Quantum computational capability of a two-dimensional valence bond solid phase

Akimasa Miyake
Perimeter Institute for Theoretical Physics, 31 Caroline Street North, Waterloo Ontario, N2L 2Y5, Canada

Quantum phases of naturally-occurring systems exhibit rich nature as manifestation of their many-body correlations, in contrast to our persistent technological challenge to build at will such correlations artificially from scratch. Here we show theoretically that quantum correlations exhibited in the two-dimensional valence bond solid phase of a quantum antiferromagnet, modeled by Affleck, Kennedy, Lieb, and Tasaki as a precursor of spin liquids and topological orders, are sufficiently complex yet structured enough to simulate universal quantum computation when every single spin can be measured individually. This unveils that an intrinsic complexity of naturally-occuring 2D quantum systems — which has been a long-standing challenge for traditional computers — could be tamed as a computationally valuable resource, regardless of our constraint not to create newly entanglement during computation. Our constructive protocol leverages a novel way to herald the correlations suitable for deterministic quantum computation through a random sampling, and may be extensible to other ground states of various 2D valence bond phases beyond the AKLT state.

In an alternative way to the conventional bottom-up idea to build up a quantum computer artificially from scratch, we suggest taking a top-down vision in that we attempt to tame a resource of suitably structured entanglement, which could either exist in nature or be simulated relatively naturally within our technology, for the sake of easier scalability. A key point of the vision is our limited ability such that once a specific natural resource of structured many-body entanglement is provided, we are supposed to utilize only operations which just consume entanglement without its new creation, such as local measurements and local turning off of an interaction.

Our target of the naturally-occuring two-dimensional (2D) system is the valence bond solid (VBS) phase of spin $\frac{3}{2}$'s on the 2D hexagonal lattice, modeled by Affleck, Kennedy, Lieb, and Tasaki (AKLT) [2, 3], which is widely recognized as a cornerstone in condensed matter physics. Their VBS construction of the ground state in terms of the distributed spin singlets (or the valence bonds) has become one of most ubiquitous insights in quantum magnetism as well as in high-T_c superconductivity, and leads to modern trends of spin liquids and topological orders. It turns out here that the 2D VBS phase, represented by the AKLT ground state, provides an ideal entanglement structure of quantum many-body systems that can be suitably *tamed* through our limited capability to the goal of universal quantum computation. Our top-down vision is materialized conveniently in taking advantage of a conventional framework of measurement-based quantum computation (MQC) whose methods have been developed to "steer" quantum information through given many-body correlations using only a set of local measurements and classical communication, under which entanglement is just consumed without new creation. Later we extend that in a wider program to tame naturally-occuring many-body correlations.

In the context of MQC, the 2D cluster state [4] is the first and canonical instance of such an entangled state that pertains to a universal quantum computational capability when every single qubit (spin $\frac{1}{2}$) is measured individually and the outcomes of the measurements are communicated classically [5, 6]. Remarkably, it was already noticed in Ref. [7] that MQC on the 2D cluster state utilizes a structure of entanglement which is analogous to that of the aforementioned VBS state. Following such an observation, the tensor network states, as a class of efficiently classically parameterizable states in extending the VBS construction, has been used in Refs. [8, 9] to construct resource states of MQC, where it was indicated that a certain set of the local matrices or tensors that describe the correlations can result in a quantum unital map through the single-site measurement. Notably, however, most known examples considered so far, including additionally those e.g. in Refs. [10–15], are constructed to have such a convenient yet artificial property — as often referred as one of peculiar properties of the correlations of the 2D cluster state — that it is possible to decouple deterministically (by measurements of only neighboring sites) a 1D-chain structure that encodes the direction of a simulated time as a quantum logical wire of the quantum circuit model. This peculiarity is said to be artifact of another less realistic feature of the 2D cluster state in that it cannot be the exact ground state of any two-body spin-$\frac{1}{2}$ Hamiltonian [16, 17], and thus one cannot expect such convenience in the correlations of a *genuine* 2D ground state of a naturally-occuring spin system.

The main result of our paper is summarized in the following (informal) theorem and illustrated in the Figure 1. As elaborated in the full paper [1], we introduce a novel way to herald the correlations suitable for *deterministic* quantum computation through a *random* sampling, to tame for the first time the genuine 2D naturally-occurring correlation, which otherwise has natural tendency to split an incoming information into two outgoing information because of certain symmetric nature of the *three* direc-

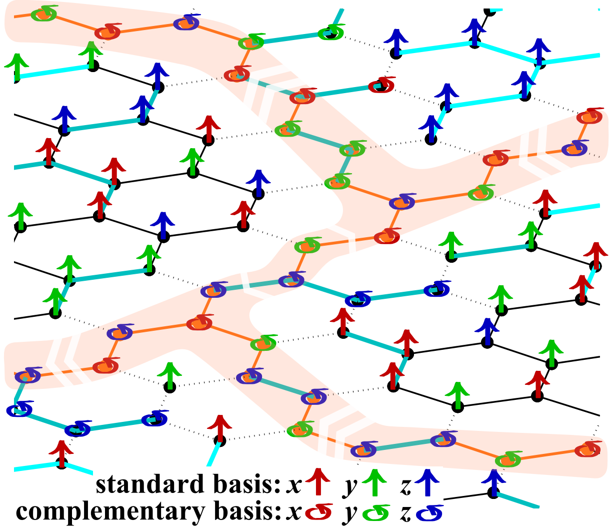

FIG. 1: A scheme of quantum computation through measuring the correlations of the 2D AKLT state, a representative state of the 2D VBS phase of spin $\frac{3}{2}$'s on the 2D hexagonal lattice. After a random sampling which assigns every spin per site to one of three axes x, y, and z, the typical configuration of the outcomes enables us to choose the backbone structure (described by a shaded region) along which quantum computation is deterministically simulated in terms of a quantum circuit. Our protocol harnesses a pair (depicted as a dotted bond of the hexagonal lattice) of neighboring sites where one is measured in a standard basis and the other is done in a complementary basis, to accommodate the desired structure of space-time along the region of the backbone. An emergence of the time is simulated if both two bits of information out of measurements per site are communicated to the same direction (as depicted as the double arrows), on the other hand, an emergence of the space is simulated if two bits of information are communicated to the opposite directions (as depicted as a pair of the single arrows pointing apart). The figure illustrates a microscopic view of the Figure 2, and the two-qubit CNOT gate is implemented in the middle region between two quantum logical wires running from the right to the left.

tions at every site of the 2D hexagonal lattice. This seems to be the reason why MQC on the 2D AKLT state has been an open question in a long time, although the AKLT state by the 1D *spin-1* chain was shown in Ref. [10] to be capable of simulating a single quantum wire of MQC. A related result about usefulness of the 2D AKLT state is recently announced independently in Ref. [18].

Theorem. *A universal quantum computation can be simulated through consuming monotonically entanglement provided as the 2D AKLT state $|\mathcal{G}\rangle$ (defined as the VBS state of a spin $\frac{3}{2}$ per site and described as a tensor network state) of the size proportional to the target quantum circuit size, in terms of single-site measurements of every individual spin $\frac{3}{2}$, a bounded amount of classical communication of measurement outcomes per site, and efficient classical side-computation.*

Insight to the MQC protocol

We intend to simulate the quantum circuit model through measuring the correlations at every site, and call the part of the 2D hexagonal lattice sites that corresponds to the quantum circuit (consisting of the quantum logical wires running almost horizontally and their entangling gates described vertically) a *backbone*, as seen in Figure. 2. The degree of the backbone site refers to how many neighbors it has *along the backbone*. The degree-3 backbone sites are used at every junction of the horizontal logical wire with a vertical entangling gate, so that they are required only occasionally.

A key insight to construct our protocol is that since the reduced density operator of every spin $\frac{3}{2}$ per site is totally mixed and isotropic, i.e., the normalized identity projector $\frac{\mathbb{1}}{4}$, we are able to extract *2 bits* of classical information by measurements per site. Then it is sensible that in stead of obtaining them at once, a part of the information, indeed $\log_2 3$ bits in our case, is first extracted and we adapt the next stage according to it. It might be surprising that the first measurement induces a kind of randomization, but intuitively speaking, this part is crucial to separate the original quantum correlation that intrinsically involves genuine 2D fluctuations into the classical correlation (or, a statistical sampling) that can be still efficiently handled by a classical side-processor and the "more rigid" quantum correlation suitable for *deterministic* quantum computation. A global statistical nature of the AKLT correlations through the first stage guarantees, in an analogous way with the classical percolation phenomenon, that an embedding of the backbone (i.e. the target quantum circuit) can be found in the *typical* configuration of a heralded, randomized distribution of entanglement. At the second stage which implements quantum computation, the measurements are invented in such a way that the standard-basis measurement and complementary-basis one, both of which are defined in Ref. [1], are always paired (as depicted by the dotted bonds in the Figure. 1).

Summary of the MQC protocol

Now we outline our MQC protocol, which consists of two stages. (i) The first stage is to apply a measurement $\{M^x, M^y, M^z\}$ which depolarizes randomly toward one of the three orthogonal axes at every site. We define a degenerate projection M^μ ($\mu = x, y, z$) as

$$M^\mu = \sqrt{\tfrac{2}{3}}(|\tfrac{3}{2}^\mu\rangle\langle\tfrac{3}{2}^\mu| + |-\tfrac{3}{2}^\mu\rangle\langle-\tfrac{3}{2}^\mu|).$$

The set of $\{M^x, M^y, M^z\}$ constitutes the positive operator value measure (POVM) by satisfying $\sum_{\mu=x,y,z} M^{\mu\dagger} M^\mu = \mathbb{1}$, so that it is a valid local measurement with three alternative, random outcomes μ.

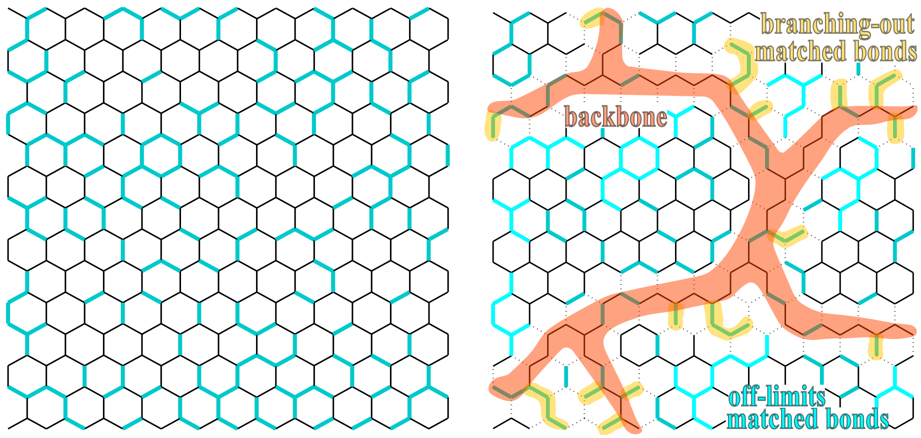

FIG. 2: (Left) Illustrated is a typical distribution of matched bonds (depicted as thicker, dark-cyan bonds) where a pair of neighboring sites are assigned to the same axis through the polarizing measurement at the first stage. (Right) The backbone (described as a shaded, orange region) is identified efficiently by analyzing classically such a distribution, so that it can skirt all pieces of "off-limits" matched bonds (depicted as thicker, light-cyan bonds) that involve either a site with triple matched bonds or a closed loop. The other matched bonds may be freely available as a part of the backbone, with an additional prescription to their "branching out" (described by a shaded, yellow region). The almost sure success of this classical side-computaton is guaranteed in an analogous way with the bond percolation phenomenon, based on the statistical property on the occurrence of matched bonds, originated from a genuine 2D nature of the correlations of the AKLT state. The microscopic view near the CNOT gate is highlighted in the Figure 1.

We must record the outcome μ_k at every site k and collect the location of the "matched" bonds such that the pair of the axes for the neighboring sites k, k' coincides, namely $\mu_k = \mu_{k'}$. Based on an occurrence of matched bonds (that need additional care in their use), we are able to determine the backbone by efficient classical side-compuation in circumventing some rare "off-limits" configurations of matched bonds (see Figure 2).

(ii) The second stage carries actual quantum computation, using further projective measurements at every site and feedforward of their outcomes. Once the backbone is identified, the computation is deterministic in a very similar way with MQC on the 2D cluster state.

Final note

The details of the work are available online in Ref. [1]. Another submission to QIP by the author about Ref. [19] is relevant to the current work in the sense that the idea of the former could be combined to make not only the representative state (i.e., the 2D AKLT state) of the 2D VBS phase but also any ground state in the 2D VBS phase ubiquitously useful for quantum computation under the similar protocol.

[1] A. Miyake, arXiv:1009.3491.
[2] I. Affleck, T. Kennedy, E.H. Lieb, and H. Tasaki, Phys. Rev. Lett. **59**, 799 (1987).
[3] I. Affleck, T. Kennedy, E.H. Lieb, and H. Tasaki, Comm. Math. Phys. **115**, 477 (1988).
[4] H.J. Briegel and R. Raussendorf, Phys. Rev. Lett. **86**, 910 (2001).
[5] R. Raussendorf and H.J. Briegel, Phys. Rev. Lett. **86**, 5188 (2001).
[6] R. Raussendorf, D.E. Browne, and H.J. Briegel, Phys. Rev. A **68**, 022312 (2003).
[7] F. Verstraete and J.I. Cirac, Phys. Rev. A **70**, 060302(R) (2004).
[8] D. Gross and J. Eisert, Phys. Rev. Lett. **98**, 220503 (2007).
[9] D. Gross, J. Eisert, N. Schuch, and D. Perez-Garcia, Phys. Rev. A **76**, 052315 (2007).
[10] G.K. Brennen and A. Miyake, Phys. Rev. Lett. **101**, 010502 (2008).
[11] D. Gross and J. Eisert, arXiv:0810.2542.
[12] S.D. Bartlett and T. Rudolph, Phys. Rev. A **74**, 040302(R) (2006).
[13] T. Griffin and S.D. Bartlett, Phys. Rev. A **78**, 062306 (2008).
[14] X. Chen, B. Zeng, Z.-C. Gu, B. Yoshida, and I.L. Chuang, Phys. Rev. Lett. **102**, 220501 (2009).
[15] J.-M. Cai, A. Miyake, W. Dür, and H.J. Briegel, arXiv:1004.1907.
[16] M.A. Nielsen, Rep. Math. Phys. **57**, 147 (2006).
[17] M. Van den Nest, K. Luttmer, W. Dür, and H.J. Briegel, Phys. Rev. A **77**, 012301 (2008)
[18] T.-C. Wei, I. Affleck, and R. Raussendorf, arXiv:1009.2840.
[19] A. Miyake, Phys. Rev. Lett. **105**, 040501 (2010).