Symmetry protection of measurement-based quantum computation in ground states

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eQus

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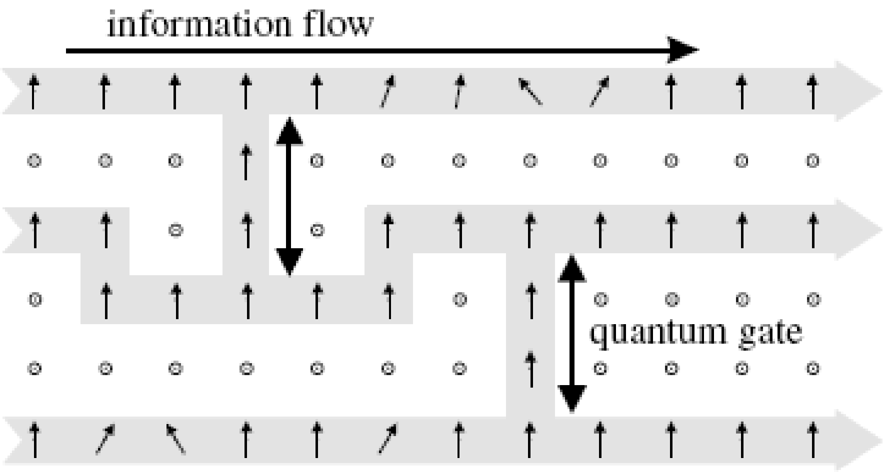
Measurement-based quantum computing

- Quantum computing can proceed through *measurements* rather than unitary evolution
- Uses a resource state such as the *cluster state*: a universal circuit board
- Resource states can be:
  - constructed with unitary gates
  - the ground state of a coupled quantum many-body system
- Computational properties = properties of states

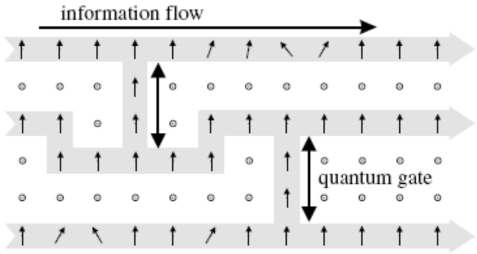


Raussendorf and Briegel, PRL (2001)

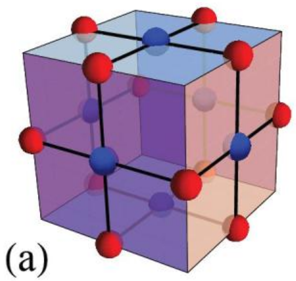
Resource states for MBQC

Q: What properties of a state are needed for MBQC?

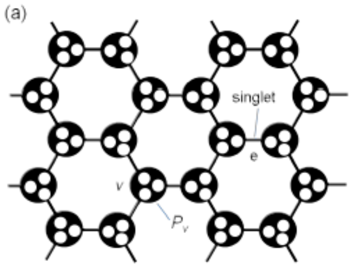
› There are a handful of proposed resource states



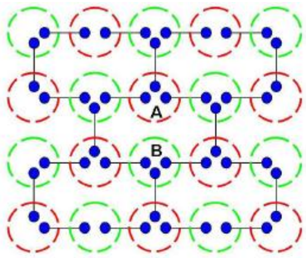
Raussendorf and Briegel



Raussendorf *et al.*



Wei, Affleck, Raussendorf Miyake



X. Chen *et al.*

› Many are PEPS (projected-entangled pair states)

- tensor network structure allows for the analytic analysis of the effect of measurements
- natural interpretation as ground states of a local Hamiltonian

› Can we identify the properties of a system that allow for MBQC as a form of robust quantum order?

Symmetry-protected topological order & MBQC

› Quantum memories

- Ground state of the toric code (a local stabilizer Hamiltonian) is a quantum memory
- Robust to local perturbations
- Topological order = quantum memory

› Measurement-based QC

- Ground state of the cluster model (a local stabilizer Hamiltonian) is a MBQC resource
- Robust to symmetric local perturbations
- Symmetry-protected topological order = MBQC resource

Symmetry-protected topological order

- (Restricted form of) topological order protects quantum info
- Symmetry-breaking measurements induce logic gates
- Can indentify families of MBQC resource states even without an analytical description of the ground state

Structure of our result

1. 1D spin chain: SPT order implies perfect identity gate with perfect measurements
   Else, Schwarz, Bartlett, Doherty, PRL (2012)
2. 1D SPT ordered spin chain: far-separated non-trivial (not identity) gates are imperfect, described by a *Markovian* noise model
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By choosing to use this MBQC resource to simulate a fault-tolerant circuit, we have:

Main Theorem

- For sufficiently small symmetry-respecting perturbations, the perturbed ground state remains a universal resource for measurement-based quantum computation.

Else, Bartlett, Doherty, NJP (2012)

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1D cluster model: the identity gate

Hamiltonian - gapped:

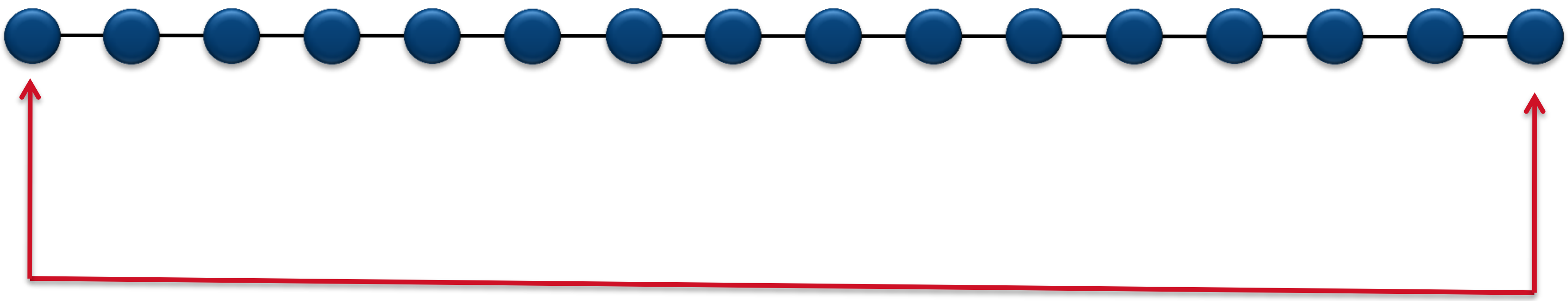
$$H = -J \sum_i Z_{i-1} X_i Z_{i+1}$$

"Frustration free" (all terms commute):

$$Z_{i-1} X_i Z_{i+1} |gs\rangle = |gs\rangle$$

Measure / apply local field

$X \quad X \quad X \quad X \quad X \quad X \quad X \quad X \quad X \quad X \quad X \quad X \quad X \quad X$



Maximally entangled state
for teleportation

$|\psi^-\rangle$

Q: What are the essential properties of a qubit wire?

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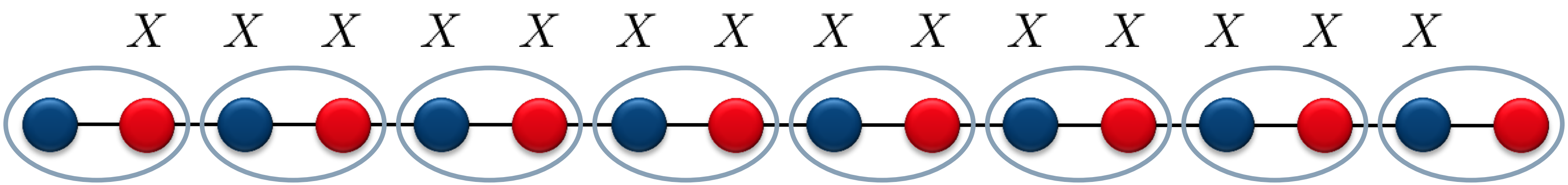
1D cluster model: a symmetry

Hamiltonian - gapped:

$$H = -J \sum_i Z_{i-1} X_i Z_{i+1}$$

"Frustration free" (all terms commute):

$$Z_{i-1} X_i Z_{i+1} |gs\rangle = |gs\rangle$$



Hamiltonian possesses a symmetry: $Z_2 \times Z_2$

i.e., 2 commuting constants of motion

Four elements:

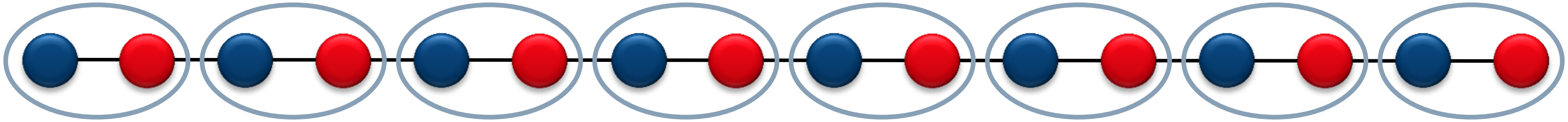
$(0,0)$ ⟵ Do nothing

$(0,1)$ ⟵ Flip red spins

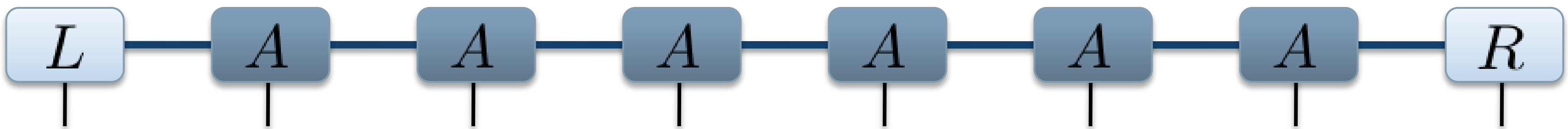
$(1,0)$ ⟵ Flip blue spins

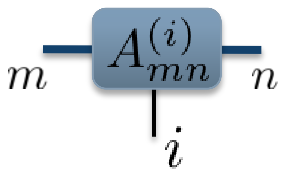
$(1,1)$ ⟵ Flip red and blue spins

Ground state as a tensor network state



tensor network state (matrix product state)





3 leg tensor

$i = 1 \ldots 4$

index for basis of spin pairs

$m, n = 1, 2$

'virtual' index - contracted

Efficient representations of ground states of 1D gapped systems

Natural language for ground-state quantum computation

Gross, Eisert, Schuch, Perez-Garcia, PRA (2007)

Goal:

Characterise properties of tensors, in terms of their symmetry, that make a good qubit wire

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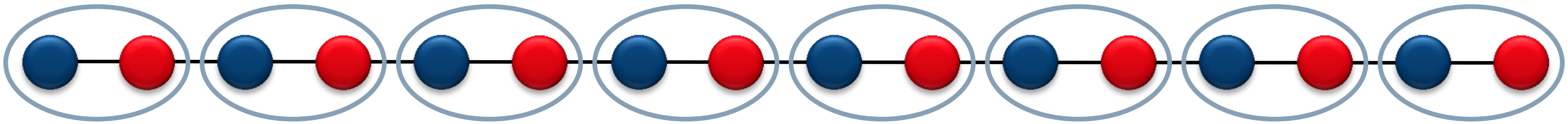
Ground state as a tensor network state



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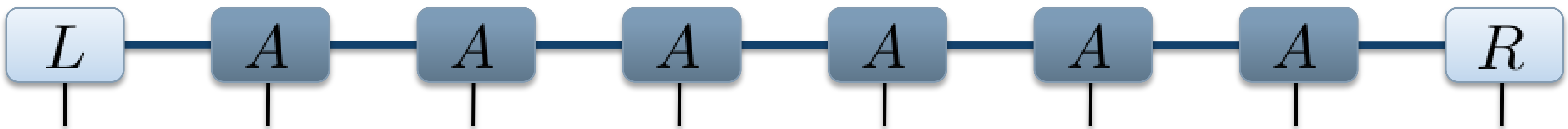
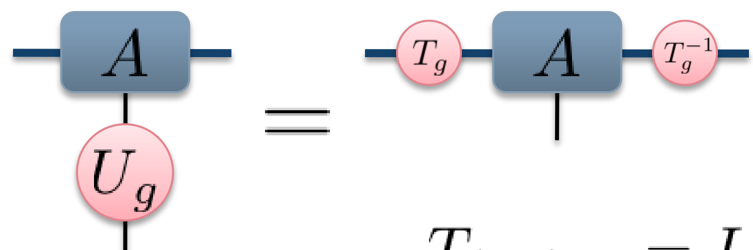


Cluster model possesses
a symmetry: $Z_2 \times Z_2$

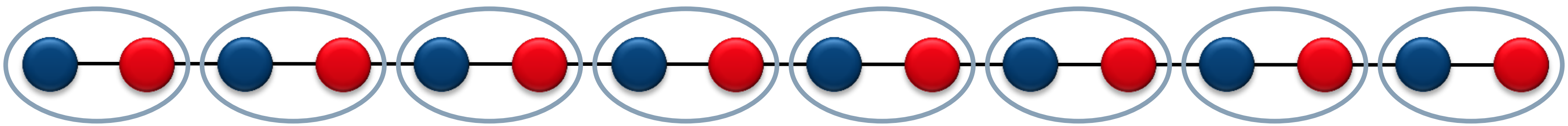
Tensors can carry a nontrivial *gauge* representation of this group

For the cluster model, T_g
is a projective representation:
the Pauli group



$$T_{(0,0)} = I$$
$$T_{(0,1)} = X$$
$$T_{(1,0)} = Z$$
$$T_{(1,1)} = Y$$

Maximally noncommutative projective representations



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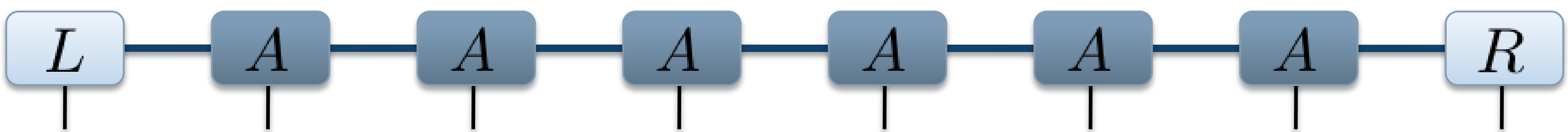
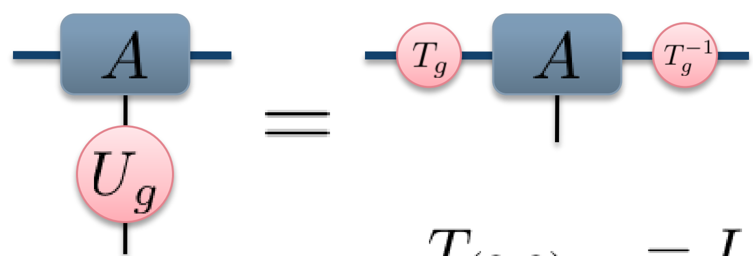


Maximally noncommutative:

This projective representation has a special property: a trivial 'projective centre'

This gives:

1. a unique projective rep
2. an isomorphism between group elements and states in a basis



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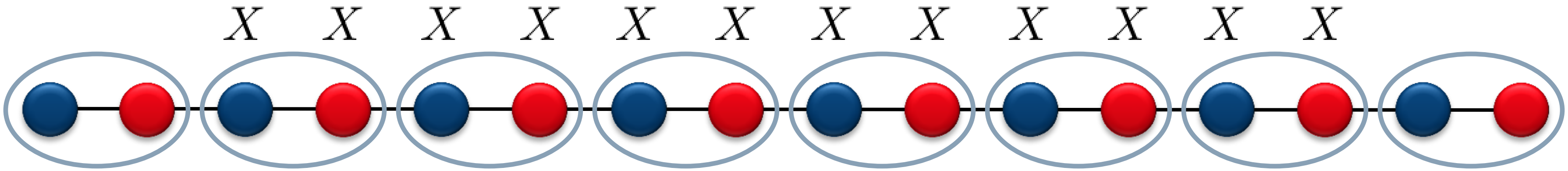
Good measurement basis



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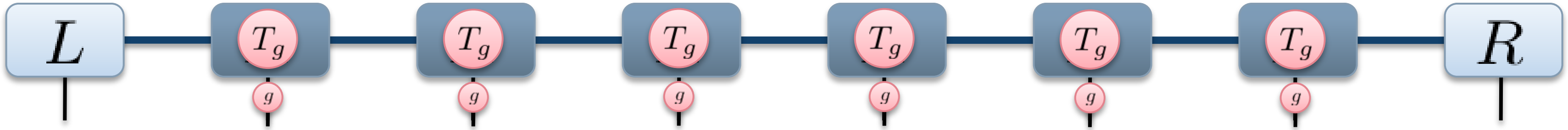


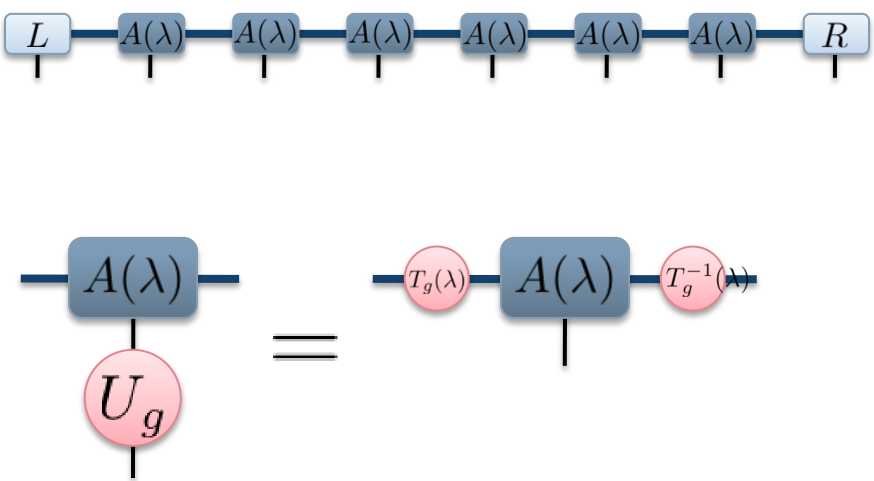
Symmetry protected topological phases

$$H = H_0 + \lambda V$$

› Symmetry-respecting perturbations V alter the ground state, but cannot change the type of representation(*)

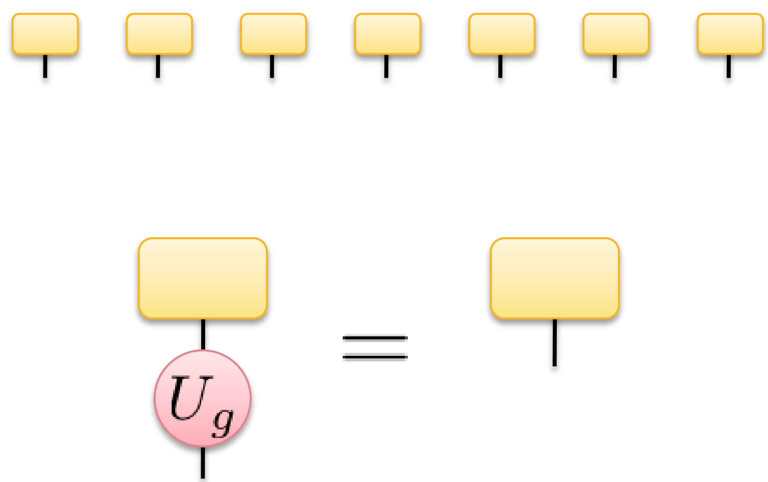
Chen, Gu, Wen, PRB (2011)

Ground state with Pauli gauge representation



Phase transition

Ground state with trivial representation



(*) technically, the second cohomology class
In this case, projective or true representation

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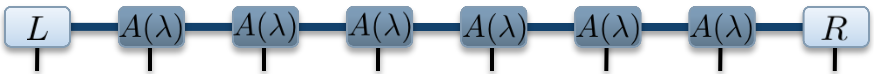
Decomposition of the MPS bond space



$$A(\lambda)^{(g)} = T_g \otimes \tilde{A}(\lambda)^{(g)}$$

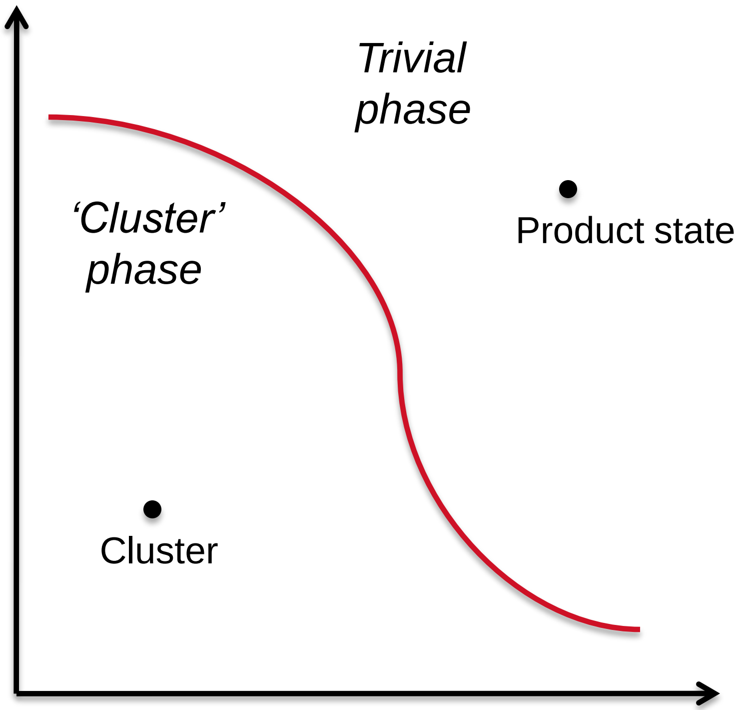
Quantum information is encoded in this subsystem:
- properties fixed throughout SPT phase
- SPT order resides here (long-ranged order)

'Junk' subsystem describes details of the specific ground state
- decoupled from the qubit
- describes a state with only short-ranged correlations

Tensor breaks up into a *structural part* (completely determined by representation) and a *'junk' part* affected by the perturbation

Singh, Pfeifer, Vidal, PRA (2010)

1D cluster model in a nontrivial SP phase



- Ground states in the 'cluster' phase possess the long-range entanglement necessary for use as a quantum wire, always with the same special basis measurement

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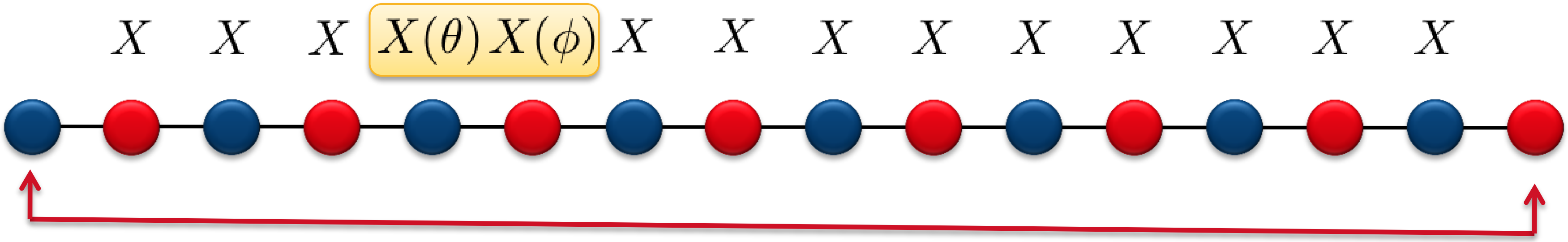
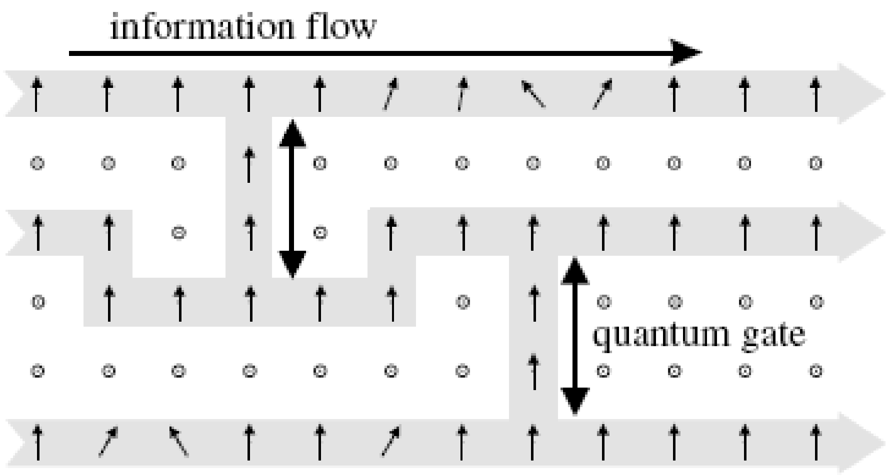
Qubit wires – 1D cluster model

Measure / apply local field



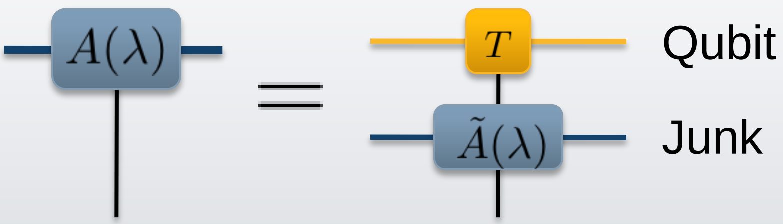
‘Rotated’ maximally entangled state for gate teleportation

$$I \otimes R(\theta, \phi)|\psi^-\rangle$$



Equivalence to local Markovian error model

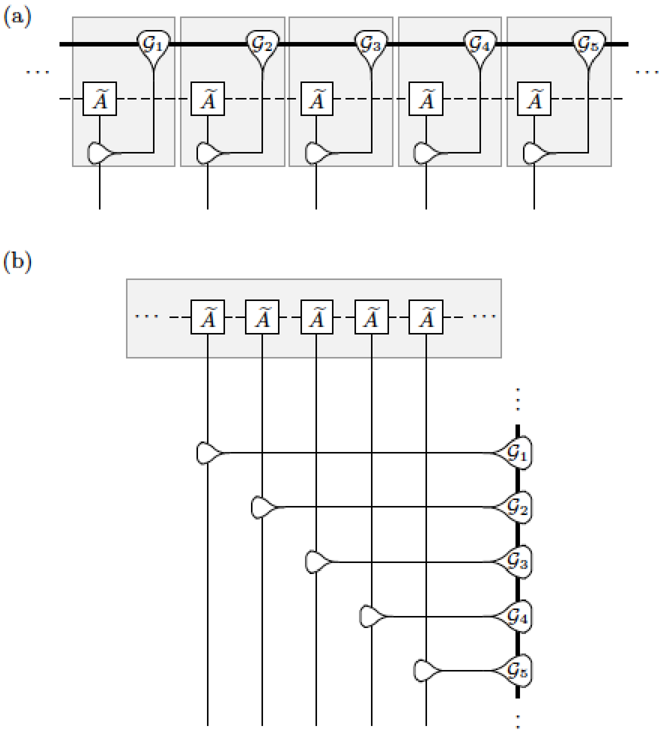
$$A(\lambda)^{(g)} = T_g \otimes \tilde{A}(\lambda)^{(g)}$$



Problem 1: adaptive measurements lead to correlations in time

Solution: work in a 'dual picture' using a nonlocal unitary to remove adaptivity

… ask me later!!



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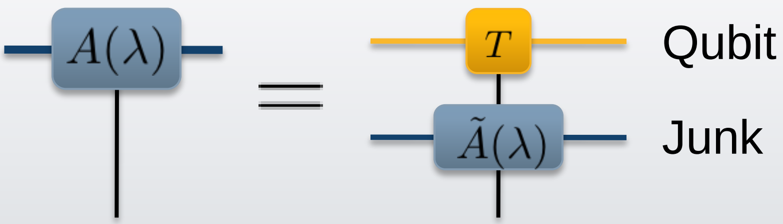
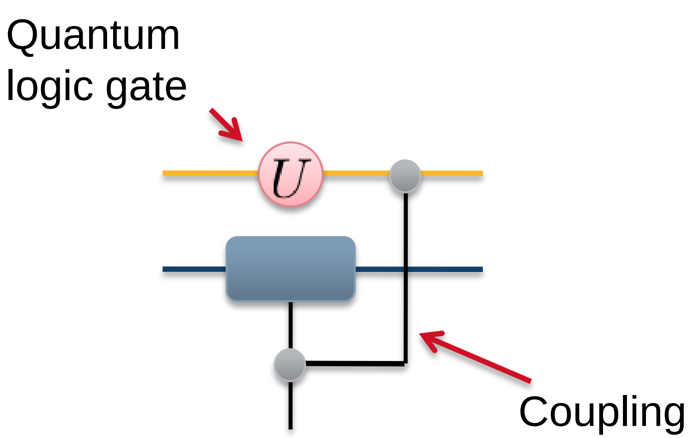


Problem 2: nontrivial gates lead to correlations between qubit and junk space

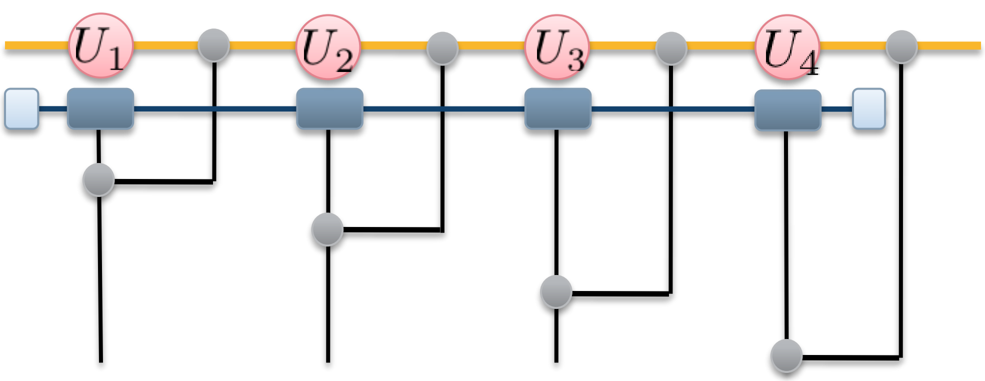


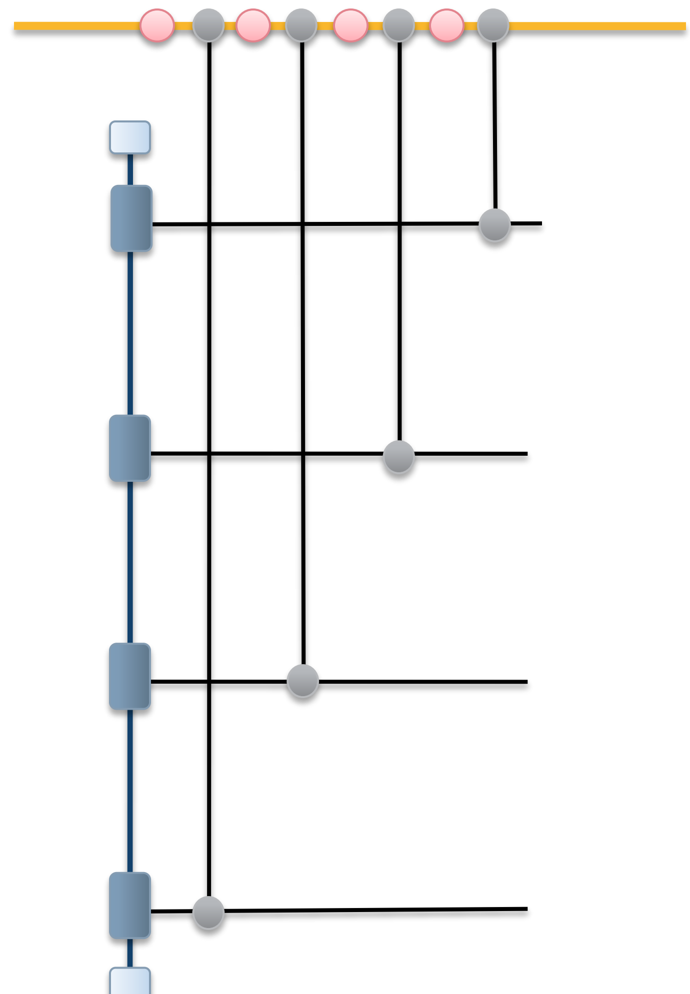
Equivalence to local Markovian error model

Solution: separate nontrivial gates beyond correlation length

The ‘junk space’ state serves as an environment

- weak coupling for small perturbations
- noise is local in ‘time’, and independent
- space gates apart beyond correlation length:
  Markovian noise model





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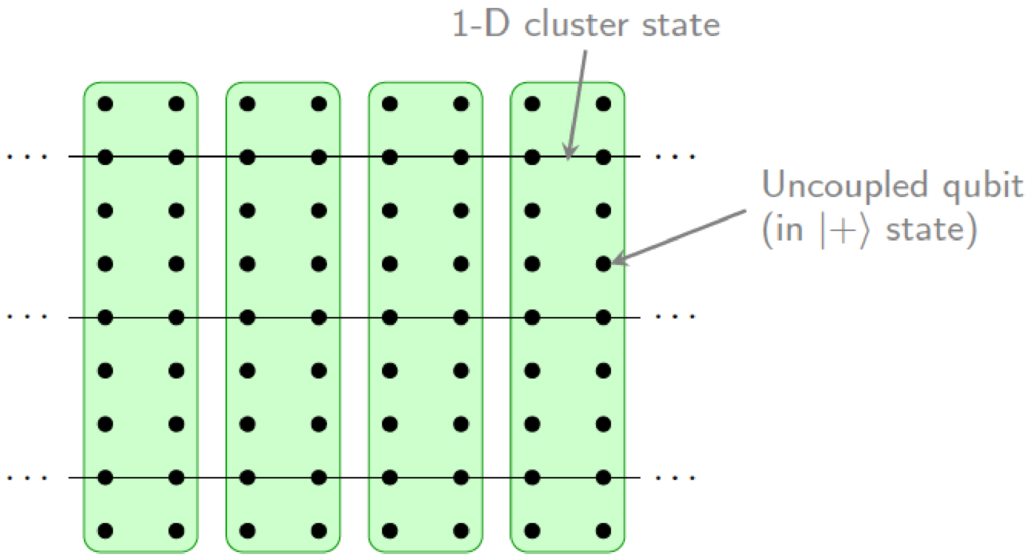
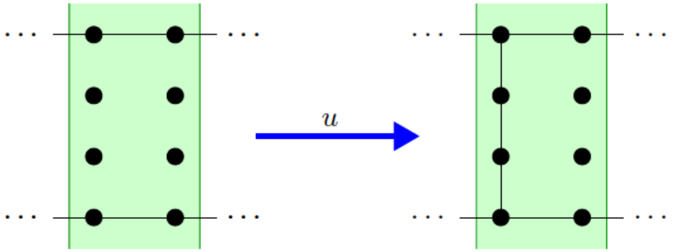
Extension to 2D



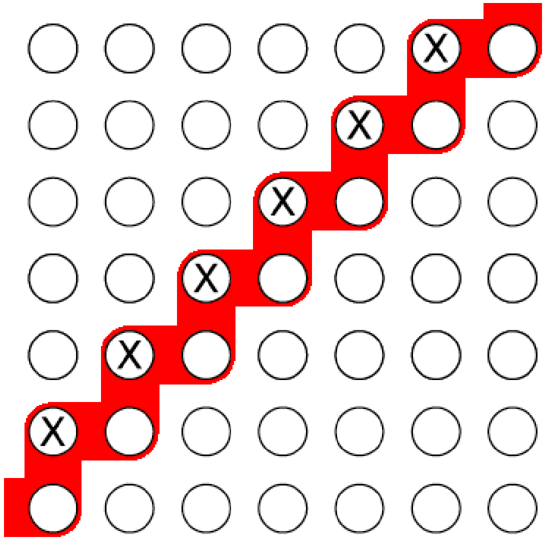


2D cluster state through ‘quasi-1D’ model

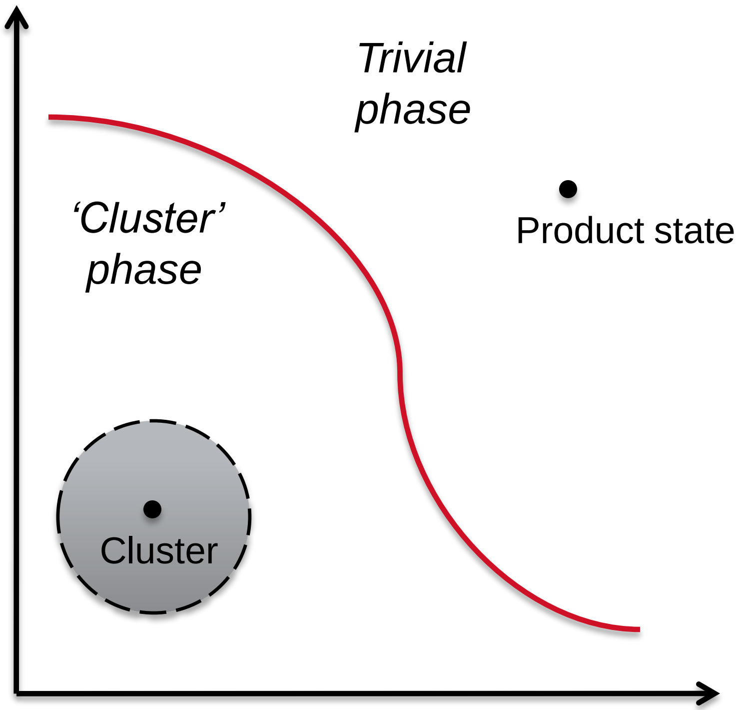
CZ gates (symmetry-respecting) couple the chains

Extensive symmetry group $(Z_2 \times Z_2)^{\times N}$

One realisation through diagonal strips of X-flips



2D cluster model in a nontrivial SP phase



- Ground states in the 'cluster' phase possess the long-range entanglement necessary for use as a qubit wire
- Quantum logic gates can be performed, with a local, independent, Markovian error model
- Apply methods of fault-tolerance

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Conclusions and future directions

- Ground-state quantum computing requires a type of ‘hidden’ long-range order:
  - symmetry-protected topological order
  - identical to a type of antiferromagnetic order, in some 1D and 2D systems

- How is this order characterised in 2D or higher-D systems?
  - Can we replace our quasi-1D symmetry with a genuine on-site symmetry in 2D?
  - Related to extensions of symmetry-protected order in 2D?

- Can we find systems allowing MBQC with a physically-motivated symmetry group? (e.g., antiferromagnets?)

- Can this order be robust at non-zero temperature?