Reliable Quantum State Tomography

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Abstract

Quantum state tomography is the task of inferring the state of a quantum system by appropriate measurements. Since the frequency distributions of the outcomes obtained from any finite number of measurements will generally deviate from their asymptotic limits, the estimation of the state can never be perfectly accurate, thus requiring the specification of error bounds. Furthermore, the individual reconstruction of matrix elements of the density operator representation of a state may lead to inconsistent results (e.g., operators with negative eigenvalues). Here we introduce a framework for quantum state tomography that enables the computation of accurate and consistent estimates and reliable error bars from a finite set of data and show that these have a well-defined and universal operational significance. The method does not require any prior assumptions about the distribution of the possible states or a specific parametrization of the state space. The resulting error bars are tight, corresponding to the fundamental limits that quantum theory imposes on the precision of measurements. At the same time, the technique is practical and particularly well suited for tomography on systems consisting of a small number of qubits, which are currently in the focus of interest in experimental quantum information science.