Complete Insecurity of Quantum Protocols for Classical Two-Party Computation

Matthias Christandl (ETH Zurich)
joint work with
Harry Buhrman (CWI, University of Amsterdam)
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arXiv:1201.0849, Phys. Rev. Lett. 109, 160501 (2012)

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thanks for reuse of slides :)

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Motivation

- ideally

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f

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$$x \longrightarrow \boxed{f} \longleftarrow y$$

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x	$\longrightarrow$	f	$\longleftarrow$	y
$f(x,y)$	$\longleftarrow$		$\longrightarrow$	$f(x,y)$

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x → f ← y

f(x,y) ← f → f(x,y)

e.g.: Yao's millionaires' problem: ≤

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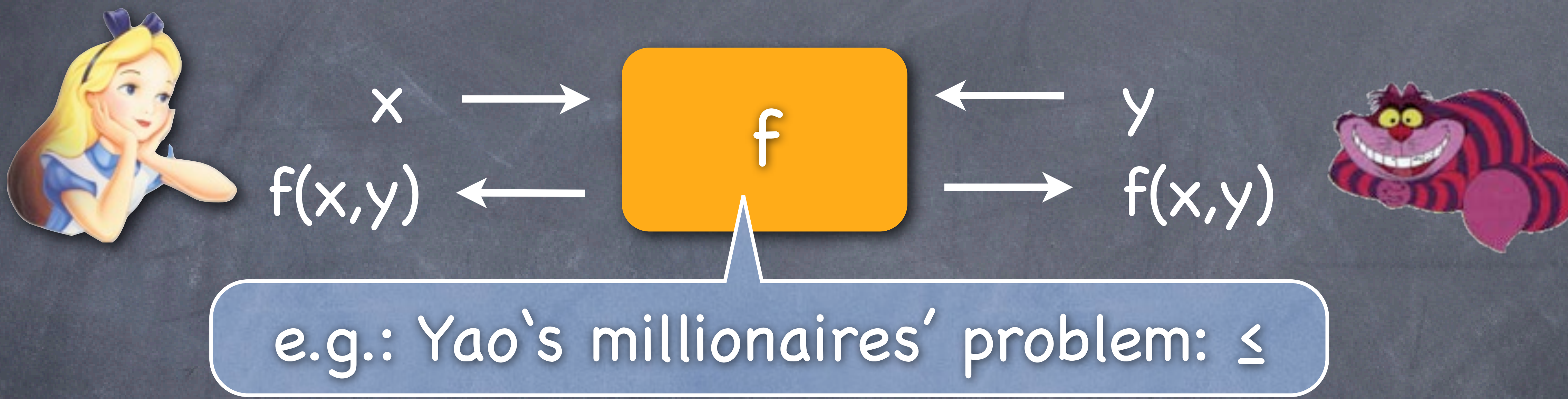

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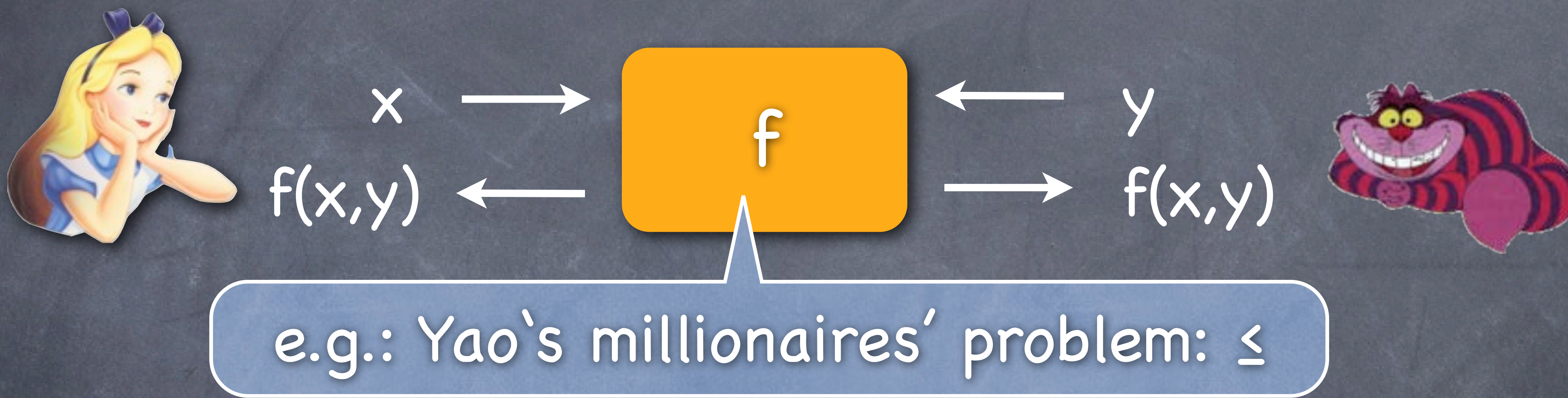

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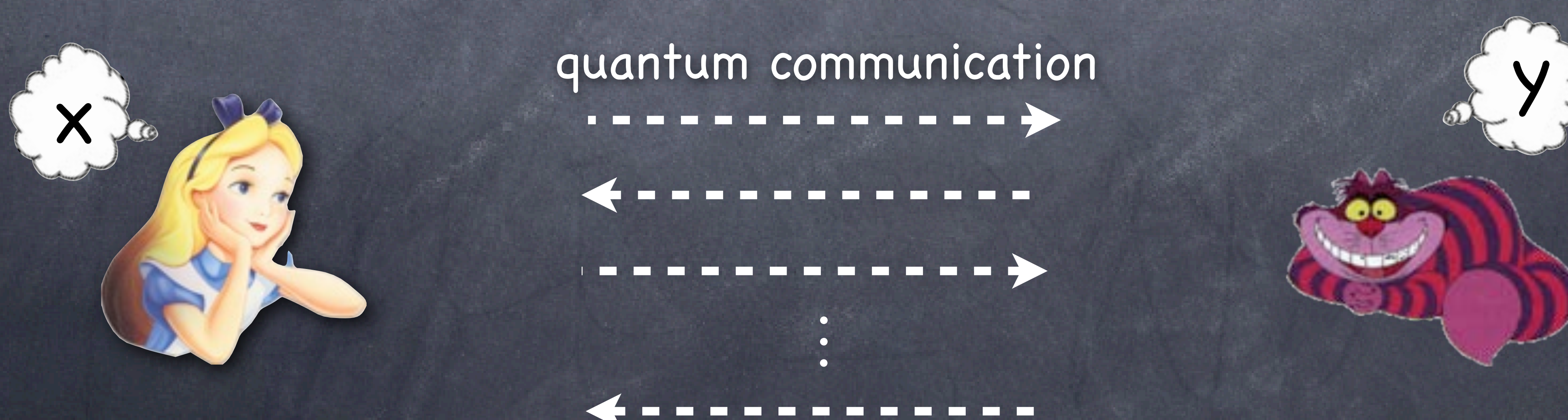

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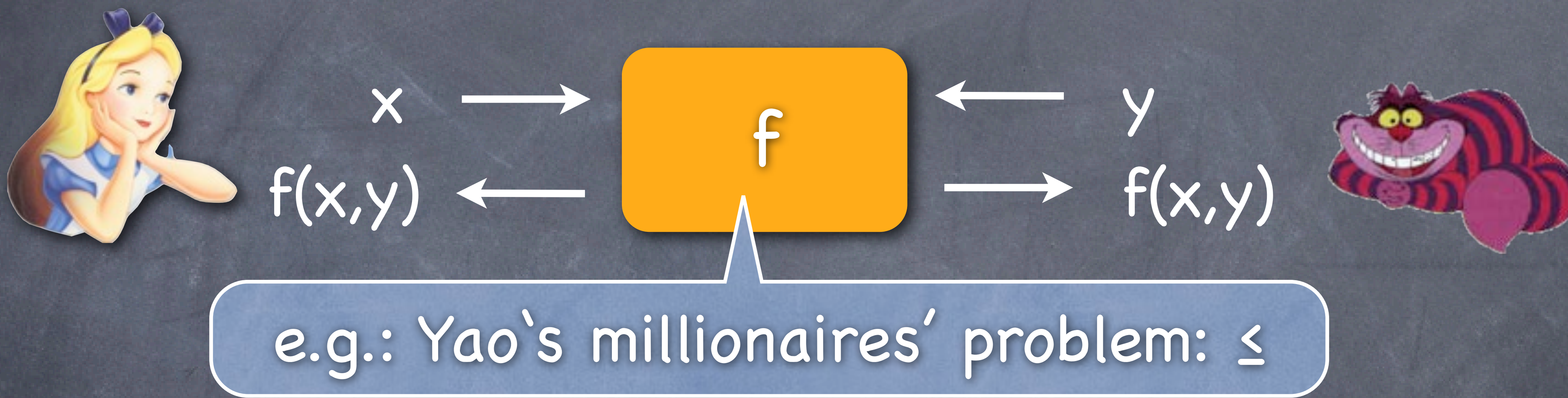

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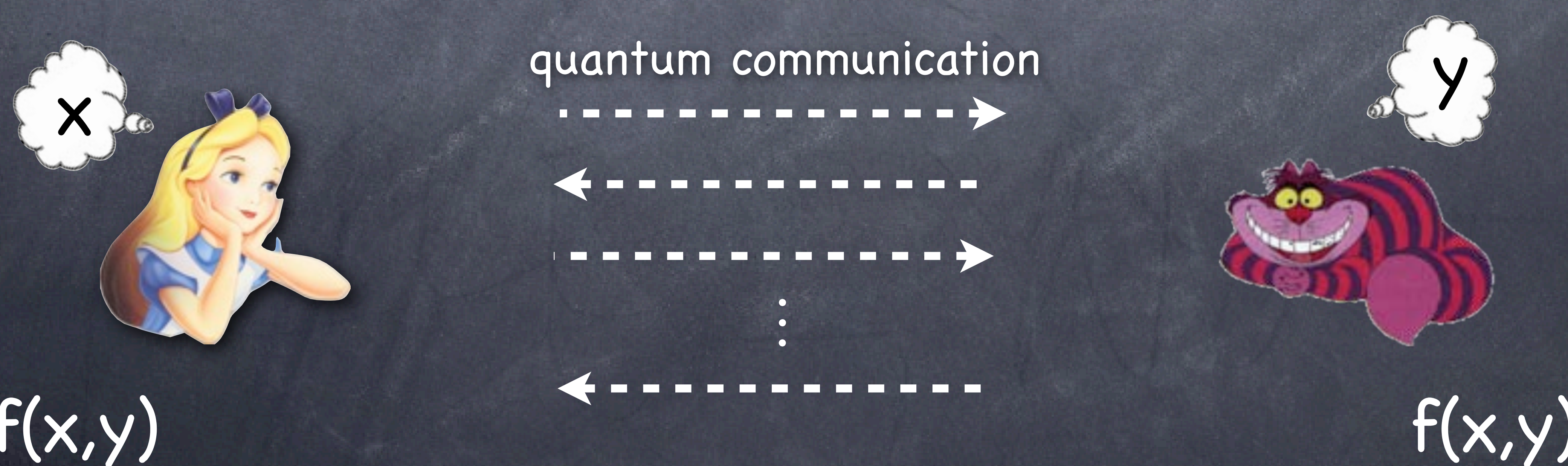

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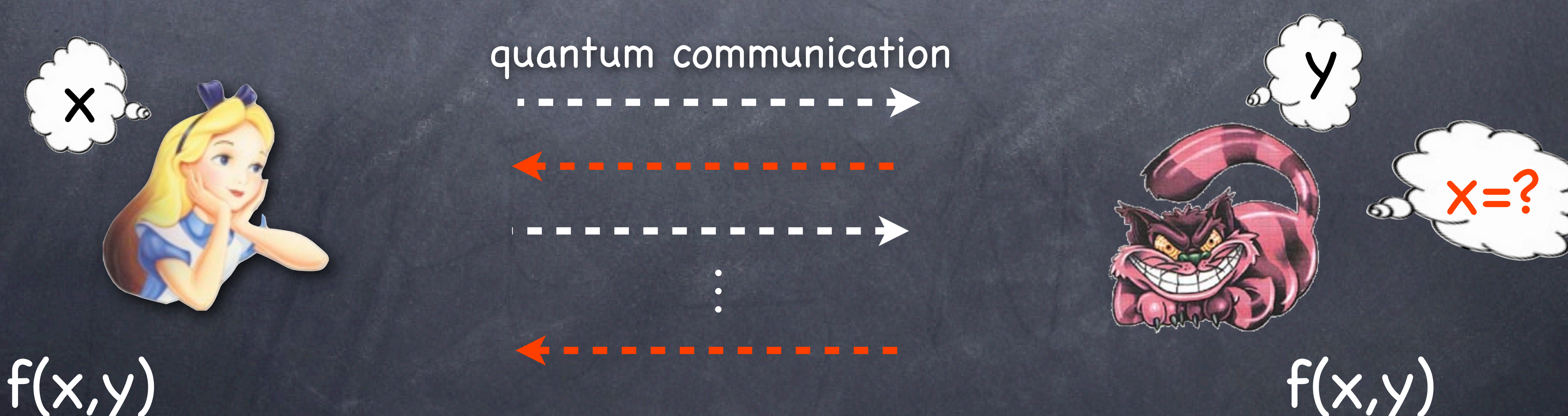

Secure Function Evaluation

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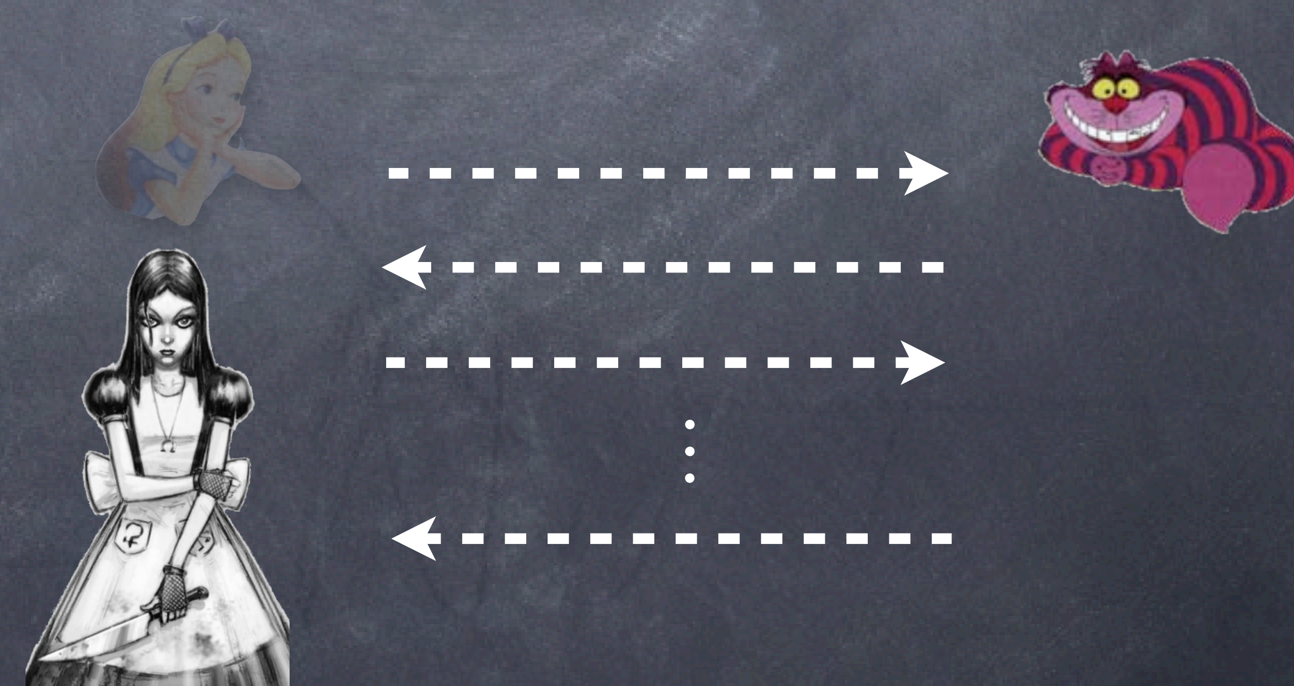

Secure Function Evaluation

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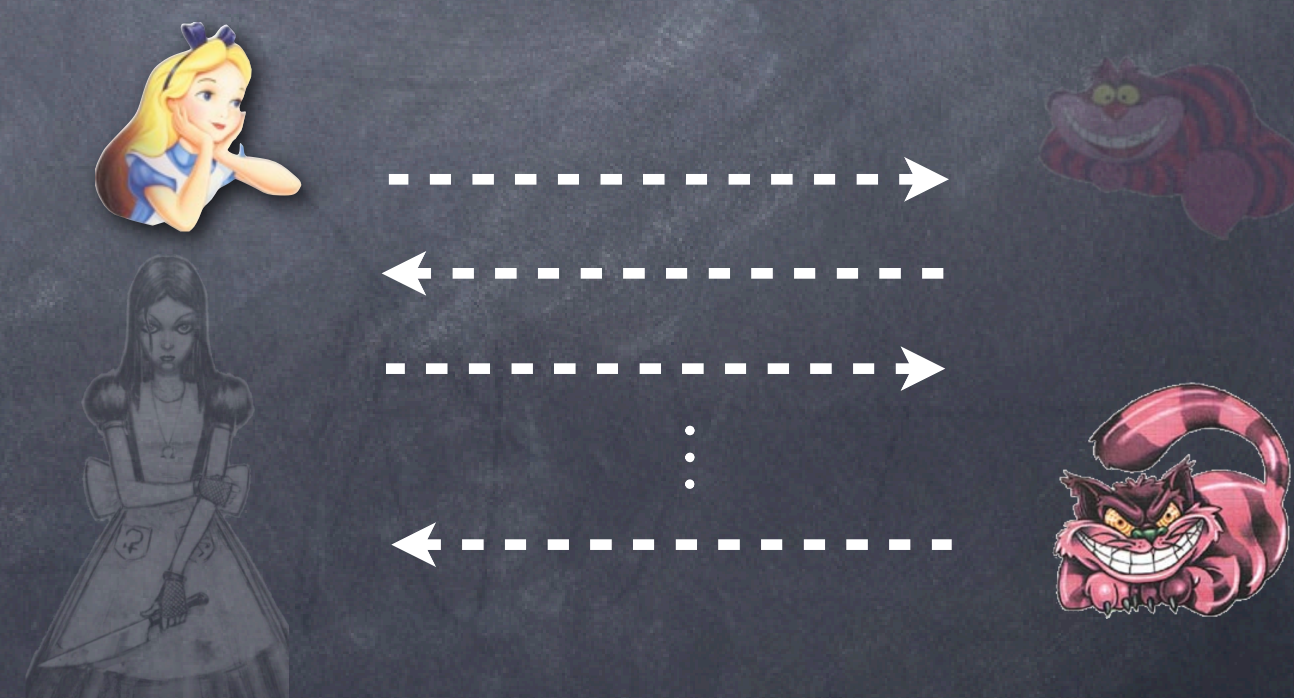
- goal: come up with protocols that are
  - correct
  - secure against dishonest Alice
  - secure against dishonest Bob



Main Result

- **Theorem**: If a quantum protocol for the evaluation of f is correct and perfectly secure against Bob, then Alice can completely break the protocol.

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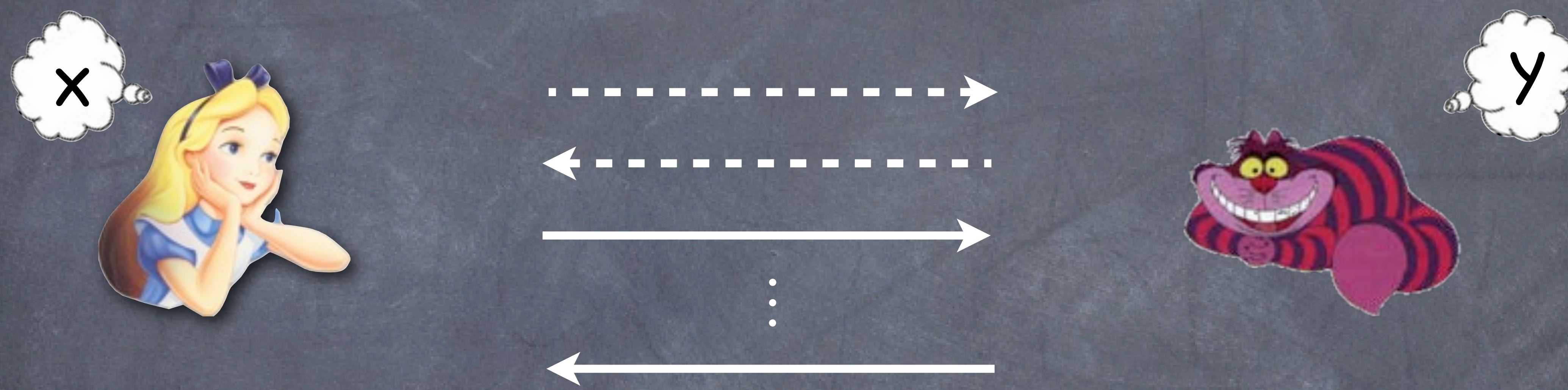

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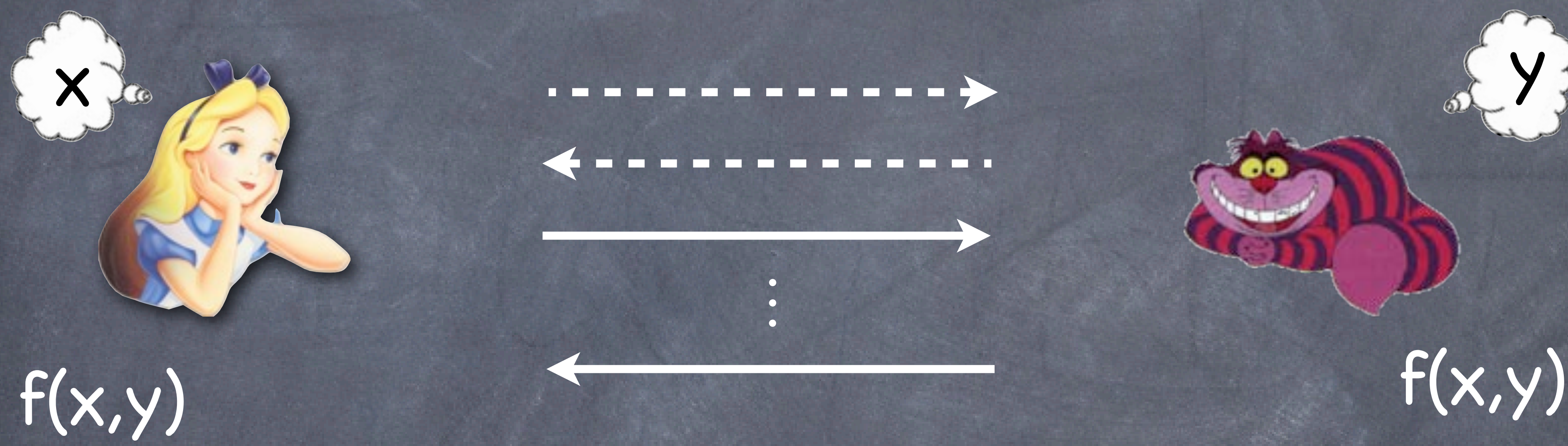

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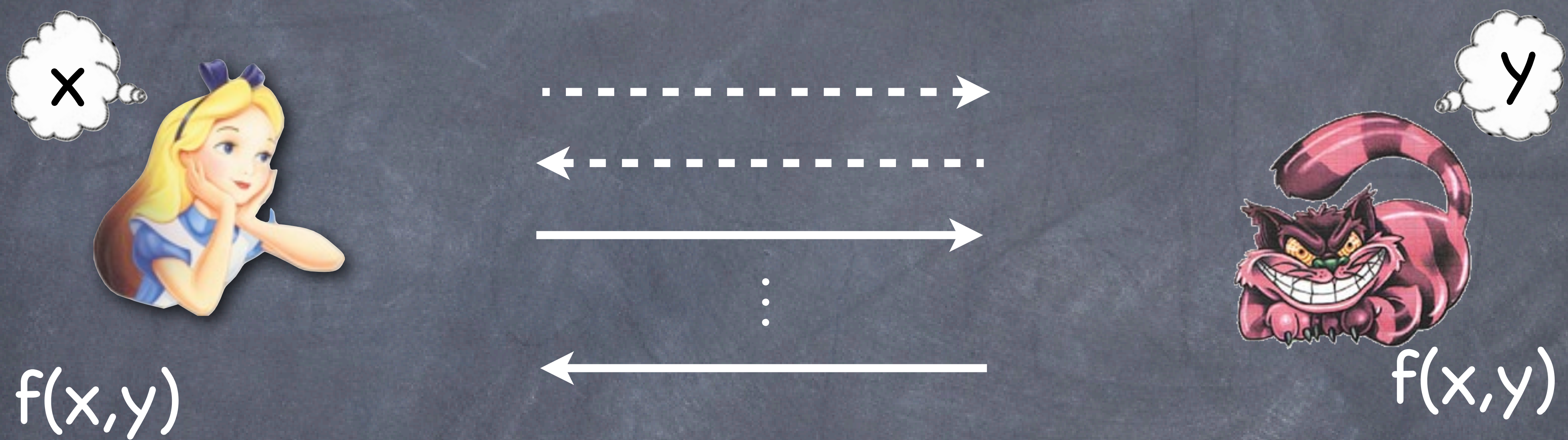

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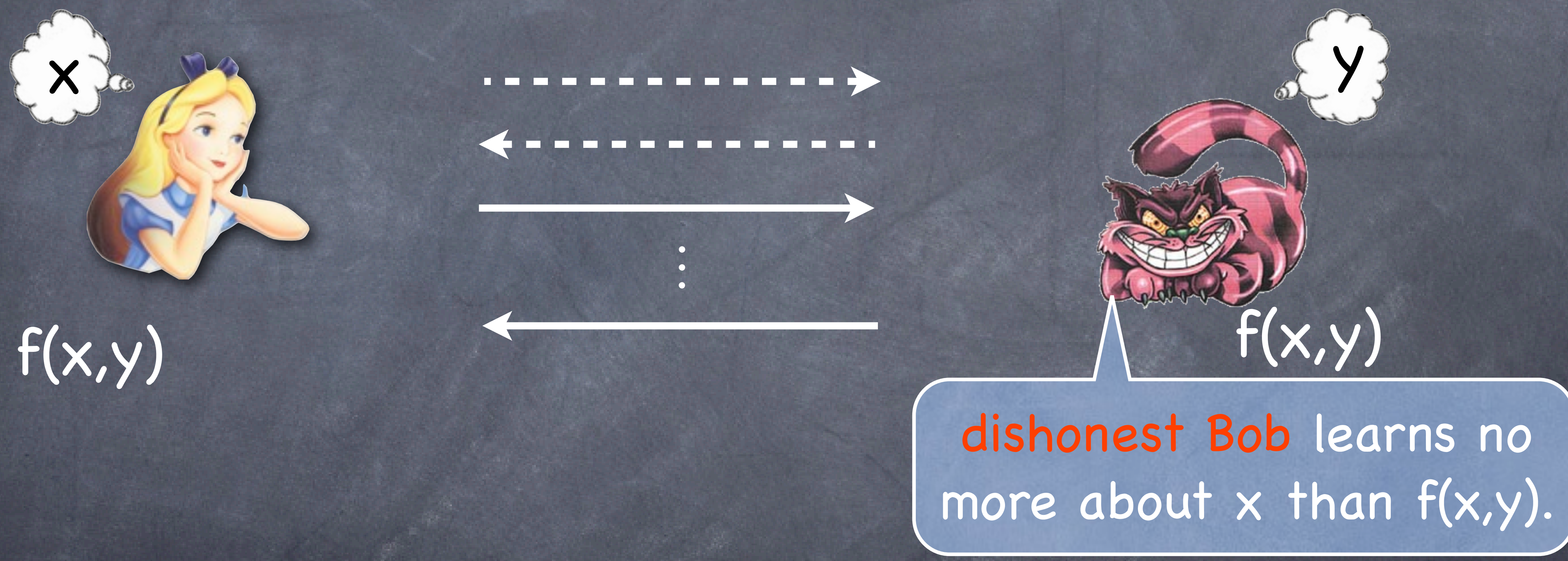

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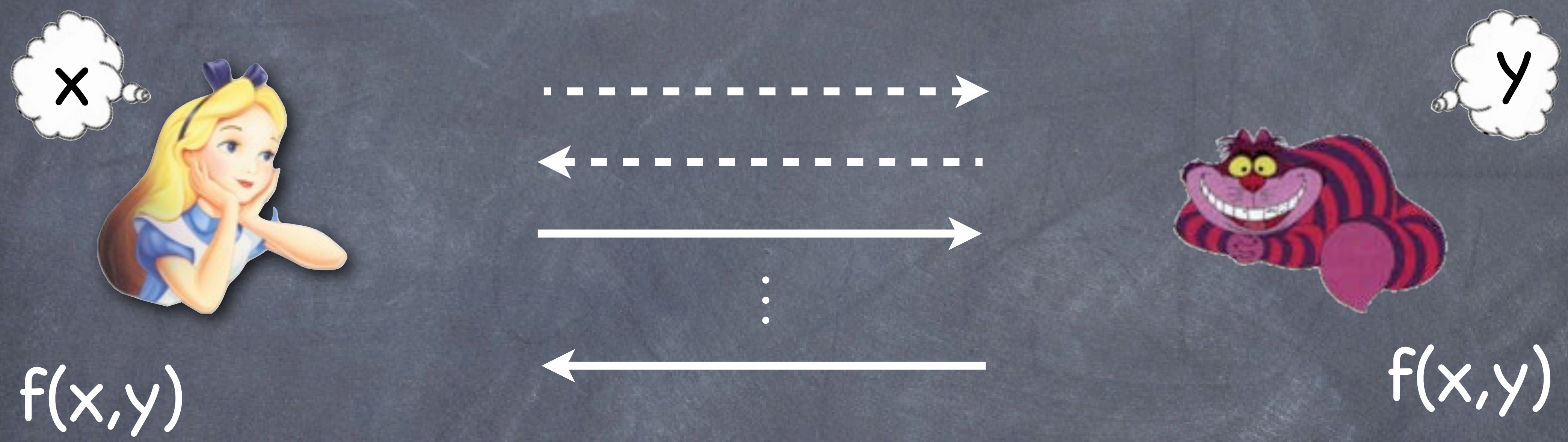

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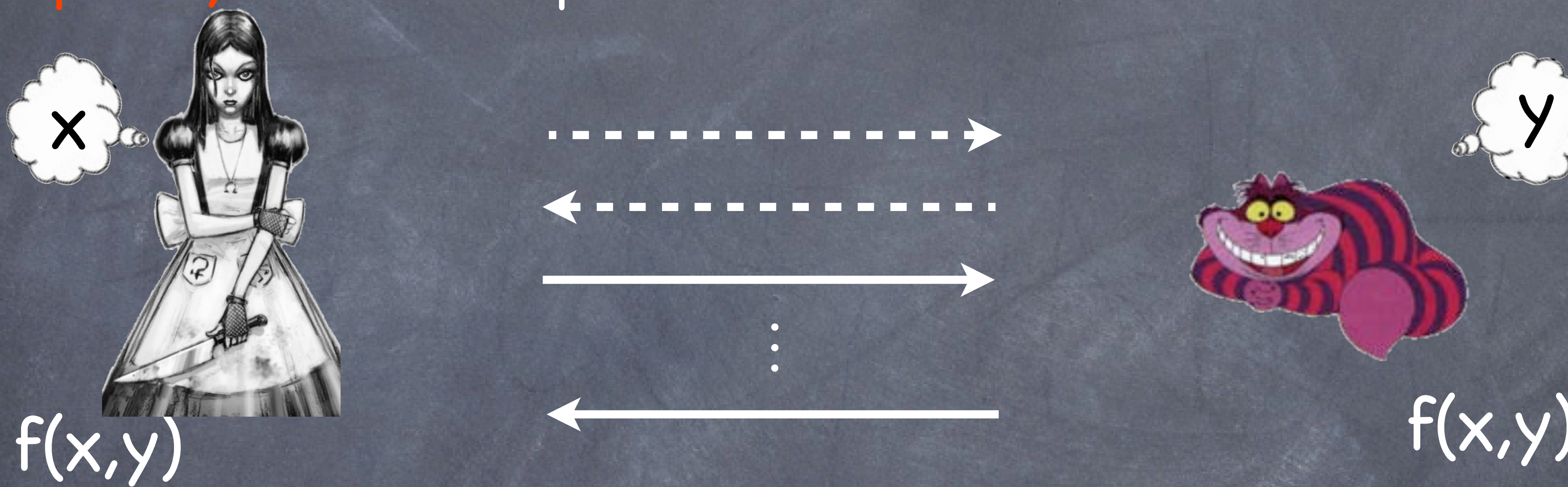

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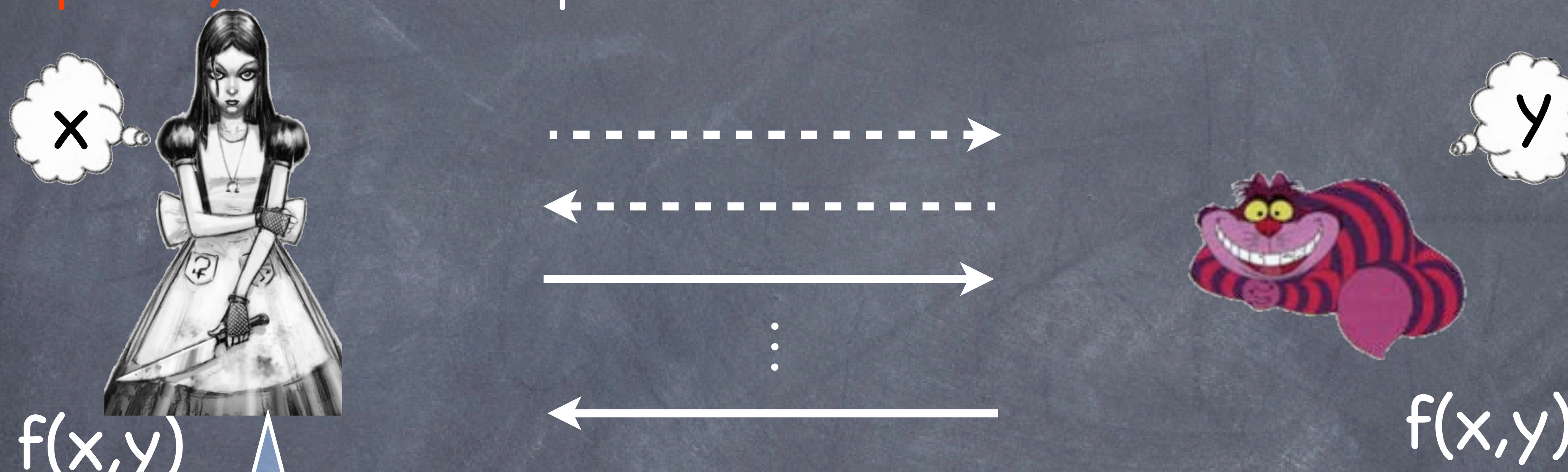

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Equivalently, she obtains y‘ s.th.
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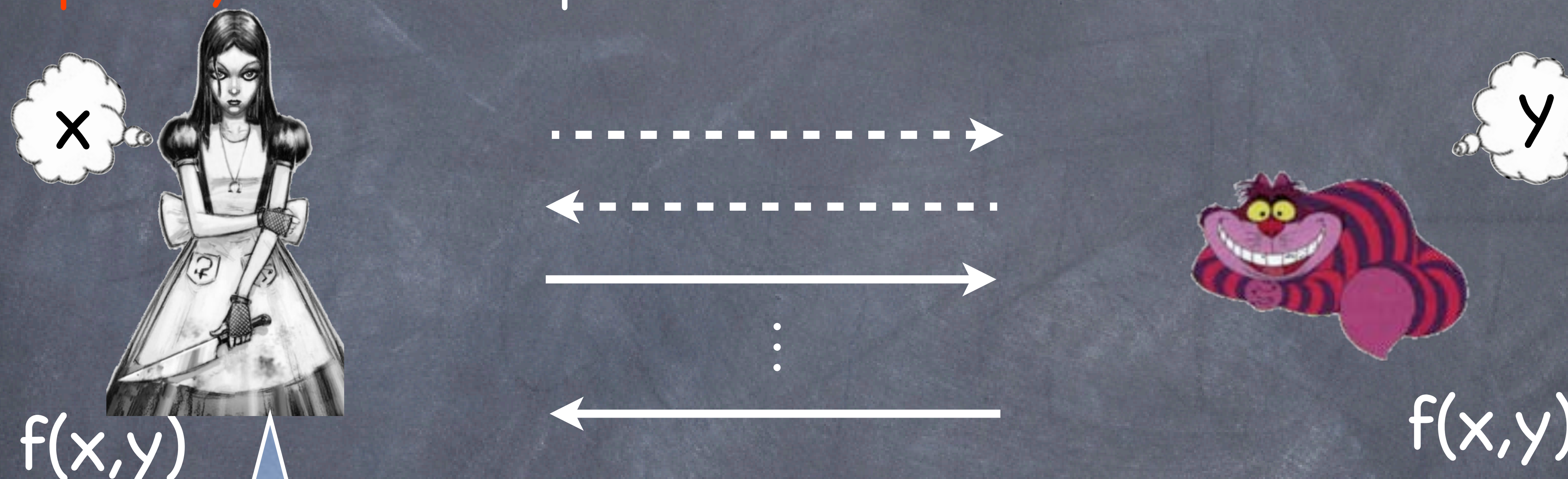

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- **Theorem:** If a quantum protocol for the evaluation of f is ε-correct and ε-secure against Bob, then Alice can break the protocol with probability 1-O(ε).

History

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- ~1970: Conjugate Coding [Wiesner]

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- this work: Complete Insecurity of Two-Sided Secure Function Evaluation (also with finite error)

Talk Outline

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- explain Lo's impossibility proof

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- problem with two-sided computation

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- explain Lo's impossibility proof
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- conclusion

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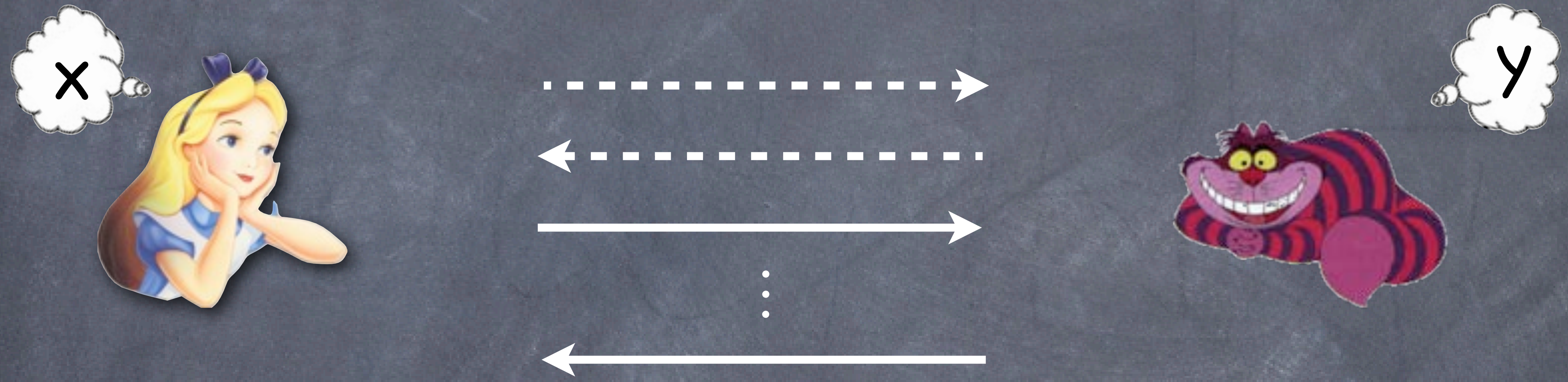

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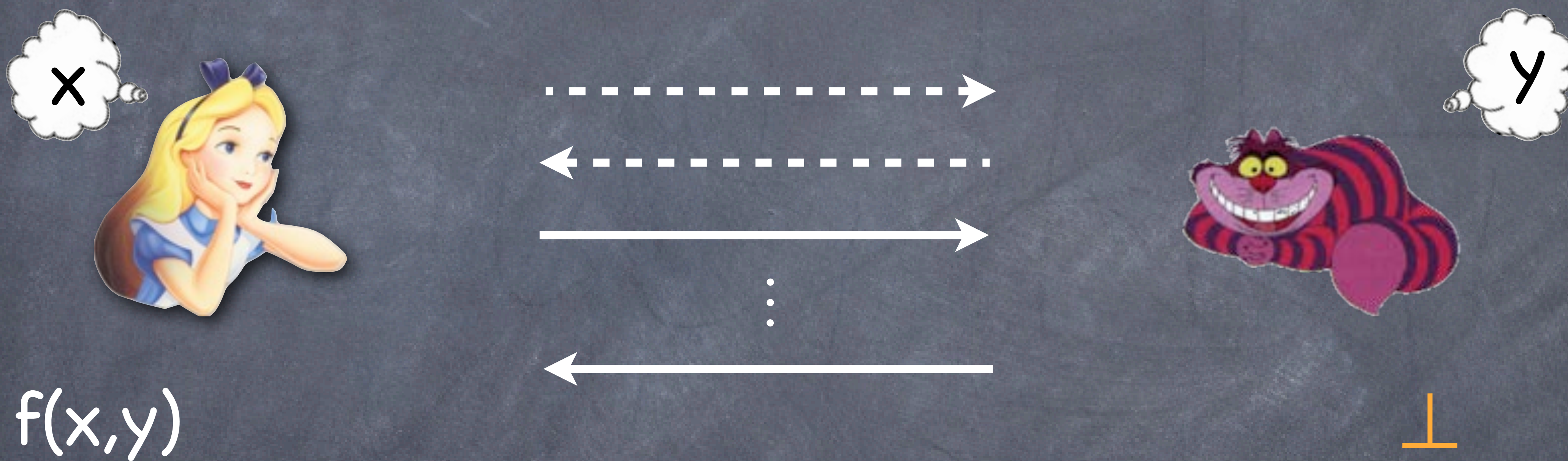

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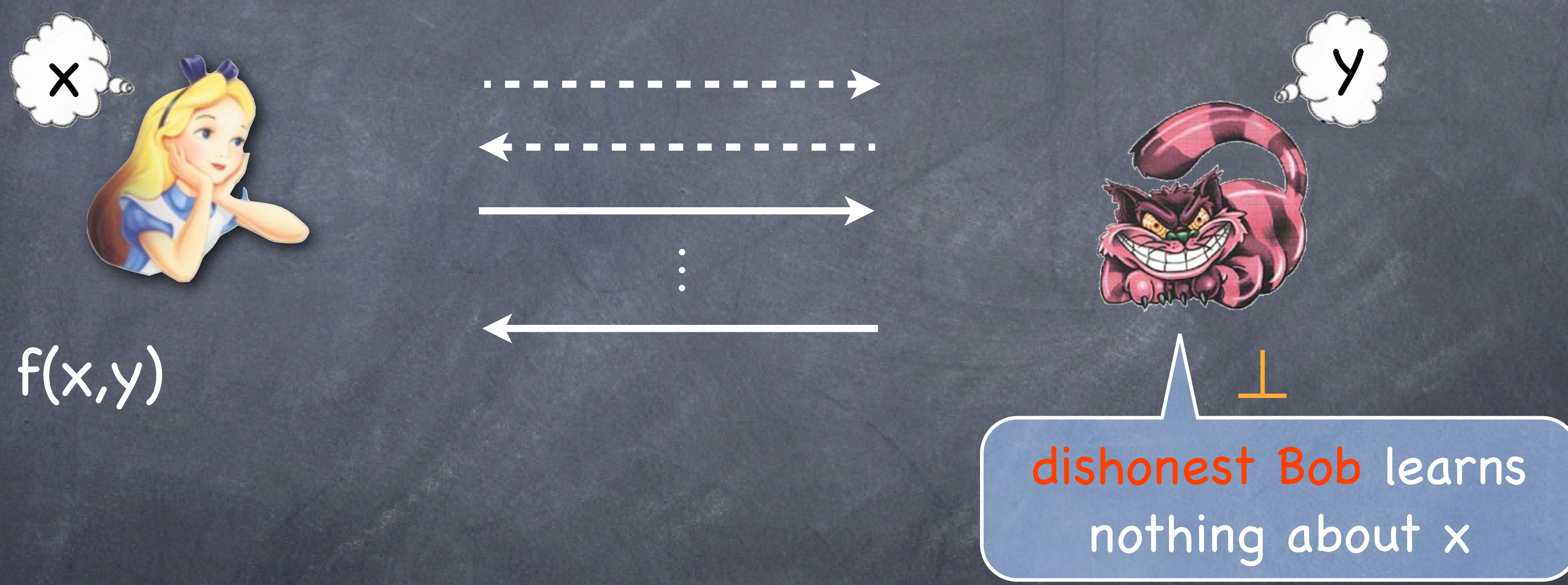

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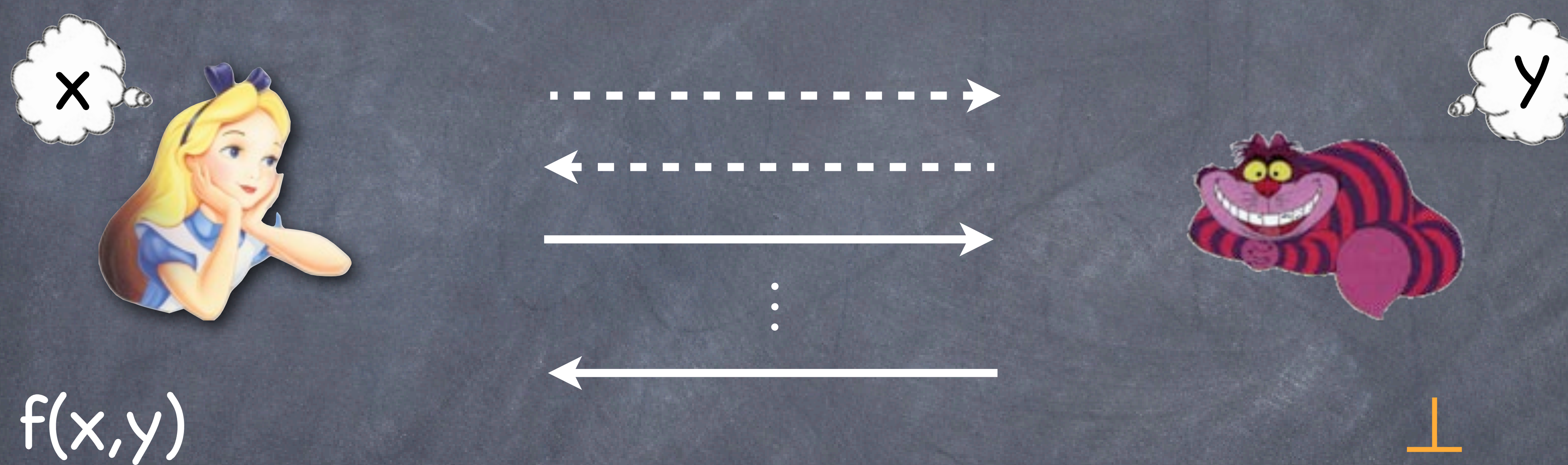

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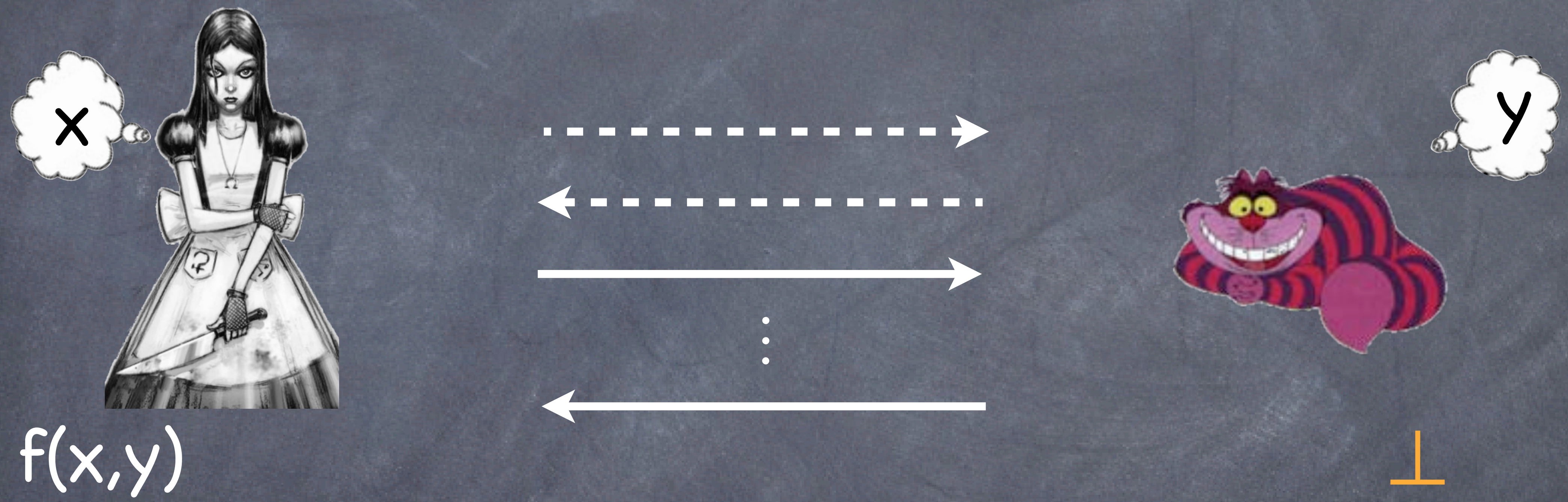

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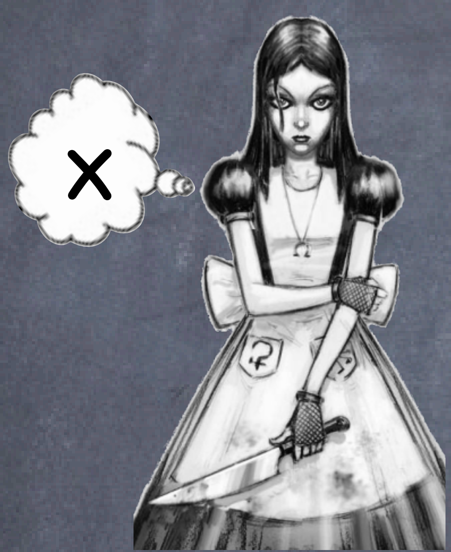

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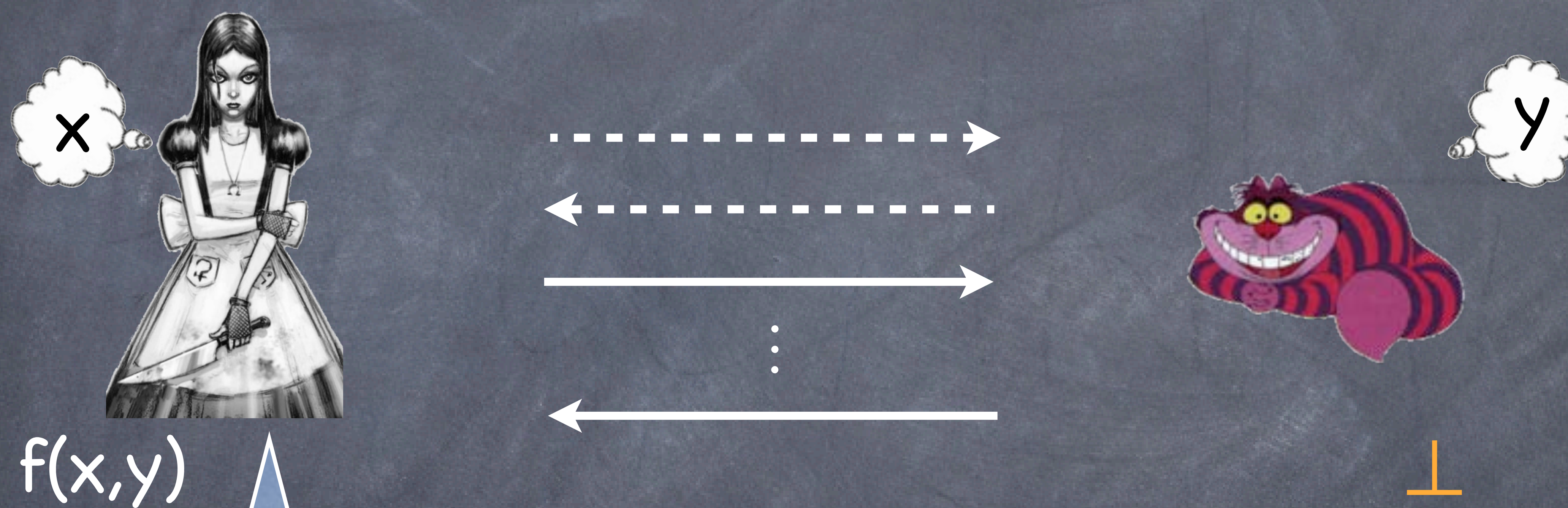

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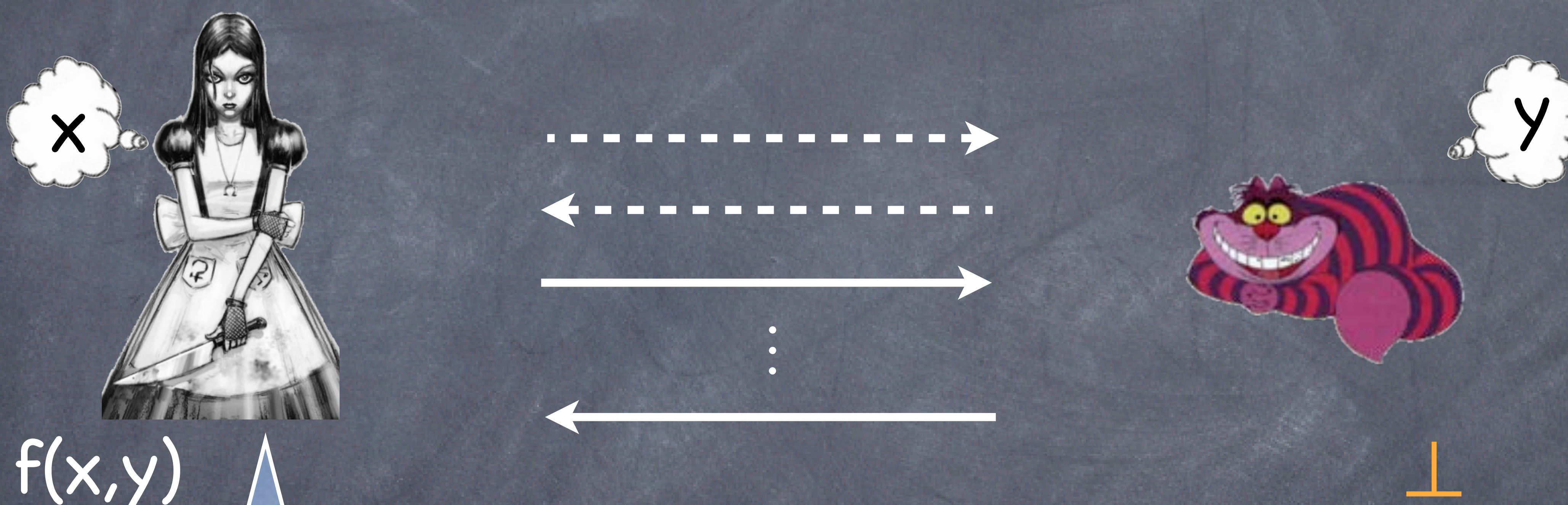

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- proof fails for two-sided computations
- error increases with number of inputs

Lo's Proof

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$f(x,y)$ $\perp$

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f(x,y) $\quad|\psi^{x,y}\rangle_{AB}\quad$ $\perp$

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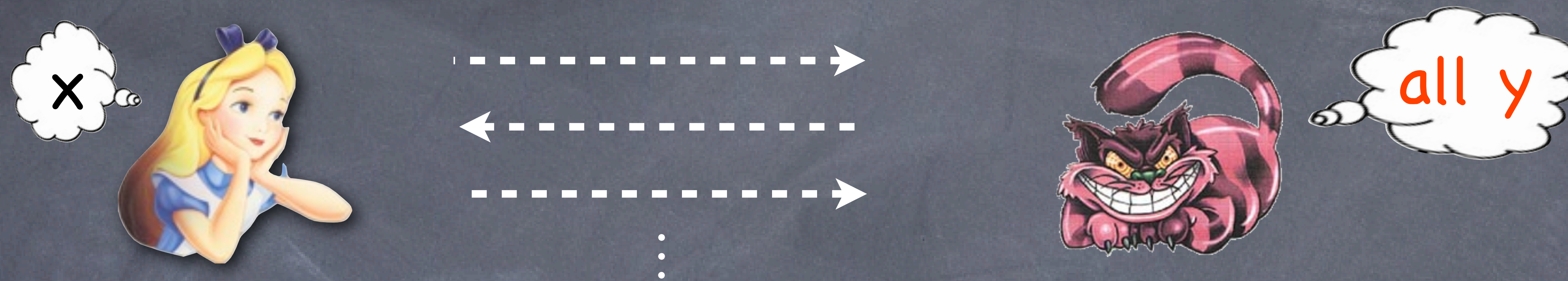

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f(x_0,y), f(x_1,y), ... $|\psi^{x,y}\rangle_{AB}$ f(x,y)

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$f(x_0,y)$, $f(x_1,y)$, ... $\quad |\psi^{x,y}\rangle_{AB} \quad$ $f(x,y)$

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crucial step!

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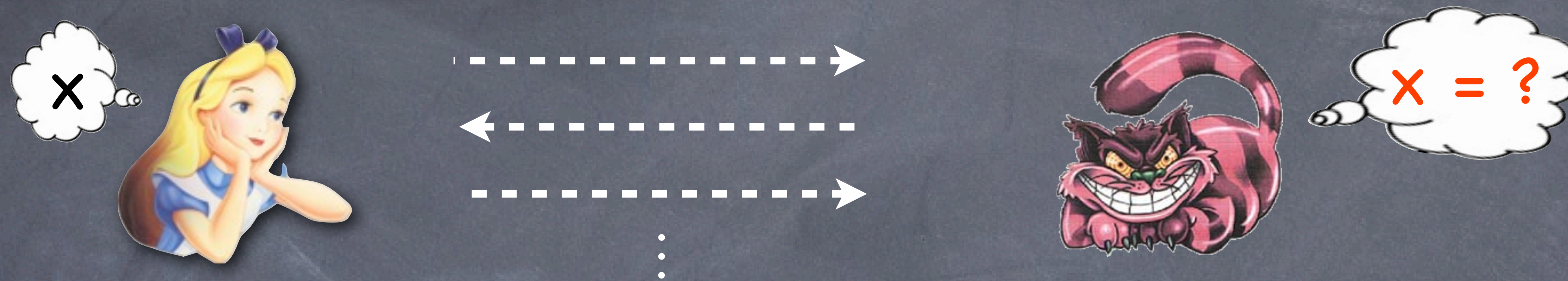

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- precise formalisation of "not learning more about x than f(x,y)" ?

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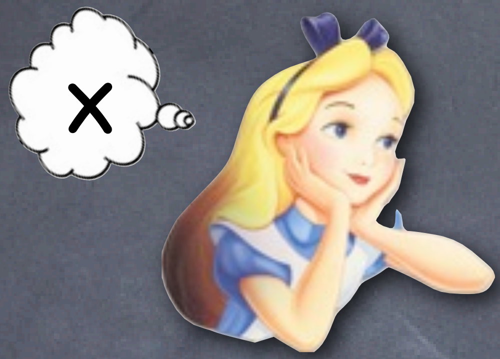

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- what if Bob has f(x,y) ? In general $\rho_B^{x_0} \neq \rho_B^{x_1}$
- precise formalisation of
  "not learning more about x than f(x,y)" ?

use the real/ideal paradigm

Informal Security Definition

- we want

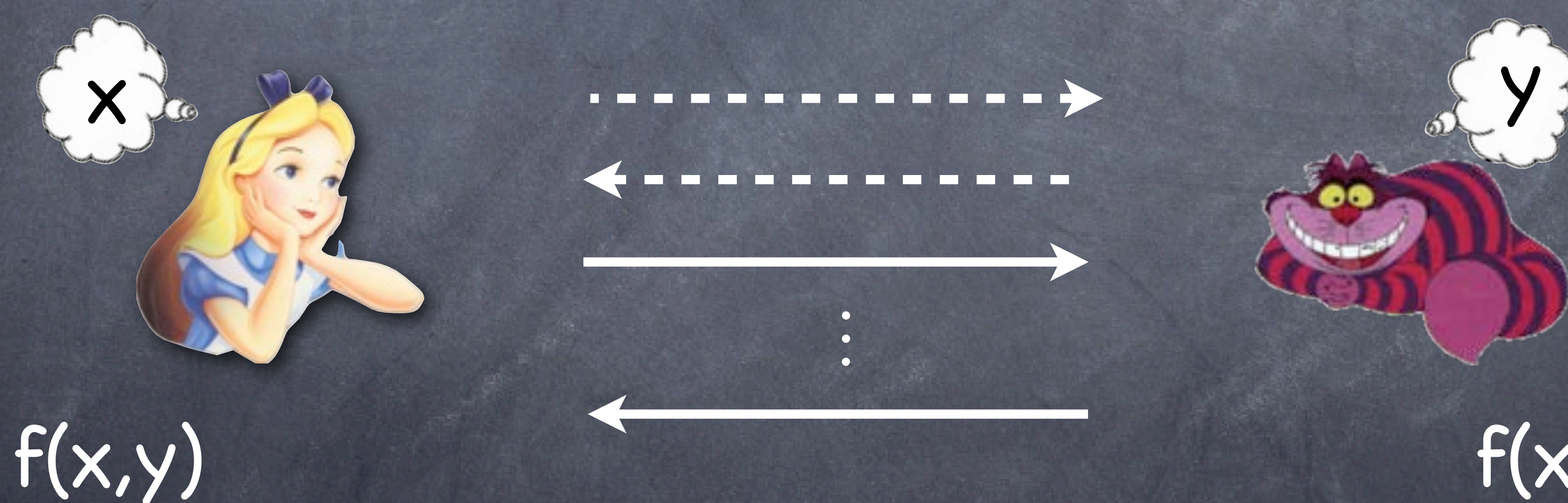
Informal Security Definition

- we want

Informal Security Definition

- we want

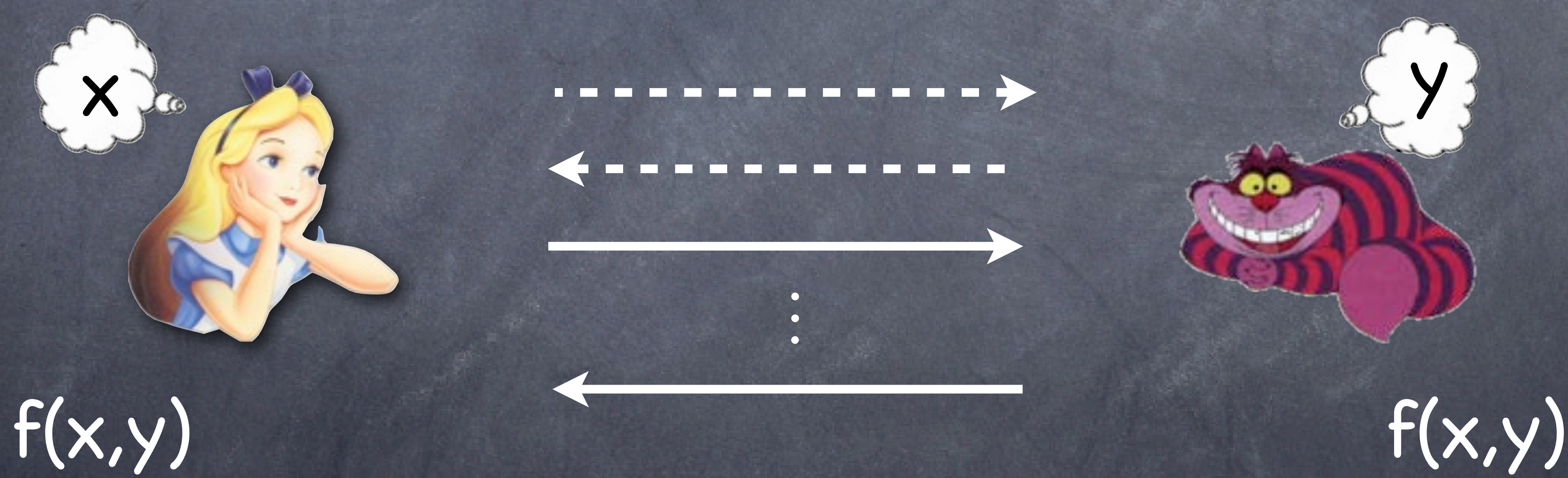
- we have



Informal Security Definition

- we want

- we have



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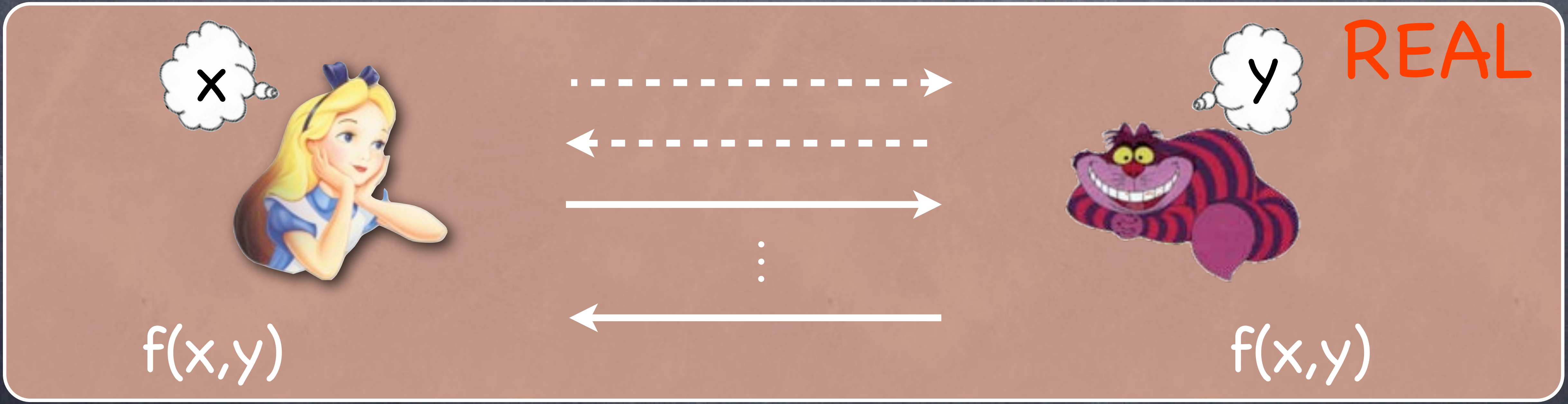

Informal Security Definition

- we want

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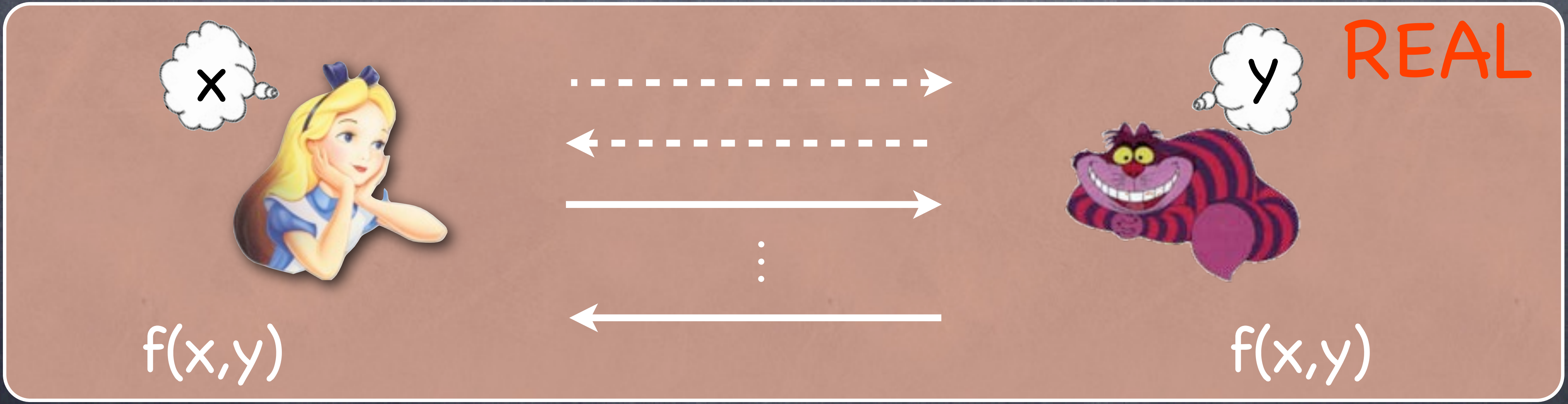
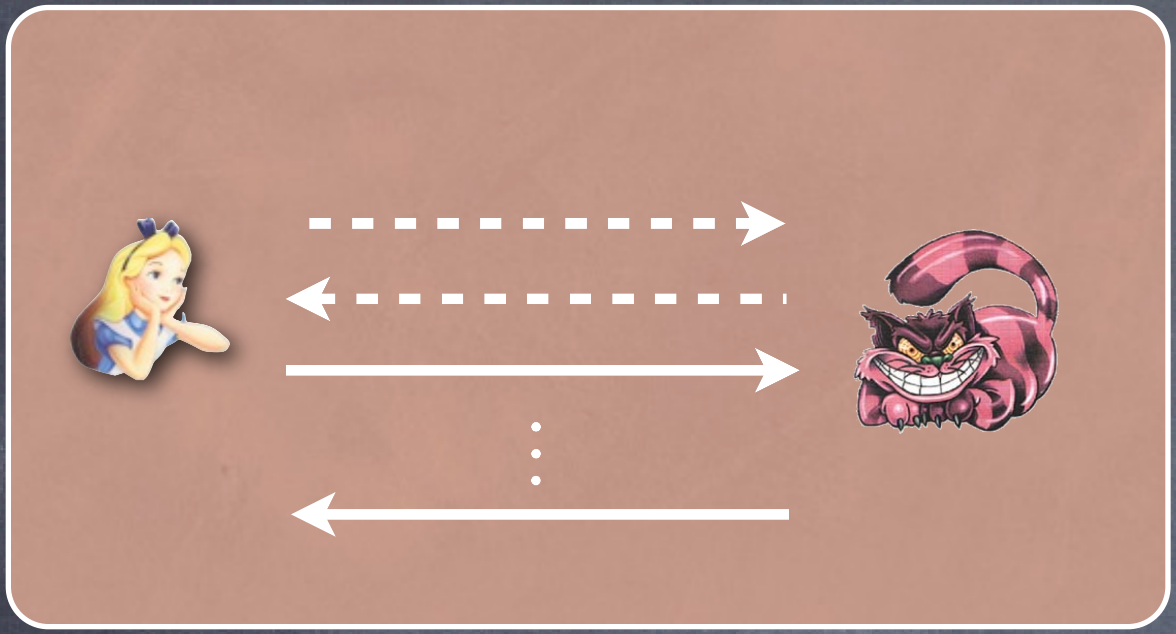
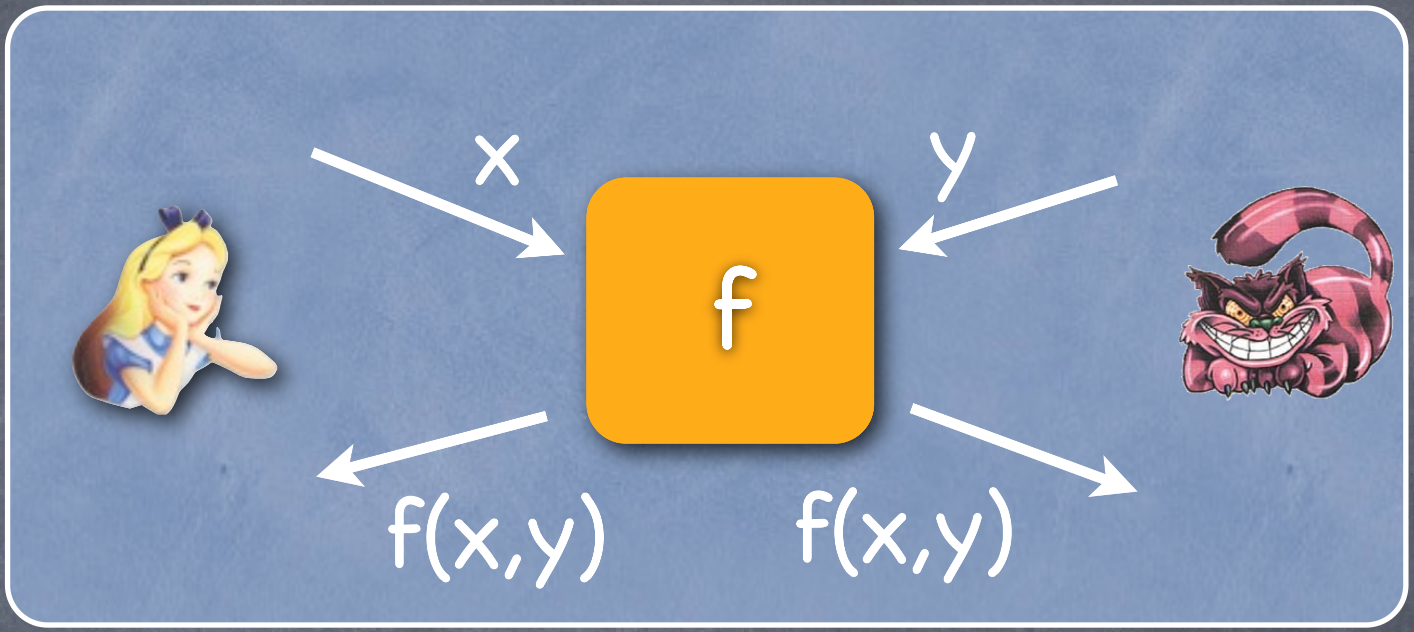


security holds if REAL looks like IDEAL to the outside world

Formal Security Definition





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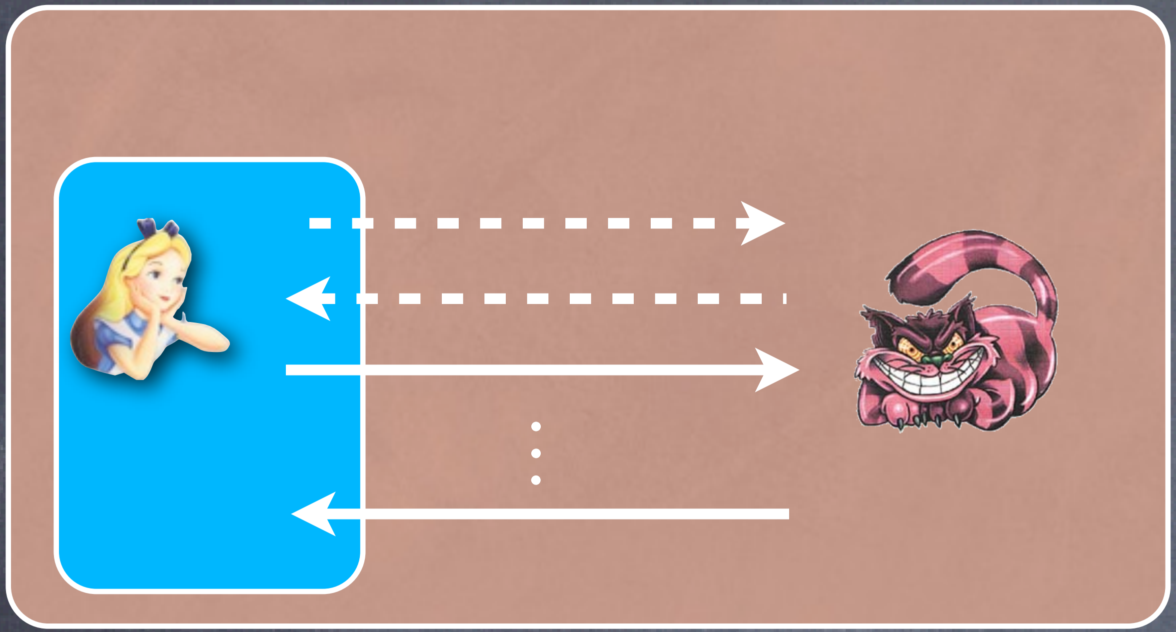

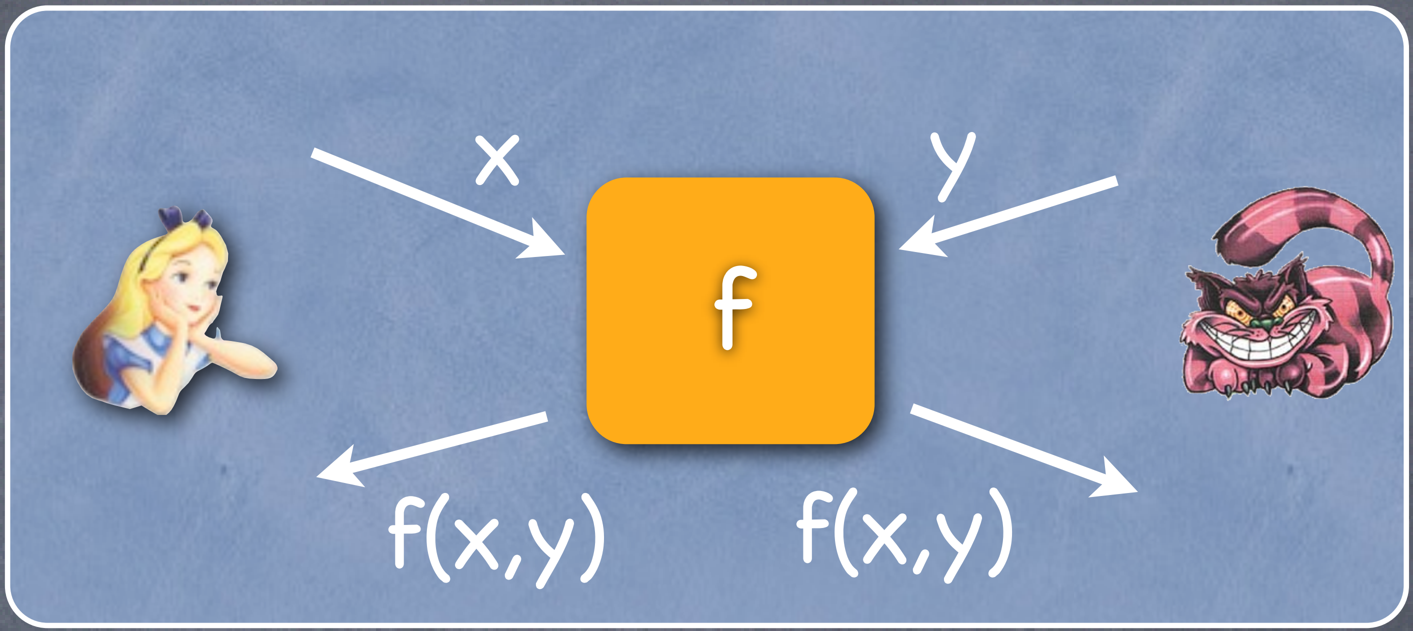

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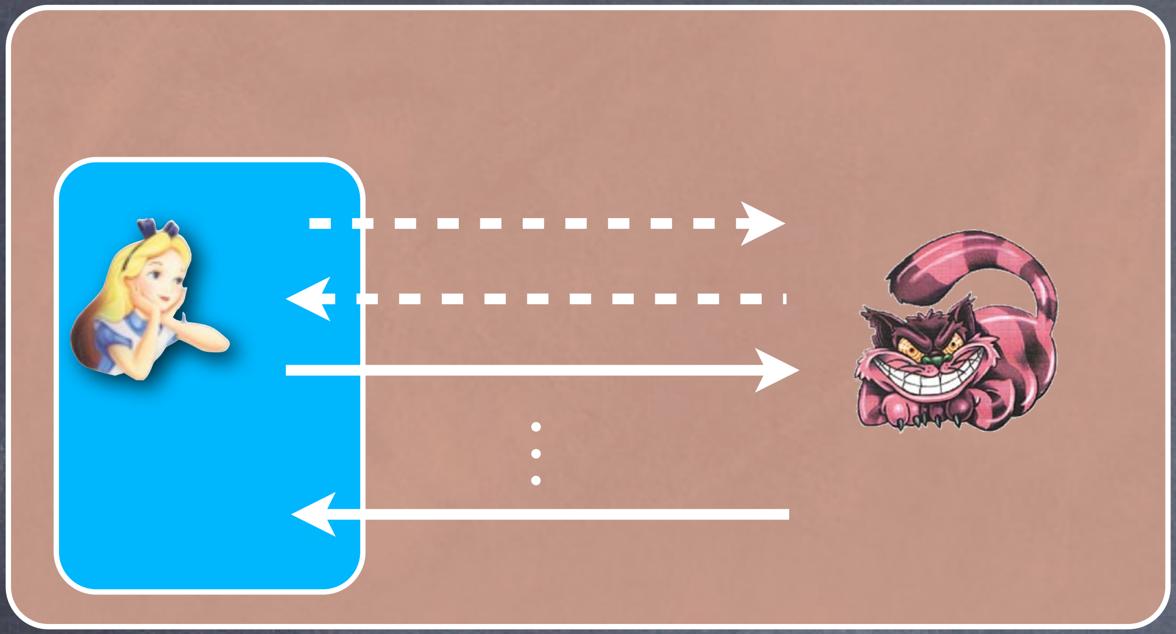

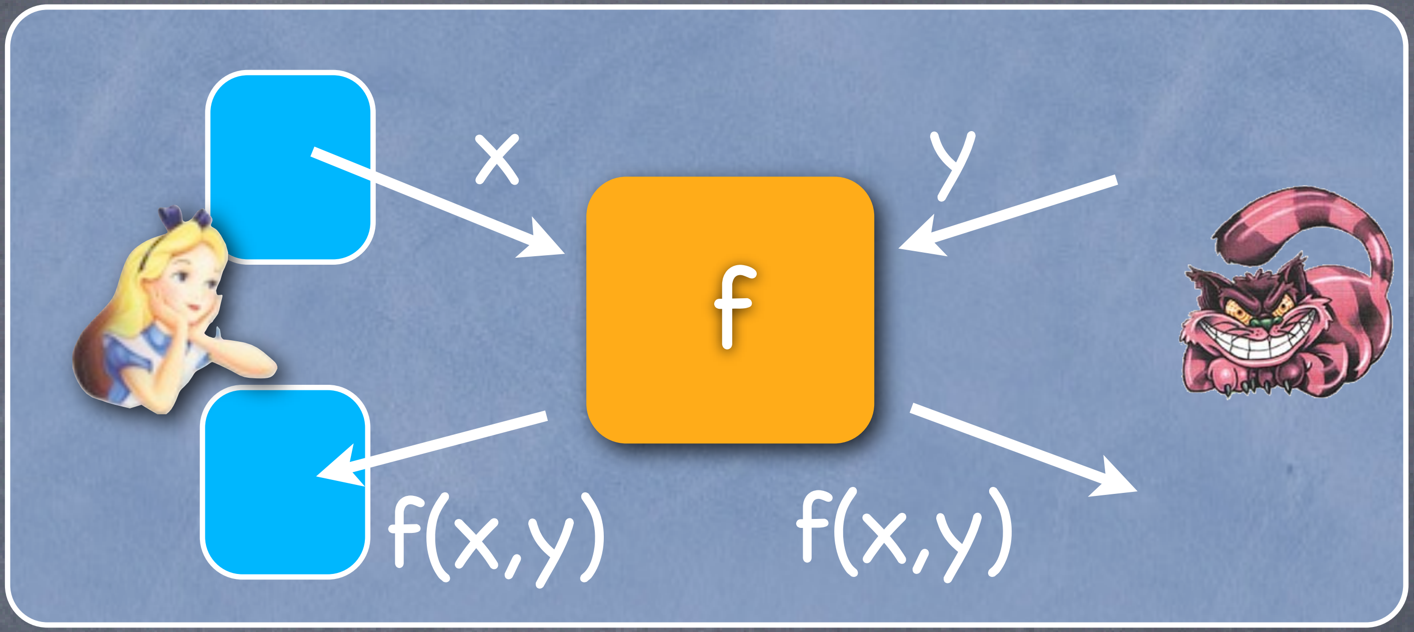

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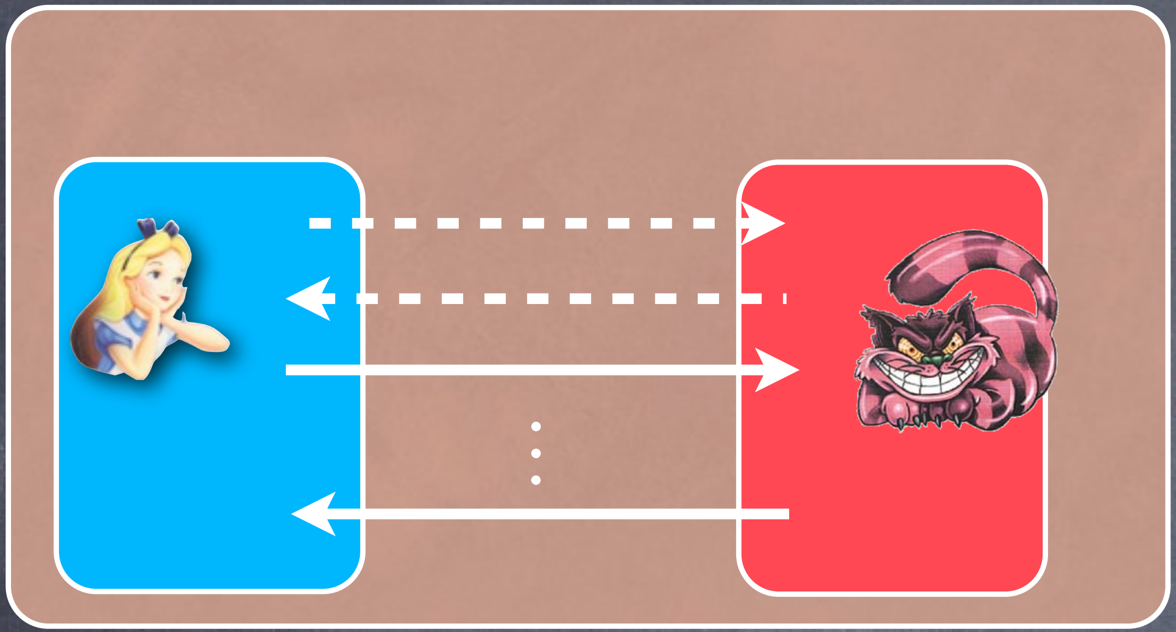

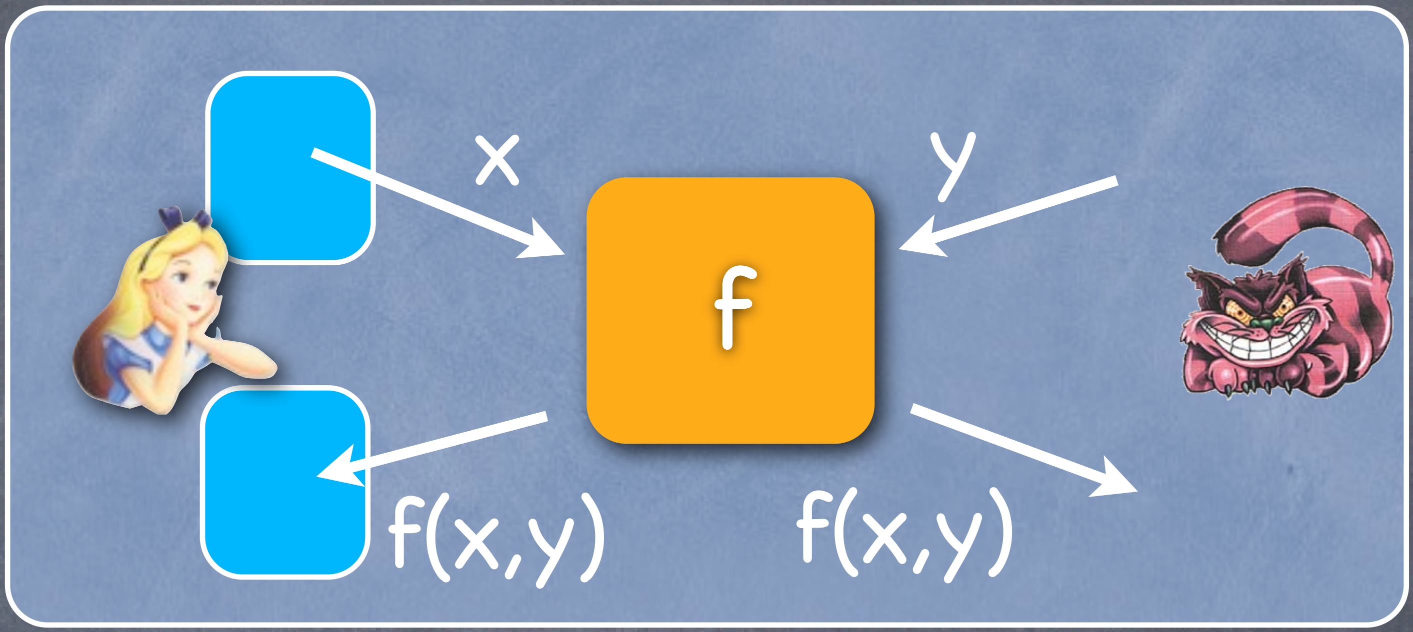

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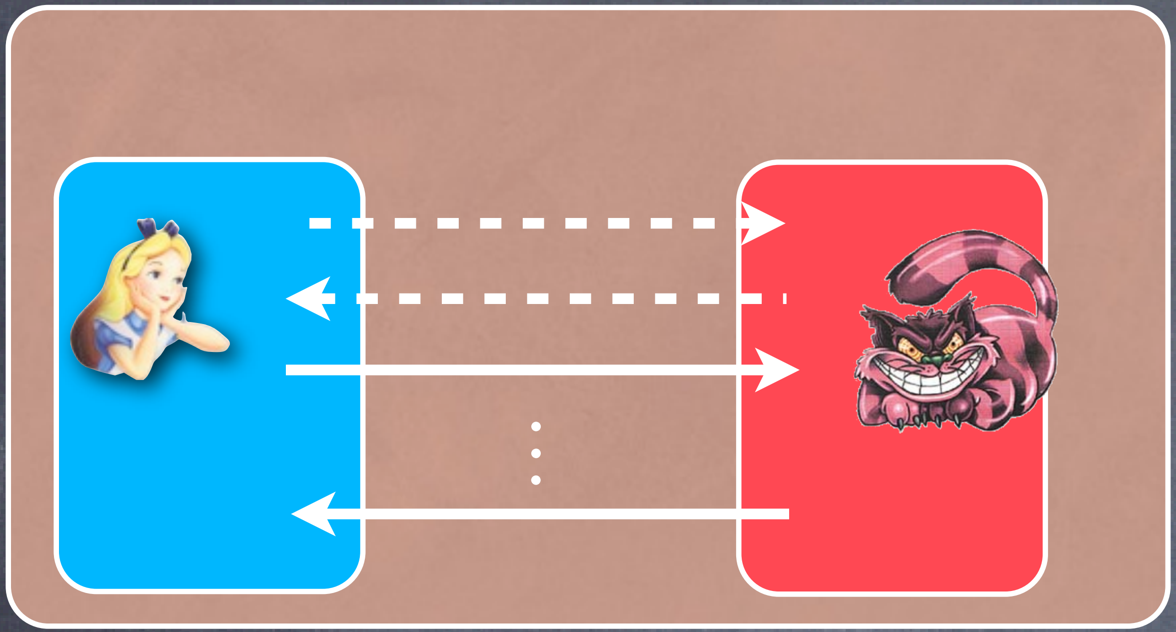

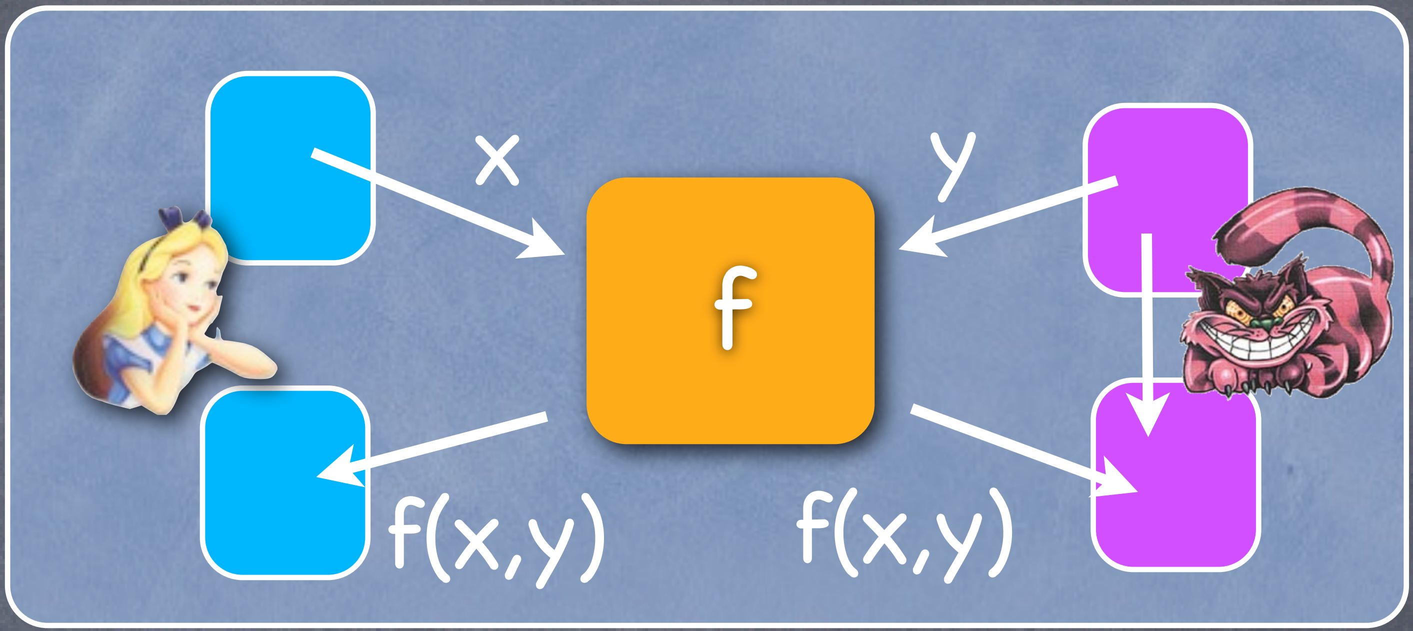

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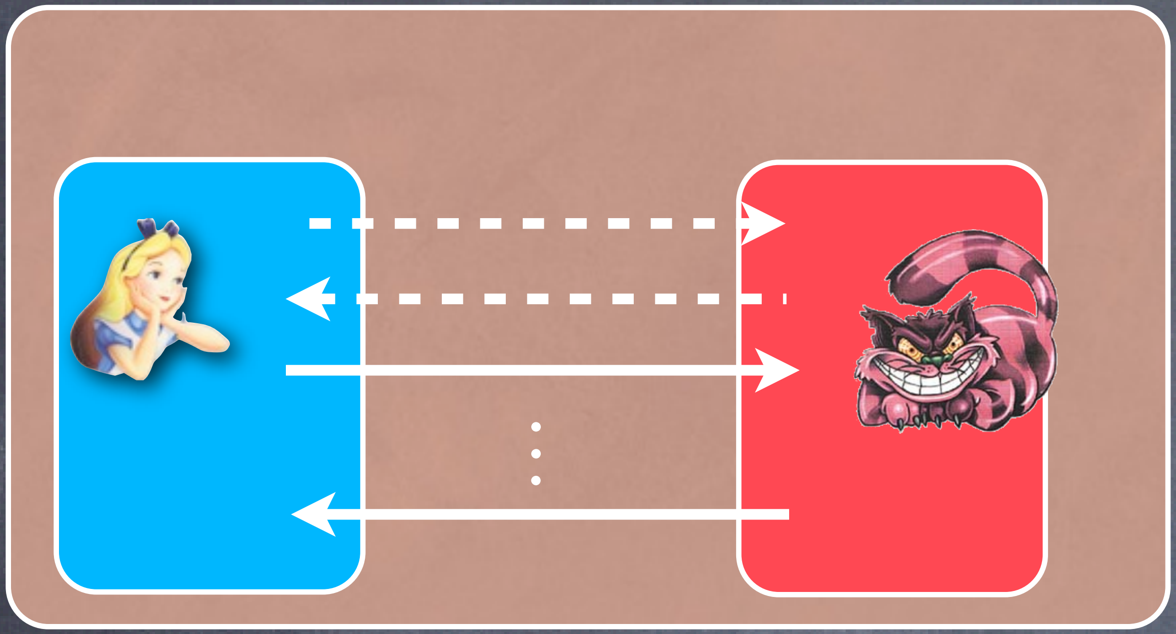
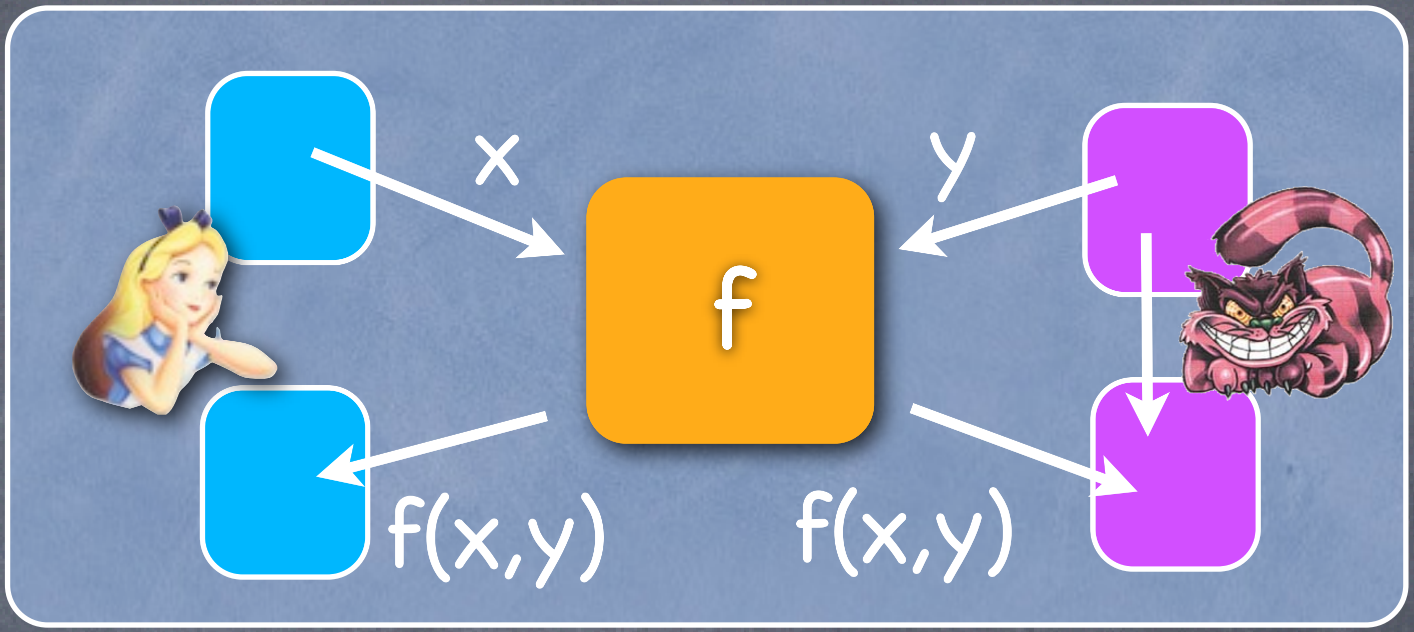
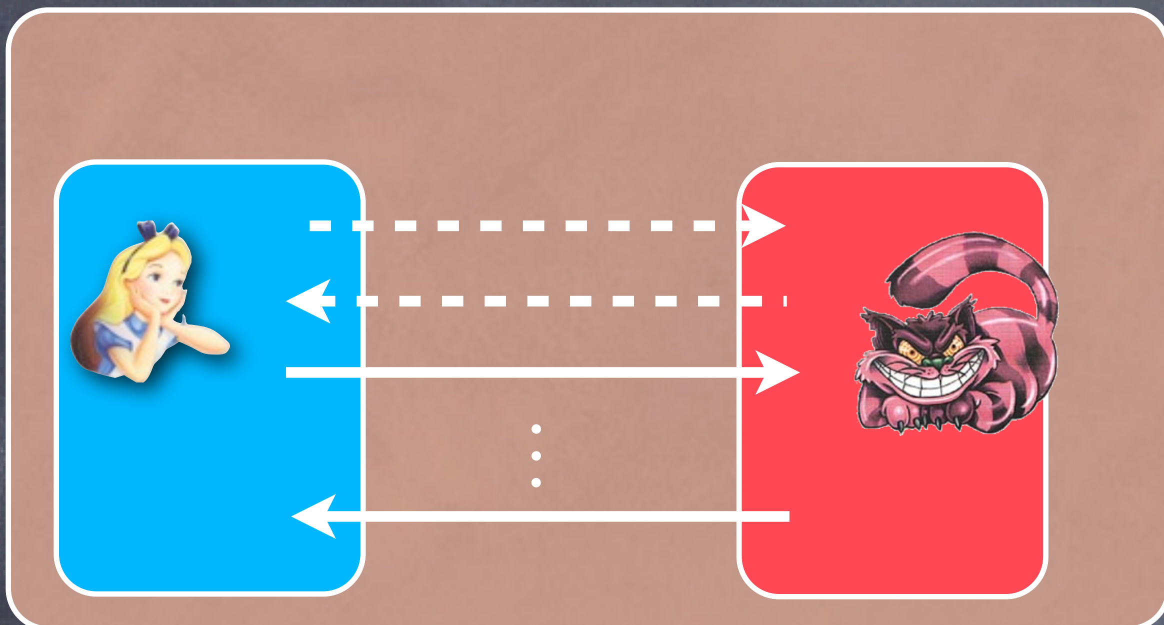


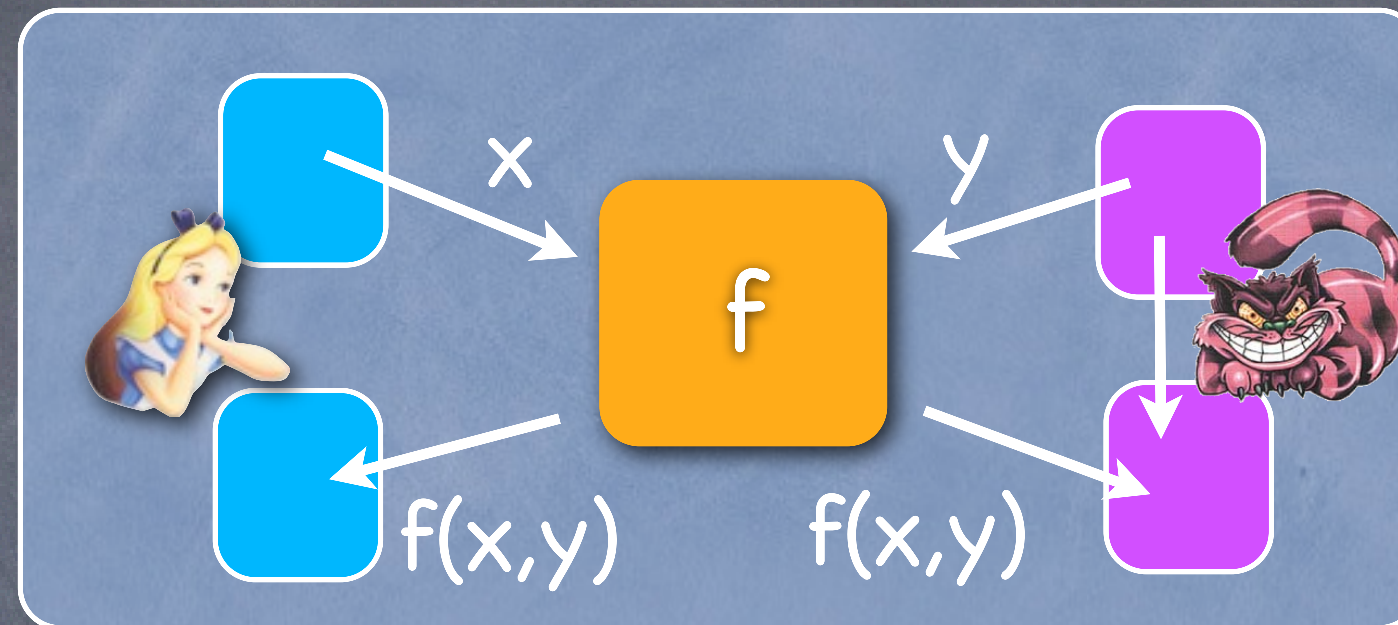


security holds if REAL looks like IDEAL to the outside world

- protocol is secure against dishonest Bob if

Formal Security Definition

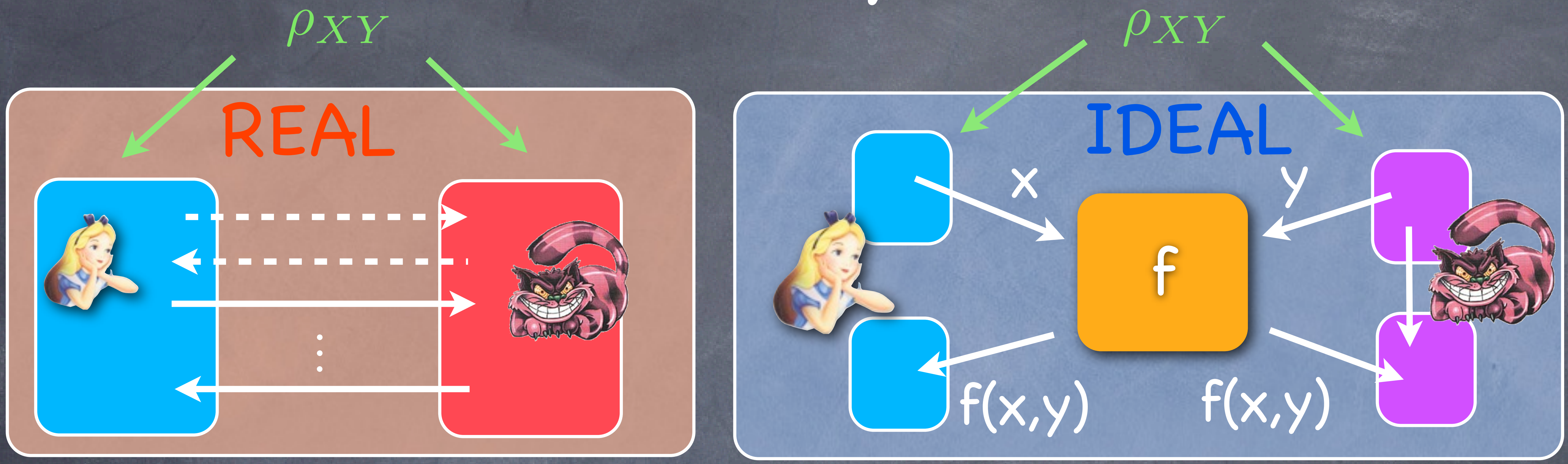




security holds if REAL looks like IDEAL to the outside world

- protocol is secure against dishonest Bob if
  - for every input distribution P(x,y), i.e. $\rho_{XY} = \sum_{x,y} P(x,y) |x\rangle\langle x|_A |y\rangle\langle y|_B$

Formal Security Definition



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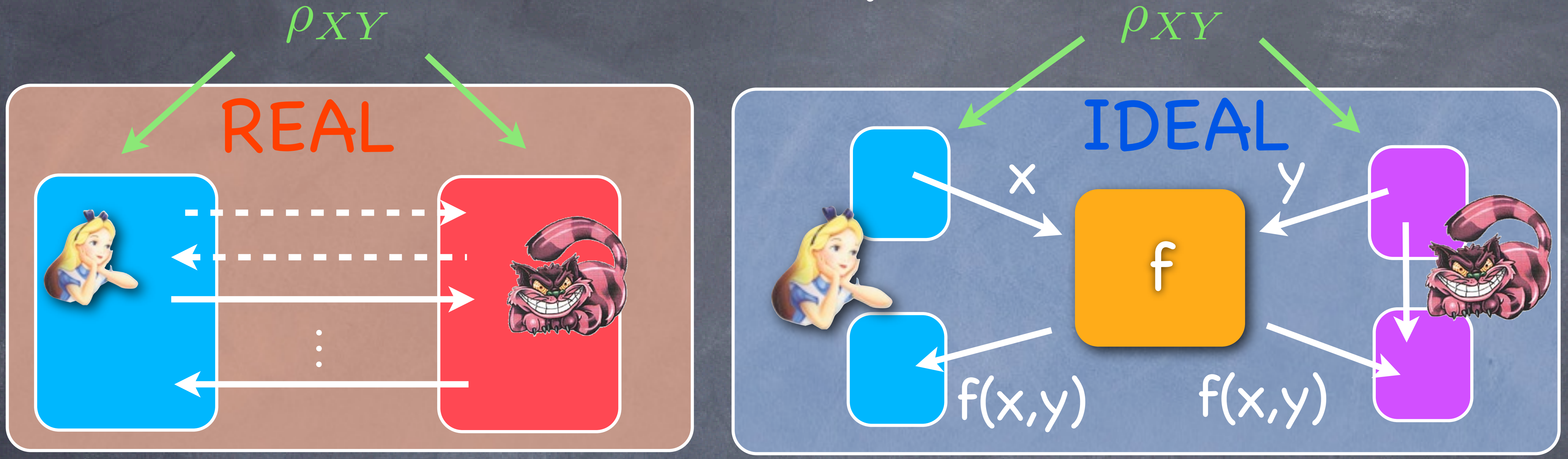

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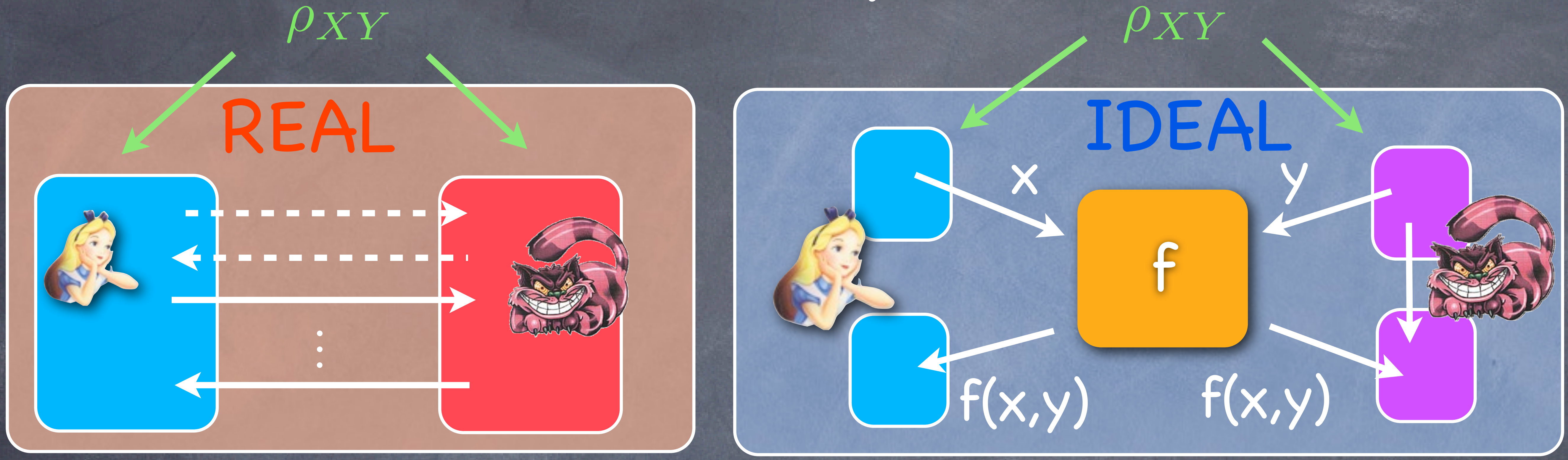
Formal Security Definition



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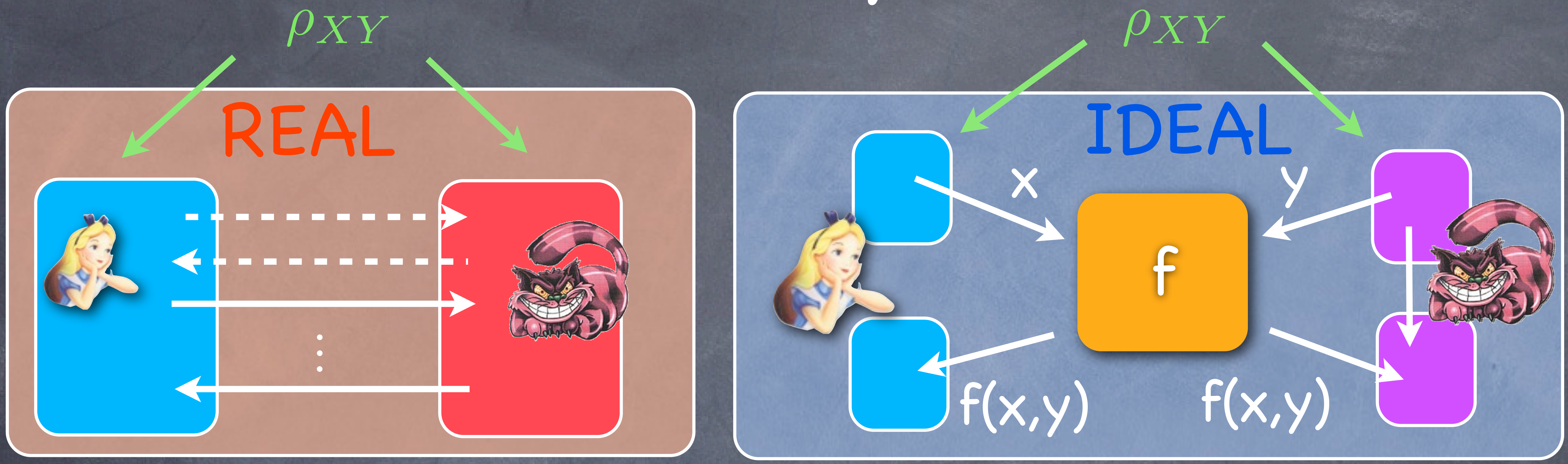
Formal Security Definition



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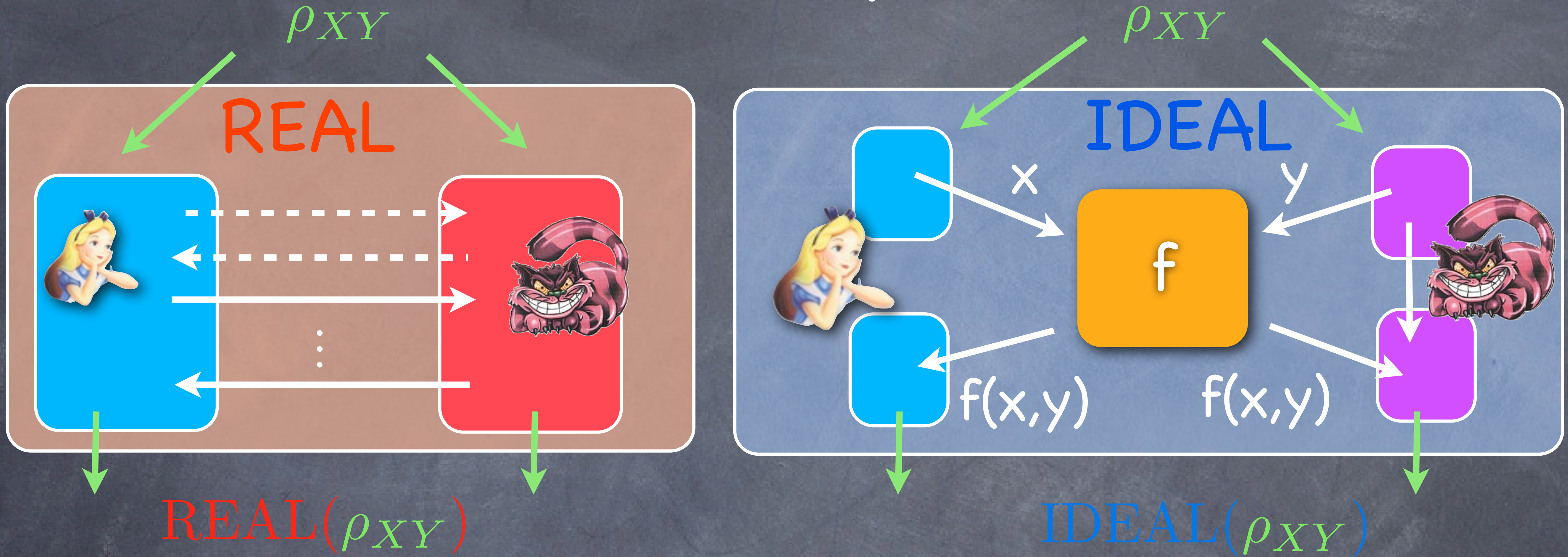
Formal Security Definition



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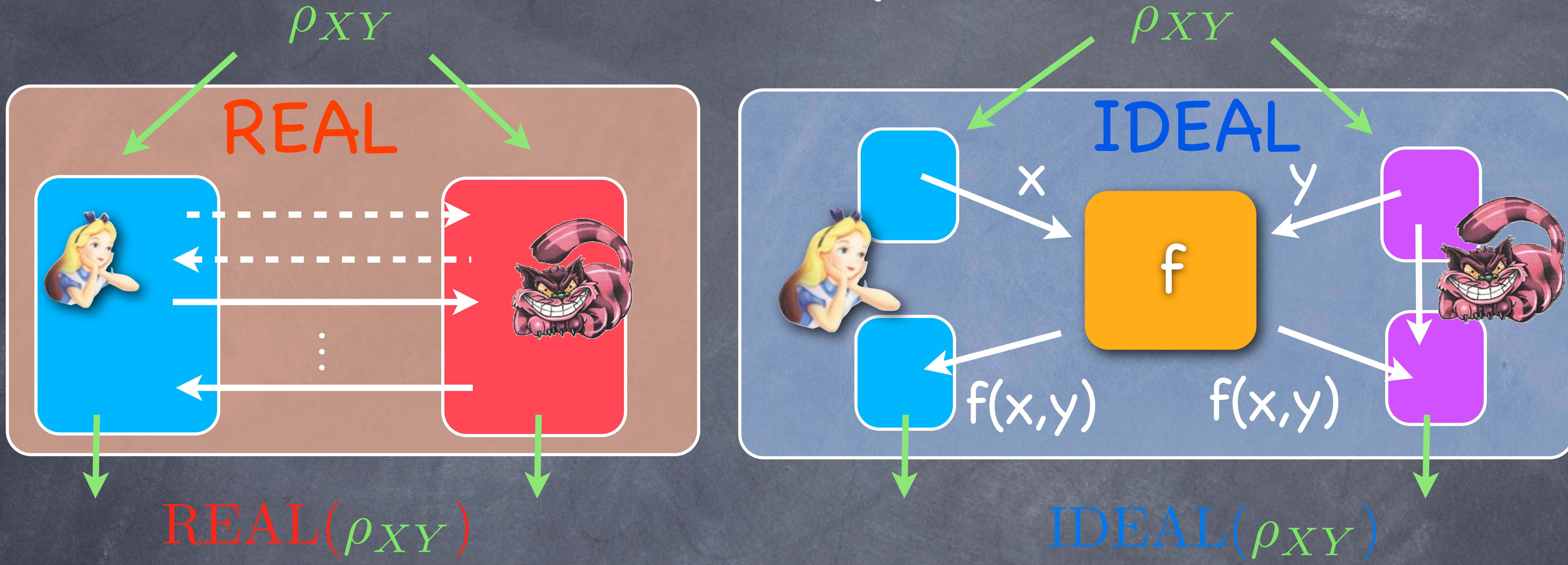
Formal Security Definition



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  - for every dishonest Bob B in the real world,
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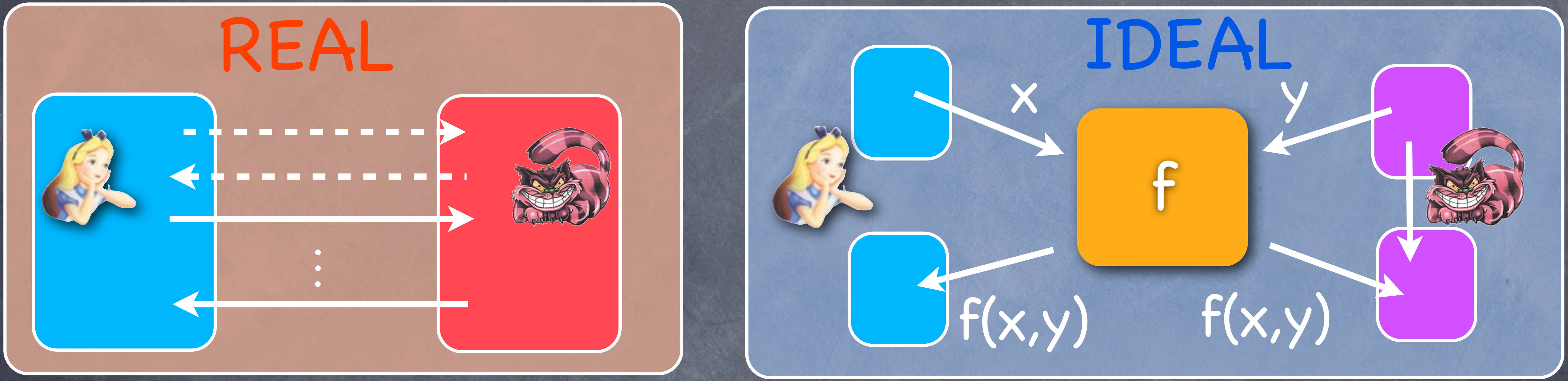
  - such that $\mathrm{REAL}(\rho_{XY}) = \mathrm{IDEAL}(\rho_{XY})$

also relative to purification

Proof of Insecurity

Security against Bob => Insecurity against Alice

security holds if REAL looks like IDEAL to the outside world



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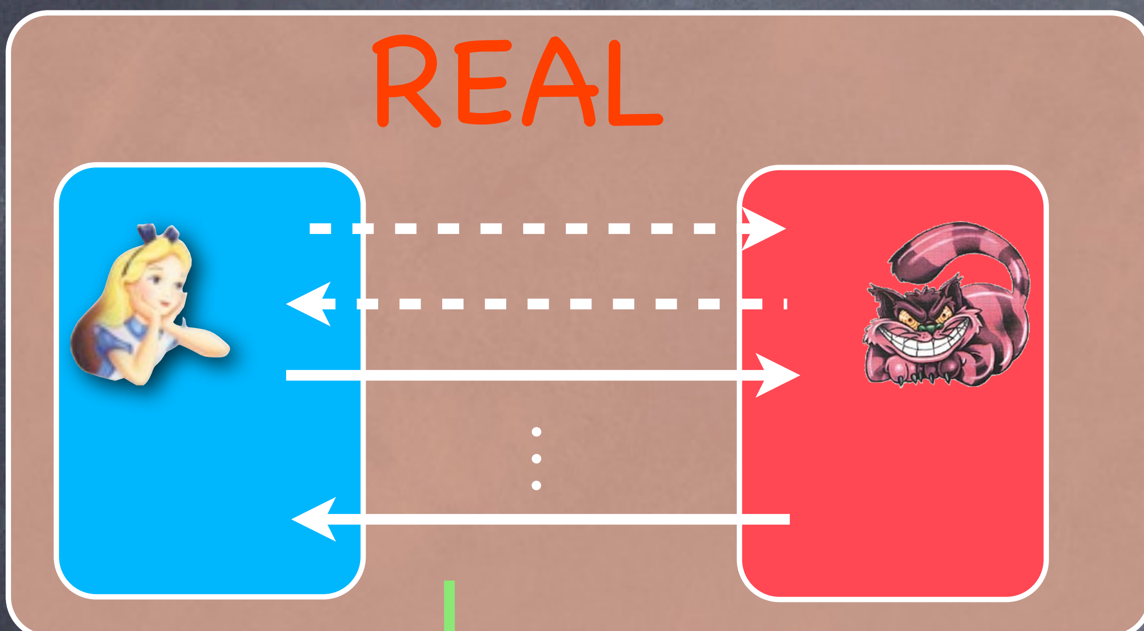
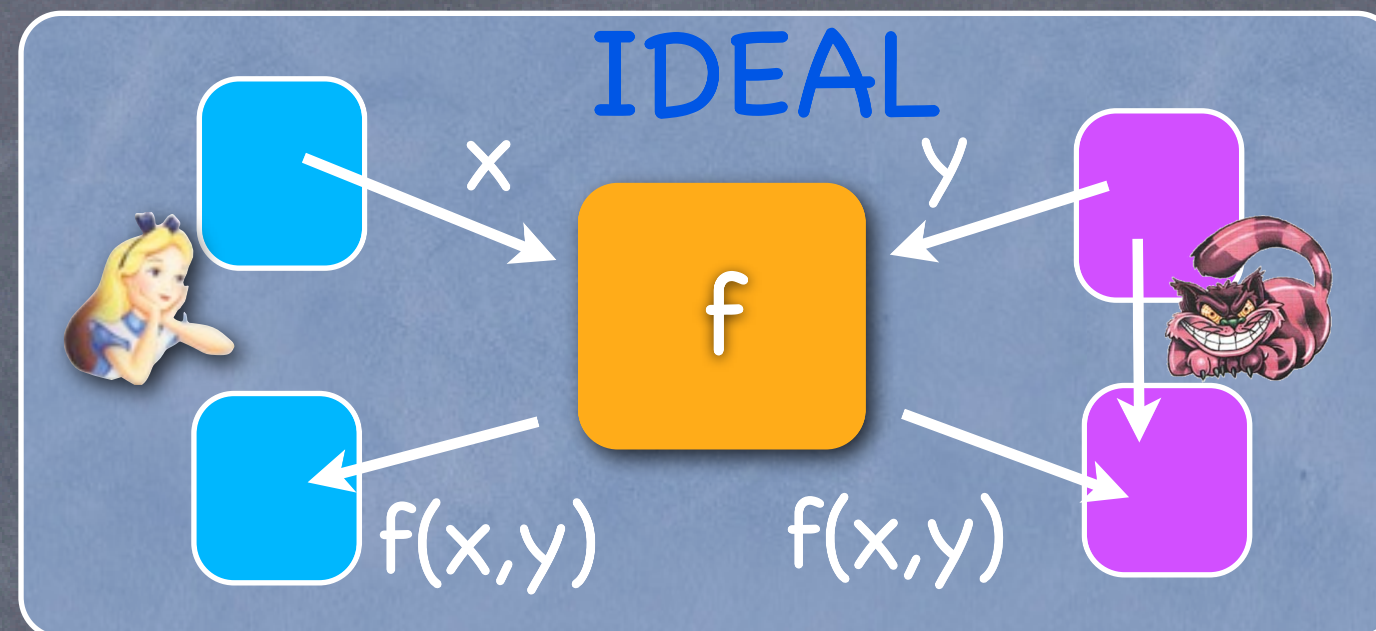




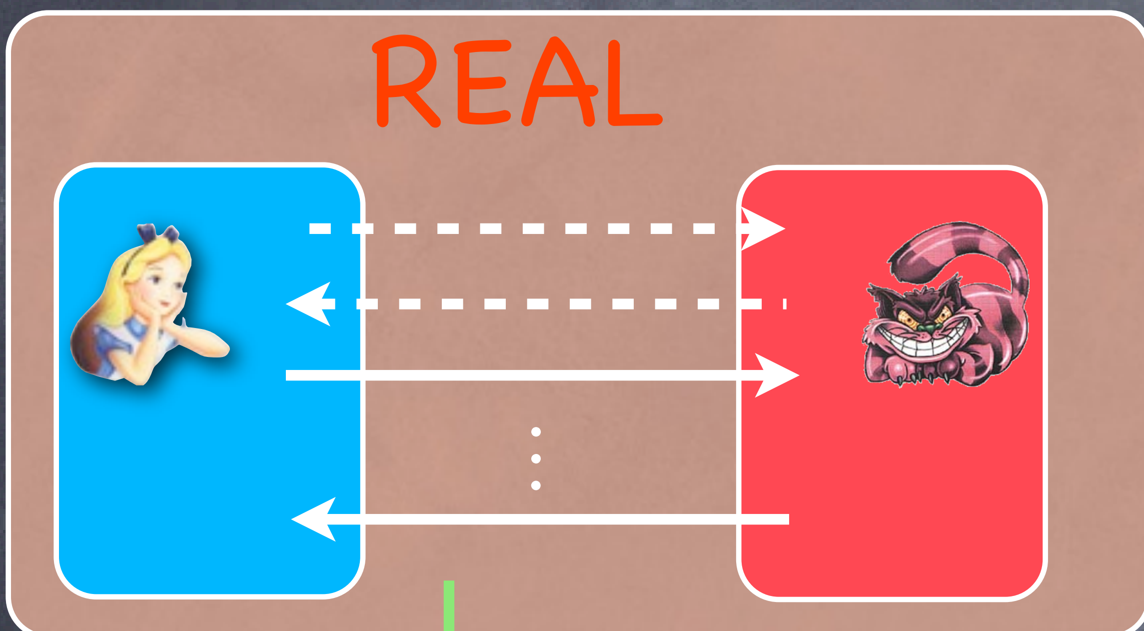
$|\psi\rangle_{A_p A B B_p}$

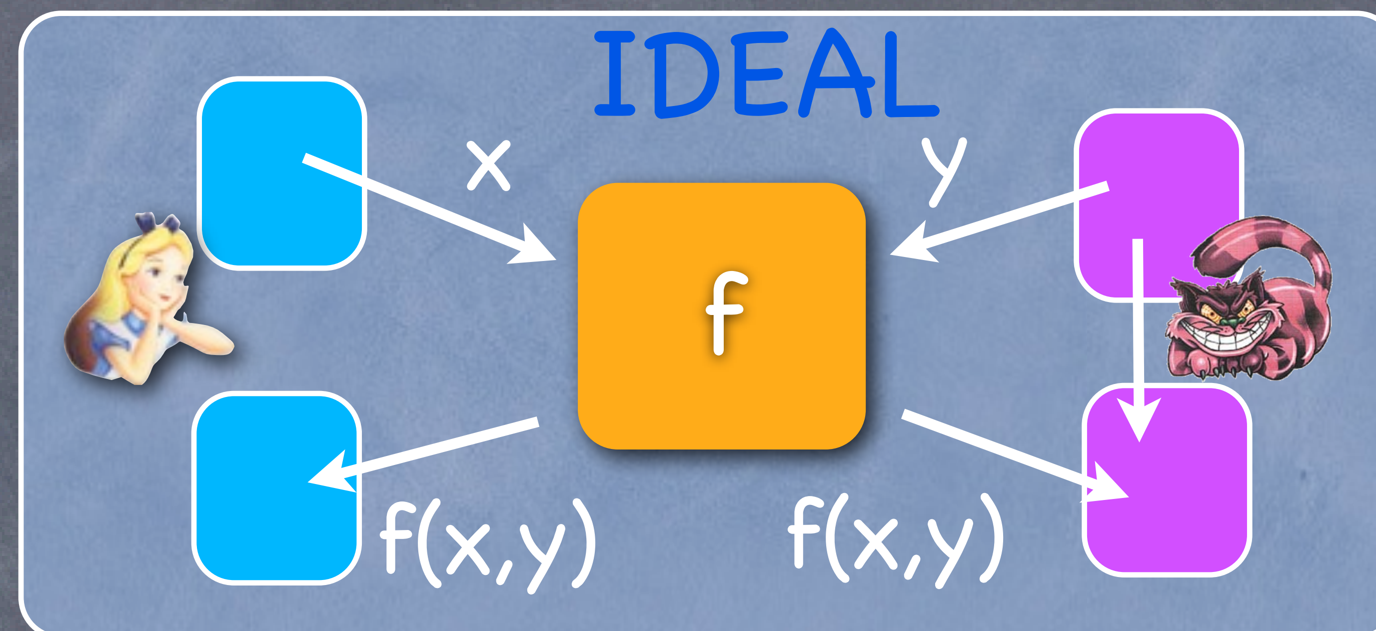
state after the real protocol if both parties play "dishonestly" by purifying their actions

Proof of Insecurity

Security against Bob => Insecurity against Alice

security holds if REAL looks like IDEAL to the outside world





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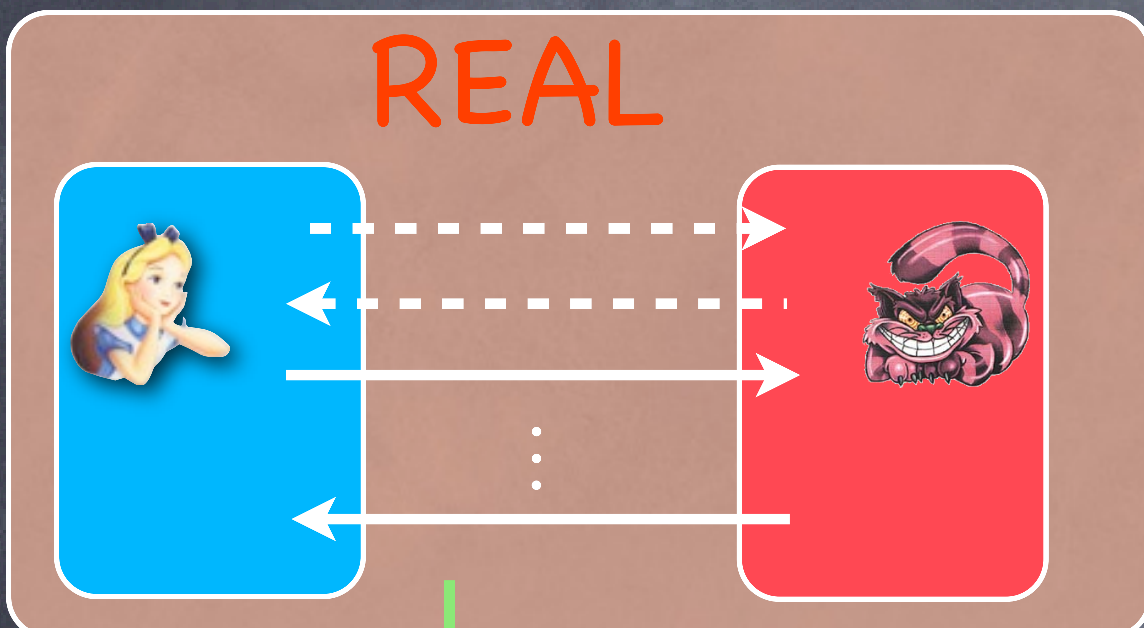

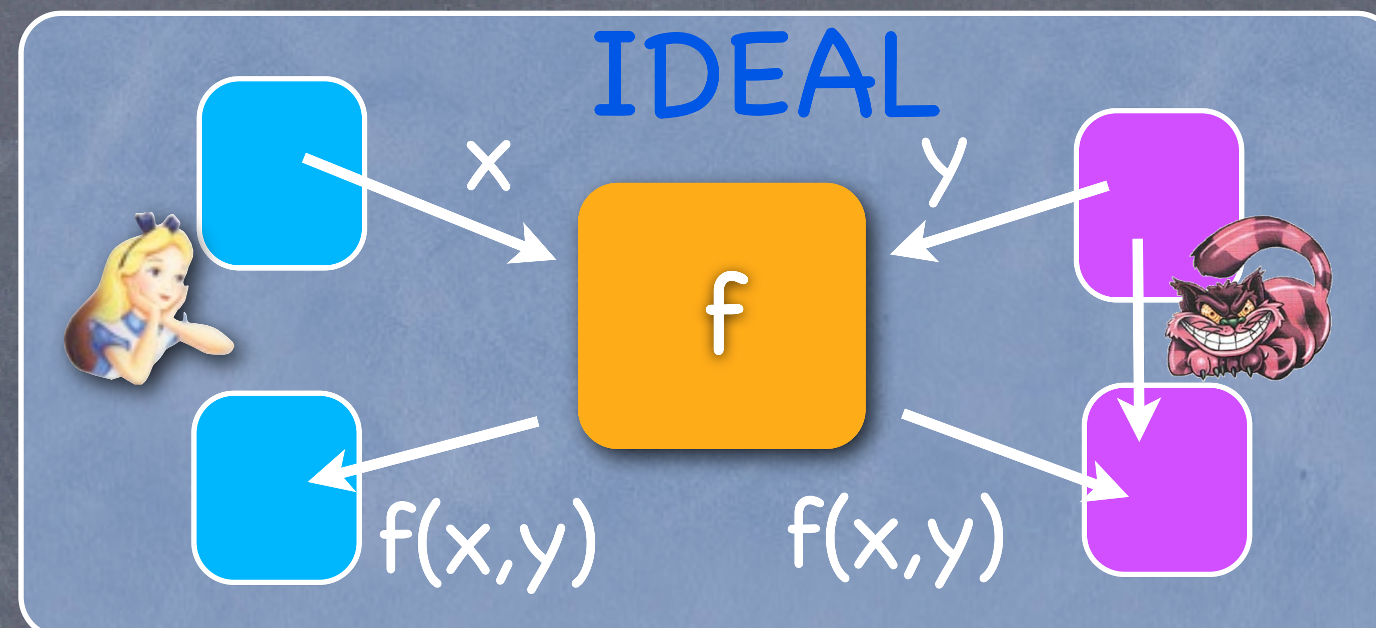

$|\psi\rangle_{A_pABB_p}$

state after the real protocol if both parties play "dishonestly" by purifying their actions

tr_{A_p}

$$\rho_{ABB_p} = \sigma_{ABB_p} = \mathrm{tr}_Y(\sigma_{ABB_pY})$$

Proof of Insecurity

Security against Bob => Insecurity against Alice

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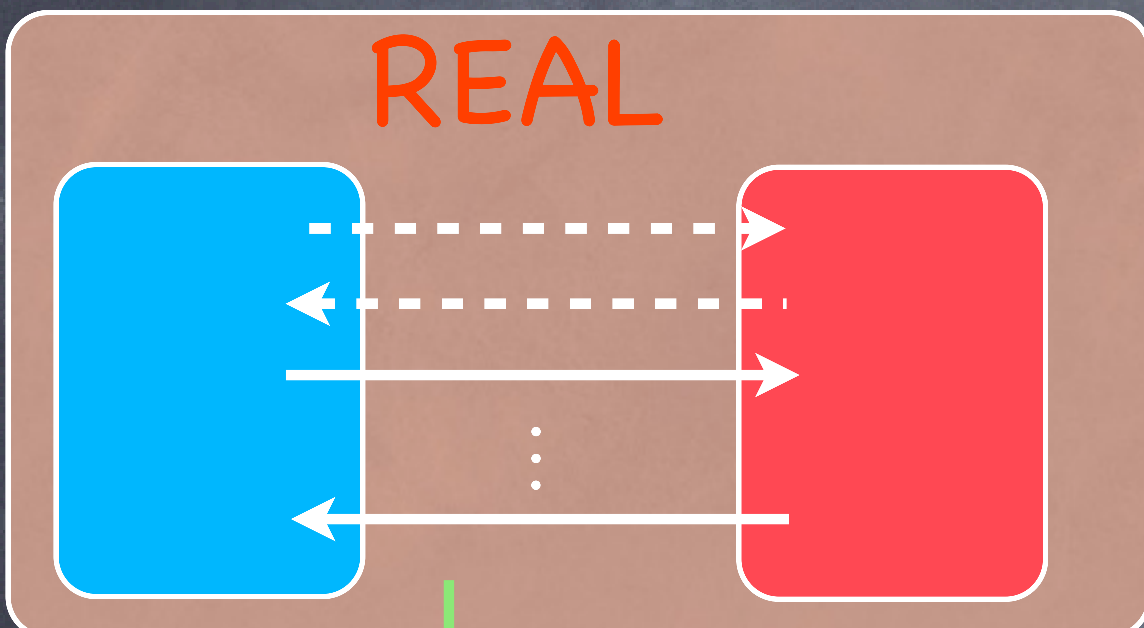

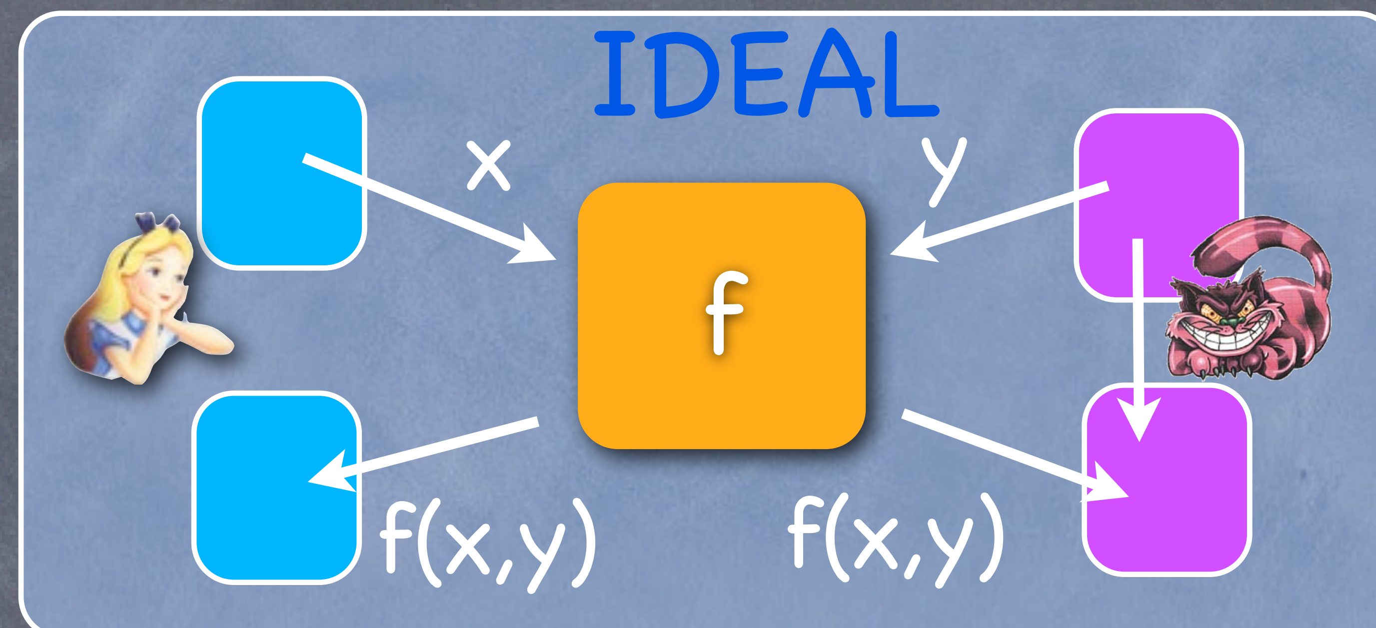

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state after the real protocol if both parties play "dishonestly" by purifying their actions

$\xrightarrow{\mathrm{tr}_{A_p}}$

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Proof of Insecurity

Security against Bob => Insecurity against Alice

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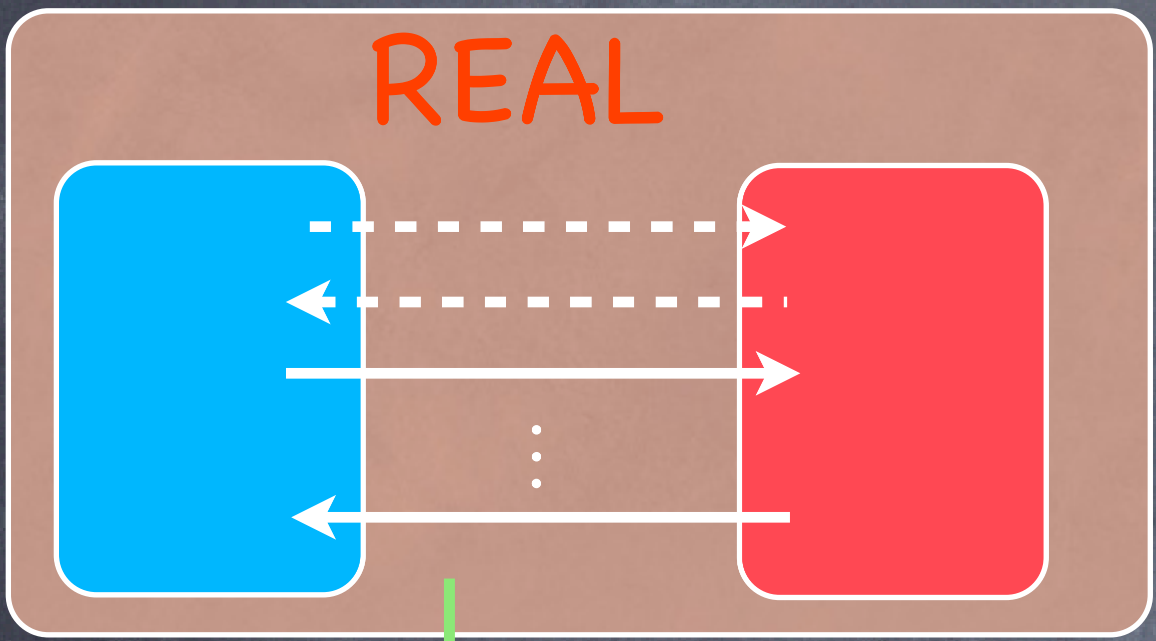

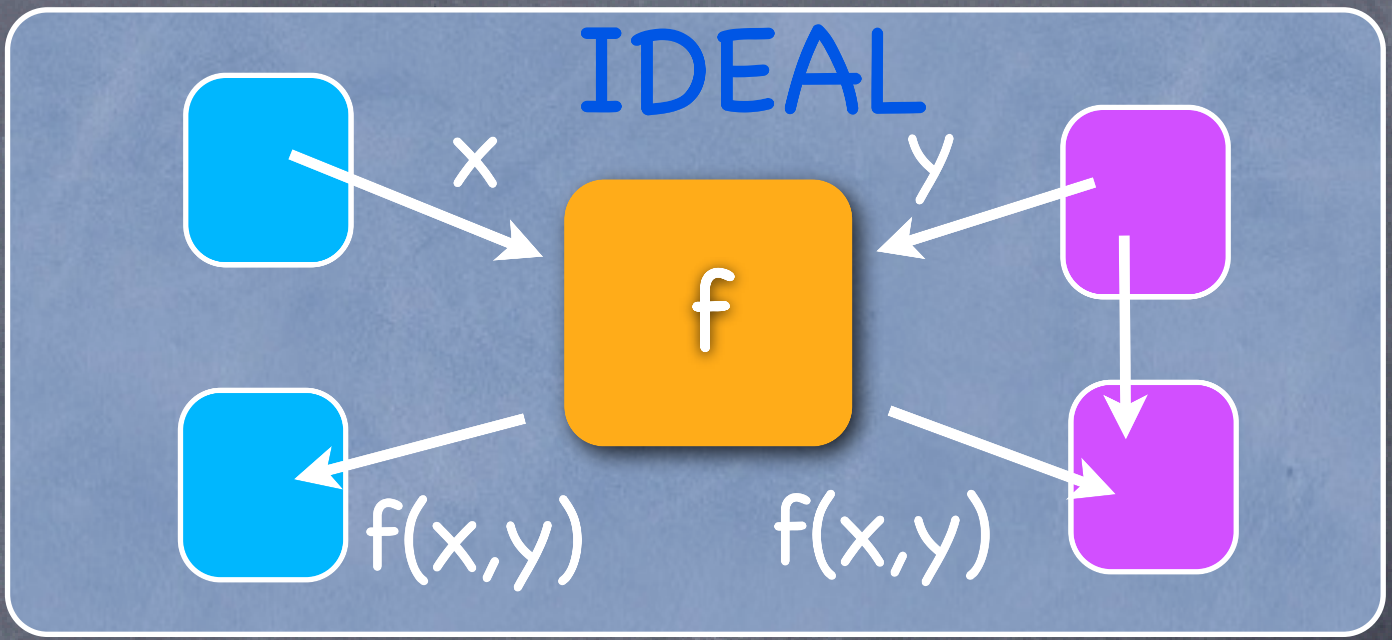

$|\psi\rangle_{A_p A B B_p}$

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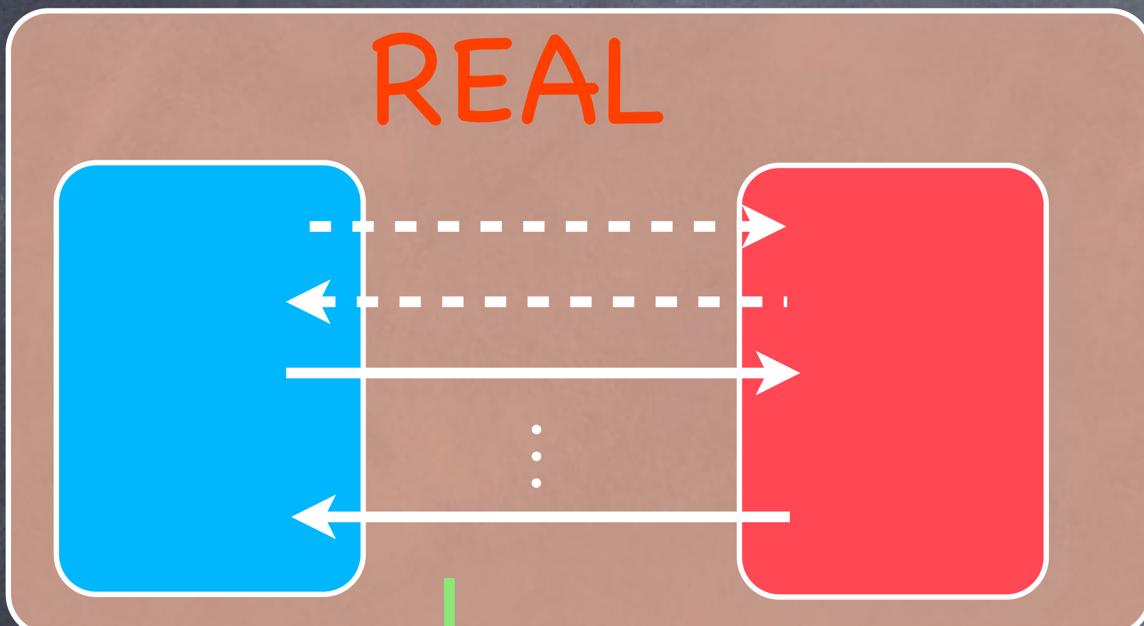

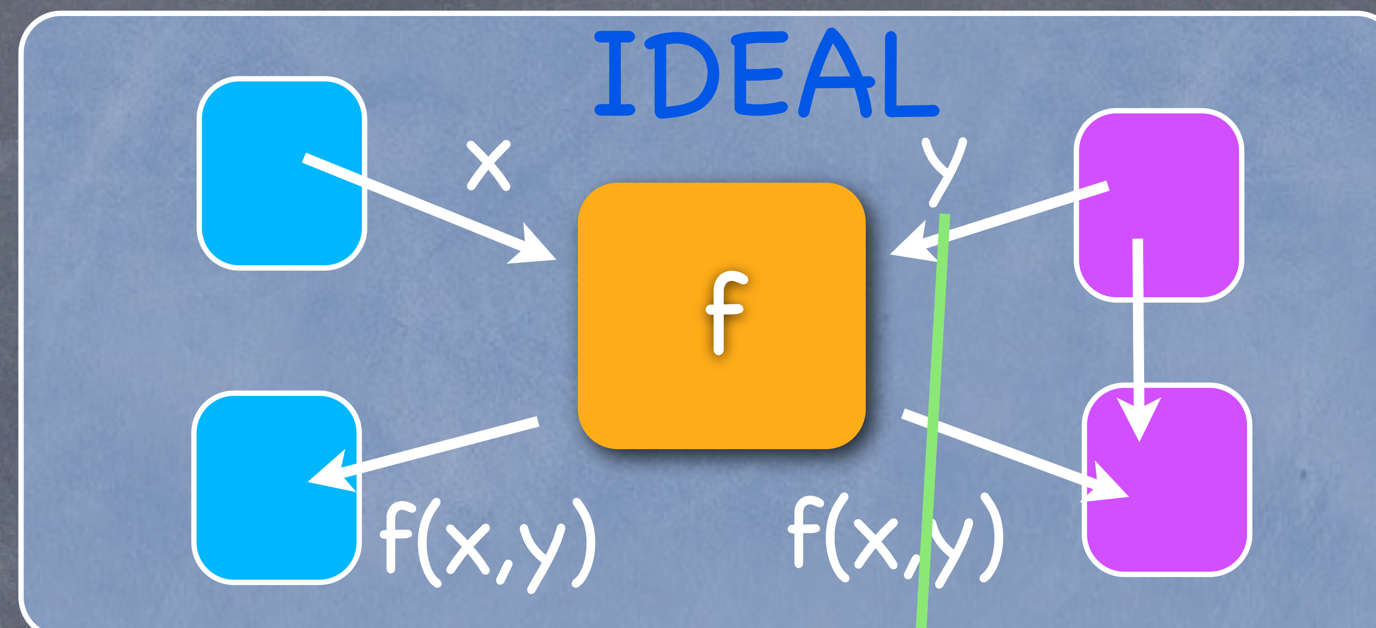

$|\psi\rangle_{A_pABB_p}$

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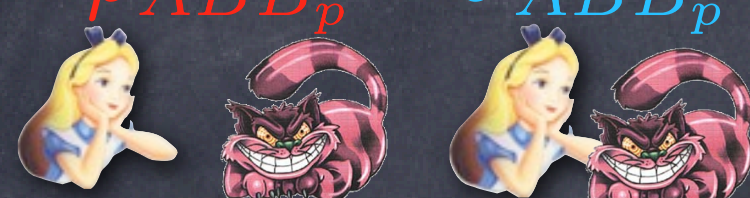

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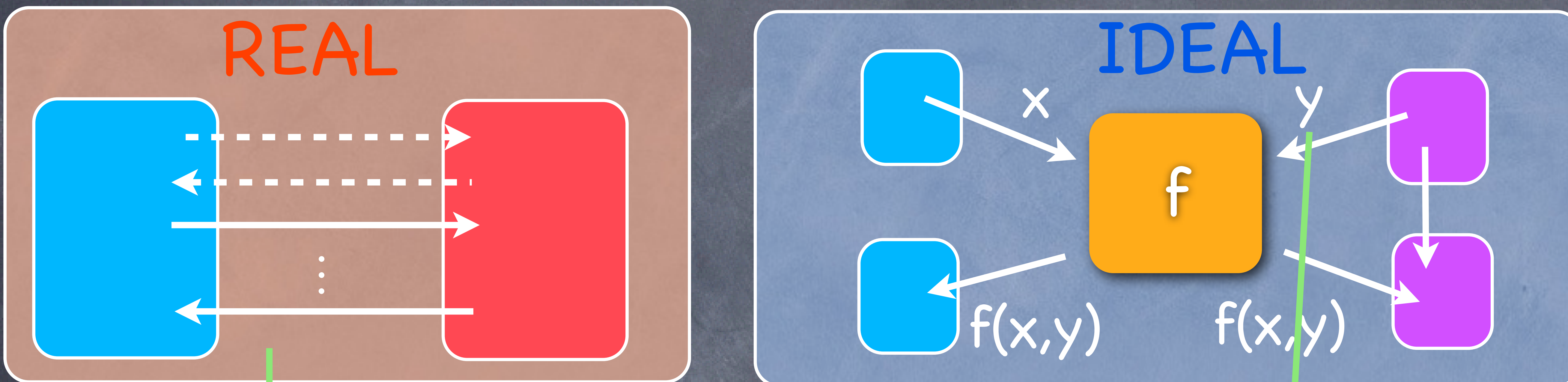


 $|\psi\rangle_{A_pABB_p}$  state after the real protocol if both parties play "dishonestly" by purifying their actions

tr_{A_p}

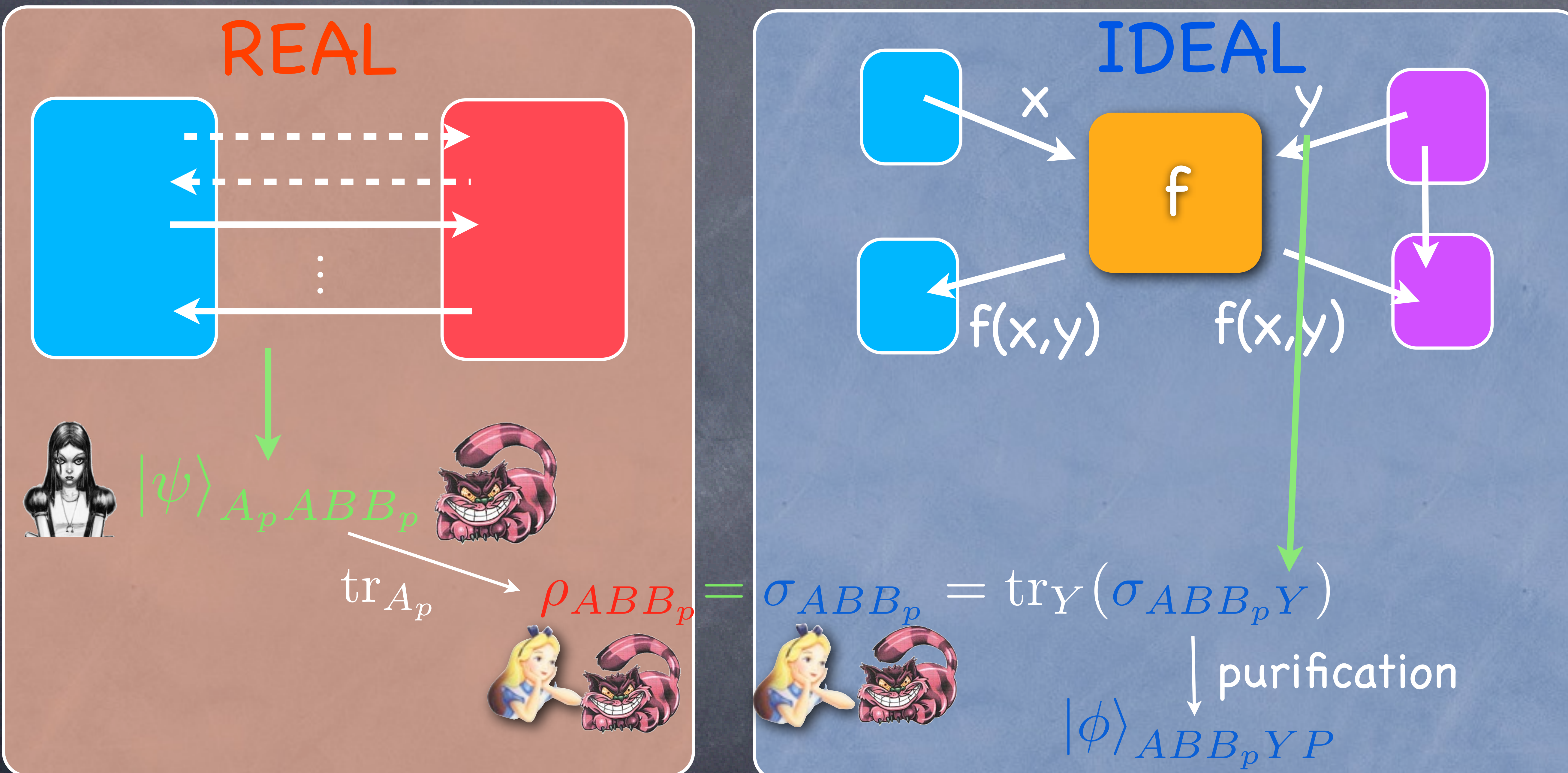
$$\rho_{ABB_p} = \sigma_{ABB_p} = \mathrm{tr}_Y(\sigma_{ABB_pY})$$

purification

$$|\phi\rangle_{ABB_pYP}$$

Proof of Insecurity

security holds if REAL looks like IDEAL to the outside world



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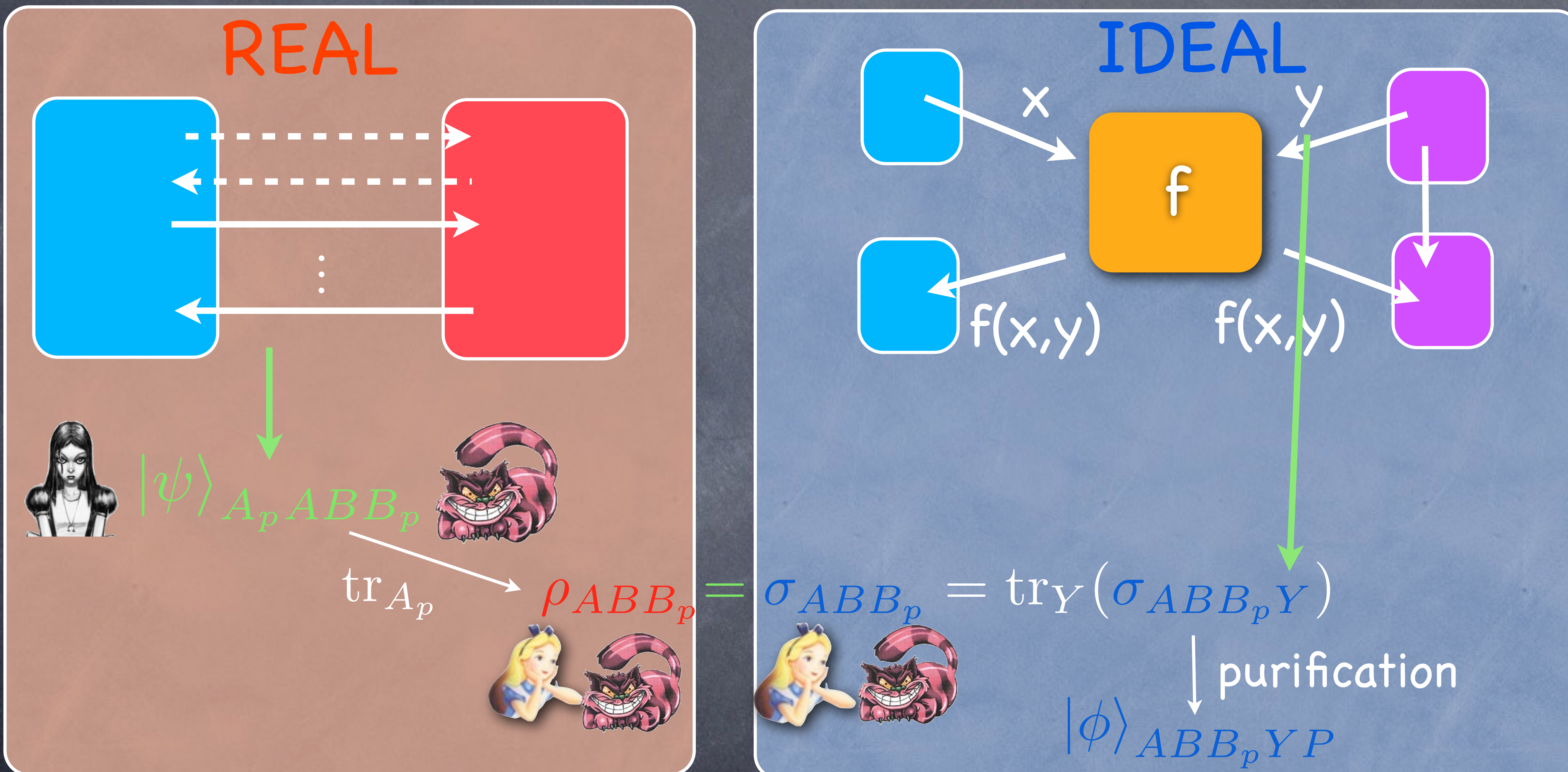
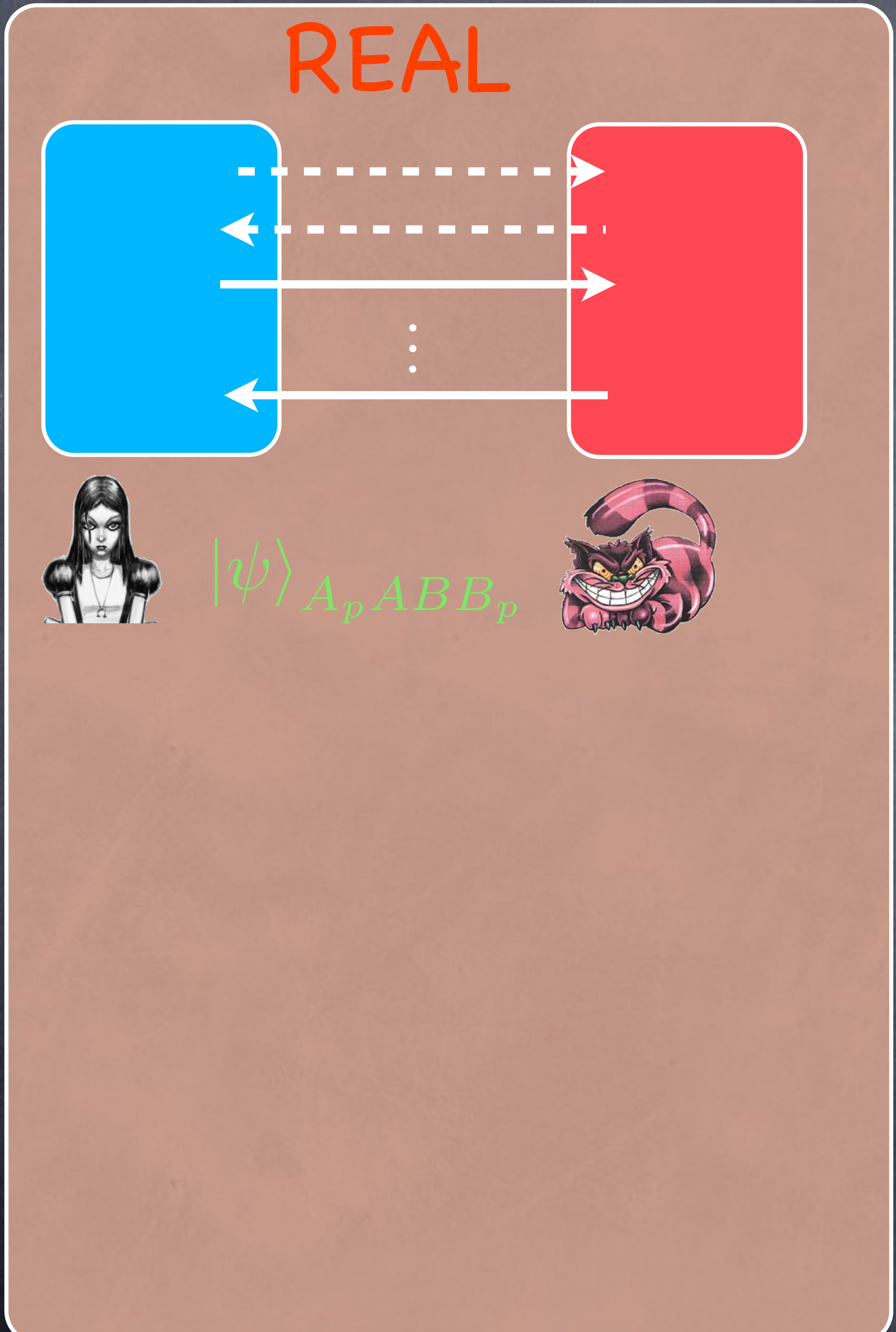
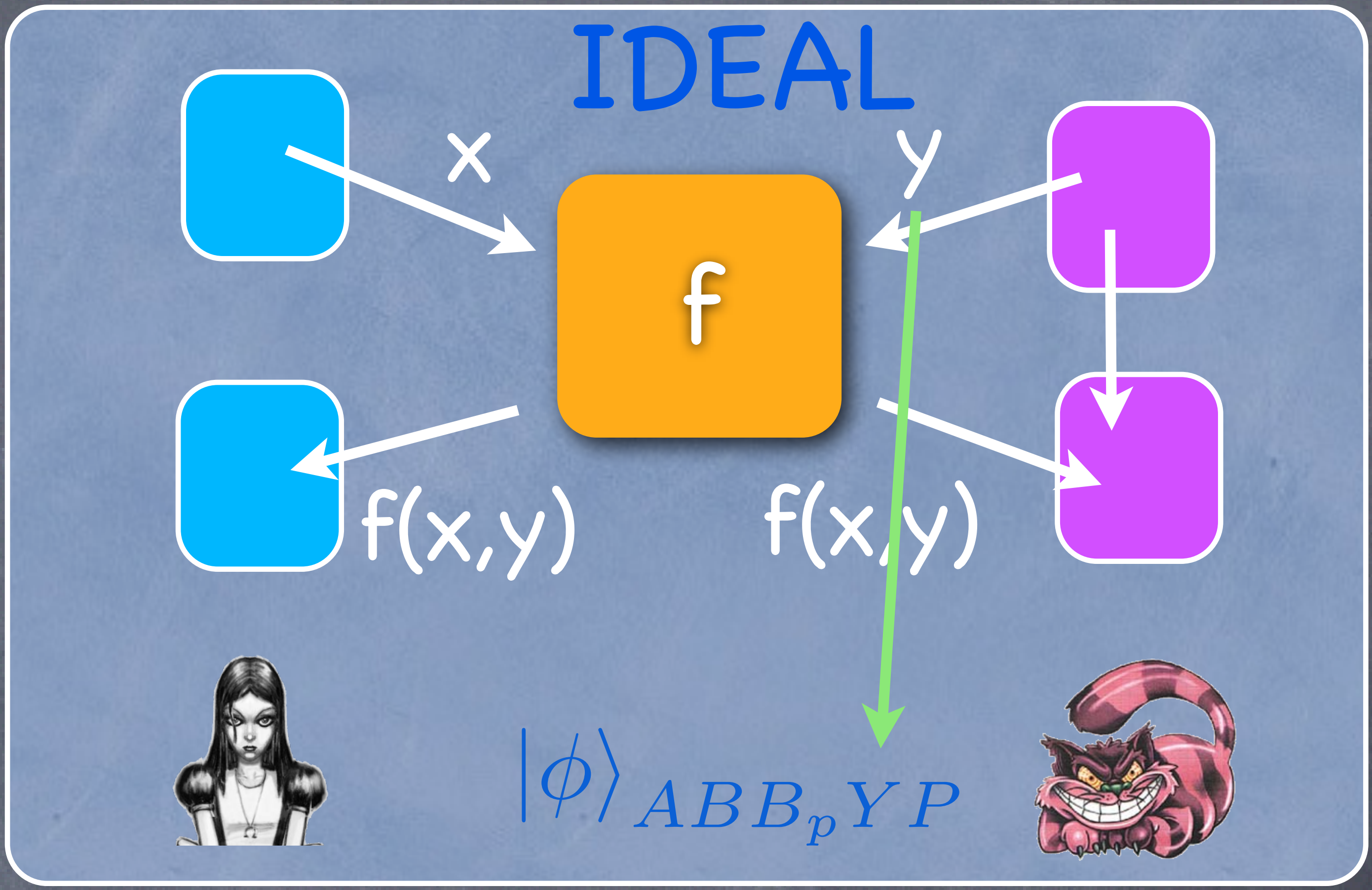


- by Uhlmann's theorem: there exists a cheating unitary U such that $U_{A_p \to YP} \, |\psi\rangle_{A_pABB_p} = |\phi\rangle_{ABB_pYP}$

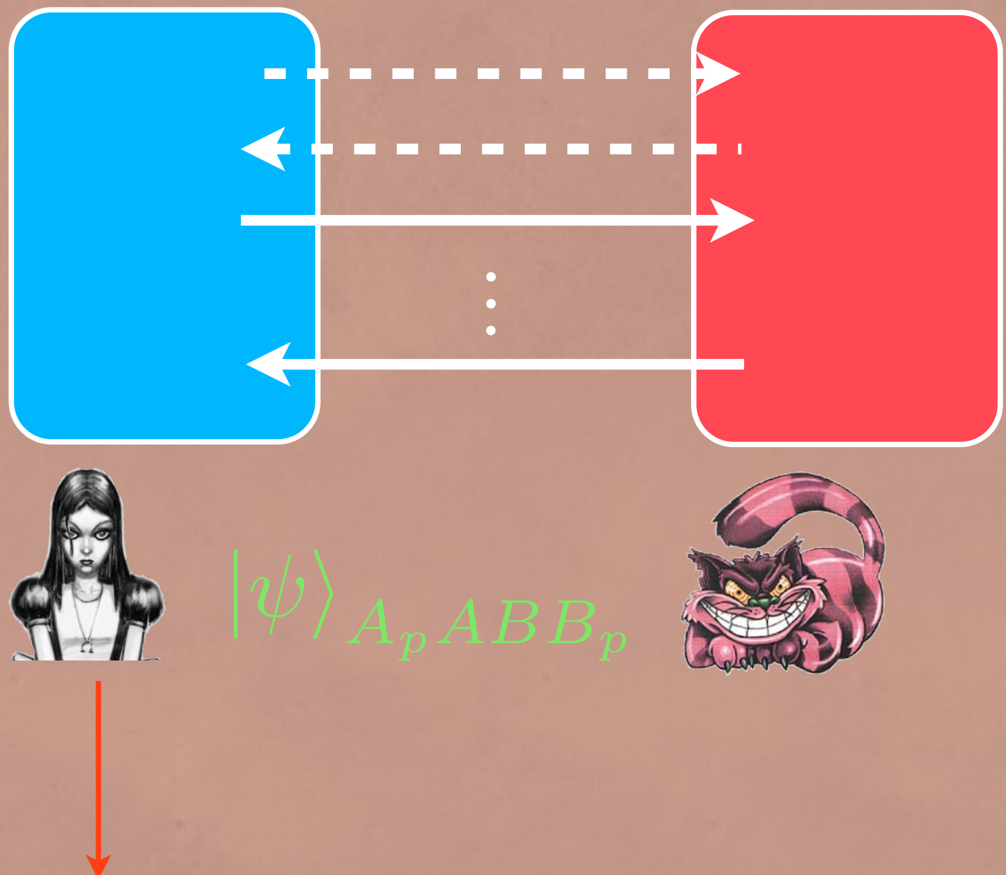
Proof of Insecurity





Proof of Insecurity

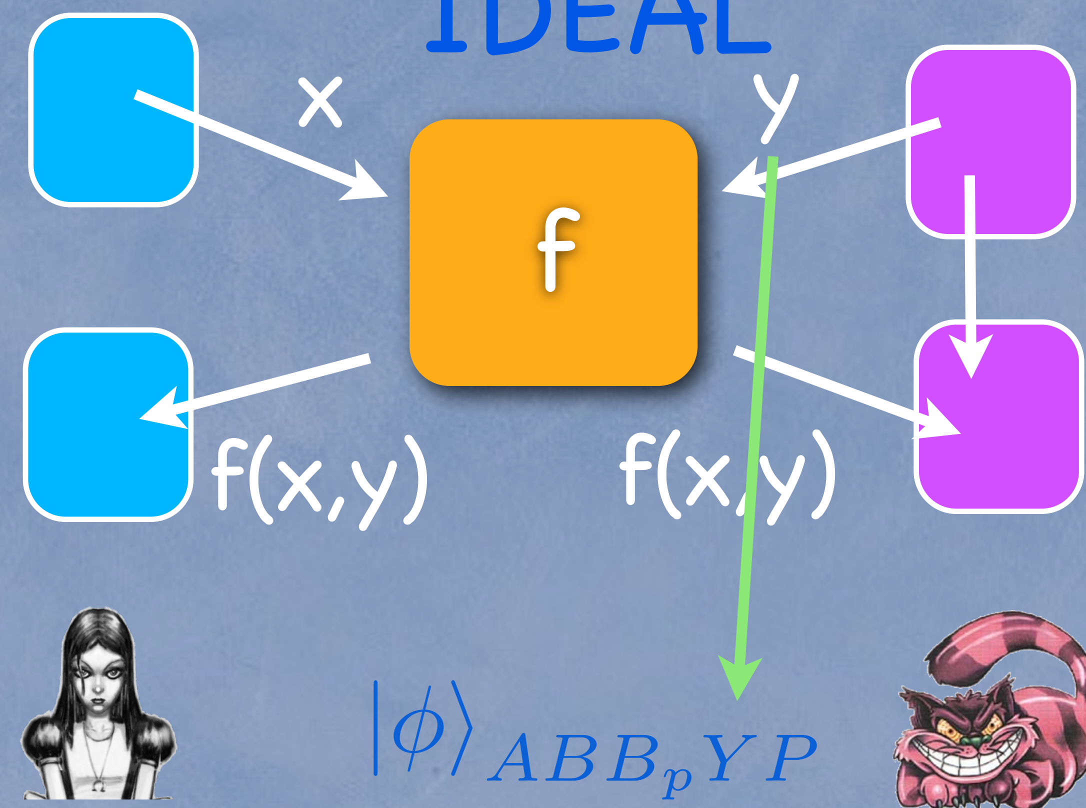
REAL



$$|\psi\rangle_{A_pABB_p}$$

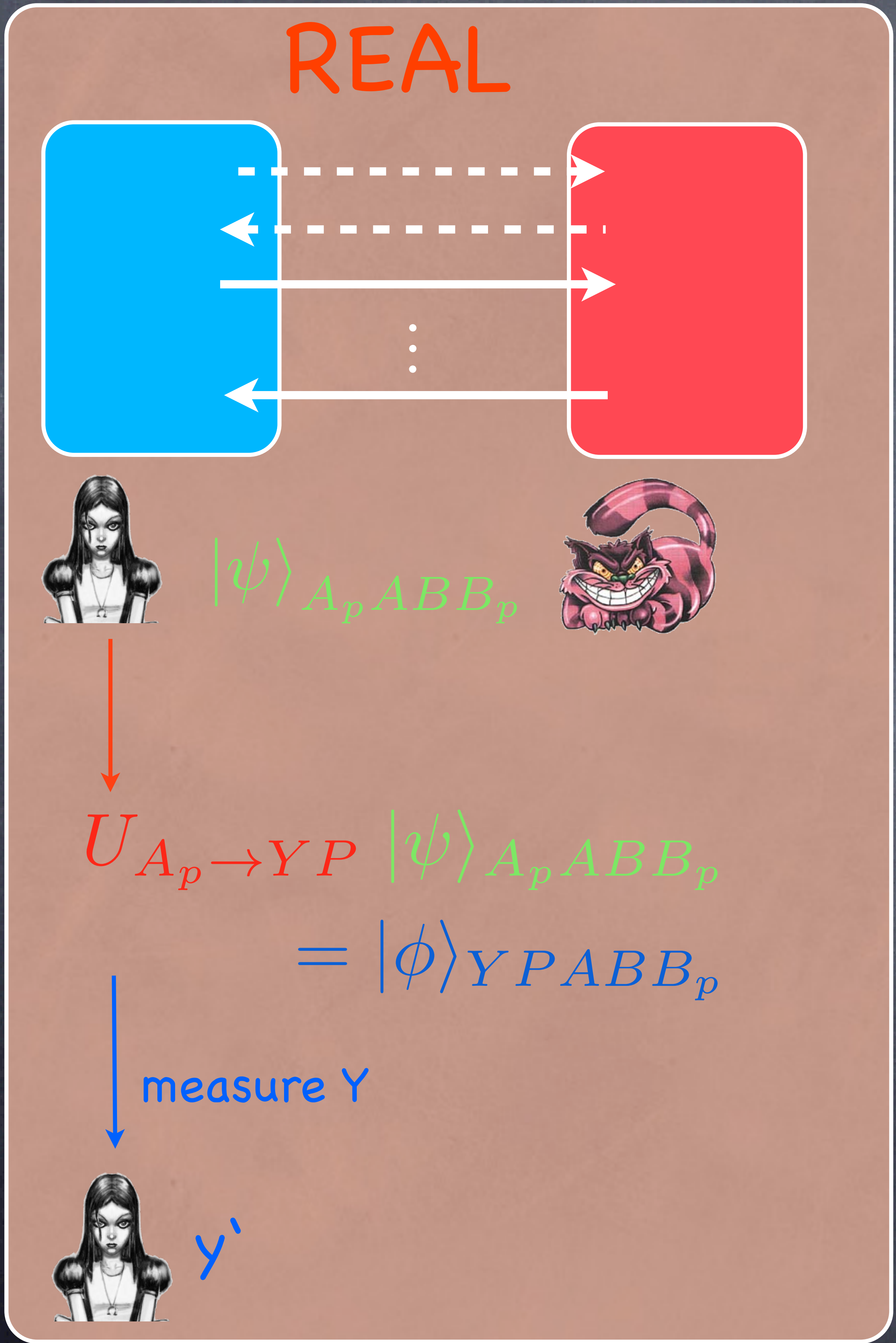
$$U_{A_p \to YP}\, |\psi\rangle_{A_pABB_p}$$
$$= |\phi\rangle_{YPABB_p}$$

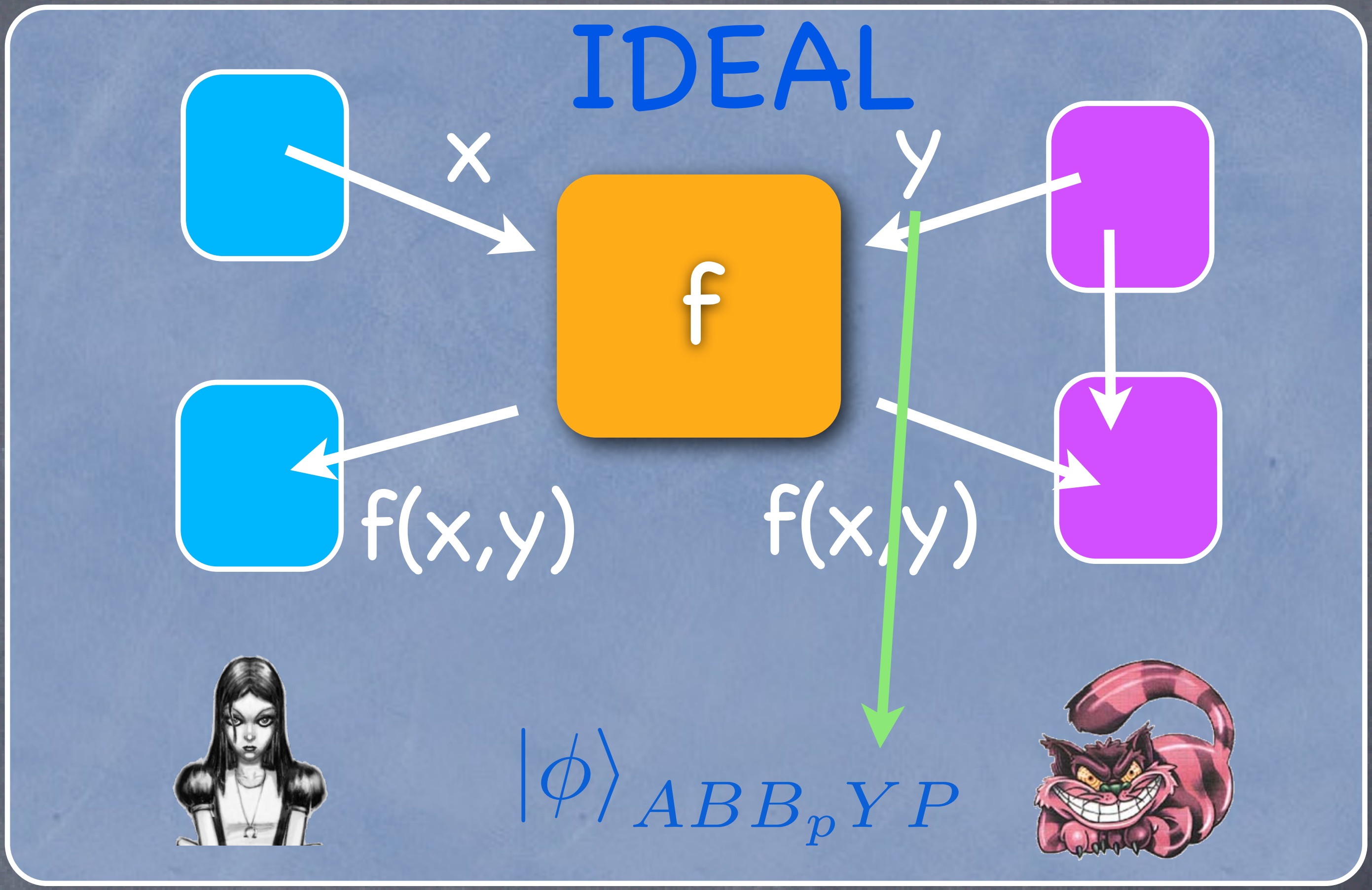
IDEAL



$$|\phi\rangle_{ABB_pYP}$$

Proof of Insecurity





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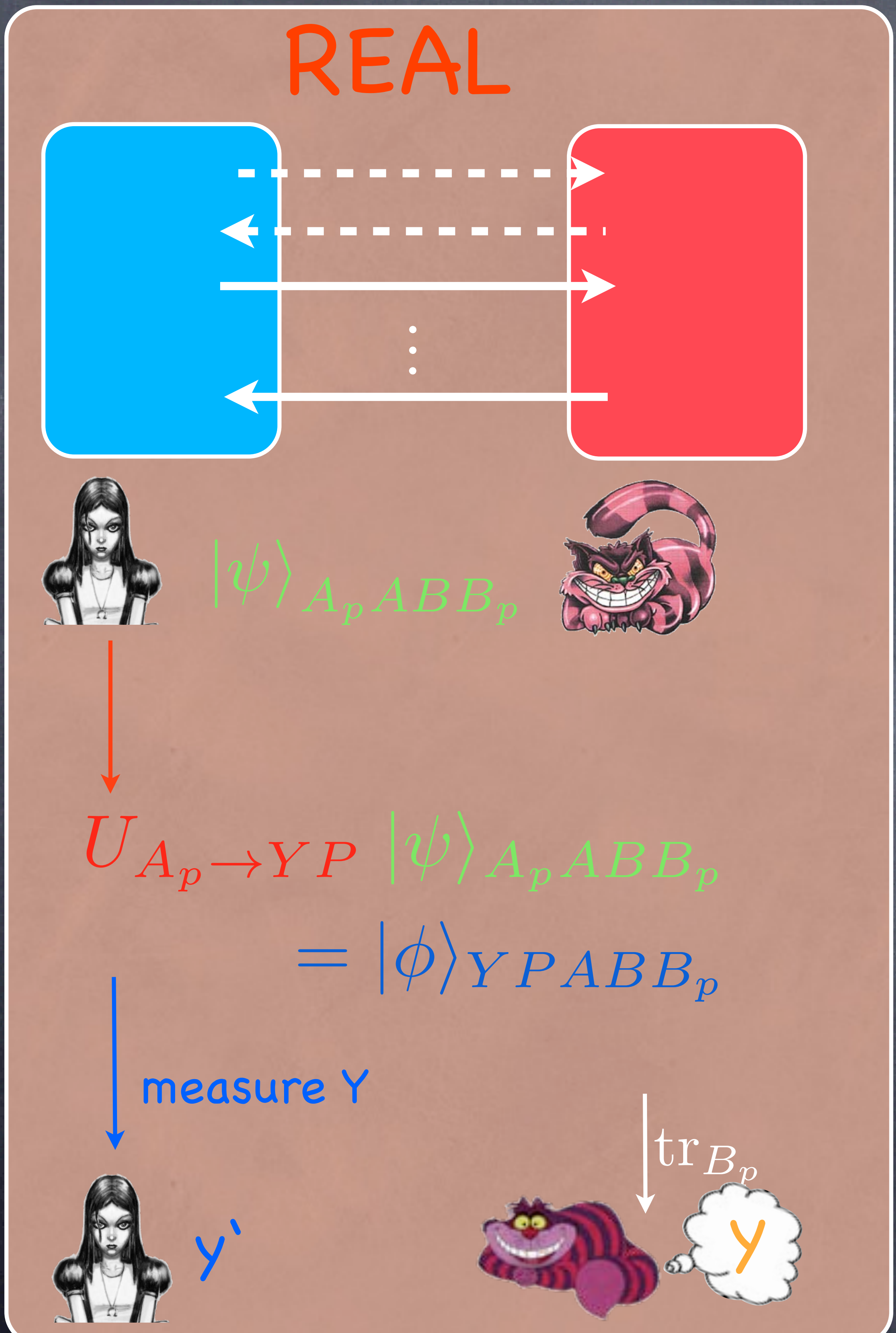

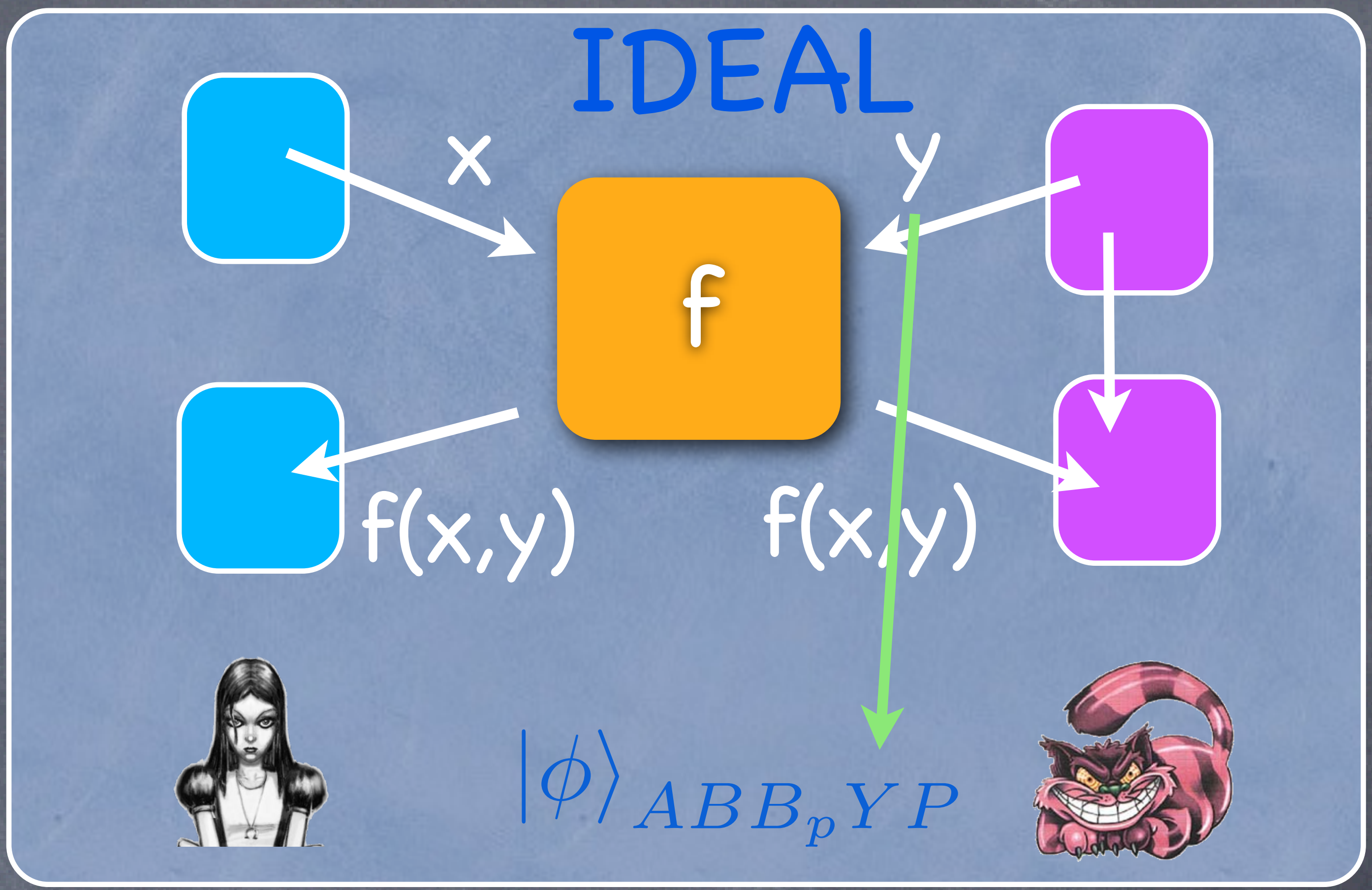

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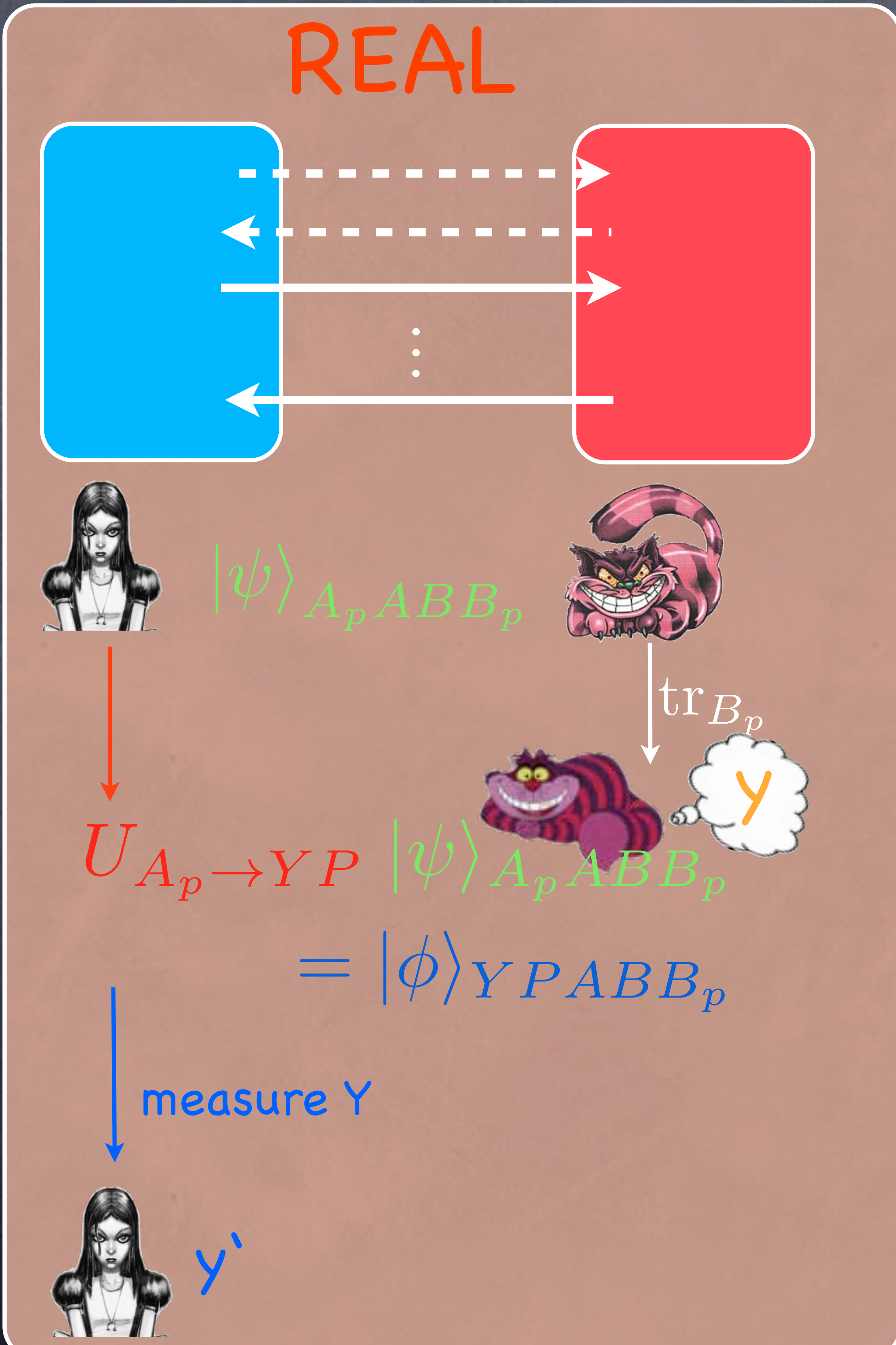

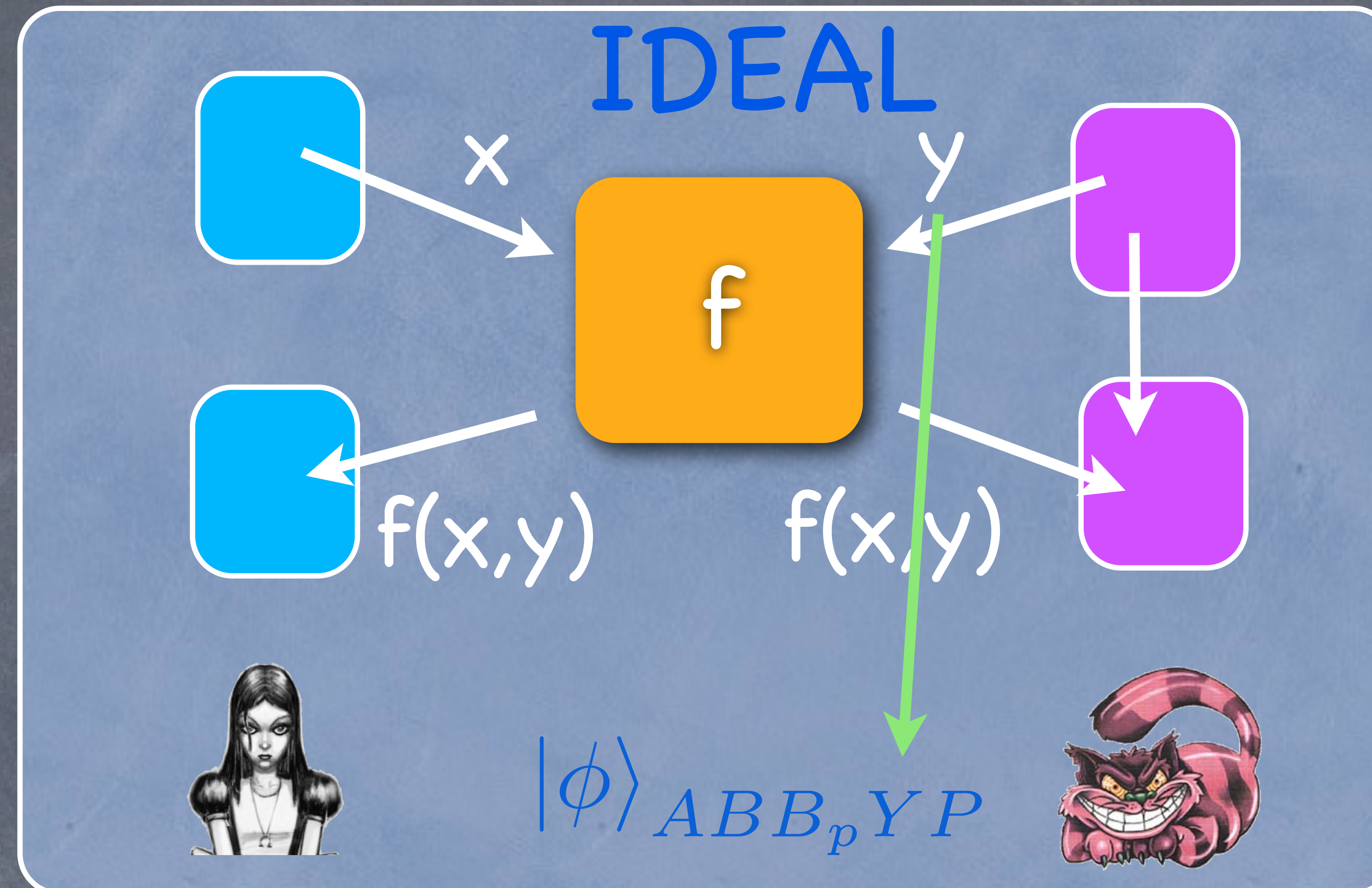

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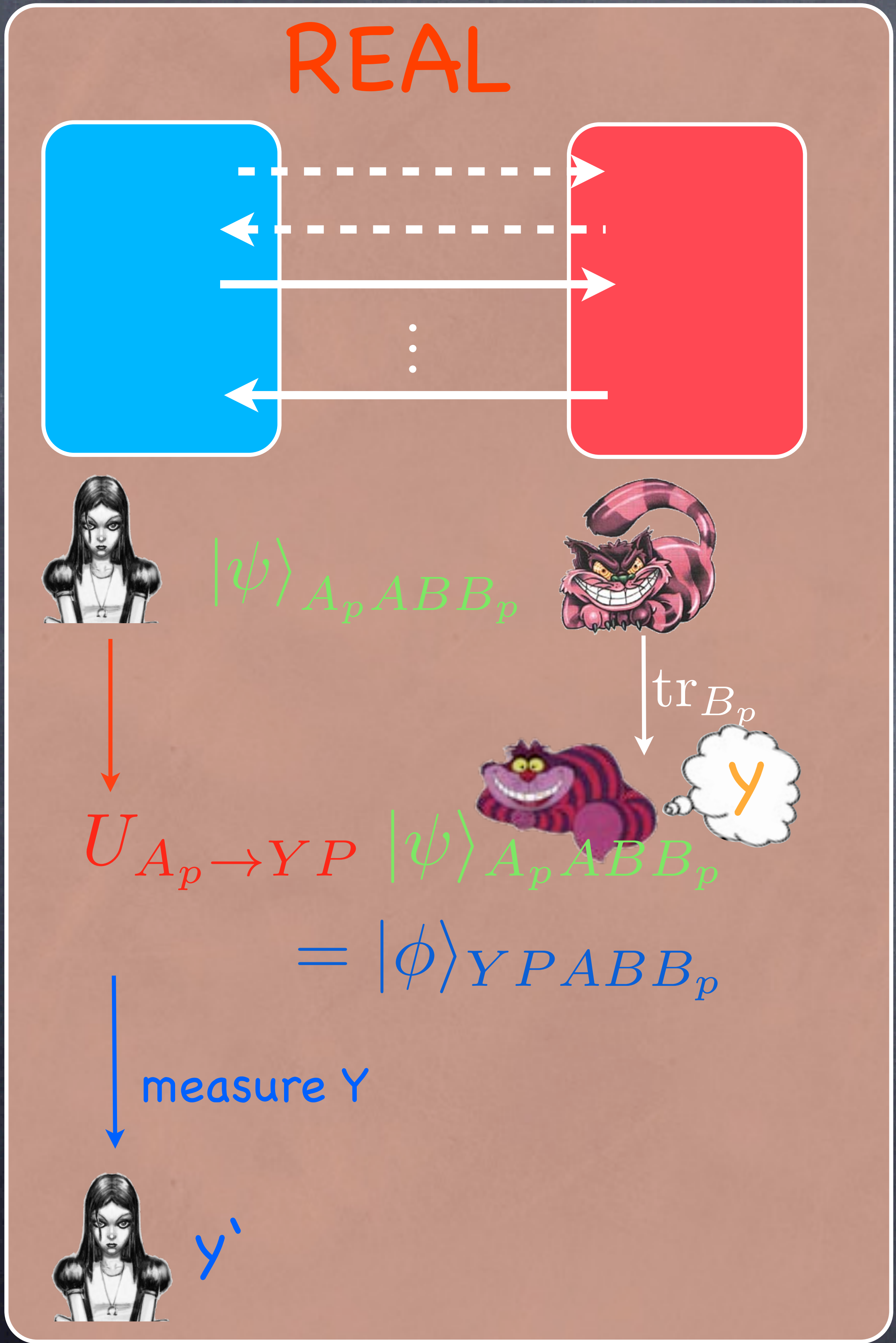
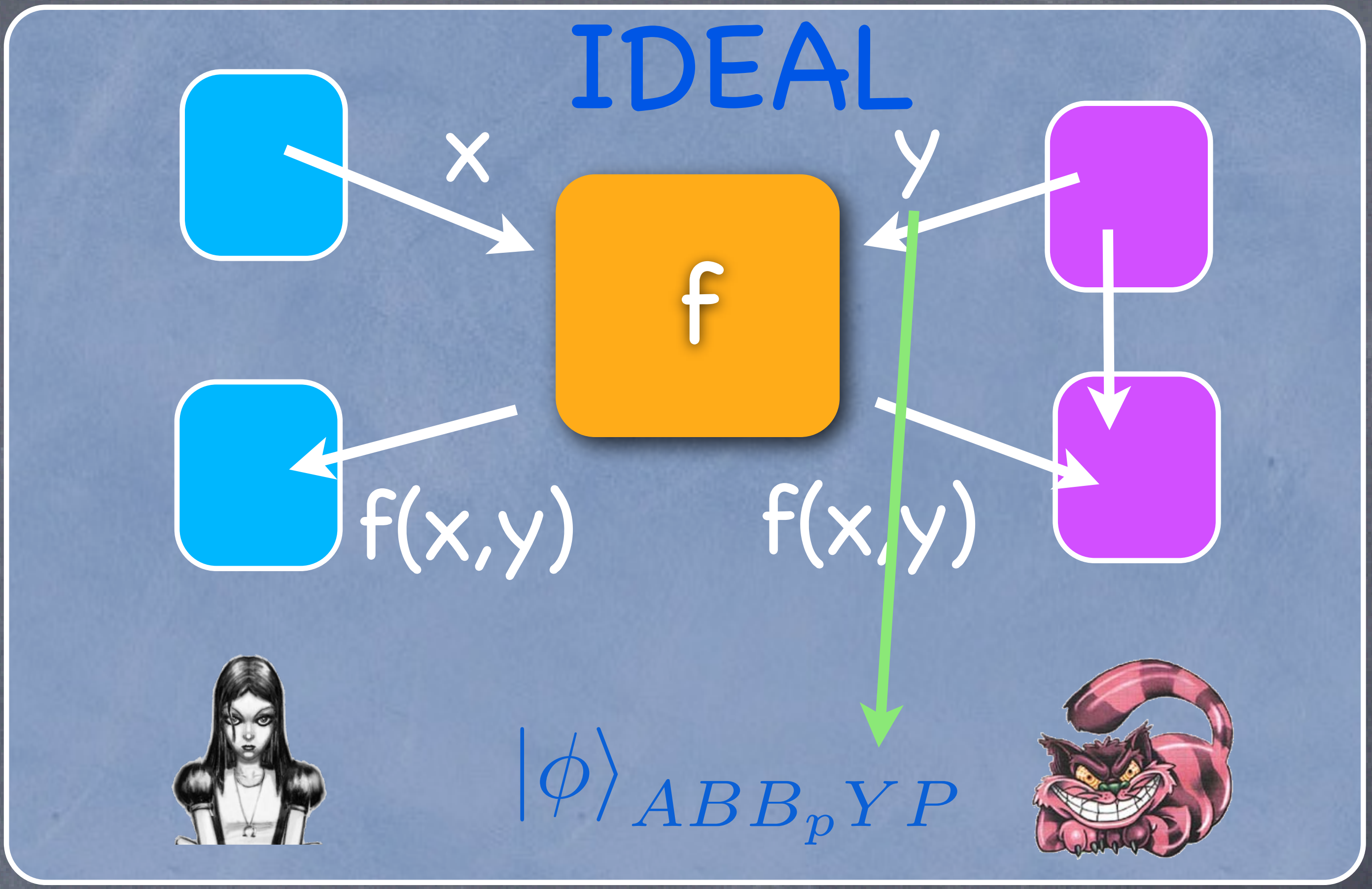
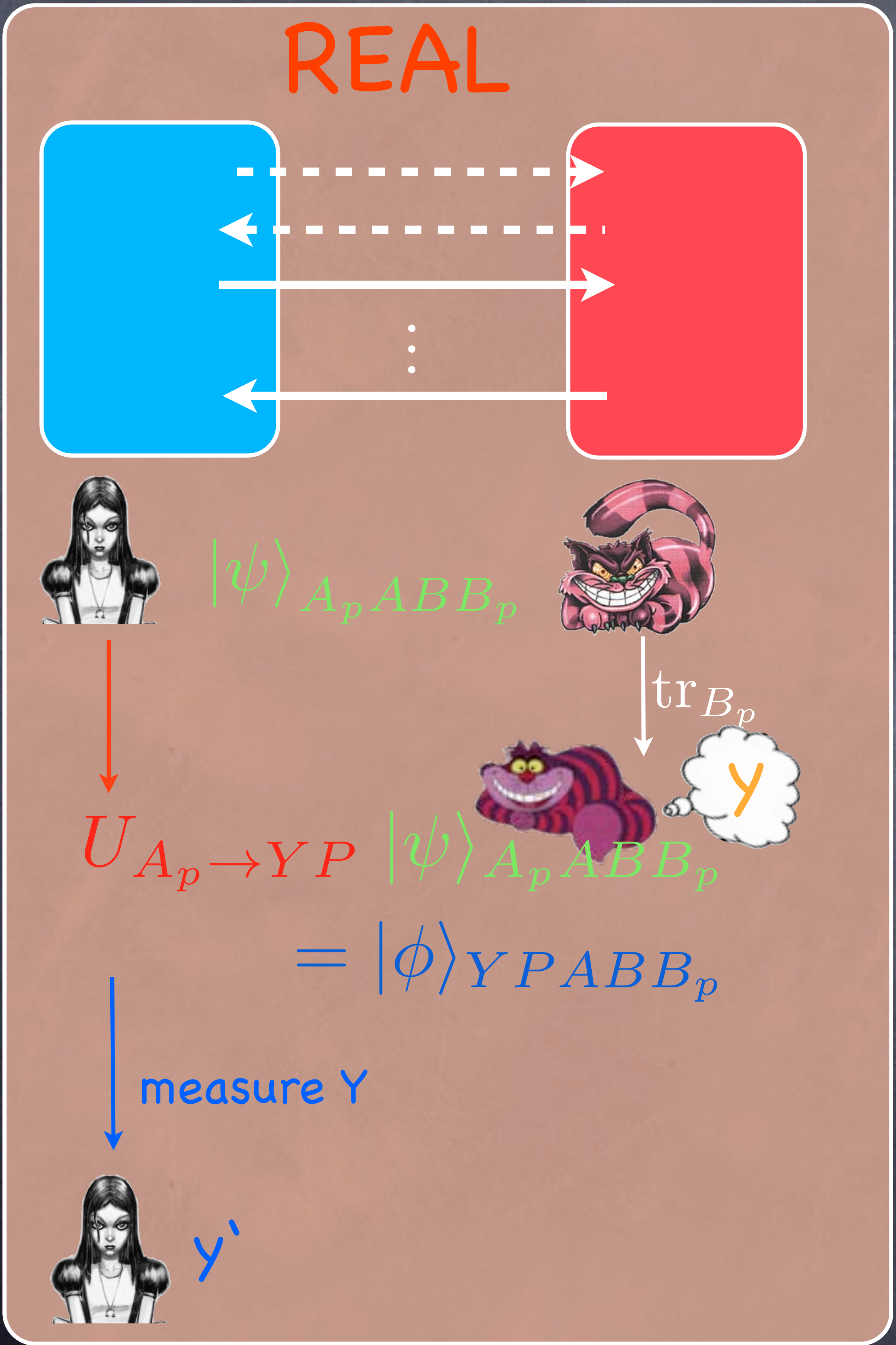
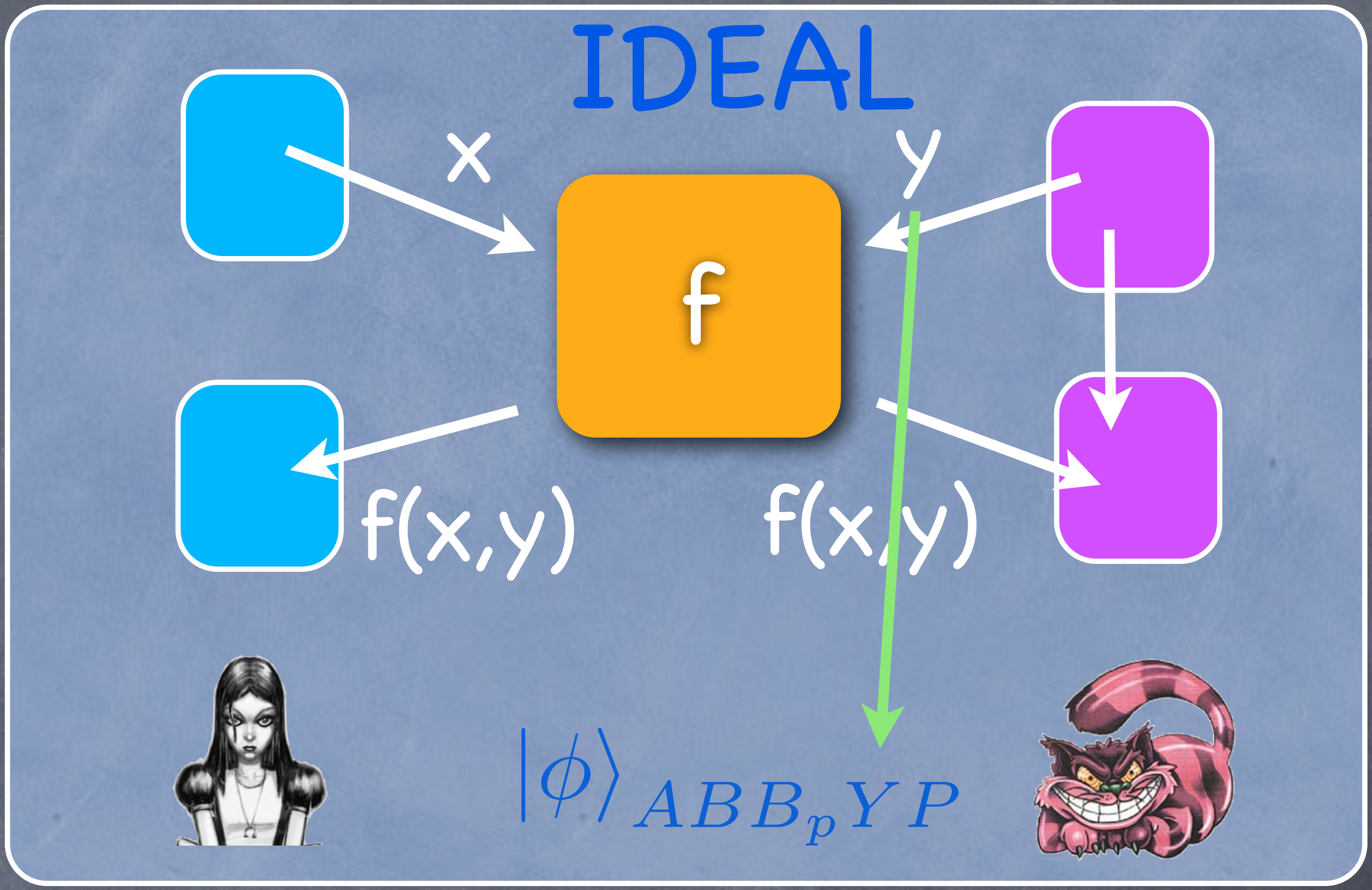




1. Alice plays „dishonestly" by purifying, Bob plays honestly

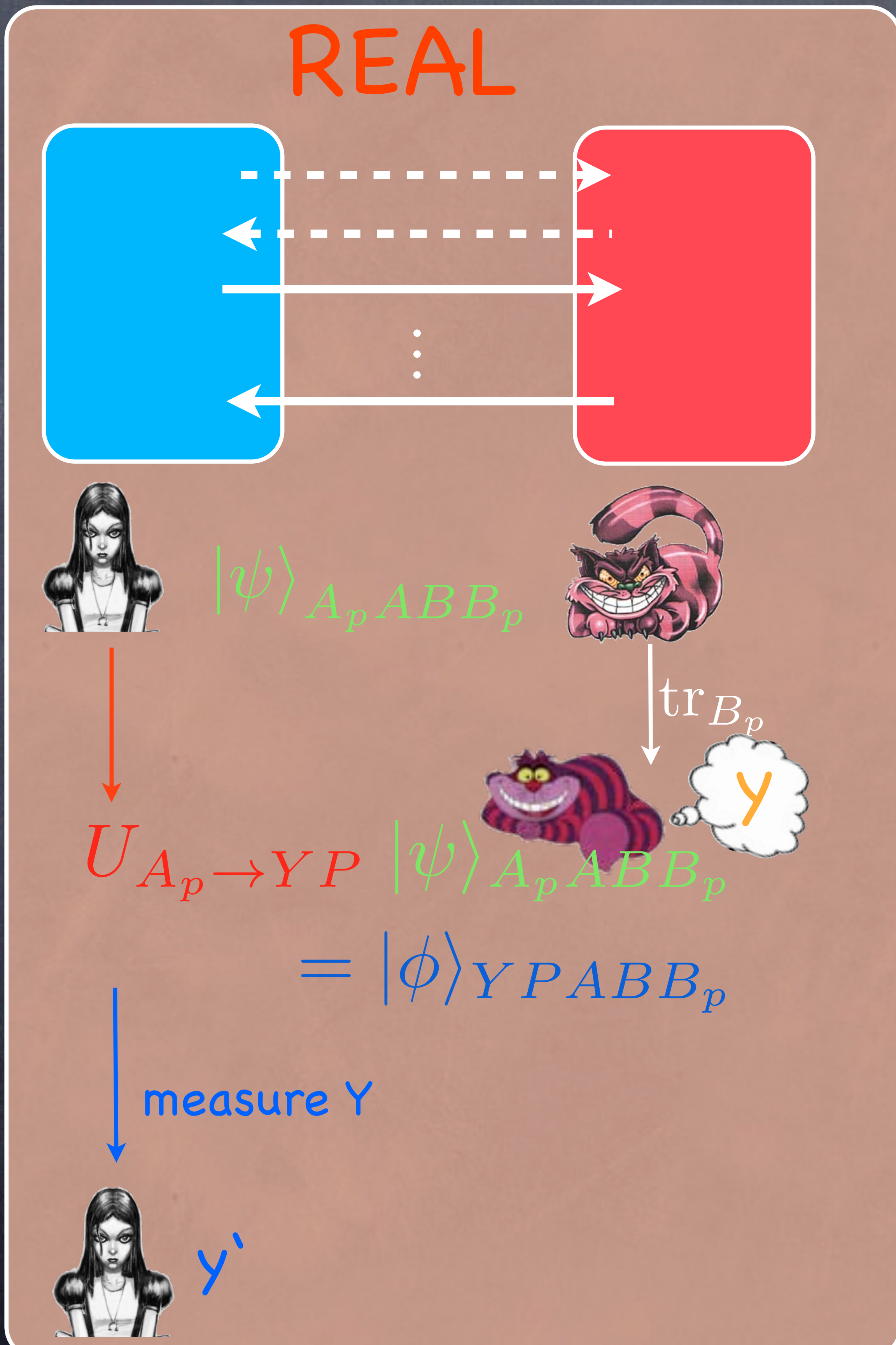
Proof of Insecurity

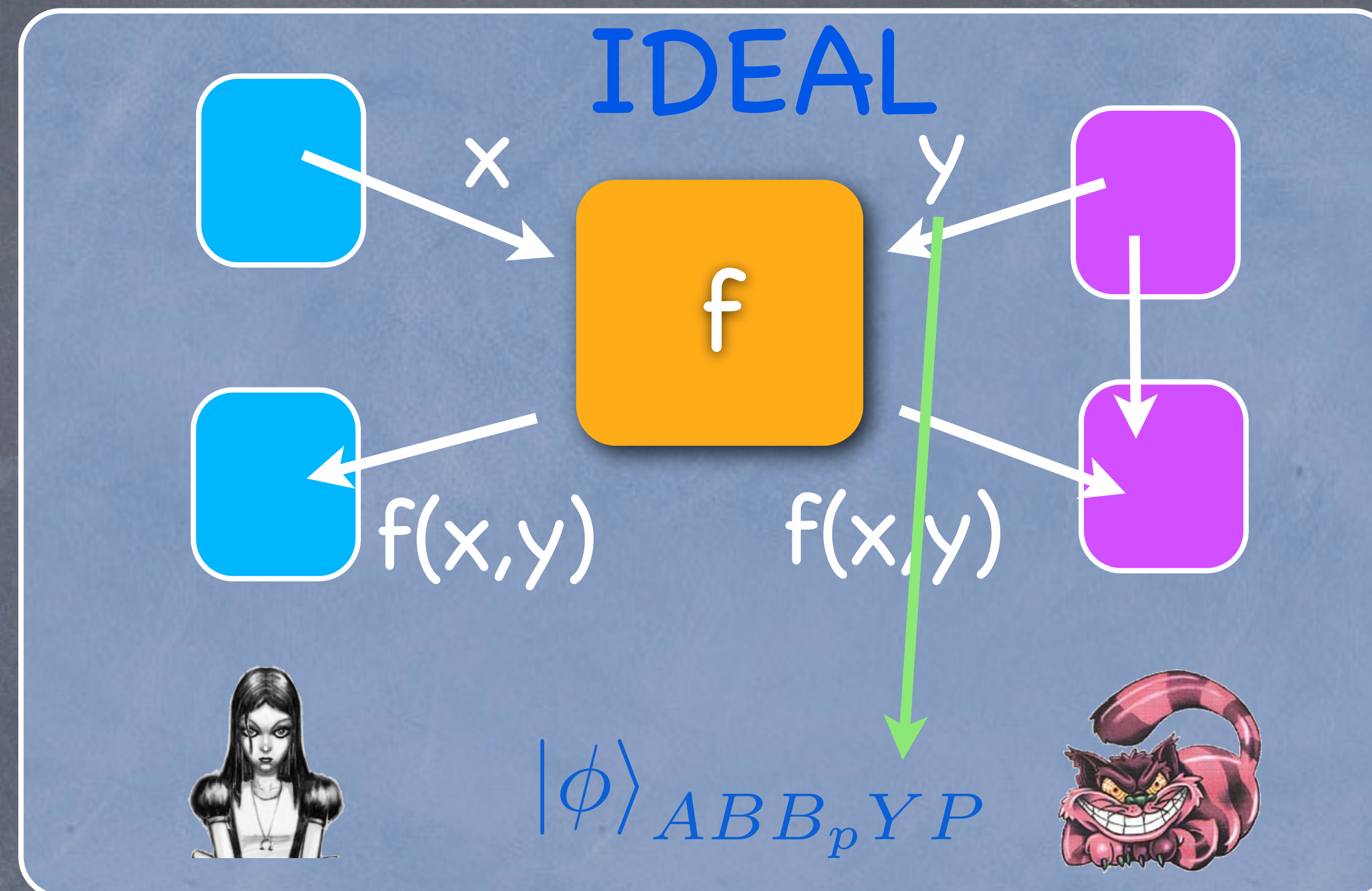




1. Alice plays „dishonestly" by purifying, Bob plays honestly
2. Alice applies cheating unitary U

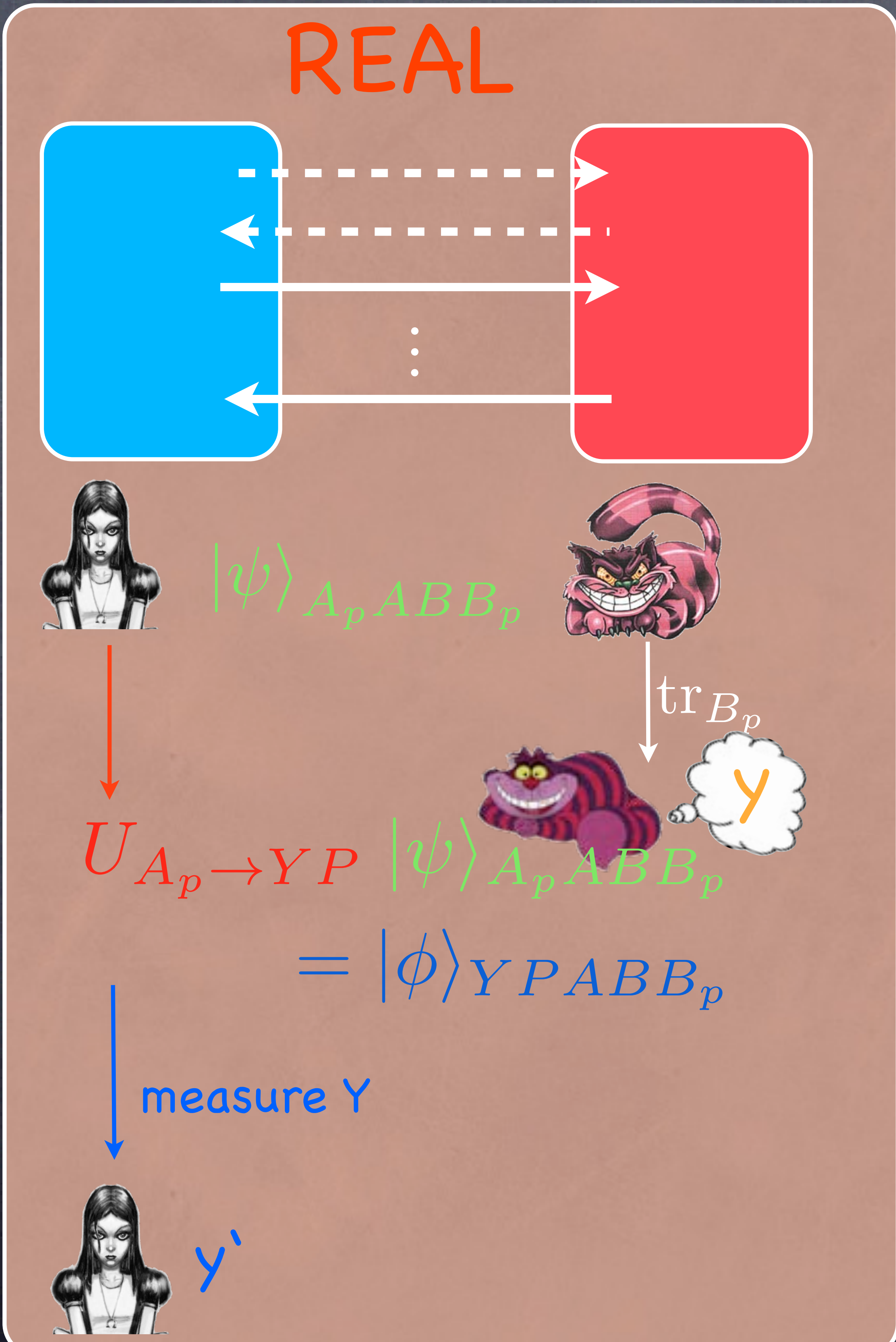
Proof of Insecurity

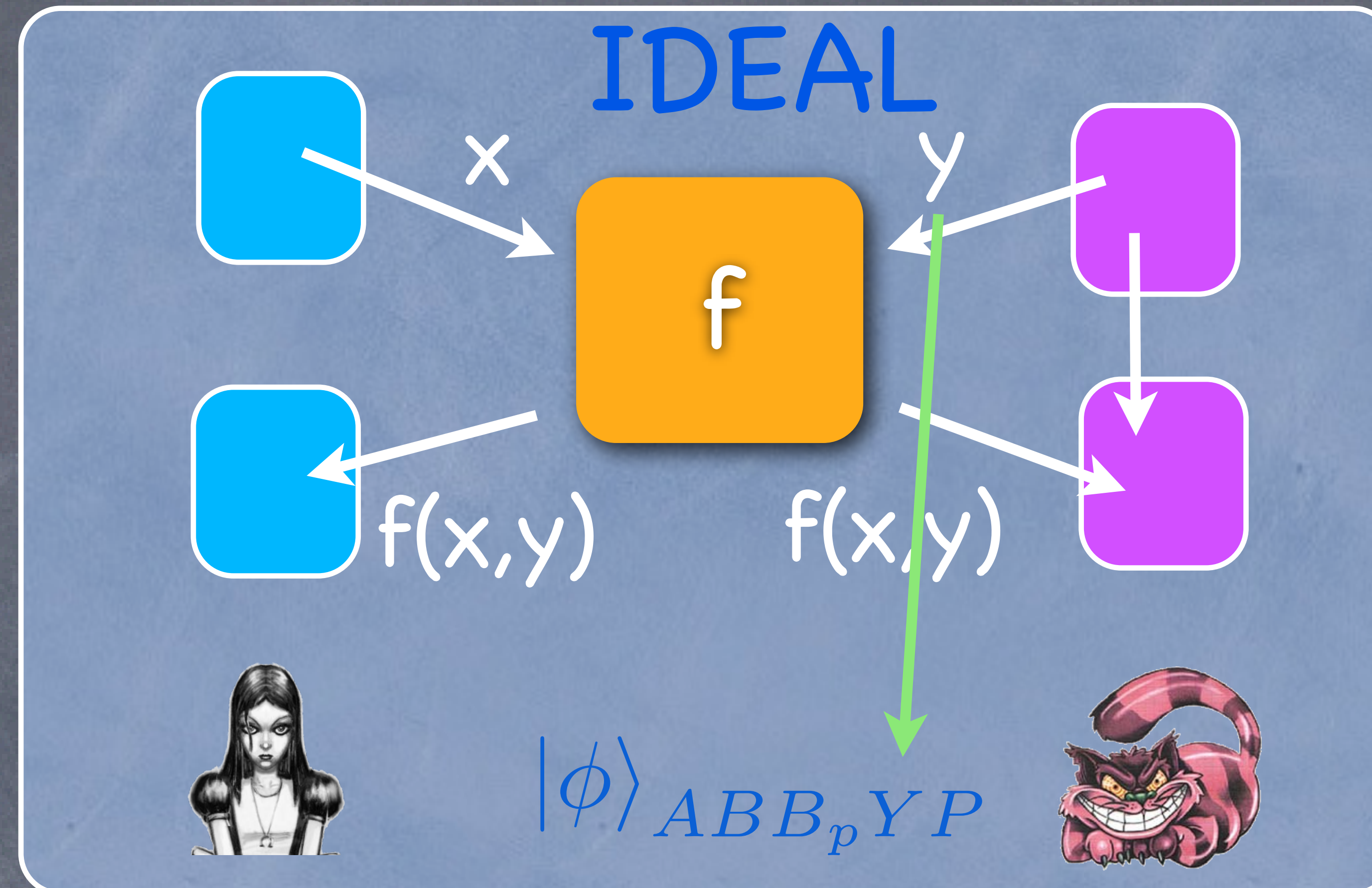




1. Alice plays „dishonestly" by purifying, Bob plays honestly
2. Alice applies cheating unitary U
3. measures register Y to obtain y'.

Proof of Insecurity





1. Alice plays „dishonestly“ by purifying, Bob plays honestly
2. Alice applies cheating unitary U
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4. since she only used purified strategy, correctness implies:
   for all x: f(x,y`) = f(x,y).

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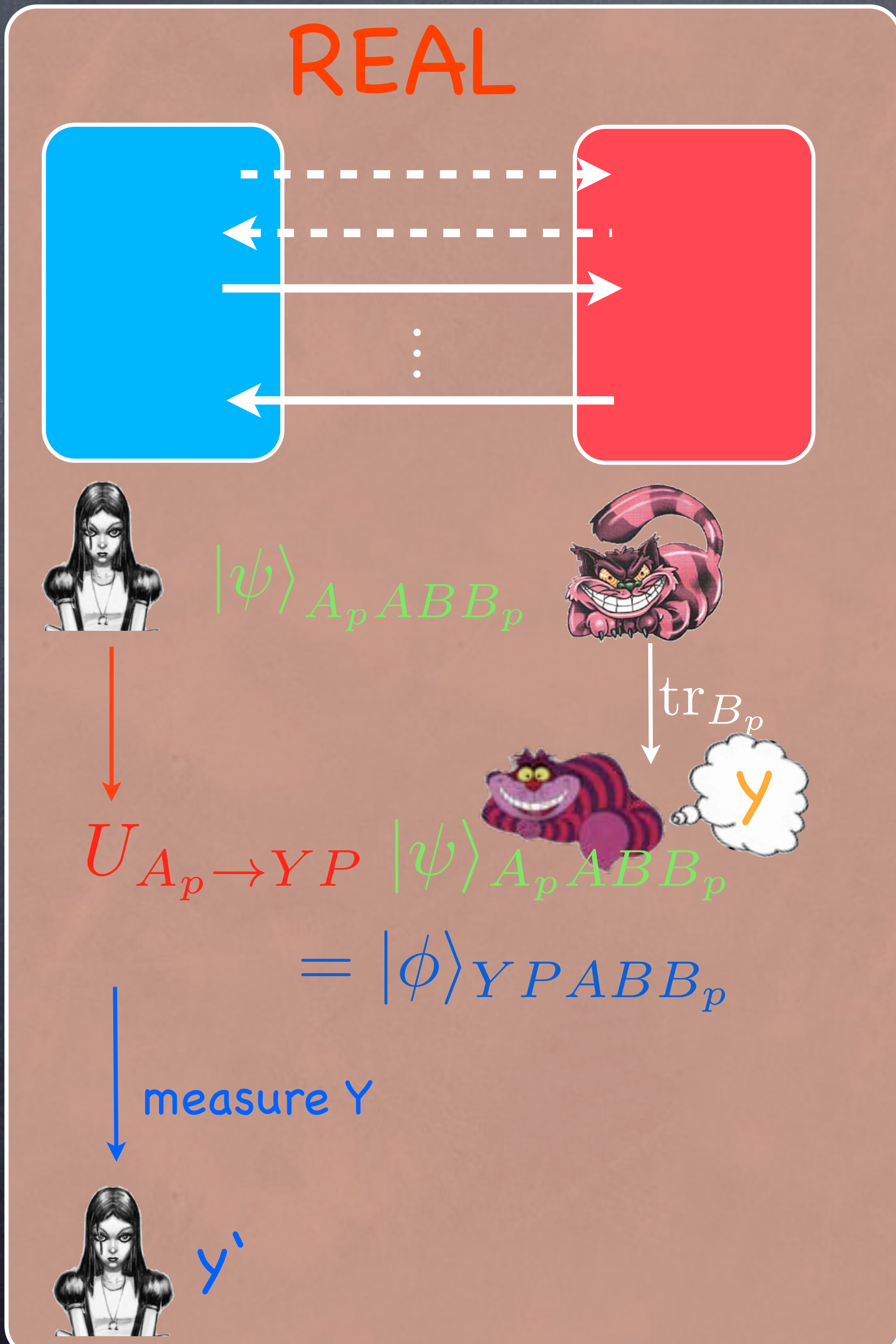

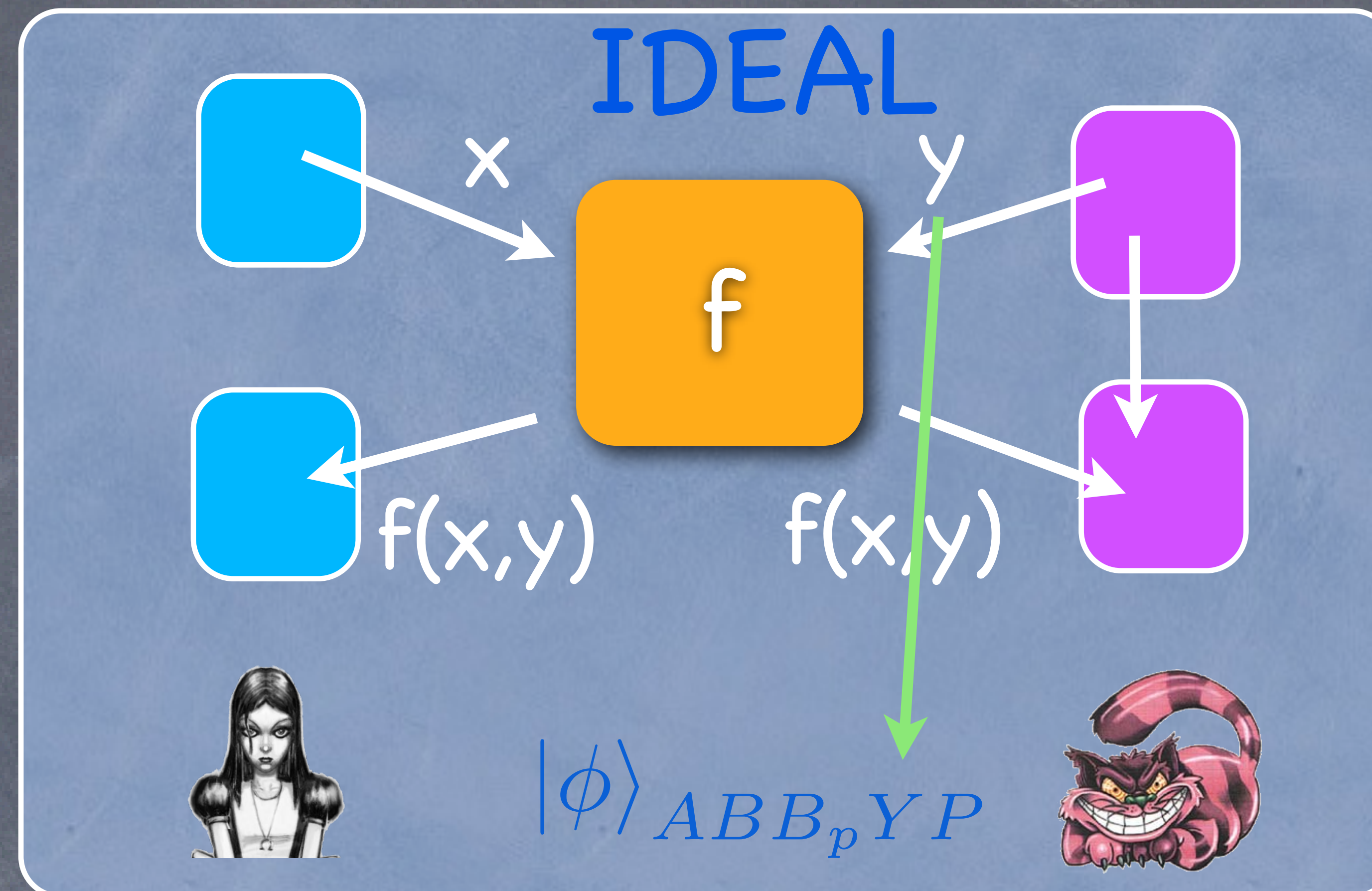

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$\square$

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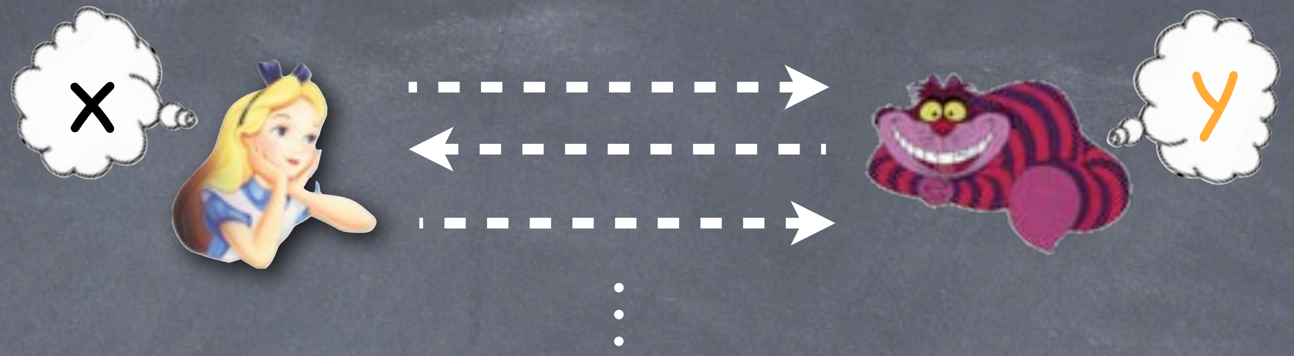

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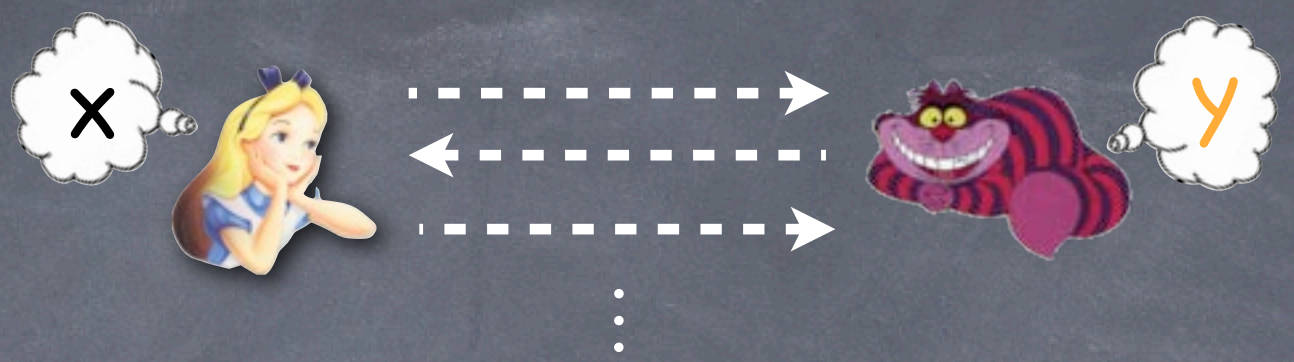

- our results also hold for ε-correctness and ε-security
- Alice gets a value y' with distribution $Q(y'|y)$ such that for all x: $\Pr_{y'}[\ f(x,y)=f(x,y')\ ] \geq 1-O(\varepsilon)$

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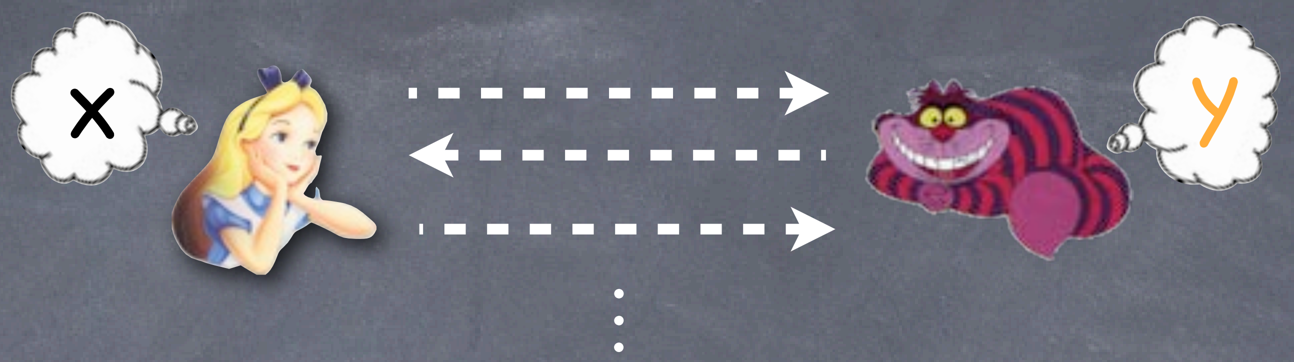

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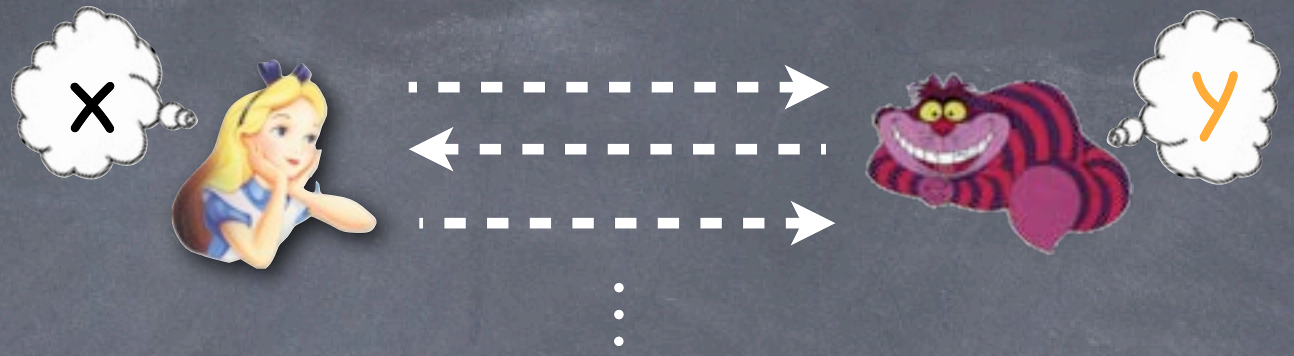

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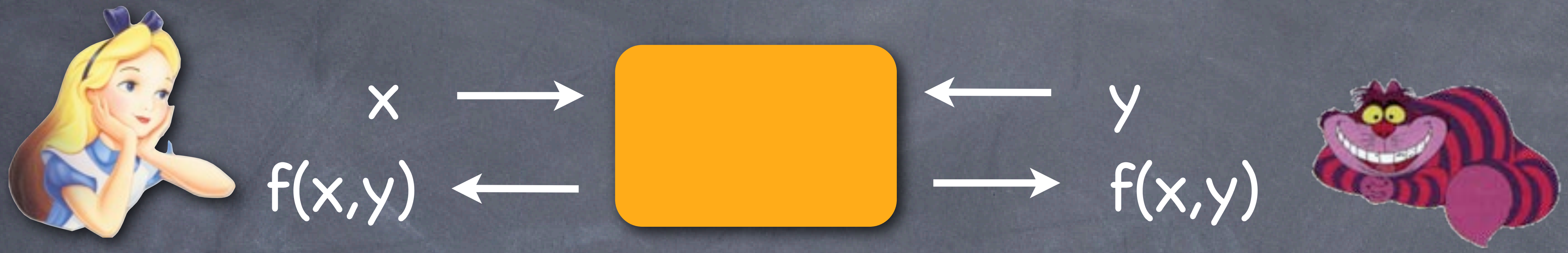
- in contrast to Lo's proof where the overall error increases linearly with the number of inputs.
- crucial use of von Neumann's minimax theorem

motivated from strong no bit commitment result
[D'Ariano Kretschmann Schlingemann Werner, 2007]

Conclusion & Open Problems



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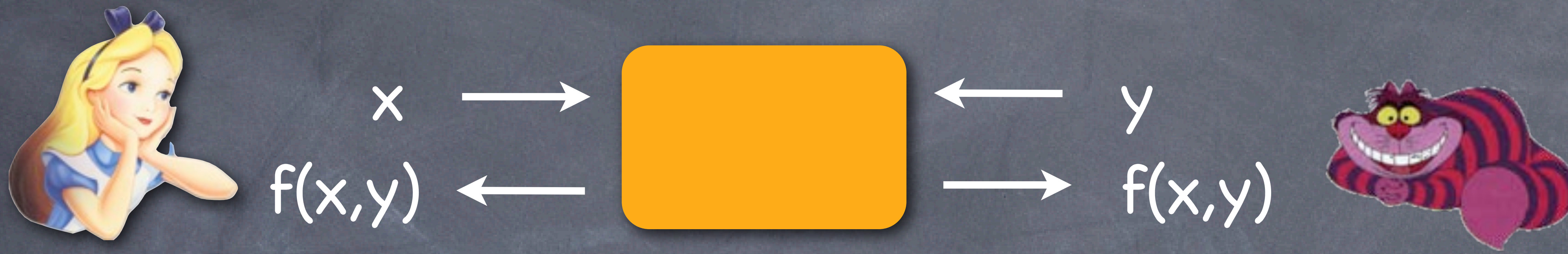

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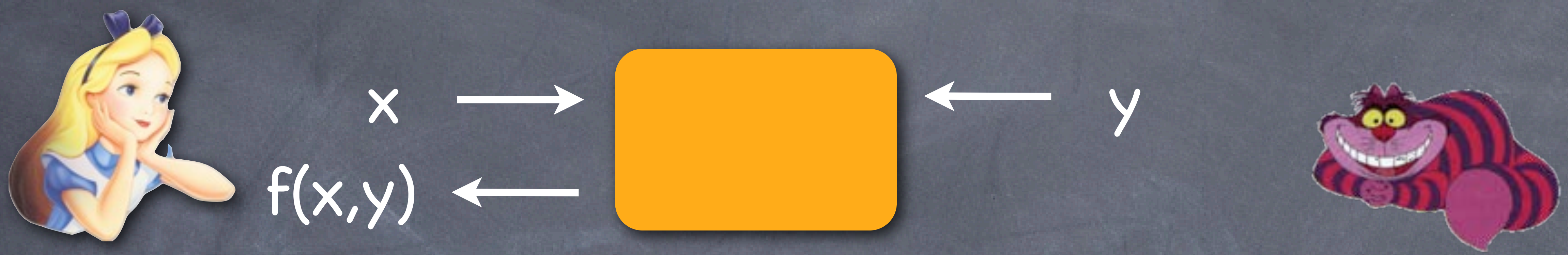
Conclusion & Open Problems



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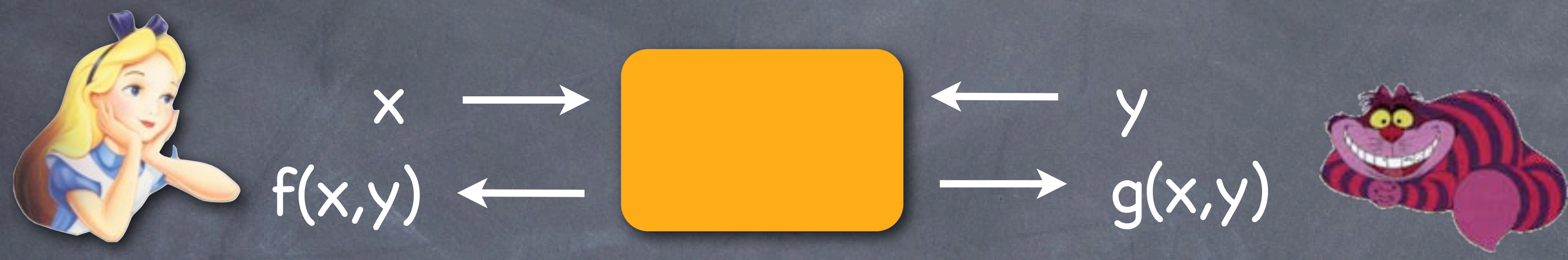

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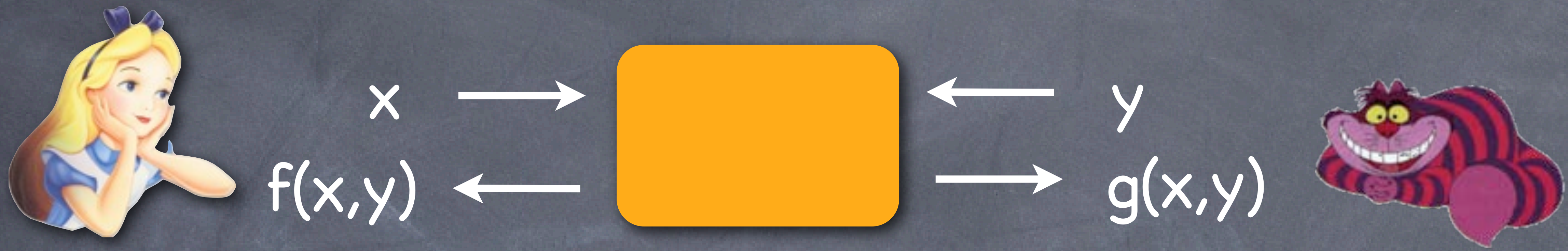

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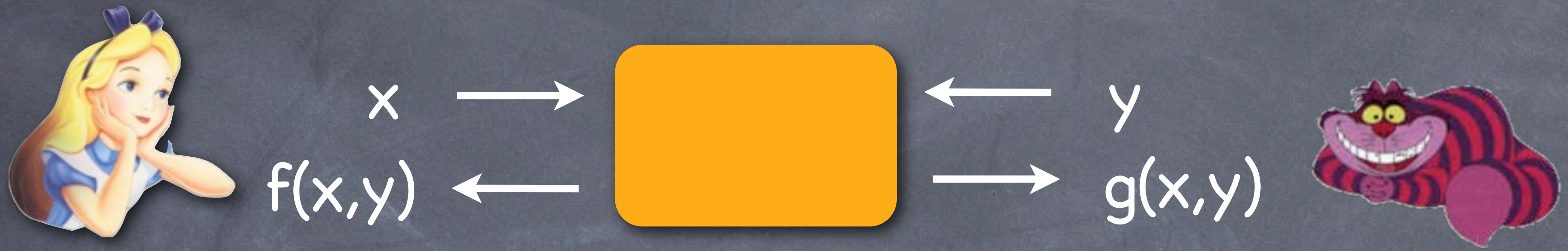

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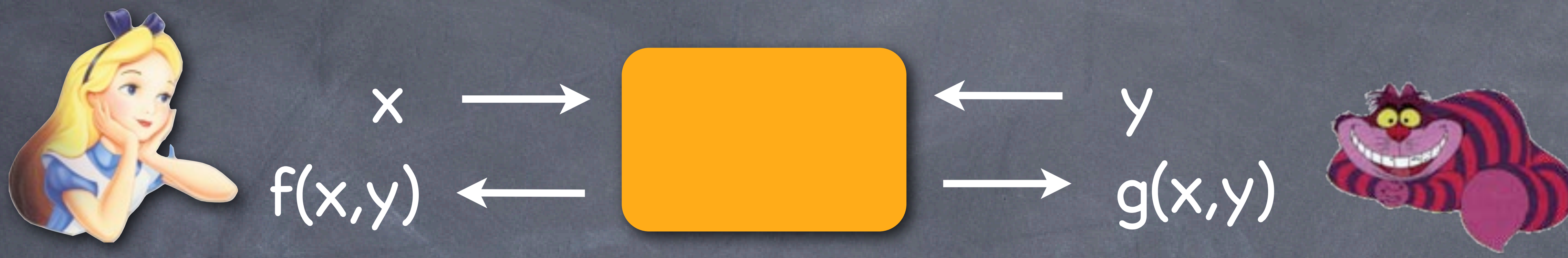

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Thank you!