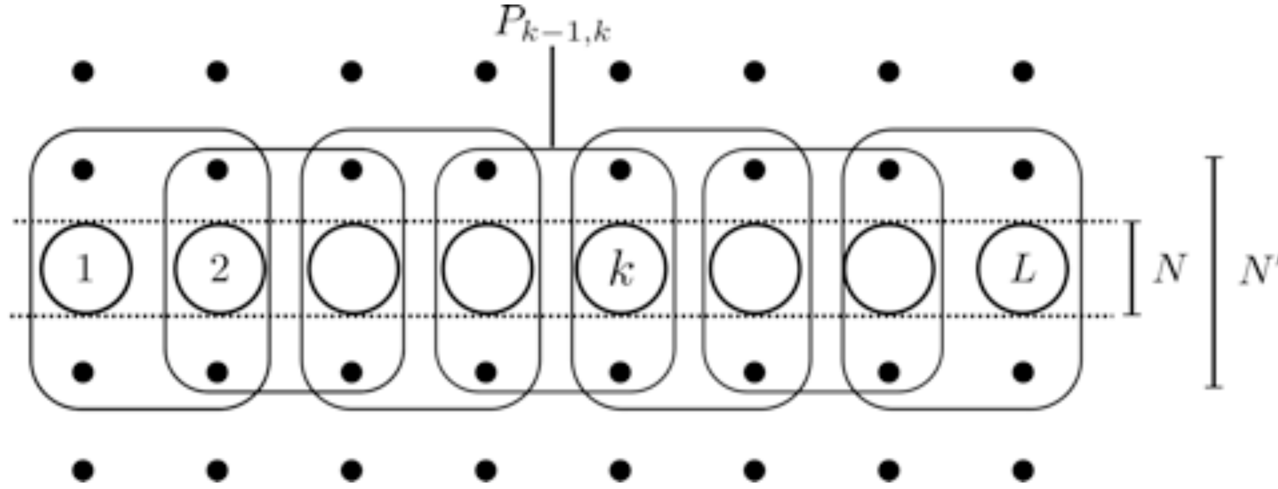


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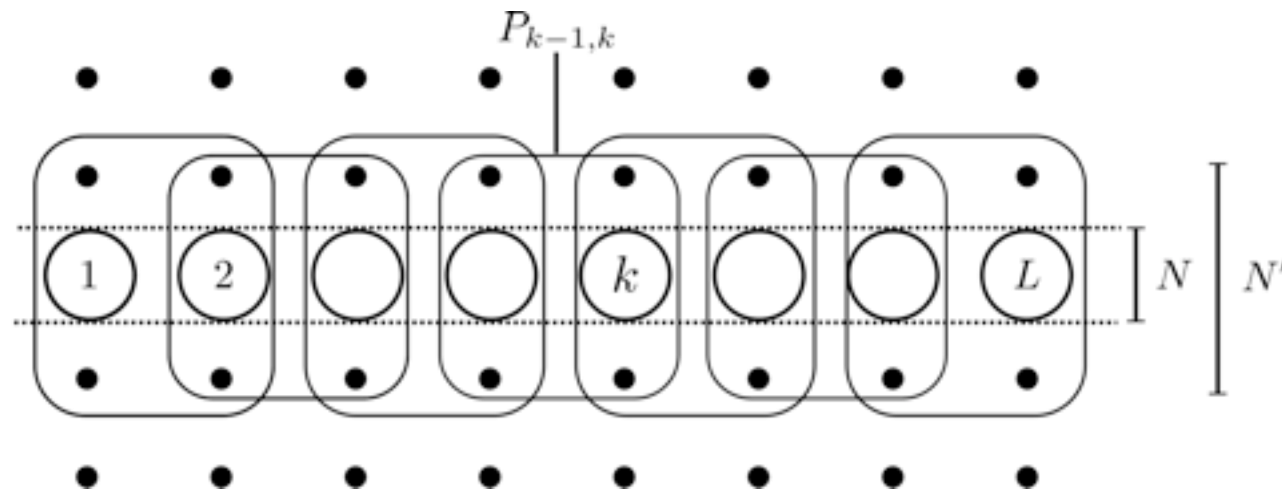


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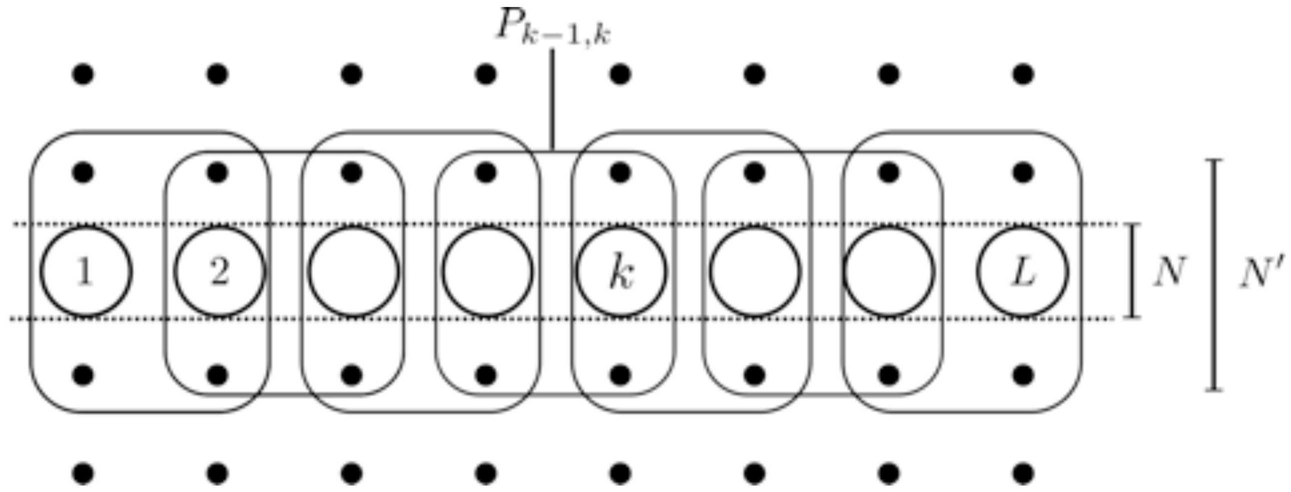


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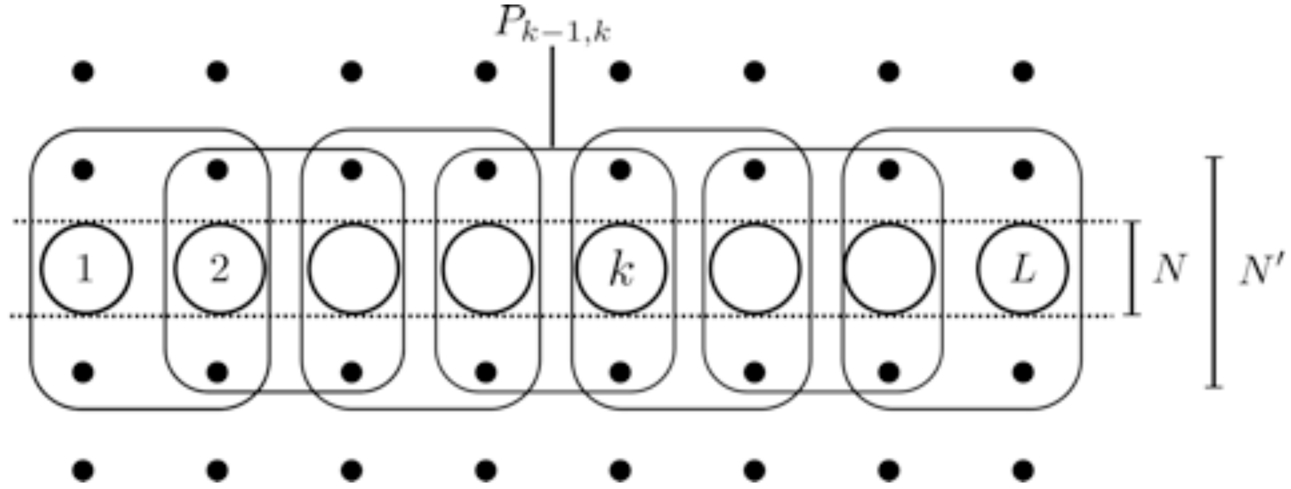


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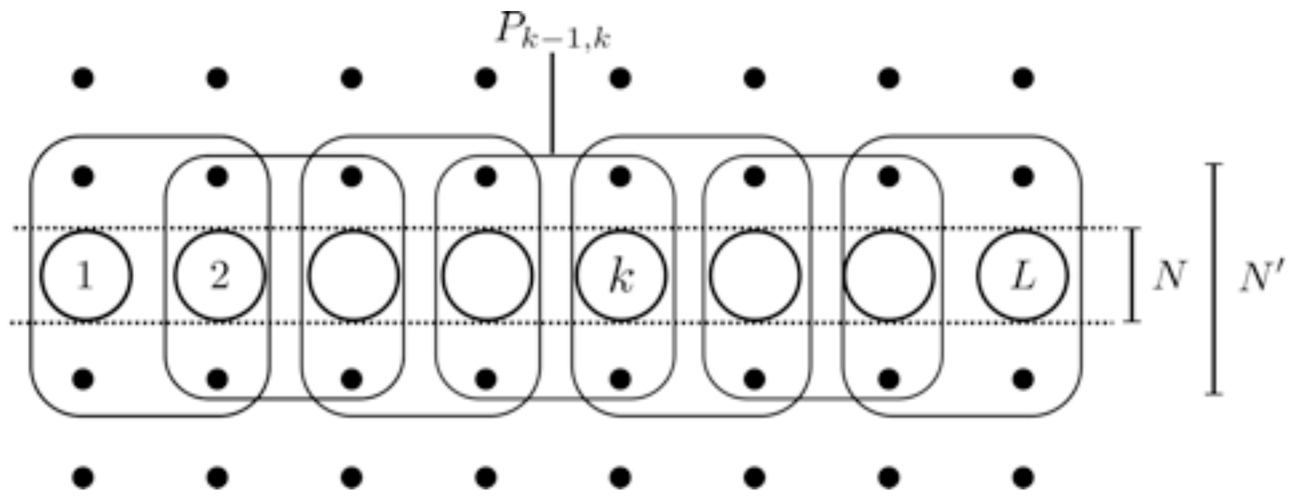


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**Thank you for your attention.**