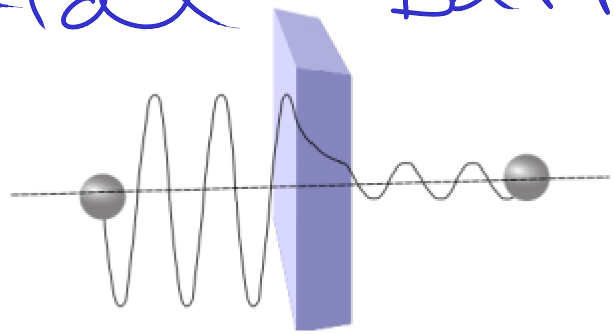
Tunneling through a potential barrier



① ssh -D 1080 username@outsidecomputer.edu

② set your browser's SOCKS proxy to 127.0.0.1 port 1080

Fundamental limits for Quantum Thermodynamics

J. Oppenheim (University College London)

quant-ph:
1111.3834 Horodecki, Oppenheim
1111.3882 Brandao, Horodecki, Oppenheim, Renes, Spekkens
1211.1037 Faist, Dupuis, Oppenheim, Renner
[see poster]

QIP2013

Its called thermodynamics because we take the thermodynamic limit!

System size
number of particles $\longrightarrow \infty$

Thermodynamics in the opposite extreme
Finite size (micro, nano)
and/or quantum

Outline

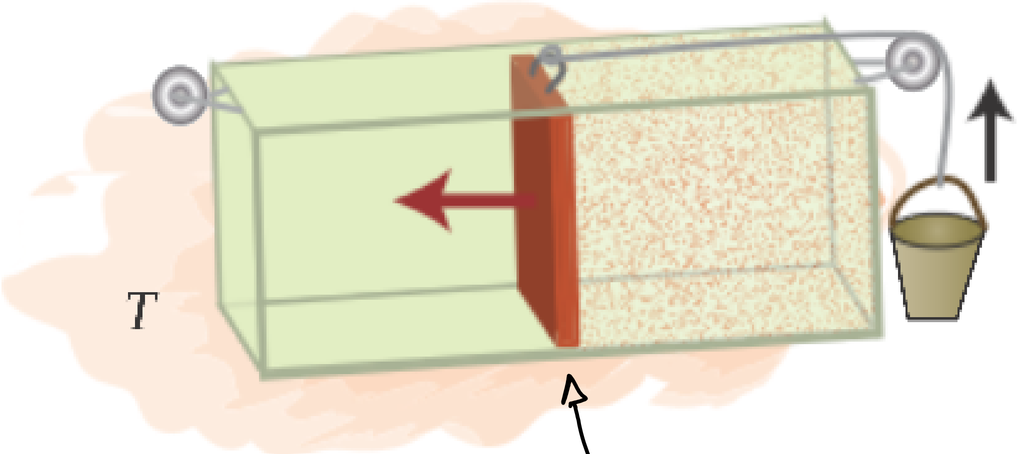
Result: two free energies
thermo-majorization

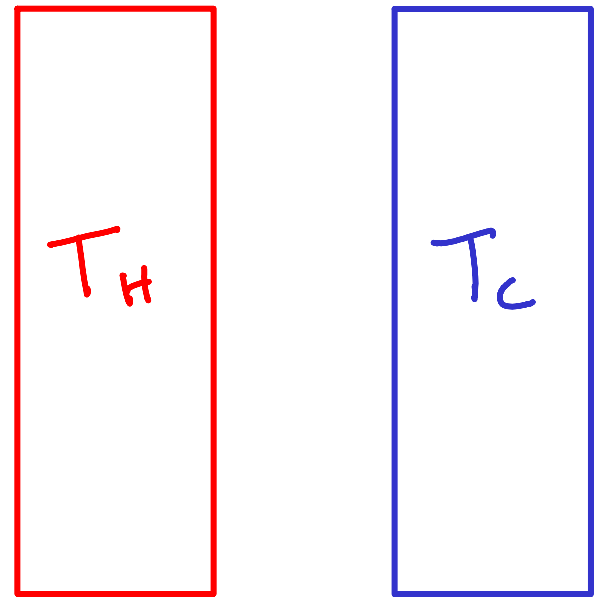
Fundamental laws of quantum Thermodynamics

Paradigm: Resource theories
Single-shot information theory

Sketch ideas: What thermodynamical transitions are possible?
When reversible?

Thermodynamics





out of equilibrium

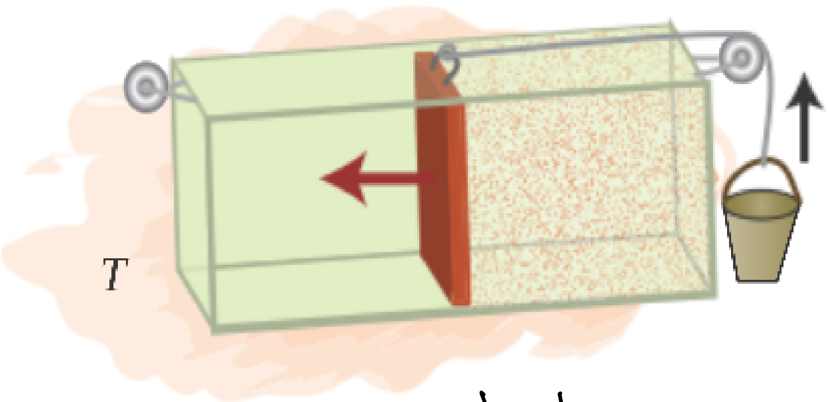
$$F = E - TS$$

$$W = F(\rho_{initial}) - F(\rho_{final})$$

$\rho_{initial} \to \rho_{final}$ only if $\Delta F \geq 0$

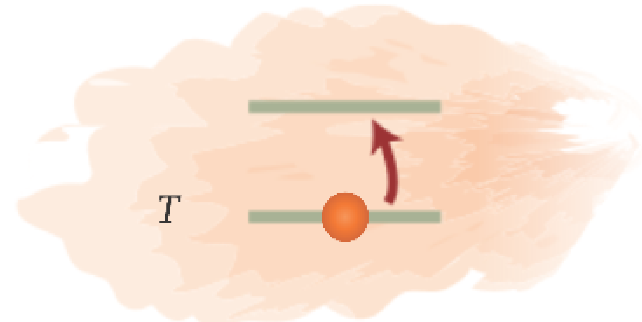
Work

Macro



weight

Micro



wit

$\cancel{F = E - TS}$

F max

F min

A combination of resources

Purity as a resource: Theory of entropy

"Single shot" purity theory

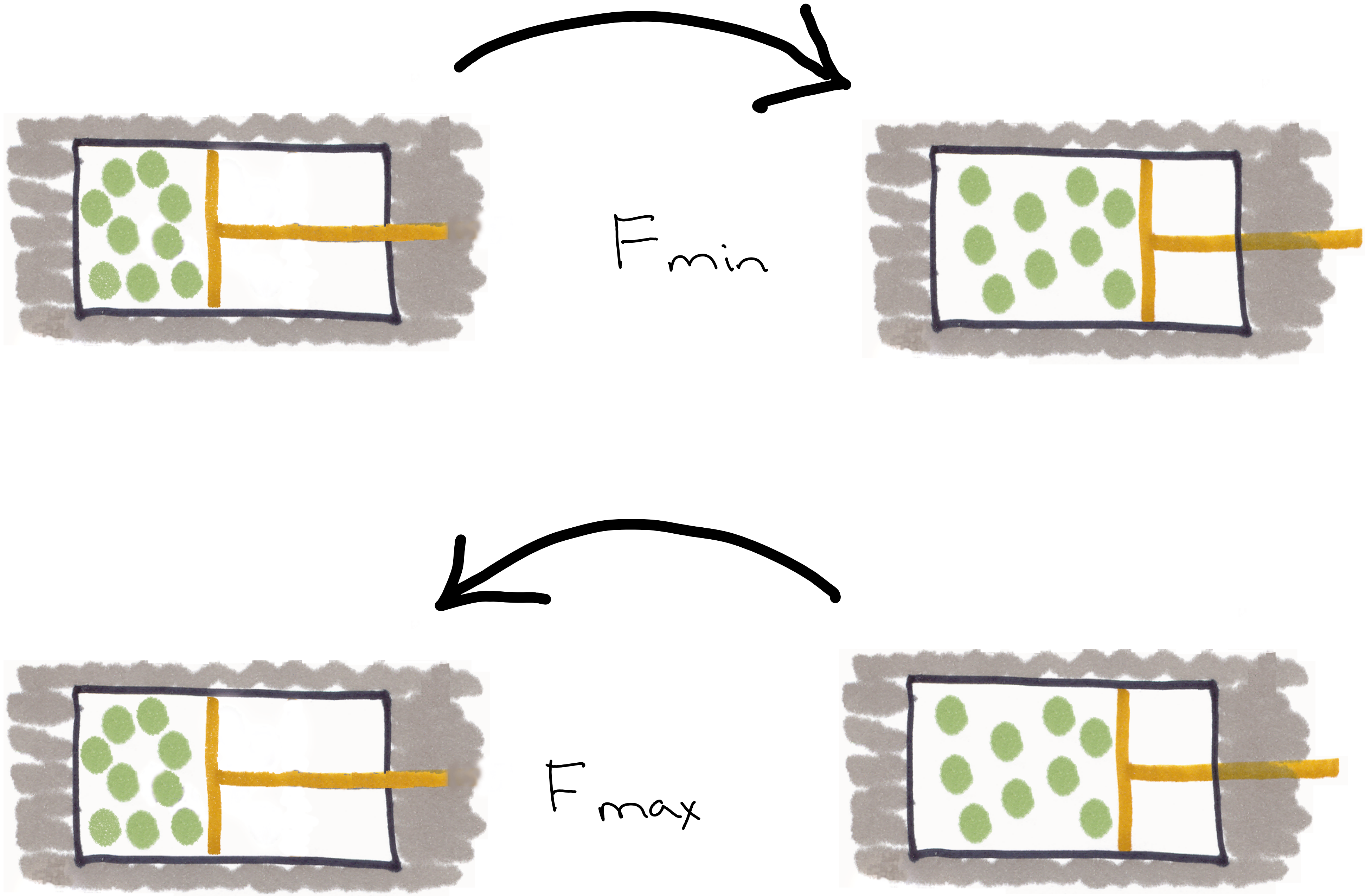
Extracting work from a single system?

- Horodecki$^{\otimes 2}$, J.O, (2003)
- Dahlsten et. al (2011) (allow ε failure)

Asymmetry: Energy

- Gour, Spekkens (2008)

Reversibility?

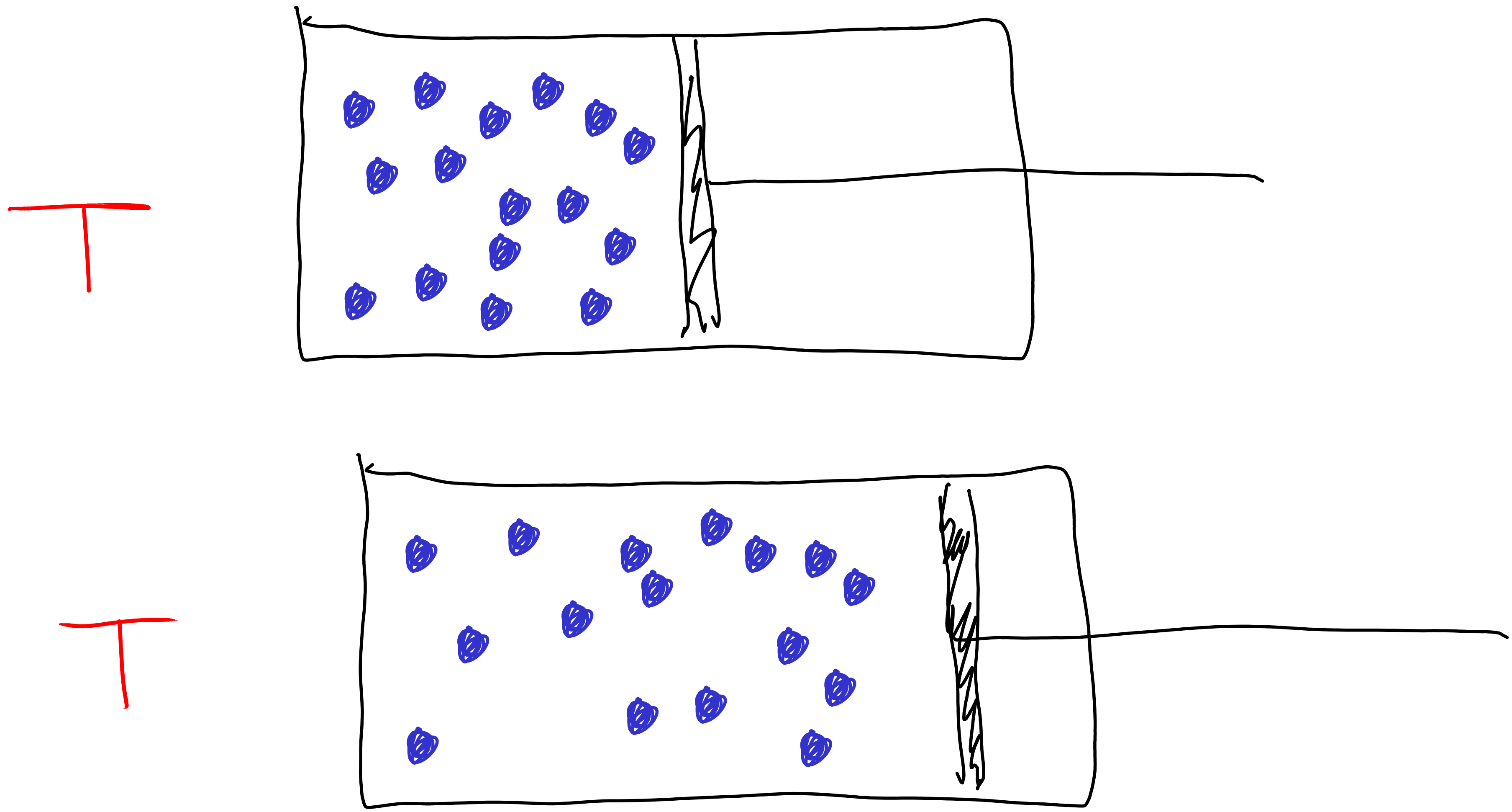


Not

Summary of Results

- Paradigm for Q.T.
- Distillation of work F^{min}
- Formation of states F^{max}
- Criteria for state-state transformations (thermo-majorization)
- irreversibility due to finite size effects
- irreversibility due to quantum coherences
- criteria for reversibility
- micro carnot cycle

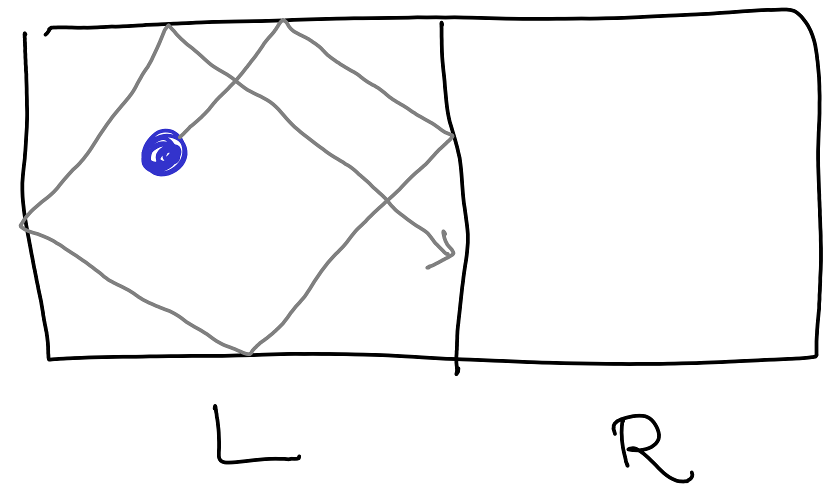
Drawing Work...

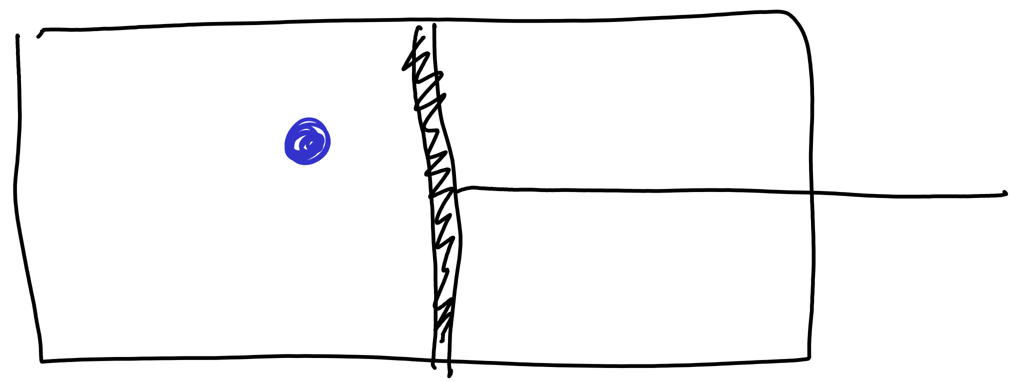


... from Information

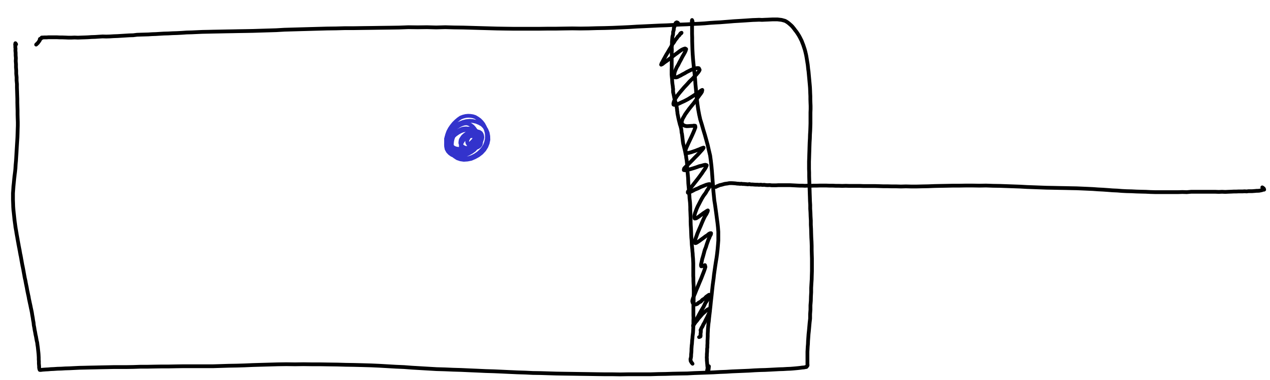
$p(L) = 1$

$p(R) = 0$



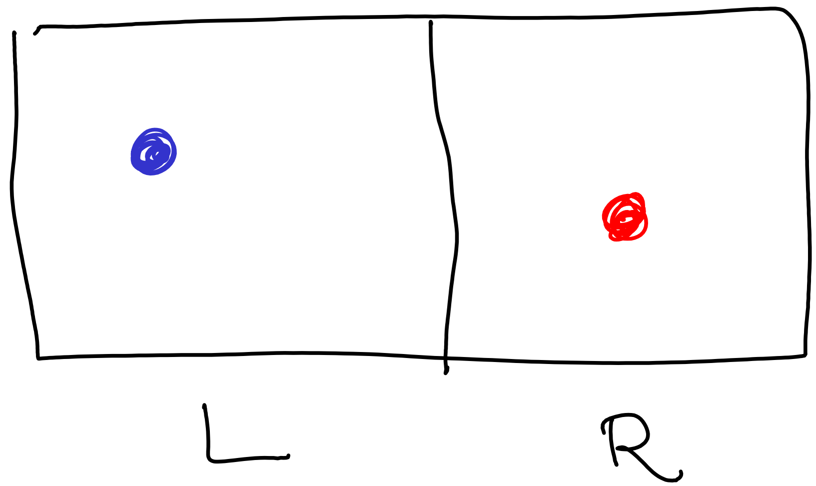


$W = kT \ln 2$

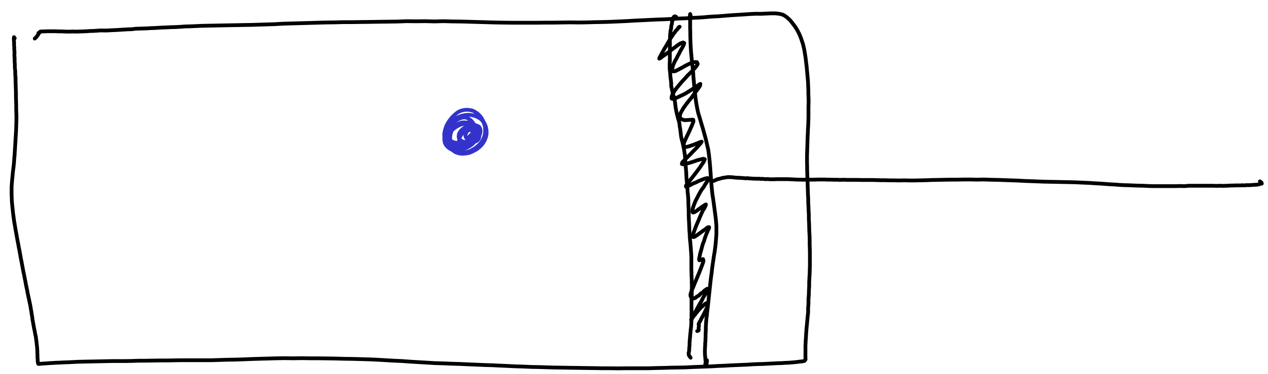


$p(L) = 2/3$

$p(R) = 1/3$

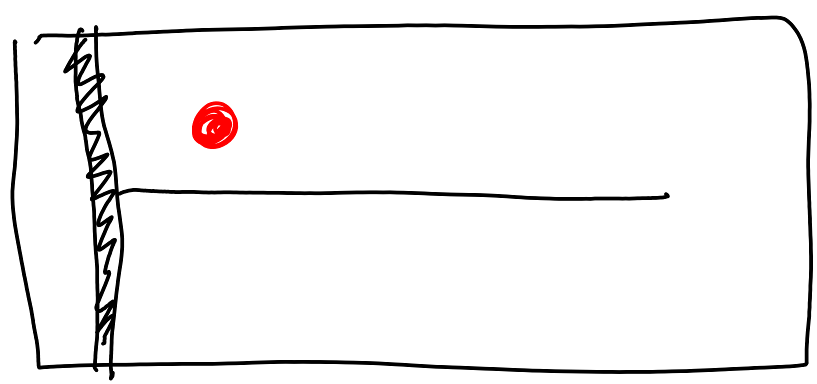


$p = 2/3$



oh no!

$p = 1/3$



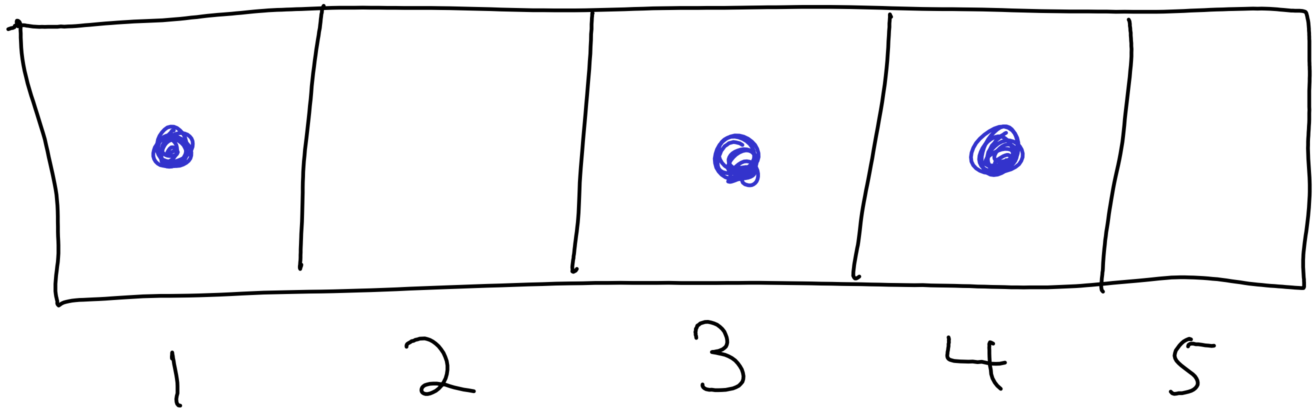
$p(2) = 0$

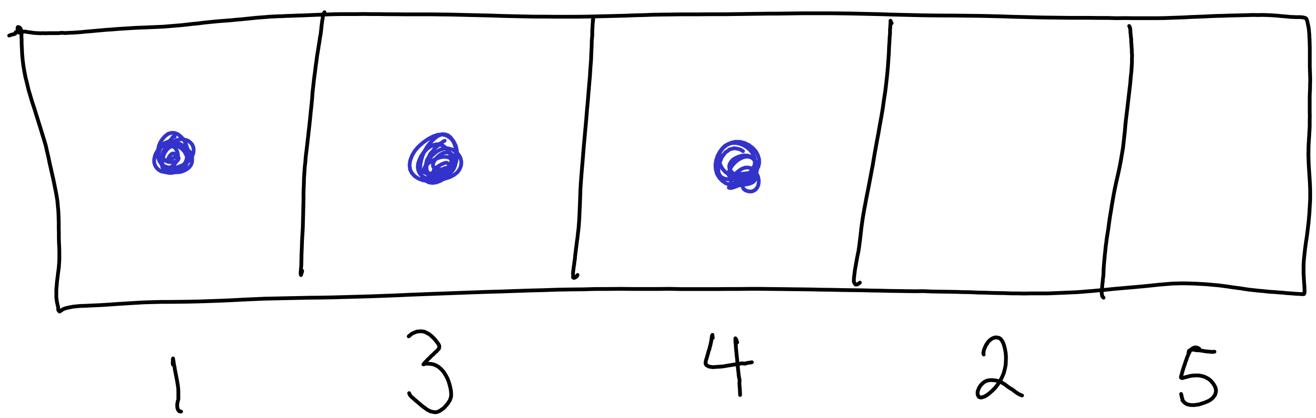
$p(5) = 0$

$p(1) = 2/3$

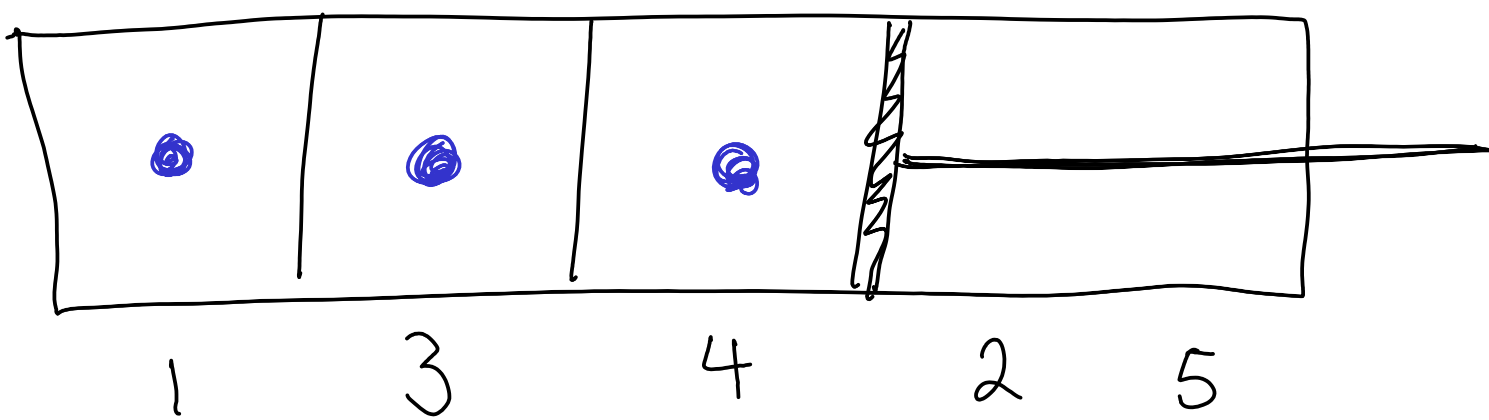
$p(3) = 1/6$

$p(4) = 1/6$





$W = kT[\ln 5 - \ln 3]$

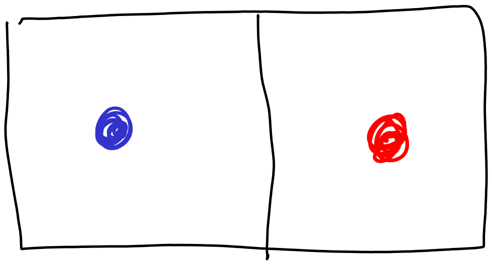


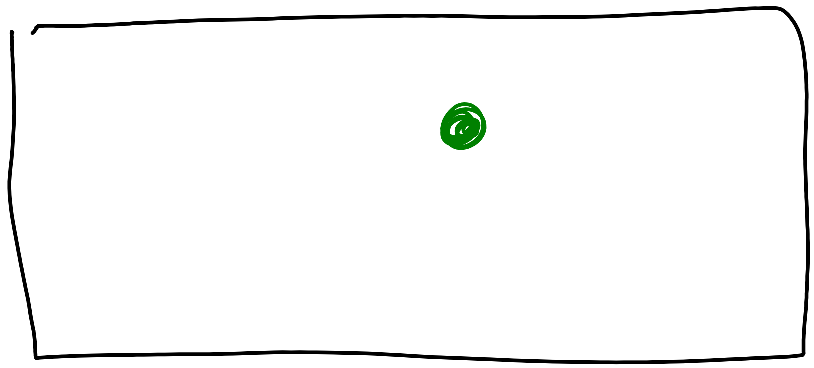
w/ ε-error (Dahlsten et. al.)

Formation

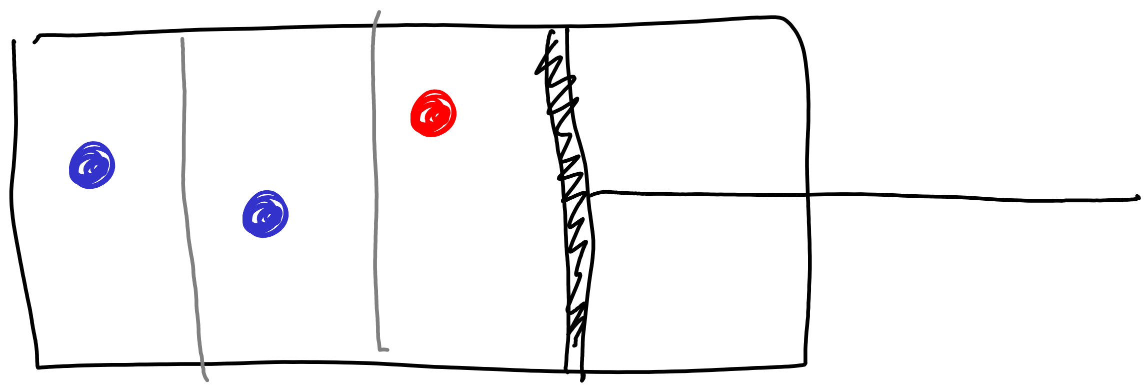
$p(L) = 2/3$

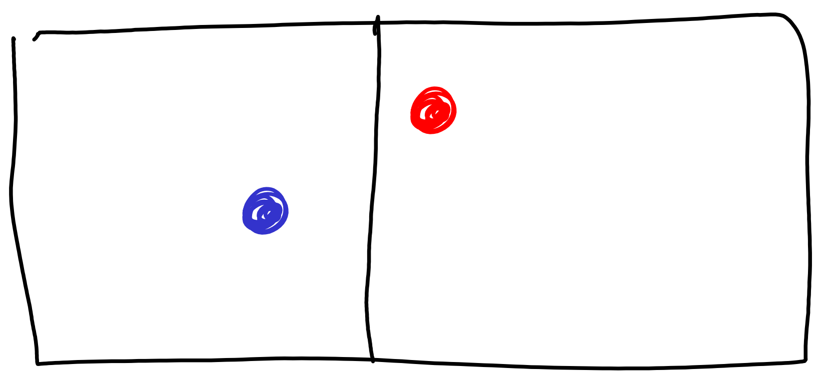
$p(R) = 1/3$



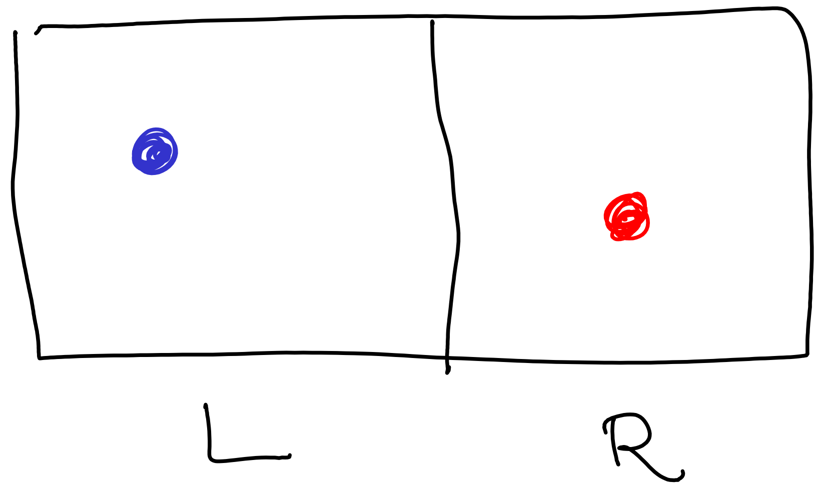


$W = -kT \ln 2/3$





$p(L) = 2/3$
$p(R) = 1/3$



$$W_{distillation} = 0$$
$$W_{formation} = kT \ln 2/3$$

irreversibility!

Thermodynamics is Information and Energy

- operations must conserve energy

- degeneracy of heat bath depends on energy

Thermodynamics as a resource theory

Class of Operations	Thermal	Energy conserving U Heat bath τ at temp T c.f. Janzig (2003) Work as $\|0\rangle \to \|W\rangle$
Closed Set	Heat bath τ at temp T	
Monotones	$F = R(\rho\|\tau)$ $\to$	F_{max} F_{min}

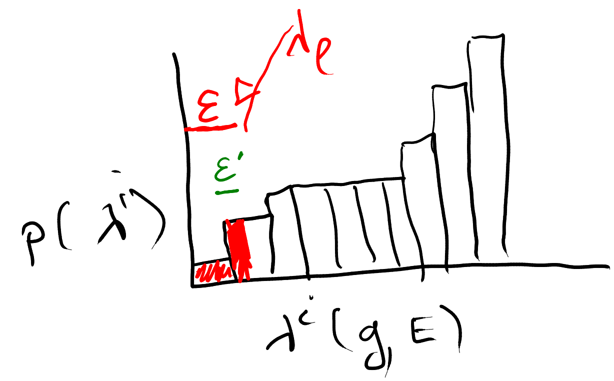
(BHORS, 2011) (HO, 2011)

Equivalent to other paradigms eg. H_{int}, $H(t)$

$$F^{\varepsilon}_{min} = -kT \ln \sum_{g,E} e^{-\beta E} h(g,E)$$

$$h(g,E) = \begin{cases} 1 & i > \ell \\ 0 & i < \ell \\ \varepsilon'/p(\lambda_\ell) & i = \ell \end{cases}$$

$$\omega = \sum_E P_E \, \rho \, P_E$$



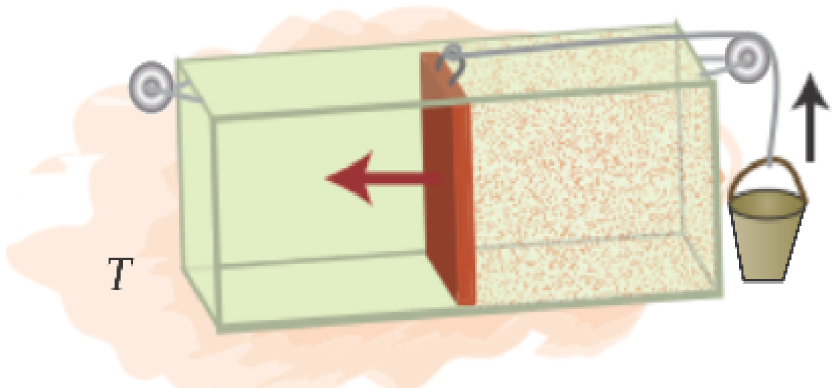
$$F_{max} = \inf_{\rho_\varepsilon} kT \ln \min\{\lambda : \rho_\varepsilon \leq \lambda \gamma\}$$

$$\gamma = \sum_{E,g} e^{-\beta E}/Z \; |E,g\rangle\langle E,g|$$

$$\|\rho_\varepsilon - \rho\| \leq \varepsilon \qquad (\rho_\varepsilon \text{ normalised})$$

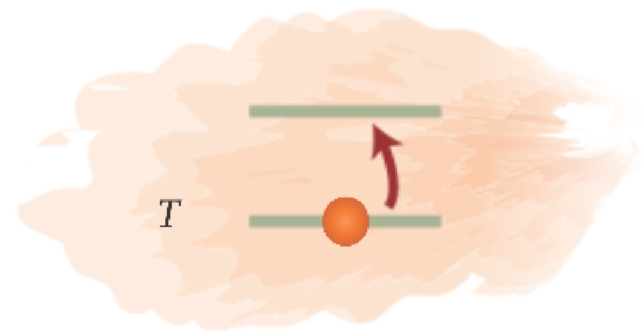
Work

Macro



weight

Micro



wit

$$\cancel{F = R(\rho\|\gamma)}$$

$$D_\varepsilon^{max}(\rho\|\gamma)$$

$$D_\varepsilon^{min}(\omega\|\gamma)$$

Donald (87)

c.f. Ahlberg (2011)

When are thermodynamical transitions possible?

$\rho \to \sigma$ iff ρ thermo-majorizes σ

recall majorization

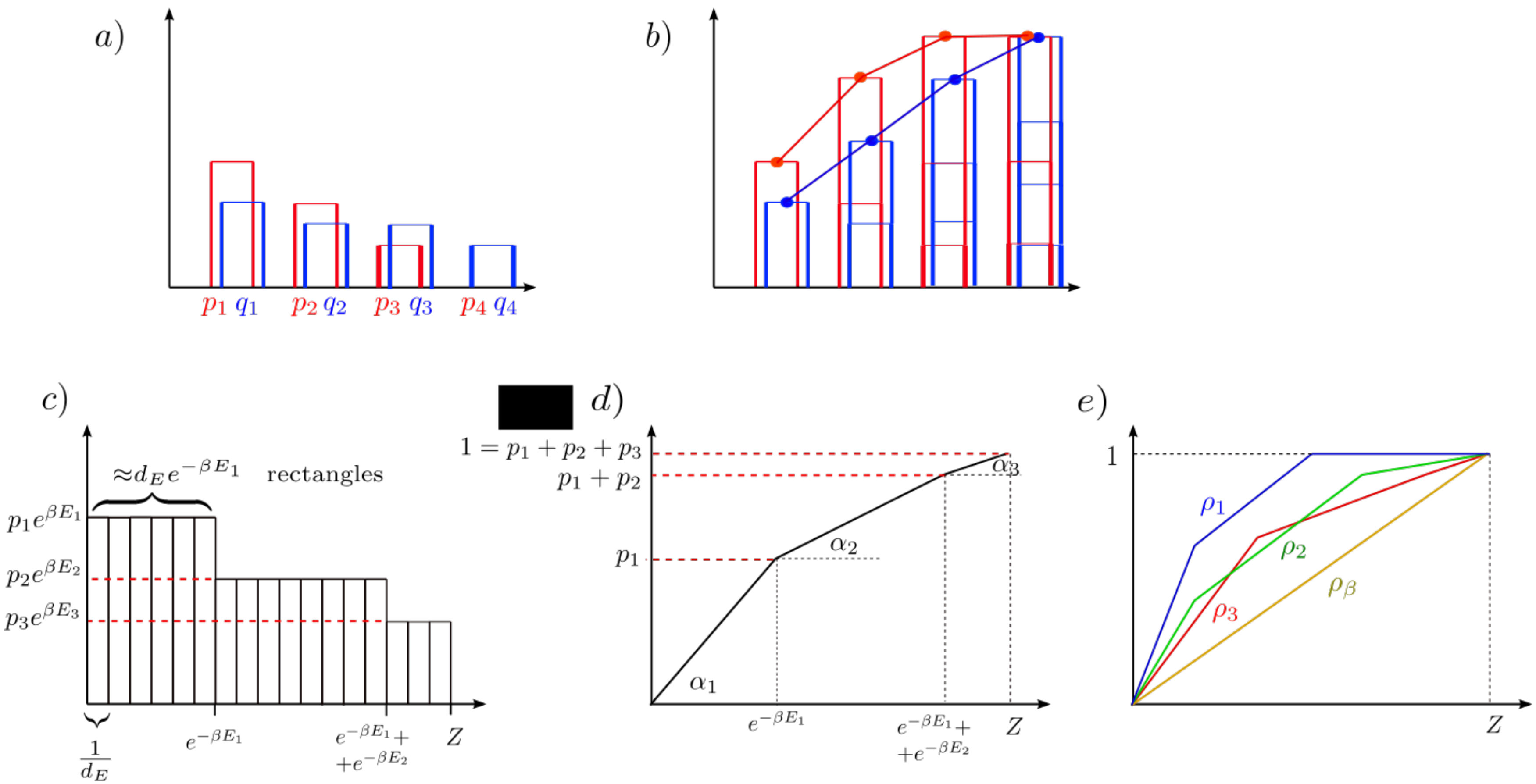
p_i of ρ in nonincreasing order

q_i of σ in nonincreasing order

i.e. $p_1 \geq p_2 \geq p_3 \ldots$

$\rho > \sigma$ if $\sum_i^k p_i \geq \sum_i^k q_i \quad \forall k$

Thermo-majorization

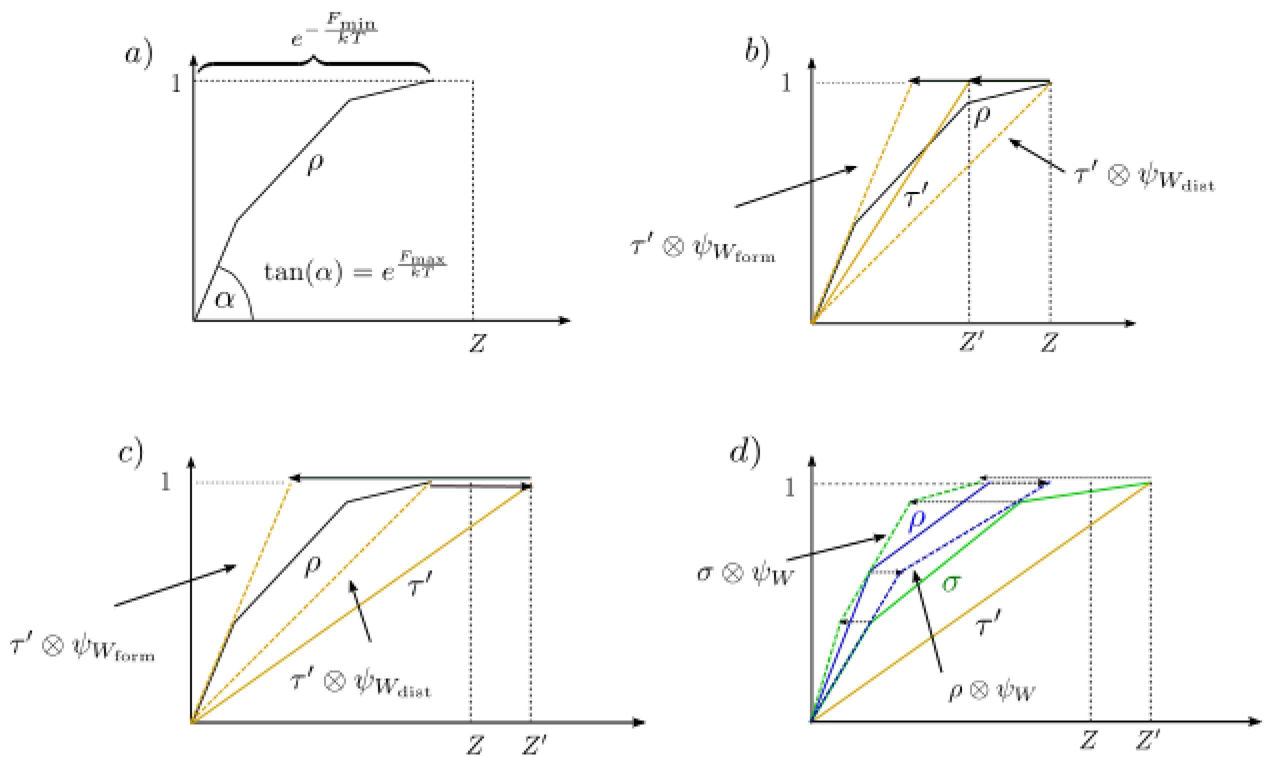


$$p(E_1, g_1)\, e^{\beta E_1} \geq p(E_2, g_2)\, e^{\beta E_2} \geq \ldots \quad \beta\text{-ordered}$$

c.f Ruch & Mead (75-77)

changing Hamiltonians

work-assisted transitions



Thermo-majorization

Majorization within energy blocks

$\gamma_R \otimes \rho_S$

$E = E_R + E_S$ fixed ie $P_E \, \gamma_R \otimes \rho_S \, P_E$

For each E_S, $g_R(E_R) = g_R(E - E_S) \simeq e^{-\beta E_S} g_R(E)$

eigenvalues of $P_E \, \gamma_R \otimes \rho_S \, P_E$ are $\dfrac{e^{\beta E_S}}{g_R(E)} \, p(E, g)$

with multiplicity $g_R(E) \, e^{-\beta E_S}$

Conclusions

- Laws of thermodynamics
  - Thermo-majorization
  - Energy Conservation

  → Heat bath preserving ops
- Two free energies → irreversibility
- Limitations due to finite size, quantumness
- Thermomajorization determines state trans
- Criteria for reversibility
- Small heat engines have prob'y to fail