QIP, Jan. 10 (2011) @ Singapore

The 2D AKLT state is universal for measurement-based quantum computation

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Outline of the talk

Introduction

- measurement-based quantum computation (MQC)
- 2D AKLT state

Part 1 (Miyake)

Simulating a quantum circuit model (arXiv:1009.3491)

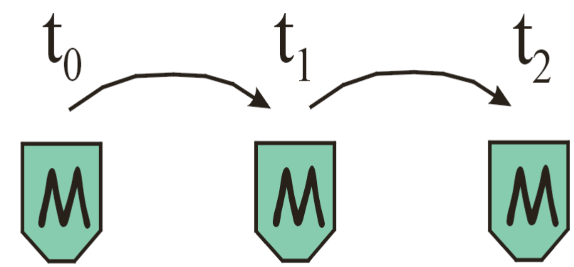
Part 2 (Wei)

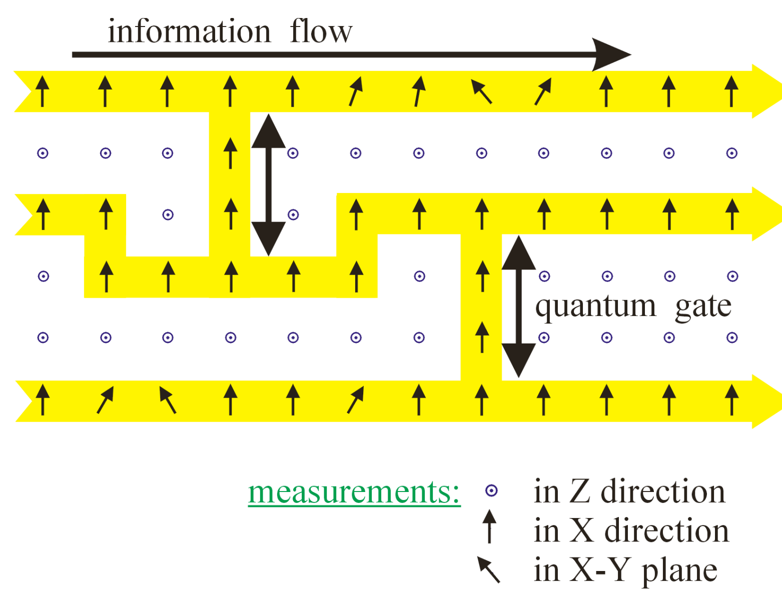
Transforming into a 2D cluster state (arXiv:1009.2840)

Measurement-based quantum computation

universal QC model by

- many-particle entanglement
- single-particle measurements
- communication of outcomes





significant questions

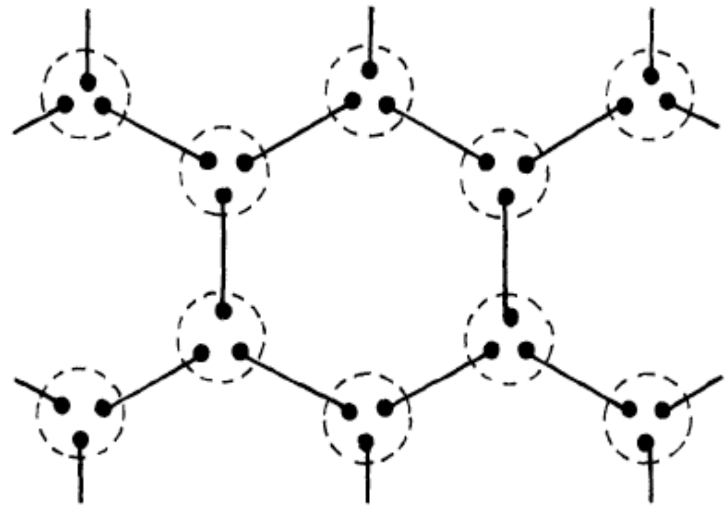
- characterization of entanglement which enables one to simulate universal quantum computation (BQP)?
- practical implementation? (large-scale entanglement)

[MQC on a 2D cluster state:
Raussendorf & Briegel, PRL '01;
Raussendorf, Browne, Briegel, PRA '03]

2D AKLT state

$$|g\rangle = \bigotimes_{v}^{\text{vertex}} P_v^{S=\frac{3}{2}} \bigotimes_{e}^{\text{edge}} |\text{singlet}\rangle_e$$

[Affleck, Kennedy, Lieb, Tasaki, PRL '87; CMP '88]



- ground state of antiferromagnetic <u>two-body</u> Hamiltonian of spin 3/2's on 2D hexagonal lattice

$$H = J \sum_{(k,k')}^{\text{n.n.}} \underbrace{\left[\mathbf{S}_k \cdot \mathbf{S}_{k'} + \frac{116}{243} (\mathbf{S}_k \cdot \mathbf{S}_{k'})^2 + \frac{16}{243} (\mathbf{S}_k \cdot \mathbf{S}_{k'})^3 \right]}_{},$$

P^3 : projector to total spin 3 for every pair

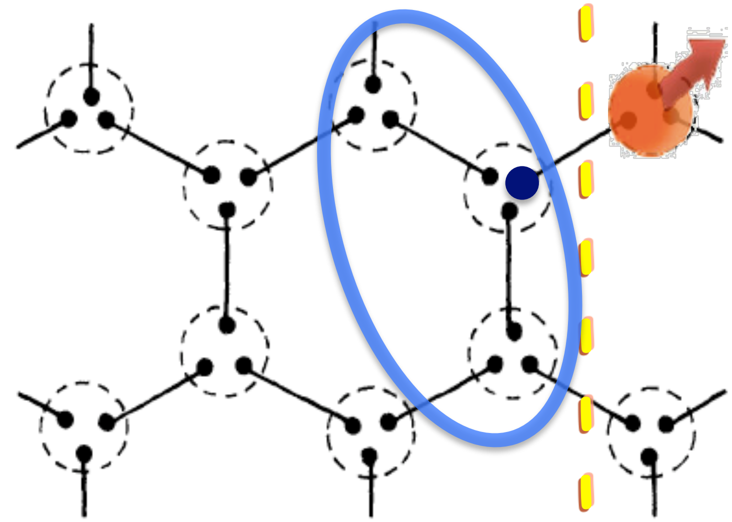
- valence bond solid state (to materialize a spin liquid)

- merits as resource: a preparation by cooling
  stability of a gapped ground state

Part 1 FAQ: where are qubits?

edge state (spin 1/2):
emergent fractional degree of freedom, localized at boundary

$\xi = 1/\ln(3/2) \approx 2.47$



- area law of entanglement
- stay in degenerate ground states (cf. topological feature)

$$|g\rangle = \bigotimes_{v}^{\text{vertex}} P_v^{S=\frac{3}{2}} \bigotimes_{e}^{\text{edge}} \underbrace{\tfrac{1}{\sqrt{2}}(|01\rangle - |10\rangle)_e}_{\text{singlet}} = \sum_{\alpha=\pm\frac{3}{2},\pm\frac{1}{2}} \mathrm{tr}\left[\prod_{v}^{\text{vertex}} A[\alpha_v]|\alpha_v\rangle\right]$$

"symmetric 3-indices tenors"

$$A[+\tfrac{3}{2}^{\mu}] \sim 000^{\mu}$$

$$A[+\tfrac{1}{2}^{\mu}] \sim \tfrac{1}{\sqrt{3}}\left(001+010+100^{\mu}\right)$$

upshot of QC: scattering process among edge states

Challenge to construct MQC Protocol

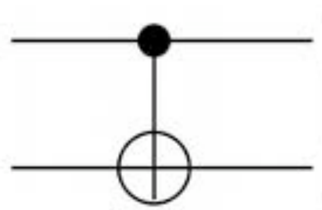
- entanglement network to gate-teleport quantum information

cluster-state has a VBS-like entanglement structure (PEPS)

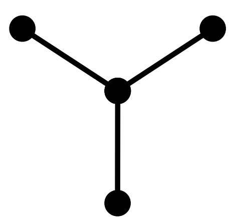
[Gottesman, Chuang, Nature'99]
[Raussendorf, Briegel, PRL'01]
[Verstraete, Cirac, PRA'04]
[Childs, Leung, Nielsen PRA'05]
[Gross, Eisert, PRL'07;
Gross, Eisert, Schuch, Perez-Garcia, PRA'07] ….

- steering quantum information in a controllable (quantum-circuit) manner

1+1D quantum circuit
= backbone



3-way symmetric !



How to get unitary maps? How to distinguish space and time?

Outline of MQC Protocol

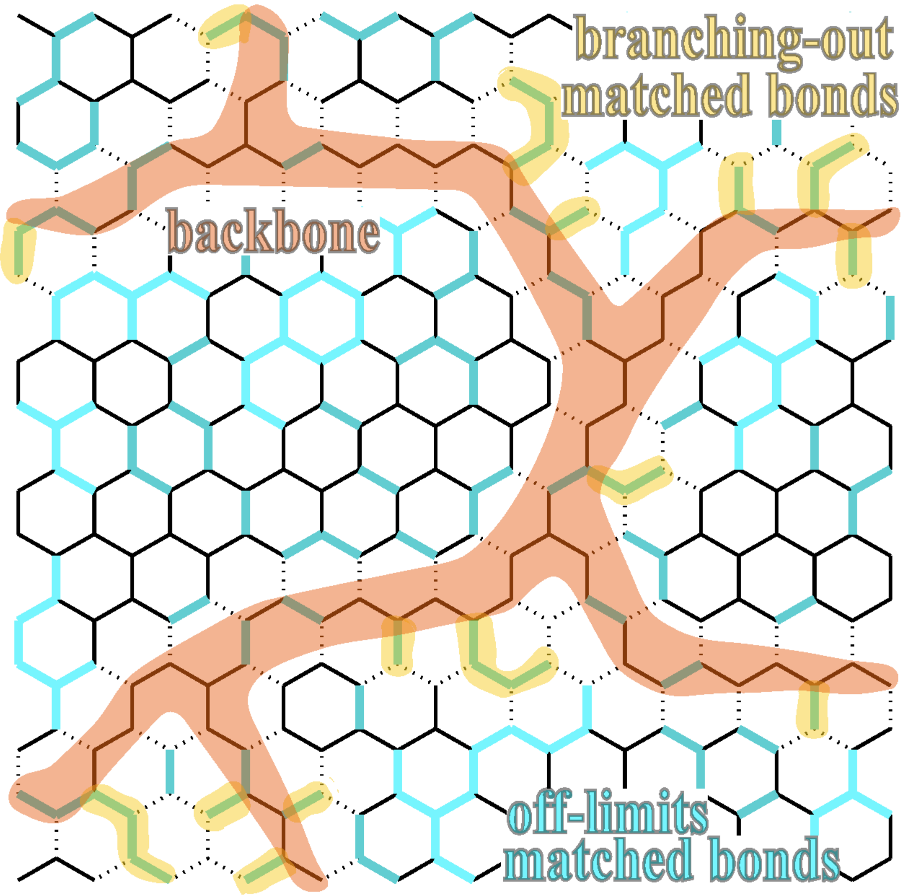
How to get unitary maps and composed them?

1. measurement at every site, depolarizing <u>randomly</u> into one of the three axes

$\{M^x, M^y, M^z\}$

$$M^\mu = \sqrt{\tfrac{2}{3}}(|\tfrac{3}{2}^\mu\rangle\langle\tfrac{3}{2}^\mu| + |-\tfrac{3}{2}^\mu\rangle\langle-\tfrac{3}{2}^\mu|)$$
$$\sum_{\mu=x,y,z} M^{\mu\dagger} M^\mu = 1$$

matched bond: $\mu_k = \mu_{k'}$



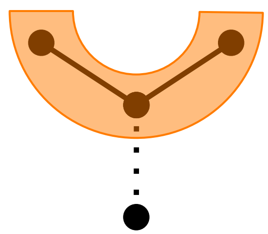
1′. classical side-computation:
in a typical configuration of matched bonds, identifying a backbone (which excludes all sites with triple matched bonds)

2. <u>deterministic</u> quantum computation

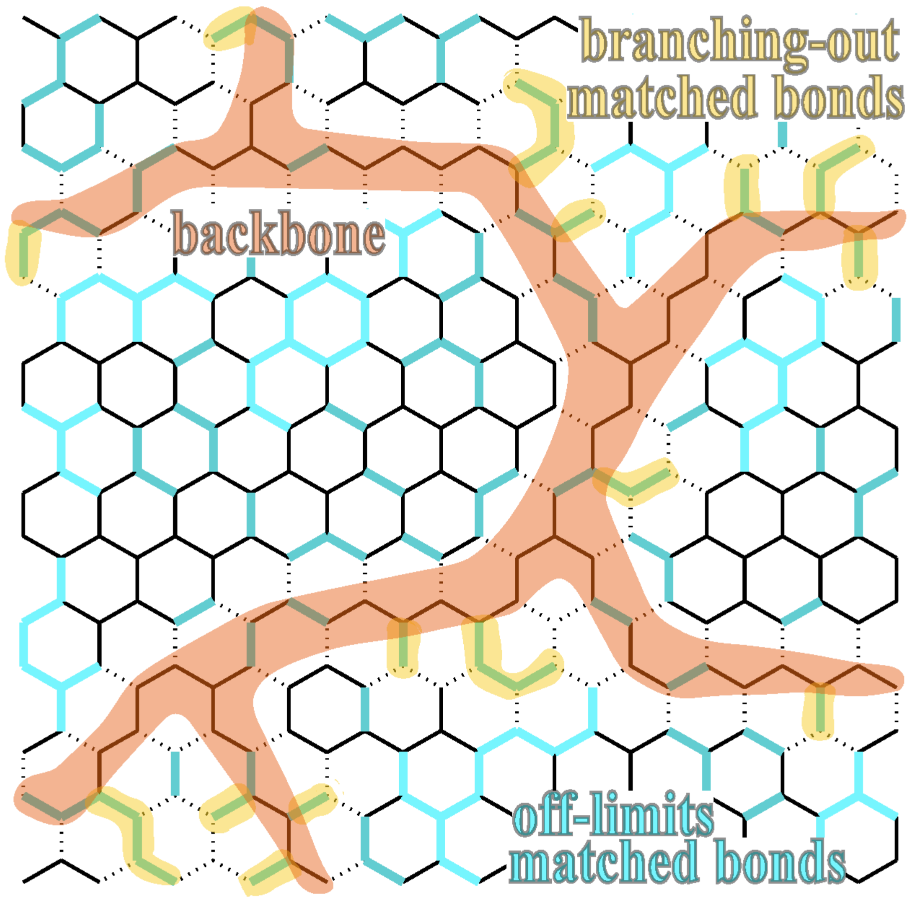
Ideas behind MQC Protocol

How to get unitary maps and composed them?

- a (mutually-unbiased) pair of standard and complementary measurements
  <= non-matched bond

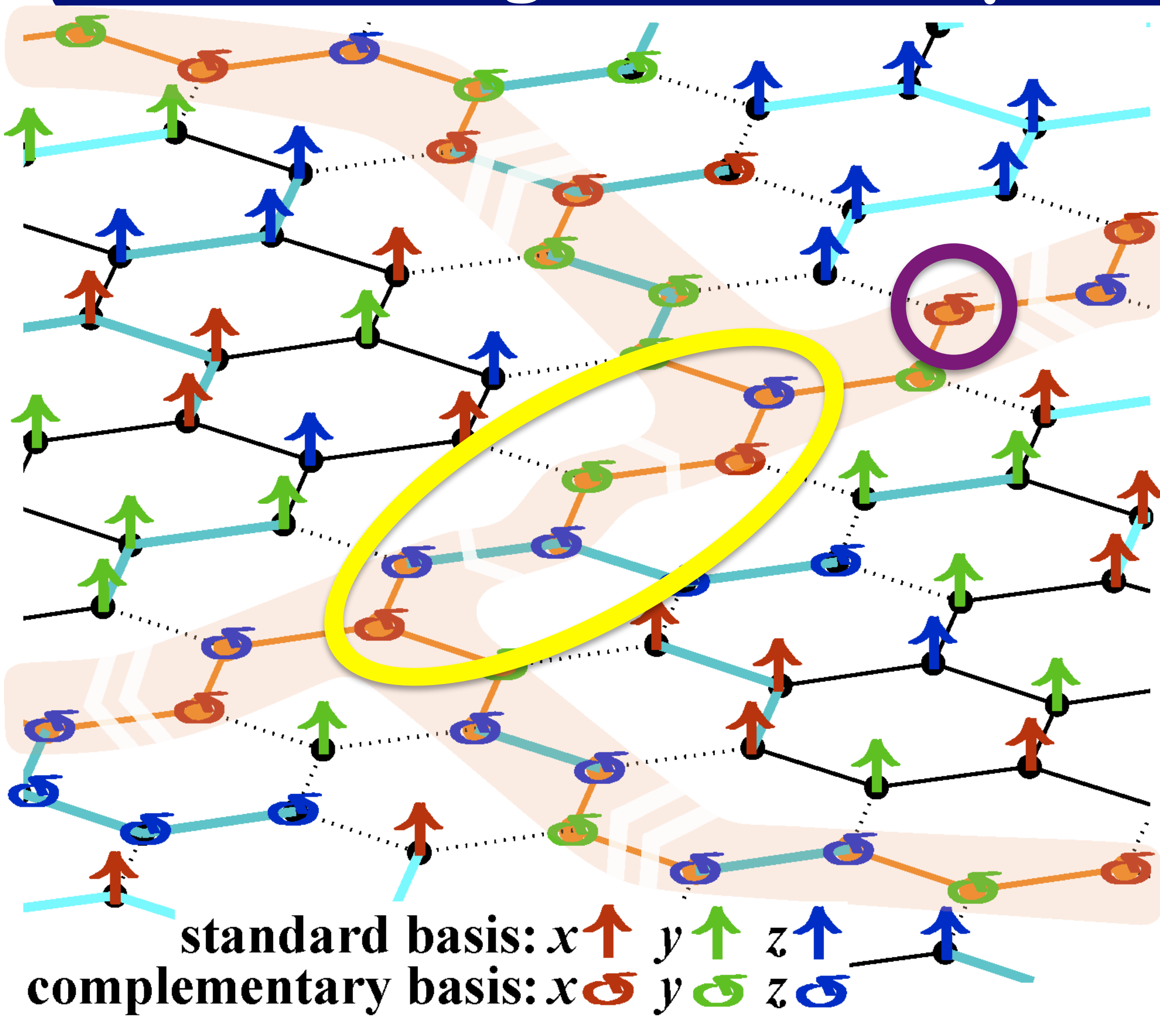


unitary!

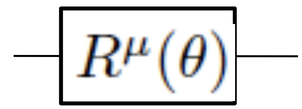


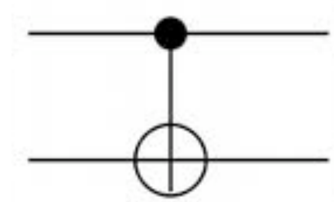
- "concentration" from 2D (3-way symmetric) correlation
  = classical statistical correlation (via random sampling)
  + "more rigid" quantum correlation

Universal gates and space-time structure



universal gates:





classical information at backbone site:

$$\Upsilon = X^{a^x} Z^{a^z}$$

time: two bits sent in the same direction

space: two bits sent in opposite directions (no net asymmetry in directions)

Summary of Part 1

ground state of a realistic 2D condensed matter system (valence bond solid phase) can be harnessed as a resource of measurement-based quantum computation.

new perspective to traditionally-intractable complexity of 2D quantum systems

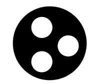
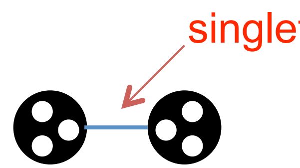
- A. Miyake, quantum computational capability of a two-dimensional valence bond solid phase, arXiv:1009.3491

Second part:
Converting AKLT state to cluster state

Tzu-Chieh Wei, Ian Affleck and Robert Raussendorf

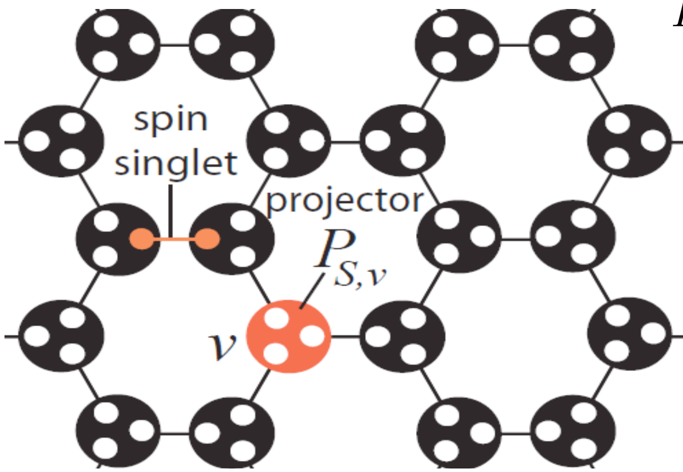
Ref: arXiv: 1009.2840

Spin-3/2 AKLT state on honeycomb

- Each site contains three virtual qubits 
- Two virtual qubits on an edge form a singlet 

$|01\rangle - |10\rangle$

- Projection ($P_{S,v}$) onto symmetric subspace of 3 qubits at each site



$$P_{S,v} = |000\rangle\langle 000| + |111\rangle\langle 111| + |W\rangle\langle W| + |\overline{W}\rangle\langle \overline{W}|$$

$$|000\rangle \leftrightarrow \left|\frac{3}{2}, \frac{3}{2}\right\rangle \qquad |111\rangle \leftrightarrow \left|\frac{3}{2}, -\frac{3}{2}\right\rangle$$

$$|W\rangle \equiv \frac{1}{\sqrt{3}}(|001\rangle + |010\rangle + |100\rangle) \leftrightarrow \left|\frac{3}{2}, \frac{1}{2}\right\rangle$$

$$|\overline{W}\rangle \equiv \frac{1}{\sqrt{3}}(|110\rangle + |101\rangle + |011\rangle) \leftrightarrow \left|\frac{3}{2}, -\frac{1}{2}\right\rangle$$

- Unique ground state of

$$H = \sum_{\text{edge } \langle i,j\rangle} \hat{P}_{i,j}^{(S=3)} = \sum_{\text{edge } \langle i,j\rangle} \left[\vec{S}_i \cdot \vec{S}_j + \frac{116}{243}(\vec{S}_i \cdot \vec{S}_j)^2 + \frac{16}{243}(\vec{S}_i \cdot \vec{S}_j)^3 + \frac{55}{108}\right]$$

The POVM in terms of virtual qubits

[*Wei,Affleck & Raussendorf* '10
Miyake '10]

- Three elements satisfy:

$$F^\dagger_{v,x}F_{v,x}+F^\dagger_{v,y}F_{v,y}+F^\dagger_{v,z}F_{v,z}=P_{v,\mathrm{sym}}$$

$$F_{v,z} = \sqrt{\frac{2}{3}}(|000\rangle\langle 000|+|111\rangle\langle 111|)$$

$$F_{v,x} = \sqrt{\frac{2}{3}}(|+++\rangle\langle +++|+|---\rangle\langle ---|)$$

$$F_{v,y} = \sqrt{\frac{2}{3}}(|i,i,i\rangle\langle i,i,i|+|-i,-i,-i\rangle\langle -i,-i,-i|)$$

$$Z|0/1\rangle \equiv \pm|0/1\rangle$$

$$|\pm\rangle \equiv (|0\rangle \pm |1\rangle)/\sqrt{2}$$

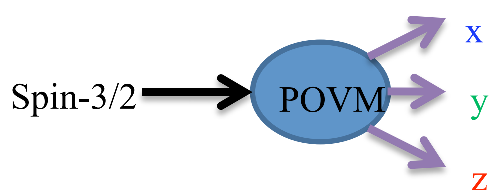
$$X|\pm\rangle \equiv \pm|\pm\rangle$$

$$|\pm i\rangle \equiv (|0\rangle \pm i|1\rangle)/\sqrt{2}$$

$$Y|\pm i\rangle \equiv \pm|\pm i\rangle$$

➔ In terms of spin-3/2, $F_{v,a}$ projects onto S_a=± 3/2 subspace

- POVM outcome (*x*,*y*, or *z*) is random: a_v ={x,y,z} ϵ *A* for all sites v



➔ new quantization axis

- Post-POVM state becomes

$$|\Psi(\mathcal{A})\rangle = \bigotimes_{v\in V(\mathcal{L})} F_{v,a_v}|\Phi_{\mathrm{AKLT}}\rangle = \bigotimes_{v\in V(\mathcal{L})} F_{v,a_v} \bigotimes_{e\in E(\mathcal{L})} |\phi\rangle_e$$

singlets

First result: the post-POVM state is an encoded graph state

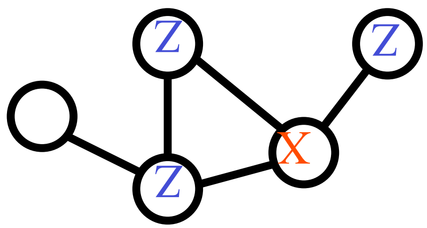
$$|\Psi(\mathcal{A})\rangle = \bigotimes_{v \in V(\mathcal{L})} F_{v,a_v} |\Phi_{\text{AKLT}}\rangle = \bigotimes_{v \in V(\mathcal{L})} F_{v,a_v} \bigotimes_{e \in E(\mathcal{L})} |\phi\rangle_e$$

Graph states: stabilizer formalism

- Stabilizer generators: $K_v = X_v \bigotimes_{u \in \mathrm{Nb}(v)} Z_u$

- On arbitrary graph: graph states

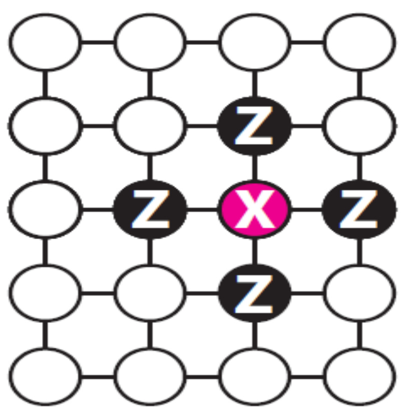
[*Hein, Eisert & Briegel* 04']

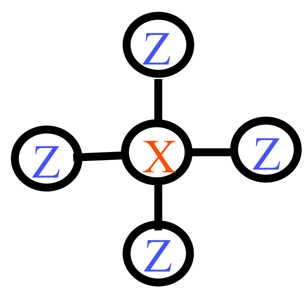


$$K_v|G\rangle = |G\rangle$$

[*Raussendorf &Briegel* 01']

- On regular lattice: Cluster states





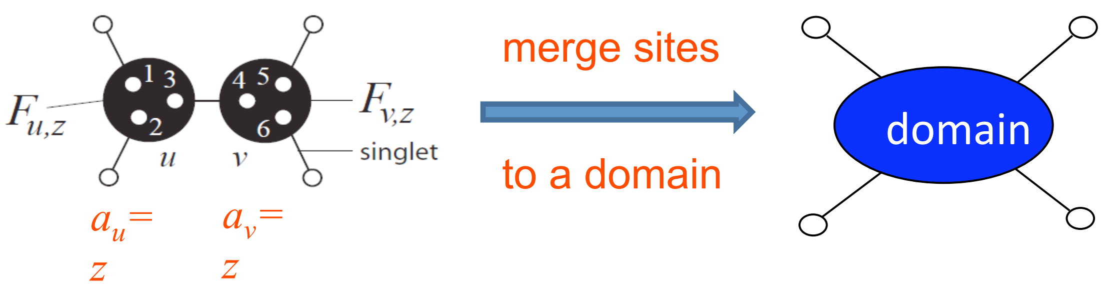
$$|\mathcal{C}\rangle = |\mathcal{C}\rangle$$

Note: $X \equiv \sigma_x, \; Y \equiv \sigma_y, \; Z \equiv \sigma_z$

First result: the post-POVM state is an encoded graph state

$$|\Psi(\mathcal{A})\rangle = \bigotimes_{v\in V(\mathcal{L})} F_{v,a_v} |\Phi_{\rm AKLT}\rangle = \bigotimes_{v\in V(\mathcal{L})} F_{v,a_v} \bigotimes_{e\in E(\mathcal{L})} |\phi\rangle_e$$

- What is the graph? Ans. Determined by two rules
  - Rule 1: merge neighboring sites of same POVM outcome



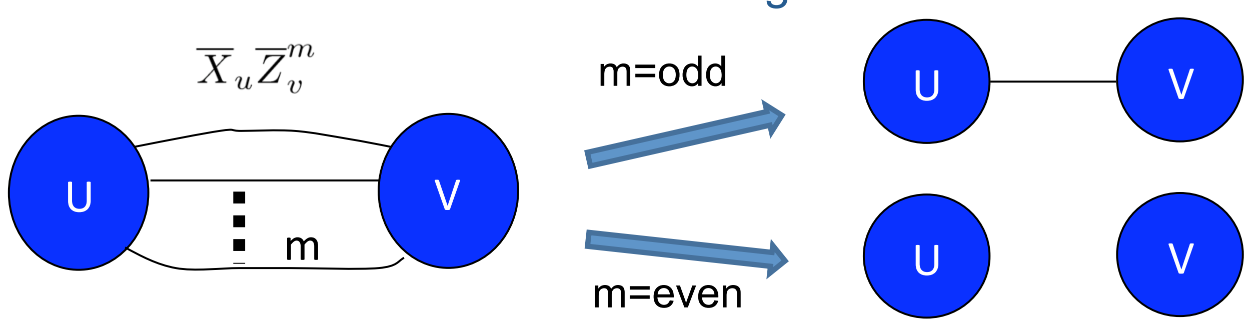
- Encoding:

$$|\bar{0}\rangle \equiv |(000),(111),\ldots\rangle \equiv |\tfrac{3}{2},\tfrac{-3}{2},\ldots\rangle$$

$$|\bar{1}\rangle \equiv |(111),(000),\ldots\rangle \equiv |\tfrac{-3}{2},\tfrac{3}{2},\ldots\rangle$$

can be reduced to single site

  - Rule 2: modulo-2 inter-domain edges



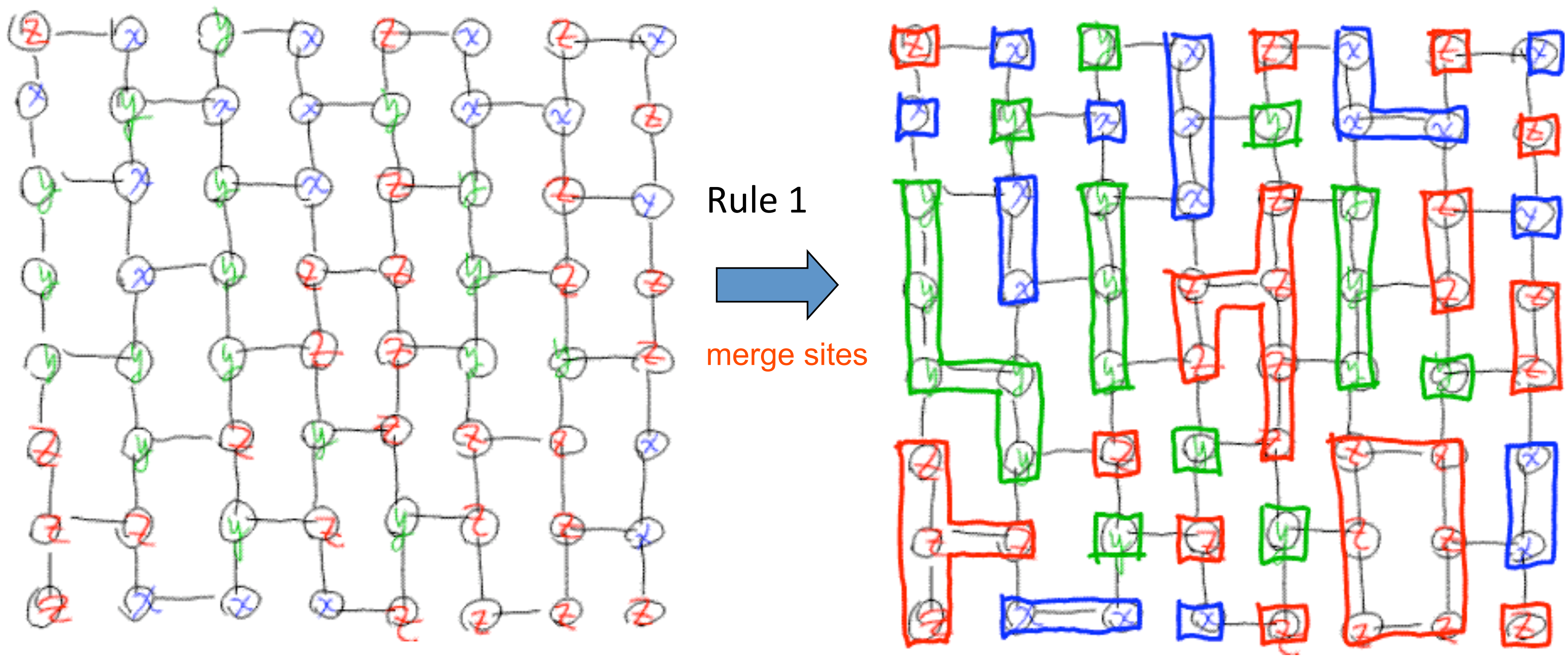
[keep any one inter-domain edge]

[remove all inter-domain edges]

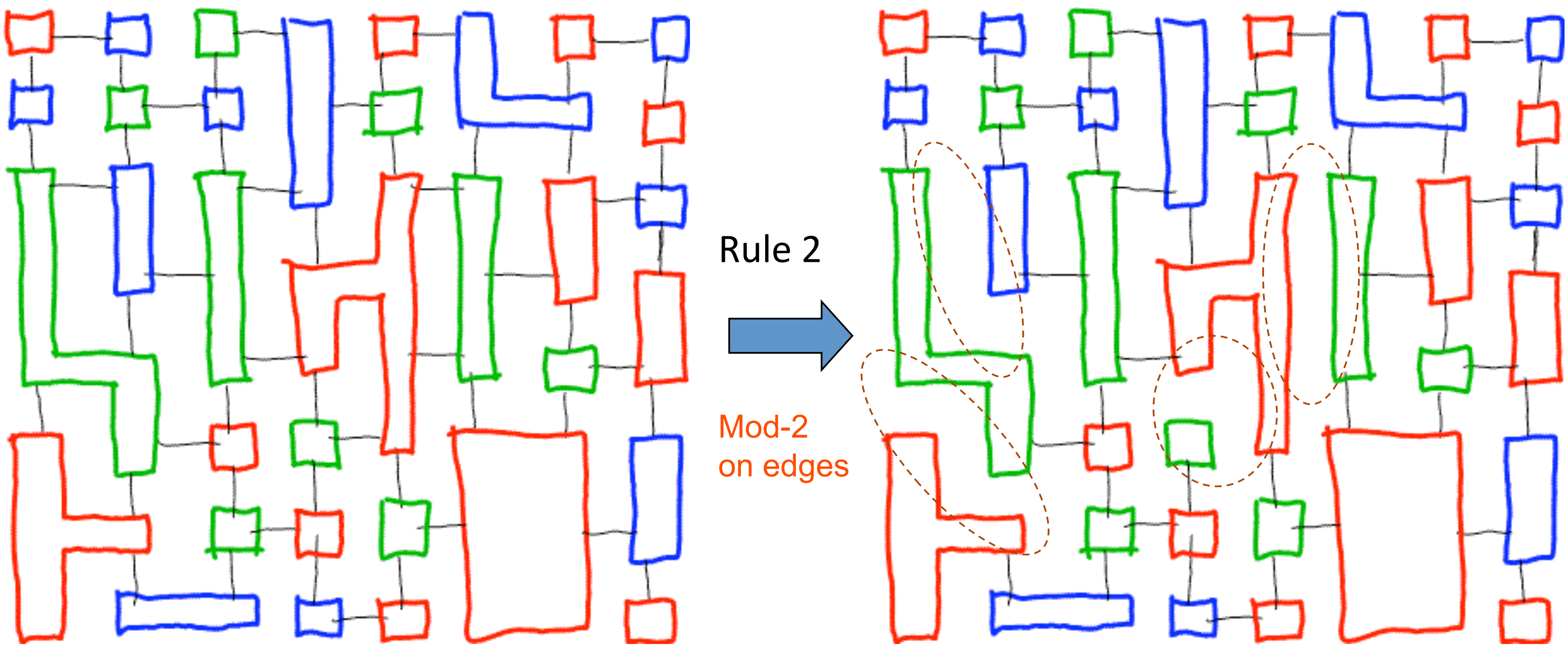
Example

- For convenience use brick-wall structure to represent honeycomb
- Use open boundary condition (terminated by spin-1/2's, not drawn)

POVM outcomes: x (blue), y (green) or z (red)

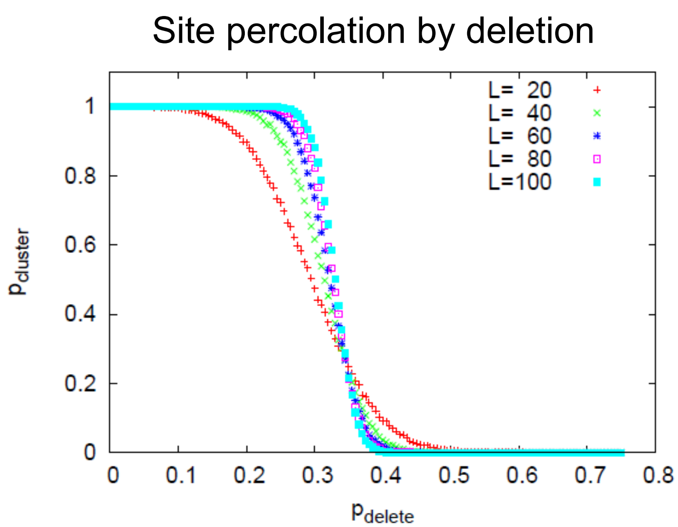


Example cont'd



Second result: can convert typical graph states to cluster states

- Typical graphs are in percolated phase (with macroscopic # of vertices |V|, edges |E|, independent loops or Betti # B)



- Honeycomb: deg=3

  |E|=1.5|V|, B=0.5|V|

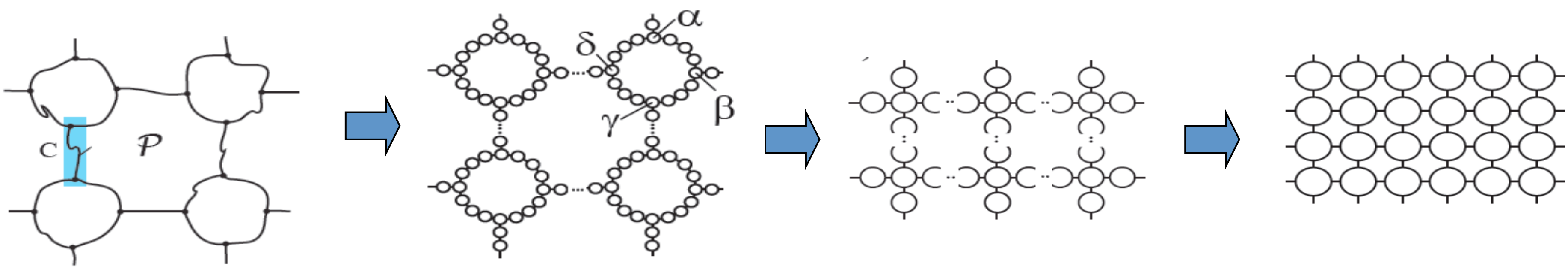
- Typical graphs: deg=3.52

  |E|=1.76|V|, B=0.76|V|

- Square lattice: deg=4

  |E|=2|V|, B=|V|

- Can identify a suitable subgraph and trim it down (by Pauli meas.) to a square lattice:



Summary

- ❑ We showed that the 2D AKLT state on the honeycomb lattice is universal for measurement-based quantum computation

I. First approach: constructed a scheme for measurement-based quantum computation (single + two-qubit gates)

✓ arXiv: 1009.3491 by Miyake

II. Second approach: showed it can be locally converted to a cluster state

✓ arXiv: 1009.2840 by Wei, Affleck & Raussendorf