Isotropic Entanglement

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Acknowledgments

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The eigenvalue distribution: Motivation

Synonymous: (Energy) Spectrum; eigenvalue distribution, density of states, level densities etc.

Note: Generally the Spectrum of QMBS is hard to find *"exactly"* [1]

- First step for all eigenvalue problems (e.g. quantum mechanics) of sums of matrices
- Physical: Partition function and therefore the thermodynamics of QMBS

<u>Goal</u>: Given the geometry, local spin states, and type of local interaction, capture the spectrum of the H.

[1] B. Brown, S. T. Flammia, N. Schuch (2010), "Computational Difficulty of Computing the Density of States"

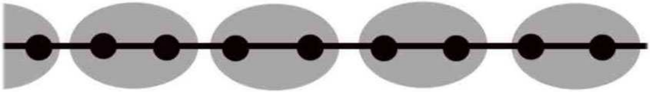
Sums of non-commuting Hamiltonians

$$H = \sum_{k=1}^{N-1} \mathbb{I}_{d^{k-1}} \otimes H_{k,k+1} \otimes \mathbb{I}_{d^{N-k-1}}.$$

- Generic local terms $\Longrightarrow$ Quantum Spin Glasses

Interactions: $H = \sum_{k=1}^{N-1} \left(\mathbb{I} \otimes H_{k,k+1} \otimes \mathbb{I} \right) = H_{\text{odd}} + H_{\text{even}}$

$H_{k,k+1}$: $d^2 \times d^2$ Generic matrix



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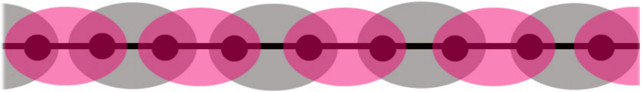


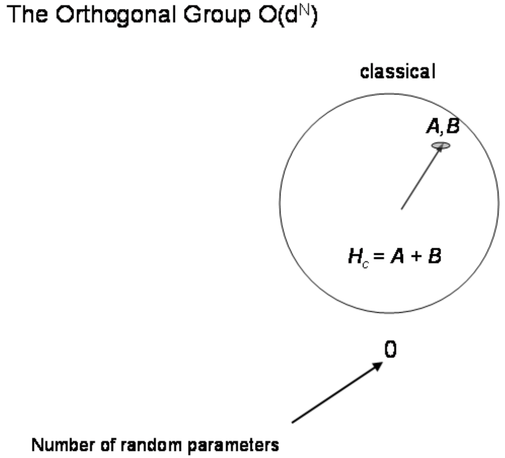
$H = H_{\text{odd}} + H_{\text{even}} = Q_A A Q_A^{-1} + Q_B B Q_B^{-1}$

Change bases such that H_{odd} is diagonal. Therefore,

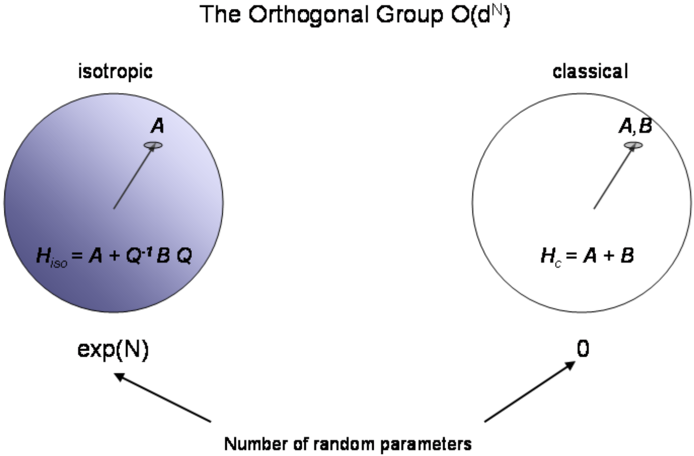
$$H = A + Q_q^{-1} B Q_q$$

$Q_q \equiv (Q_B)^{-1} Q_A \qquad \sim N$ random parameters

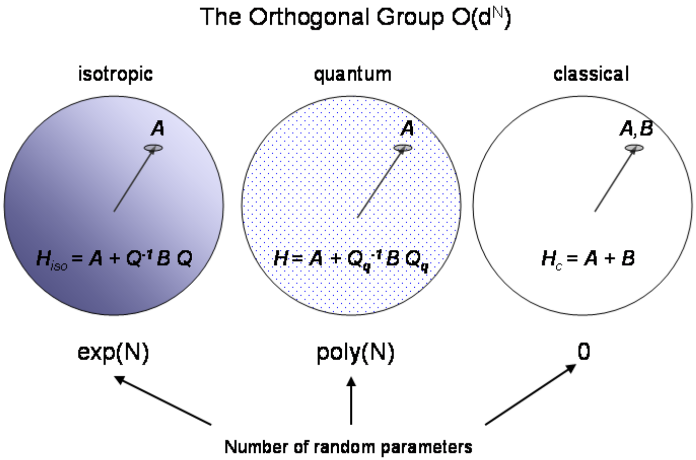
Classical sum: $p = 1$



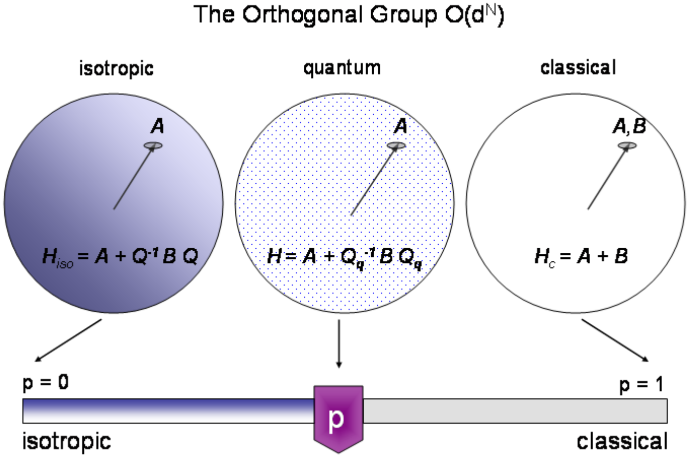
Isotropic (Free) sum: $p = 0$



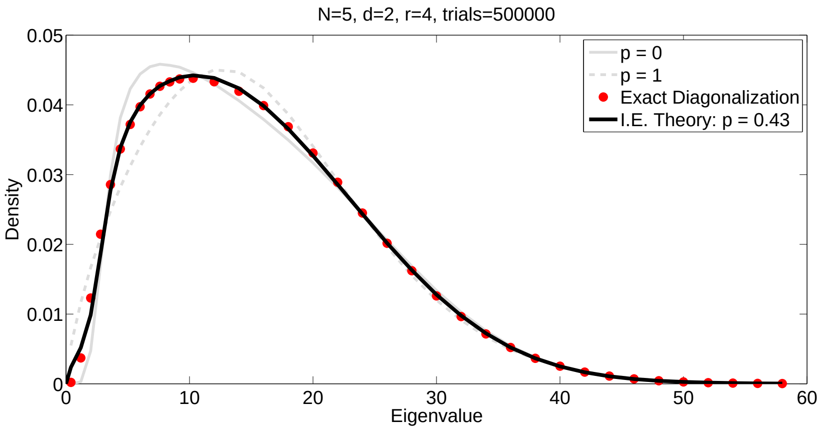
Isotropic, Quantum, and Classical



Quantum as a "sliding" sum of classical and iso



Local terms: Wishart matrices



The action starts at the fourth moment

Theorem

(The Matching Three Moments Theorem) *The first three moments of the quantum, iso and classical sums are equal.*

The Departure Theorem

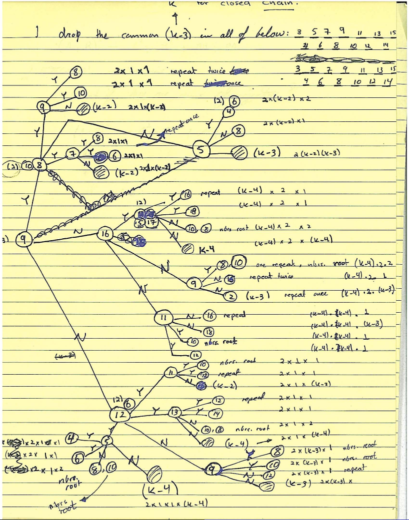
The Departure Theorem

$$m_4^{iso}=\frac{1}{d^N}\mathbb{E}\left\{\mathrm{Tr}\left[A^4+4A^3Q^{-1}BQ+4A^2Q^{-1}B^2Q+4AQ^{-1}B^3Q+\underline{\mathbf{2}\left(\mathbf{AQ^{-1}BQ}\right)^2}+B^4\right]\right\}$$

$$m_4^{q}=\frac{1}{d^N}\mathbb{E}\left\{\mathrm{Tr}\left[A^4+4A^3Q_q^{-1}BQ_q+4A^2Q_q^{-1}B^2Q_q+4AQ_q^{-1}B^3Q_q+\underline{\mathbf{2}\left(\mathbf{AQ_q^{-1}BQ_q}\right)^2}+B^4\right]\right\}$$

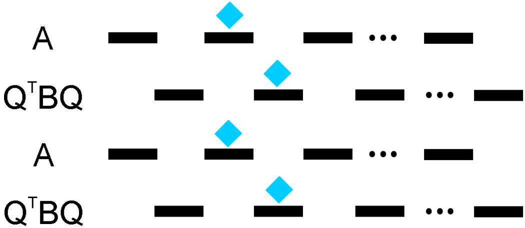
$$m_4^{c}=\frac{1}{d^N}\mathbb{E}\left\{\mathrm{Tr}\left[A^4+4A^3B+4A^2B^2+4AB^3+\underline{\mathbf{2}(\mathbf{AB})^2}+B^4\right]\right\}$$

Quantum agony



Resolving the agony

Lemma: Only these matter



$$\frac{1}{m}\mathbb{E}\mathrm{Tr}\Big[\big(H^{(3)}\otimes\mathbb{I}_{d^{N-2}}\big)\big(\mathbb{I}\otimes H^{(4)}\otimes\mathbb{I}_{d^{N-3}}\big)\big(H^{(3)}\otimes\mathbb{I}_{d^{N-2}}\big)\big(\mathbb{I}\otimes H^{(4)}\otimes\mathbb{I}_{d^{N-3}}\big)\Big]$$

$$= \frac{1}{d^3}\left\{\mathbb{E}\left(H^{(3)}_{i_3 i_4, j_3 j_4} H^{(3)}_{i_3 p_4, j_3 k_4}\right)\mathbb{E}\left(H^{(4)}_{j_4 i_5, k_4 k_5} H^{(4)}_{i_4 i_5, p_4 k_5}\right)\right\},$$

Quantum as a convex combination of *classical* and *iso*

- Use fourth moments to form a hybrid theory

$$\gamma_2^{q} = p\gamma_2^{c} + (1-p)\,\gamma_2^{iso}$$

$\gamma_2^{(\bullet)}$ is found from the fourth moments

The Slider Theorem

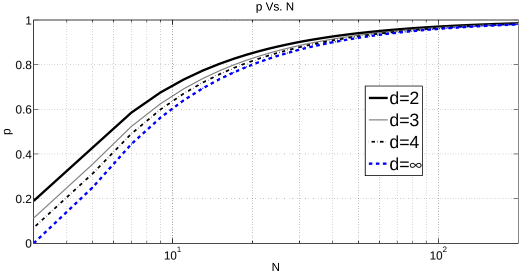
Theorem

(The Slider Theorem) *The quantum kurtosis lies in between the classical and the iso kurtoses,* $\gamma_2^{iso} \leq \gamma_2^{q} \leq \gamma_2^{c}$. *Therefore there exists a* $0 \leq p \leq 1$ *such that* $\gamma_2^{q} = p\gamma_2^{c} + (1-p)\,\gamma_2^{iso}$. *Further,* $\lim_{N\to\infty} p = 1$.

Universality of p

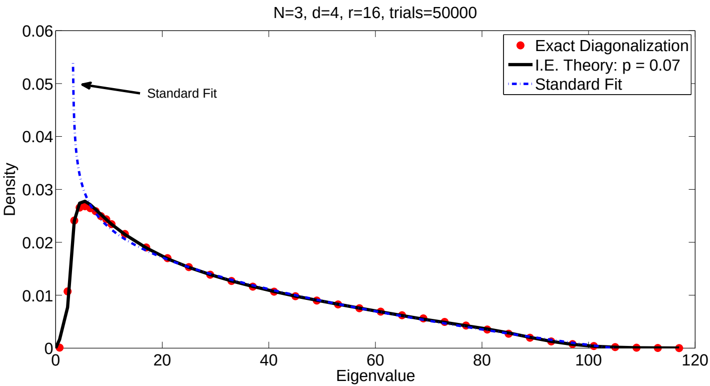
Corollary

(Universality) $p \mapsto p(N, d, \beta)$, *namely, it is independent of the distribution of the local terms.*

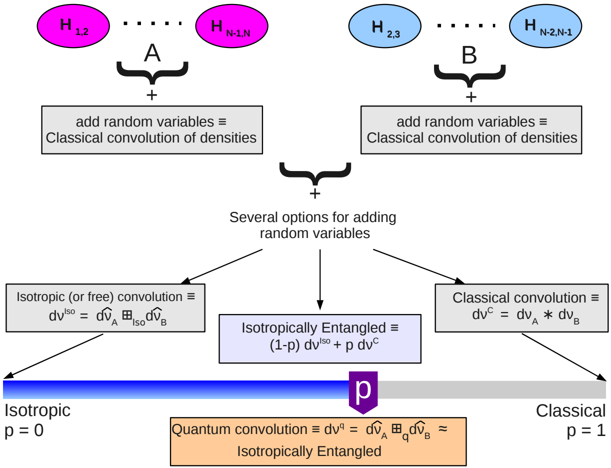


Here $\beta = 1$. Therefore, p only depends on eigenvectors!

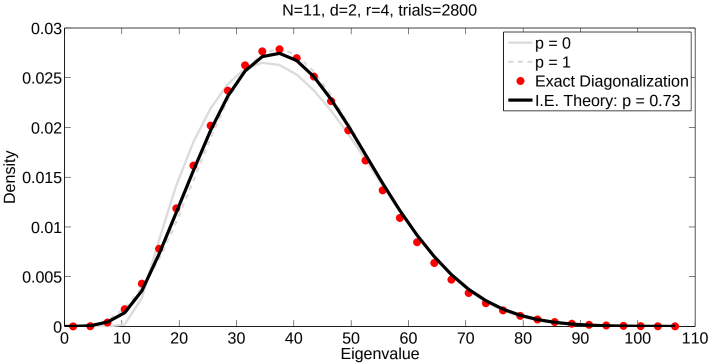
Suppose you have the first four moments



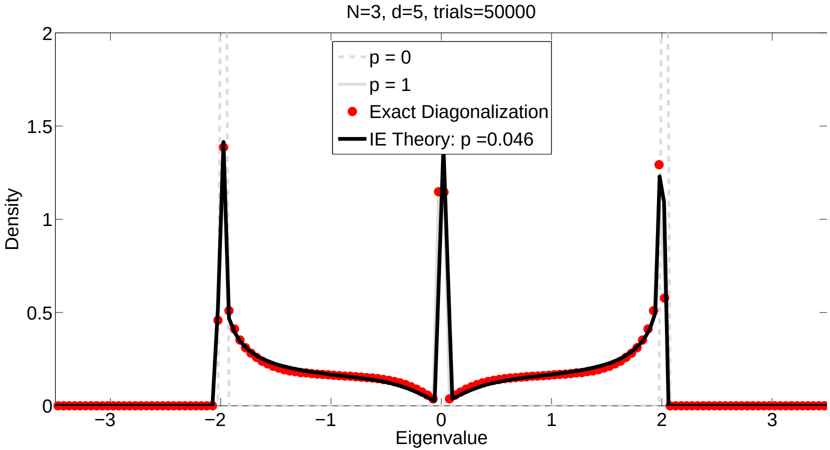
Summary: Method of Isotropic Entanglement



Local terms: Wishart matrices



Local terms: sign $\left[\text{rand}(d^2, 1)\right]$



Lastly...

This is just the beginning

Thank$Q$

QMBS: Eigenvectors (States)

Capture using: *MPS, DMRG, TPS, PEPS, MERA etc.*

$$|\Psi\rangle = \sum_{i_1,i_2,\ldots,i_N=1}^{d} c_{i_1,i_2\ldots,i_N} |i_1\rangle \otimes |i_2\rangle \otimes \ldots \otimes |i_N\rangle$$

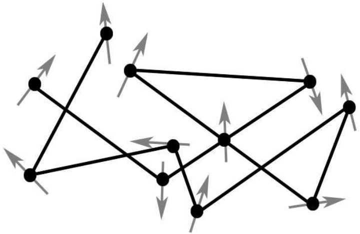


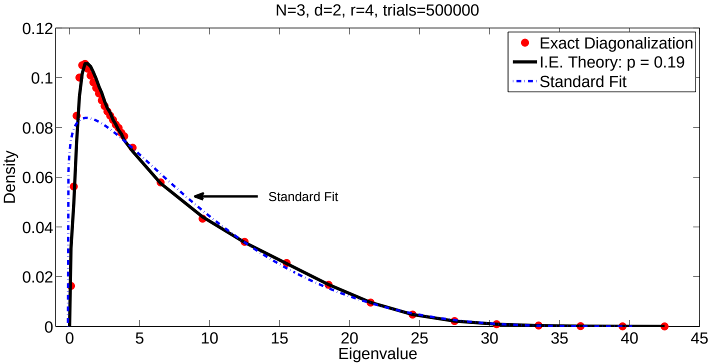
Figure: Each arrow represents a d dimensional spin. Each edge represents an interaction.

Do we do better than four moment accuracy?

Because of the departure theorem.

$$m_5 = \frac{1}{m}\mathbb{E}\mathrm{Tr}\Big(A^5 + 5A^4 Q_\bullet^T B Q_\bullet + 5A^3 Q_\bullet^T B^2 Q_\bullet + 5A^2 Q_\bullet^T B^3 Q_\bullet + \underline{\mathbf{5A}\left(\mathbf{A Q_\bullet^T B Q_\bullet}\right)^2} + \underline{\mathbf{5}\left(\mathbf{A Q_\bullet^T B Q_\bullet}\right)^2 \mathbf{Q_\bullet^T B Q_\bullet}} + 5A Q_\bullet^T B^4 Q_\bullet + B^5\Big) \quad (1)$$

Suppose you have the first four moments



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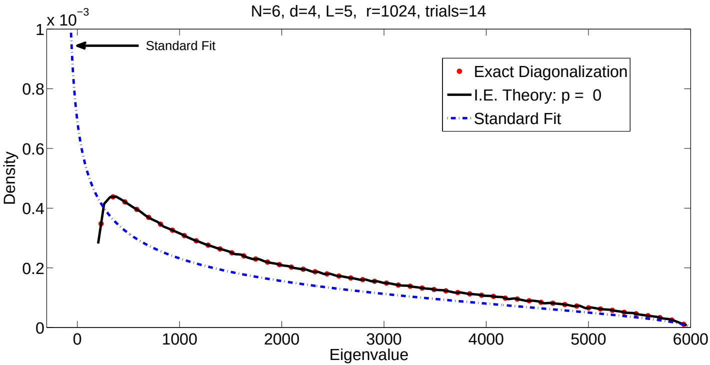

Figure: Wisharts