The uncertainty principle determines the non-locality of Quantum Mechanics

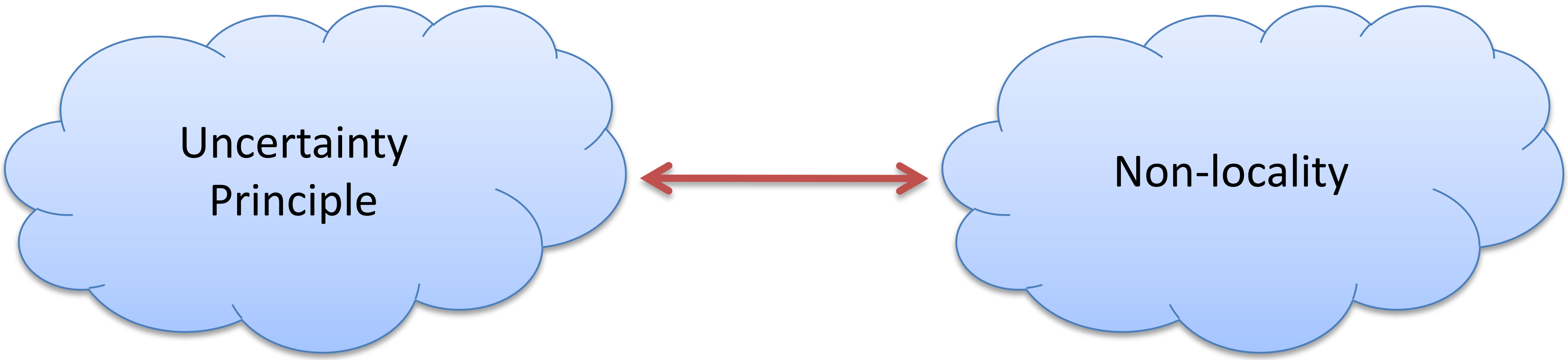
Stephanie Wehner

Joint work with Jonathan Oppenheim
Science, vol 330, no 7006, pp 1072-1074

Centre for Quantum Technologies

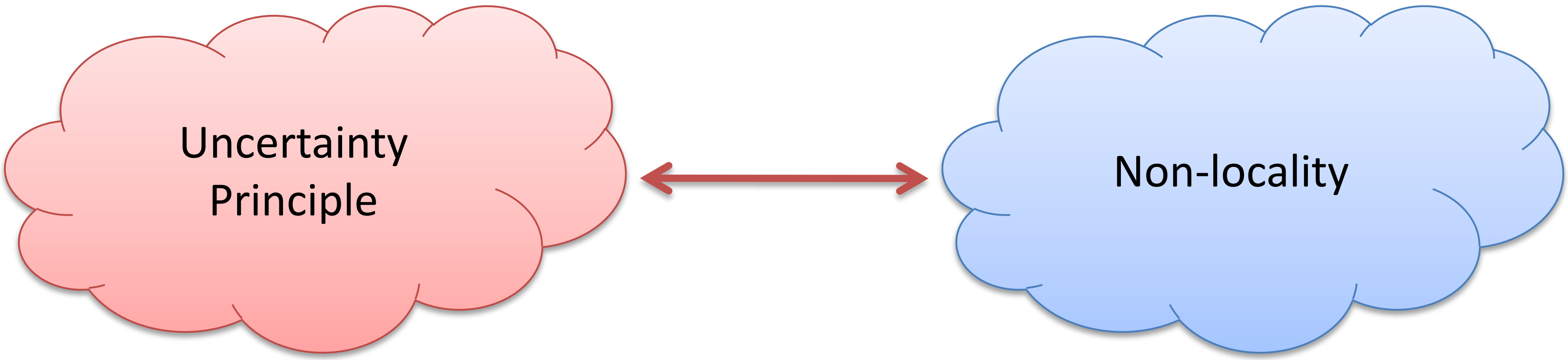
NUS National University of Singapore

Linking two fundamental concepts



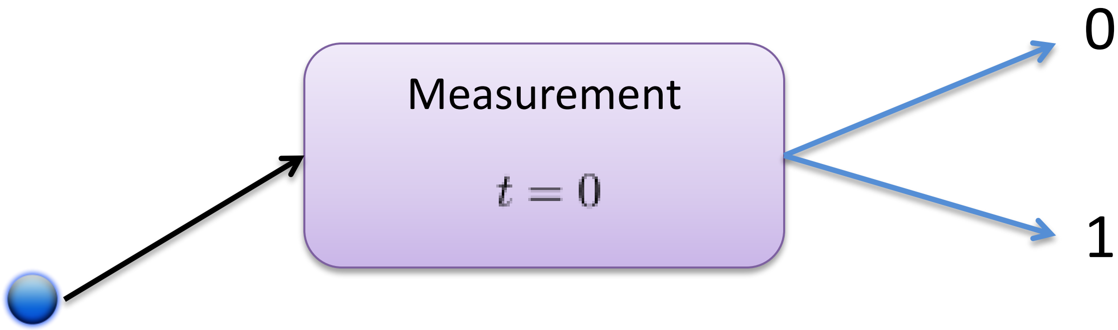
Steering

Linking two fundamental concepts



Steering

What is uncertainty?



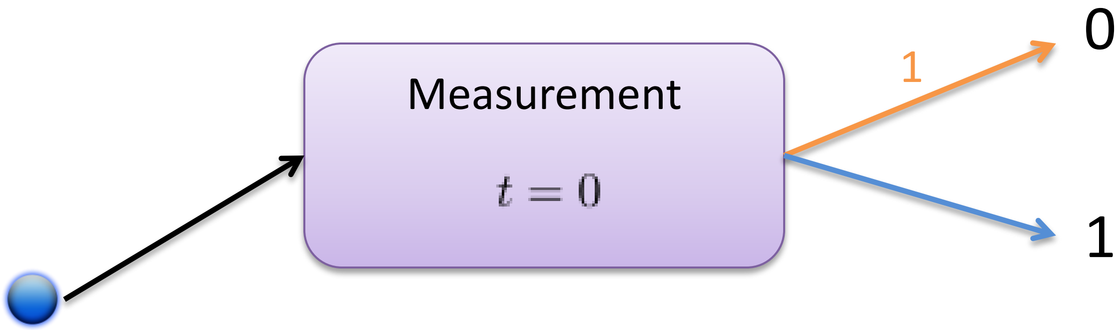
p(b|t)

Probability of obtaining outcome bit **b**

when performing measurement **t**

“Cannot know both the particle’s position and its momentum at the same time” observed by Werner Heisenberg in 1927.

What is uncertainty?



p(b|t)

Probability of obtaining outcome bit **b**

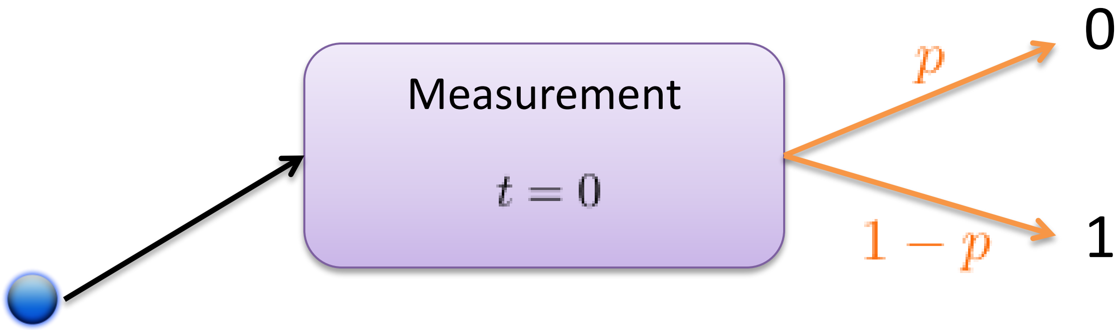
when performing measurement **t**

Certainty:
We know the property to be determined perfectly.

For some outcome b (here $b = 0$) we have $p(b|t) = 1$

“Cannot know both the particle’s position and its momentum at the same time” observed by Werner Heisenberg in 1927.

What is uncertainty?



p(b|t)

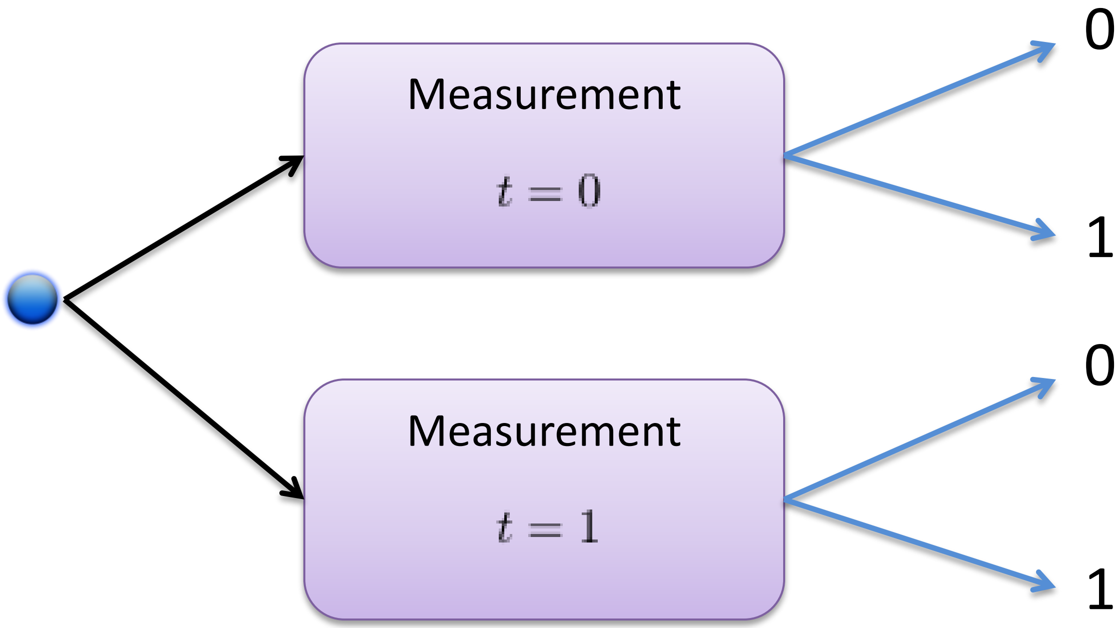
Probability of obtaining outcome bit **b**

when performing measurement **t**

Uncertainty:
We do *not* know the property to be determined perfectly.
$(0 < p < 1)$

"Cannot know both the particle's position and its momentum at the same time" observed by Werner Heisenberg in 1927.

Uncertainty relations



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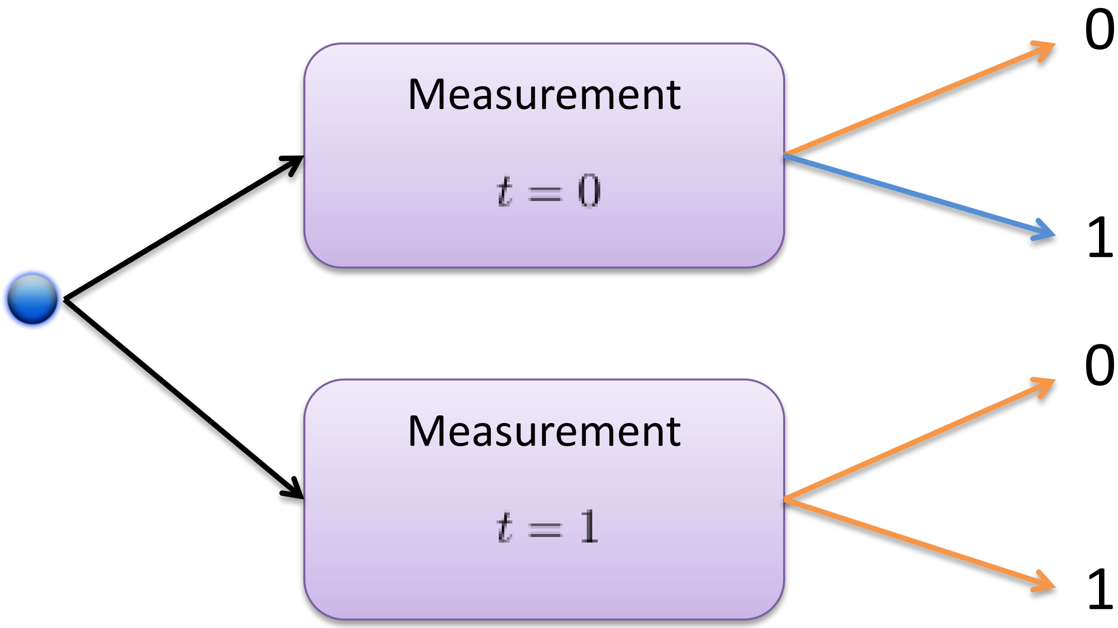
Uncertainty relations



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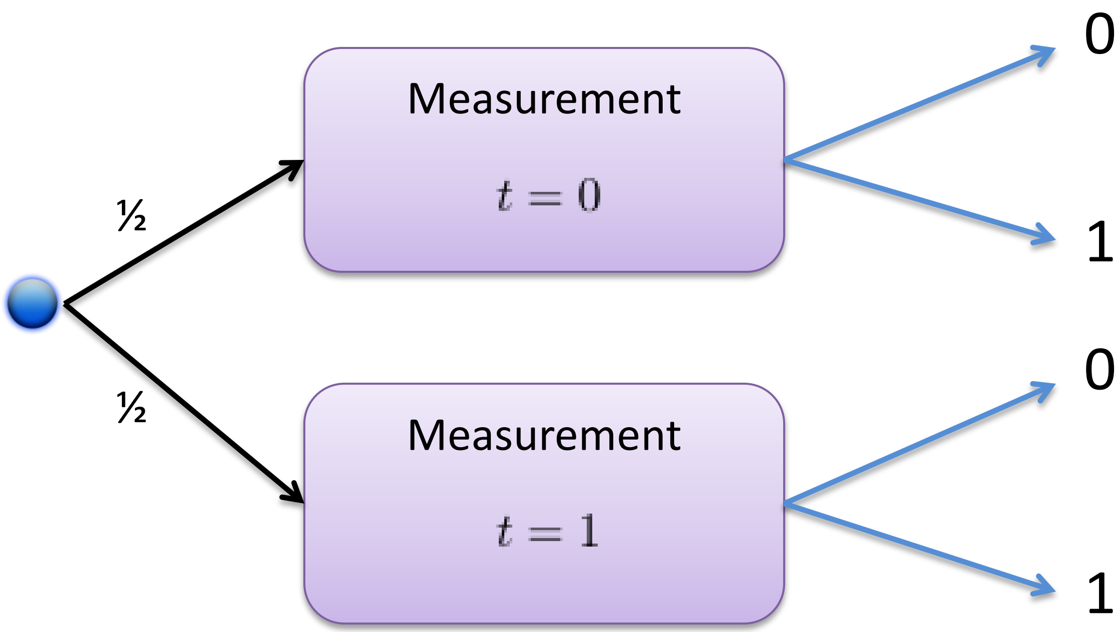
Choice of measurements



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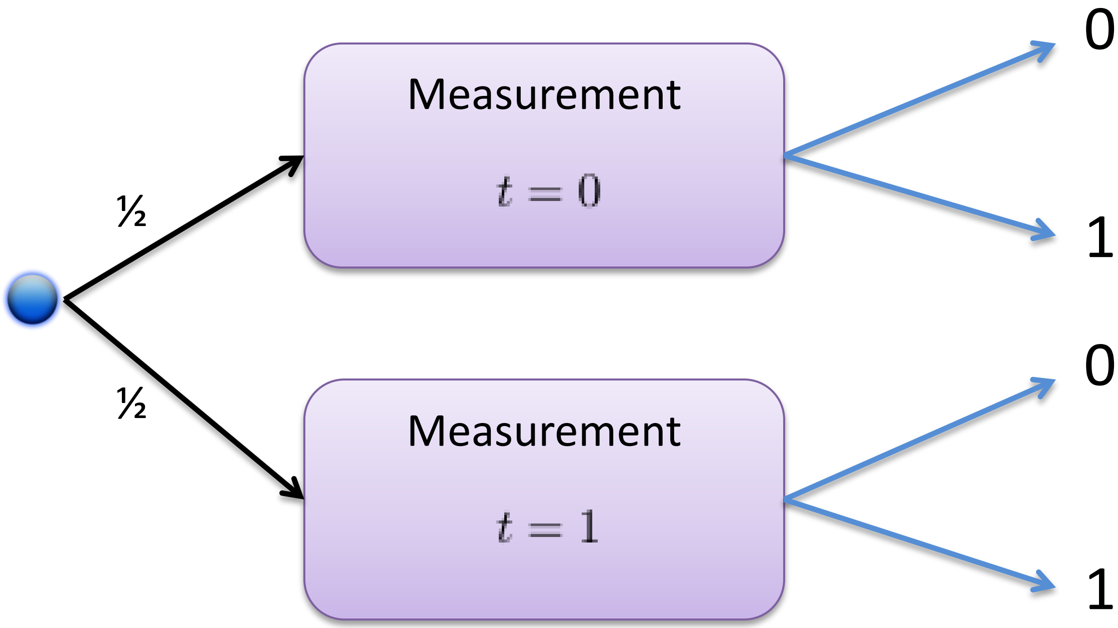
Fine-grained uncertainty relations



p(b|t)

Probability of obtaining outcome bit **b**

when performing measurement **t**

Suppose that for **any** quantum state:

$$\frac{1}{2}\left(p(0|t=0) + p(0|t=1)\right) \leq c$$
$$\frac{1}{2}\left(p(0|t=0) + p(1|t=1)\right) \leq c$$
$$\frac{1}{2}\left(p(1|t=0) + p(0|t=1)\right) \leq c$$
$$\frac{1}{2}\left(p(1|t=0) + p(1|t=1)\right) \leq c$$

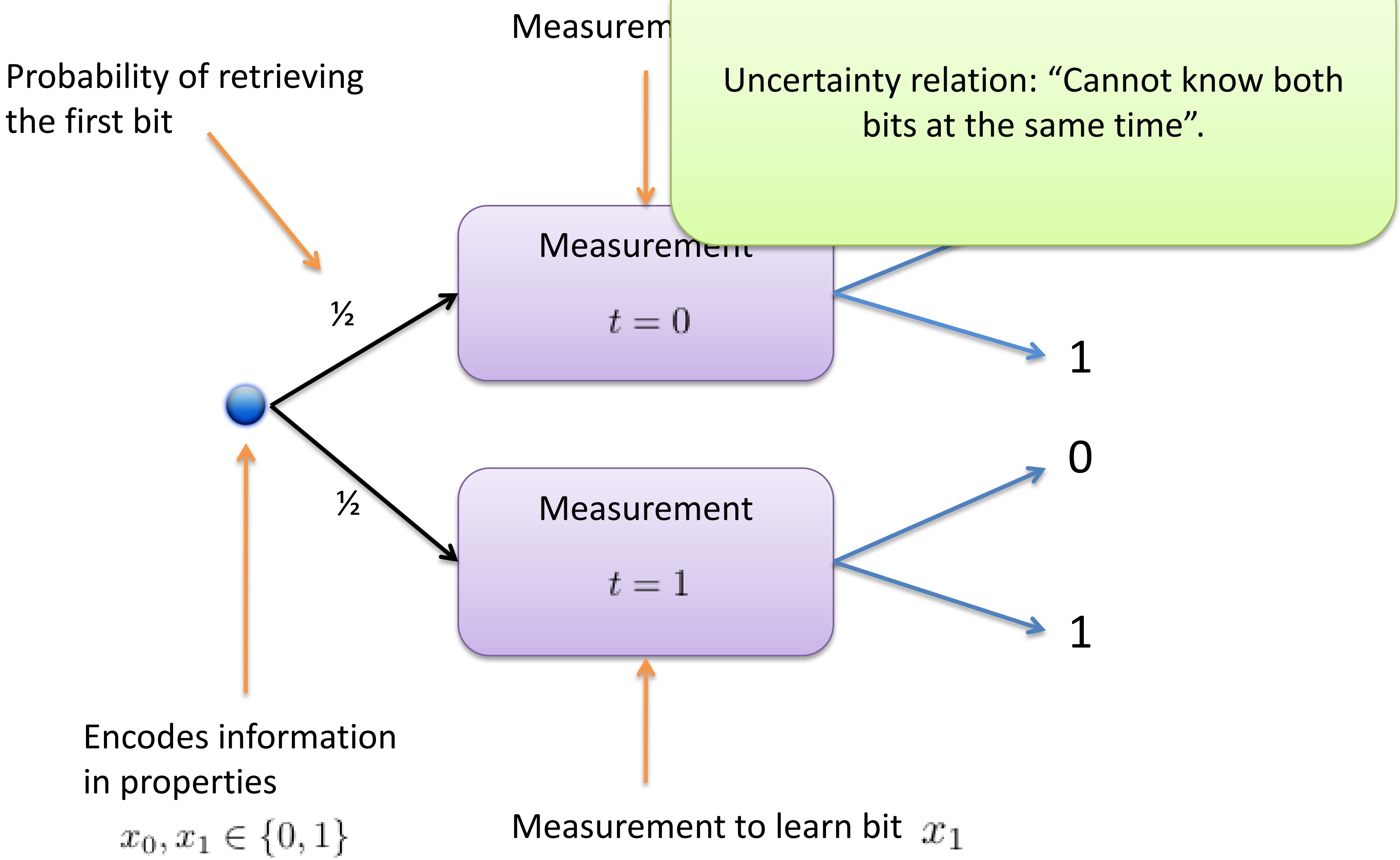
Fine-grained uncertainty relation:

Imagine c< 1.
Cannot have

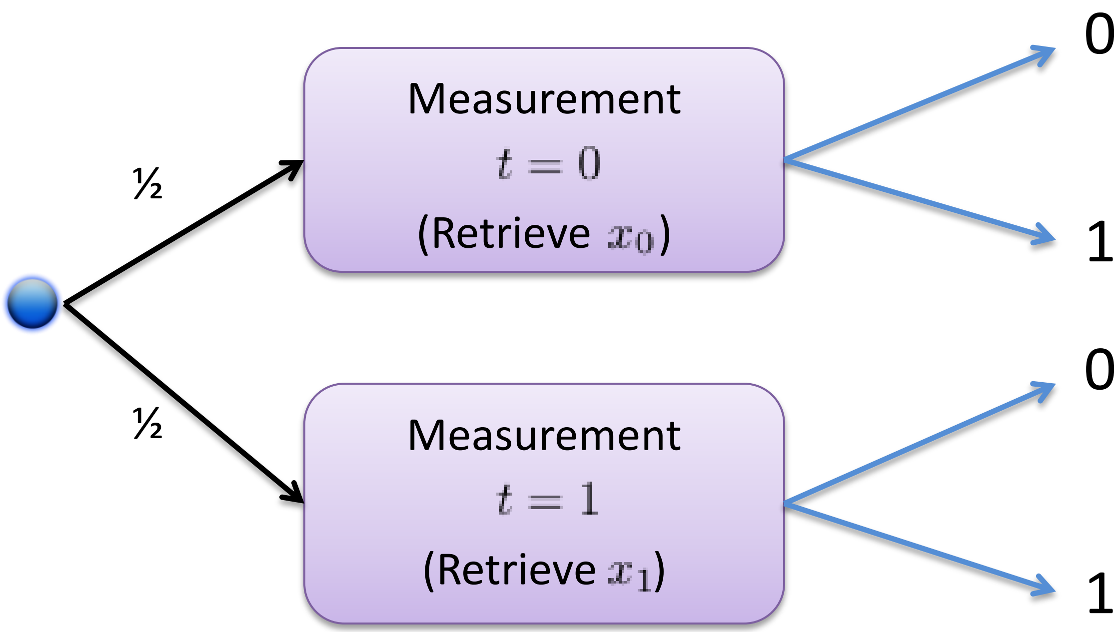
$$p(0|t=0) = p(1|t=1) = 1$$

Cannot know both properties!

A change of perspective



Encoding and decoding information



p(b|t)

Probability of obtaining outcome bit **b**

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Suppose that for **any** quantum state:

$$\frac{1}{2}\left(p(0|t=0)+p(0|t=1)\right)\leq c$$

$$\frac{1}{2}\left(p(0|t=0)+p(1|t=1)\right)\leq c$$

$$\frac{1}{2}\left(p(1|t=0)+p(0|t=1)\right)\leq c$$

$$\frac{1}{2}\left(p(1|t=0)+p(1|t=1)\right)\leq c$$

Coding interpretation:

Choose to retrieve x_0 or x_1 with probability ½

Average probability we correctly retrieve the desired bit from $x_0 x_1 = (0,0)$ for **any** encoding using those measurements.

Maximally certain states

Suppose that for **any** quantum state:

$$\frac{1}{2}\left(p(0|t=0)+p(0|t=1)\right) \leq c$$

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Maximally certain states

For the **maximally certain states**

$\frac{1}{2}\left(p(0|t=0)+p(0|t=1)\right)=c$ $\xleftarrow{\rho_{0,0}}$ Encoding of $x=x_0x_1=(0,0)$

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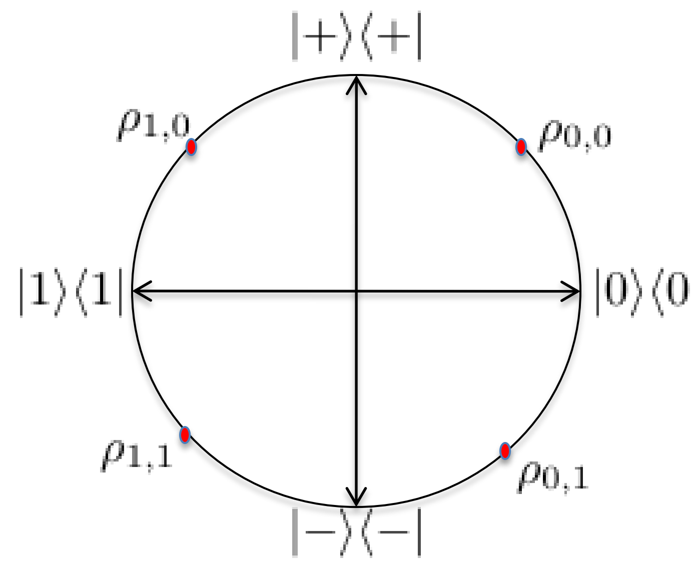
An example:
Measurements in Z and X basis

$$\frac{1}{2}\text{tr}\left[\rho\left(|0\rangle\langle 0|+|+\rangle\langle +|\right)\right]\leq\frac{1}{2}+\frac{1}{2\sqrt{2}}$$
$$\frac{1}{2}\text{tr}\left[\rho\left(|0\rangle\langle 0|+|-\rangle\langle -|\right)\right]\leq\frac{1}{2}+\frac{1}{2\sqrt{2}}$$
$$\vdots$$



General form

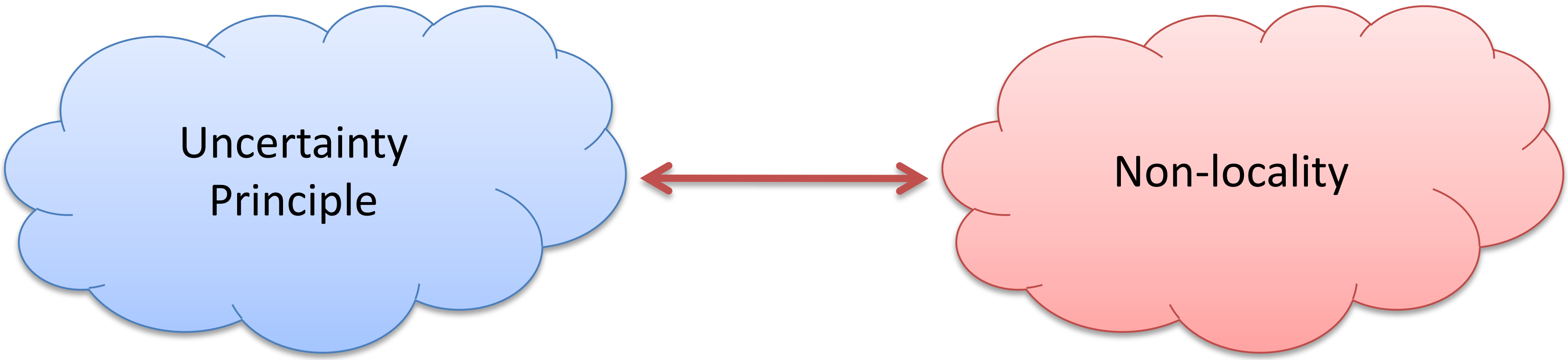
Set of measurements T
Set of outcomes B
Probability distribution over measurements $p(t)$

$$\mathcal{F}_{\mathrm{UR}} = \left\{ \sum_t p(t) p(x_t|t) \leq \zeta_{\vec{x}} \mid \vec{x} = (x_1, x_2, \ldots, x_{|T|}) \in B^{\times |T|} \right\}$$

Uncertainty relations have an operational interpretation telling us how well we can retrieve information using two particular measurements.

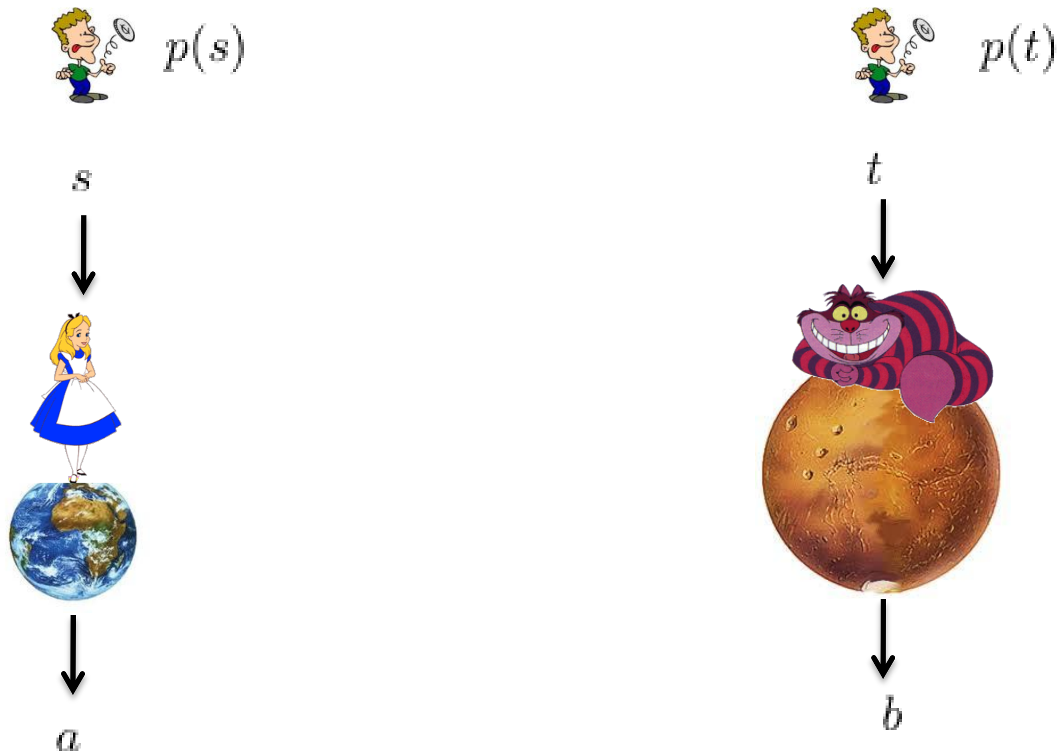
Maximally certain states are the best encoding with respect to those measurements.

Linking two fundamental concepts



Steering

Non-locality as a Game



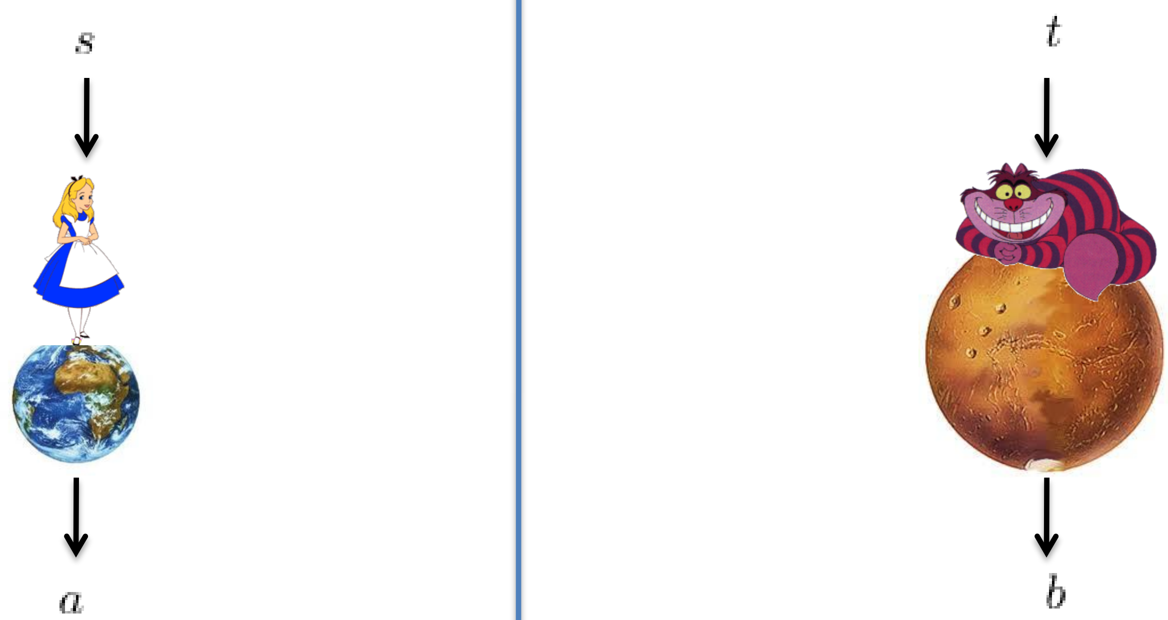
Assumption about rules:

For all questions s, t and all answers a there exists exactly one winning answer b for Bob

Rules written as strings (W,Christandl, Doherty, '08): $\vec{x}_{s,a} = (x_{s,a}^{t=0}, x_{s,a}^{t=1}, \ldots)$

Correct answer for $t = 0$ Correct answer for $t = 1$

Making Alice and Bob's life difficult..



1. Can agree on any strategy beforehand
2. Once the game starts, they can no longer communicate

Amount of non-locality measured by winning probability

$$p_{\text{win}} = \sum_{s,t} p(s,t) \sum_{a,b} p(a, b = x^t_{s,a} | s, t)$$

Example: CHSH as a game

$s \in \{0,1\}$

$p(s) = \frac{1}{2}$ $\downarrow$

$\downarrow$

$a \in \{0,1\}$

$t \in \{0,1\}$

$p(t) = \frac{1}{2}$ $\downarrow$

$\downarrow$

$b \in \{0,1\}$

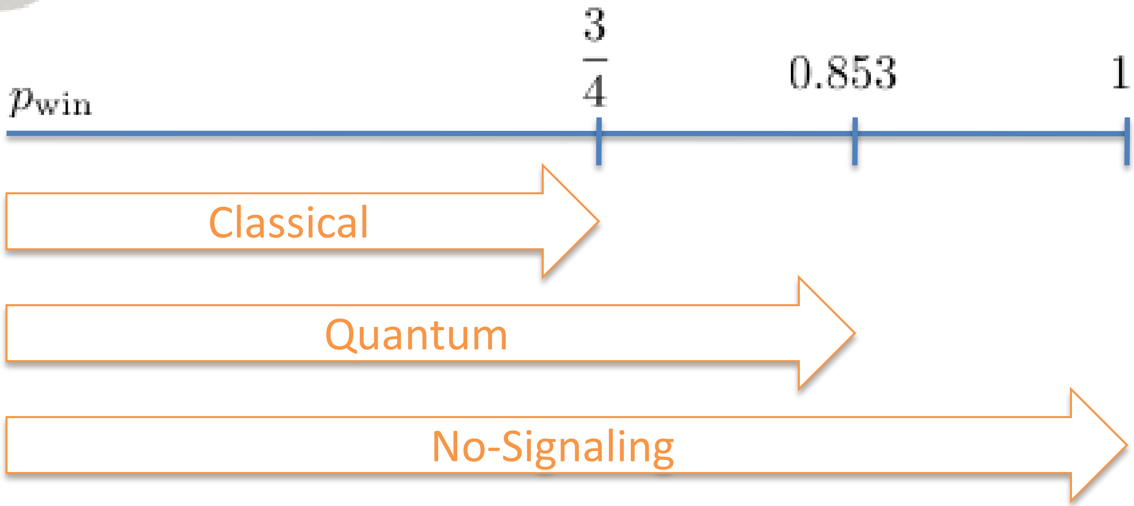
Rules as an equation $s \cdot t = a + b \mod 2$

Rules as strings

$$\left.\begin{array}{l} x_{0,0} = (0,0) \\ x_{0,1} = (1,1) \end{array}\right\} s = 0$$

$$\left.\begin{array}{l} x_{1,0} = (0,1) \\ x_{1,1} = (1,0) \end{array}\right\} s = 1$$

Why is non-locality limited?



Popescu and Rohrlich '96: Why is nature not more non-local?

More non-locality would allow for much better information processing:
Less communication (van Dam'00), store more information (verSteeg, W '08),
send more information per bit (Pawlowski, Paterek, Kaszlikowski, Scarani, Winter, Zukowski '09),…
Being locally quantum implies quantum correlations (Barnum, Beigi,Boixo,Elliot,Wehner '09)…

Would need to break the uncertainty principle for a set of measurements to have more non-locality.

Producing answers

s

a

Measurement

s

Outcome

a

t

b

Encoding of

$$x_{s,a} = (x_{s,a}^{t=0}, x_{s,a}^{t=1})$$

$$x_{0,0} = (0,0)$$
$$x_{0,1} = (1,1)$$
$$x_{1,0} = (0,1)$$
$$x_{1,1} = (1,0)$$

To give the right answer Bob needs to retrieve bit $x_{s,a}^{t}$ from the encoding

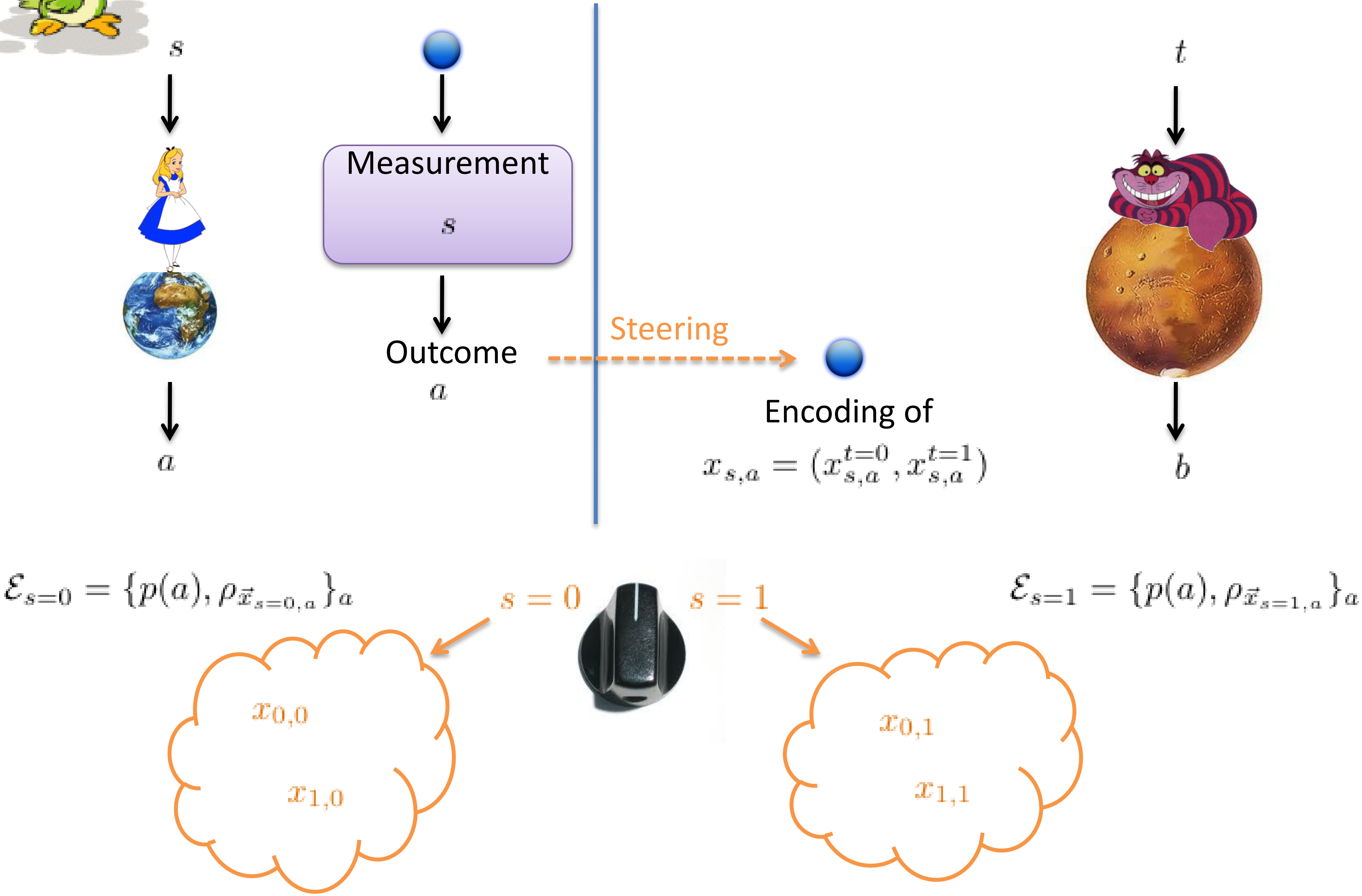
Linking two fundamental concepts

Uncertainty Principle

Non-locality

Steering

Steering (Schroedinger, 1935)



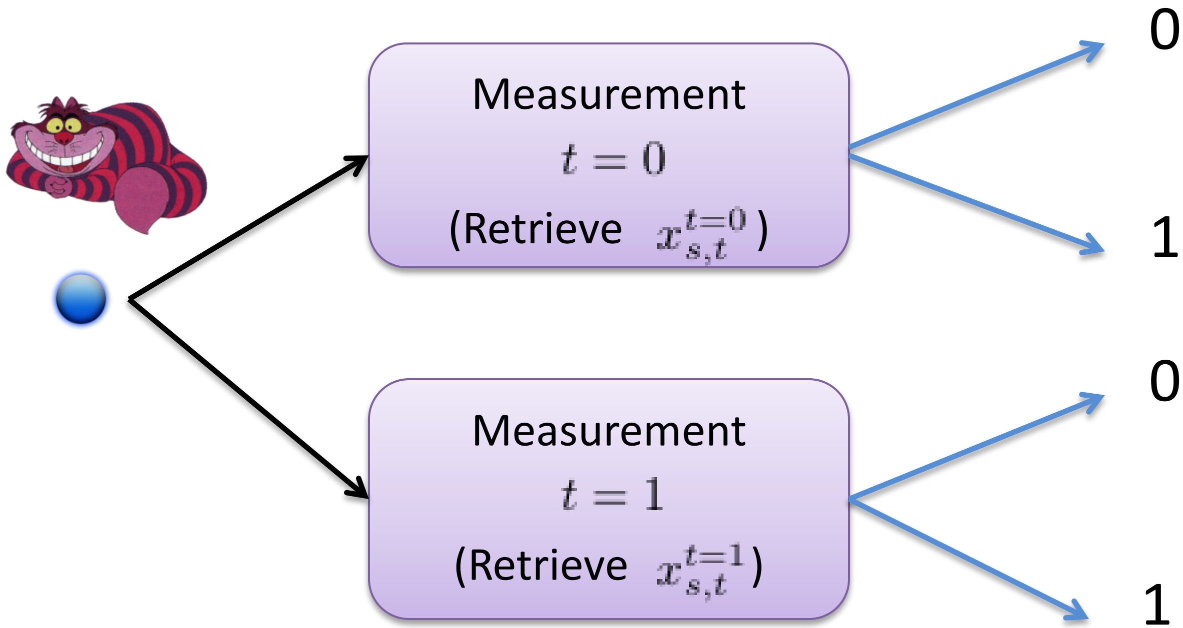
Linking two fundamental concepts

Uncertainty Principle

Non-locality

Steering

Producing answers



Uncertainty principle limits non-locality

$$p_{\text{win}} = \sum_{s,t} p(s,t) \sum_{a,b} p(a, b = x^t_{s,a} | s,t)$$

$$= \sum_{s,a} p(s)p(a|s) \left[\sum_t p(t) p(x^t_{s,a} | s,t,a) \right]$$

$$\leq \sum_{s,a} p(s)p(a|s) \zeta_{\vec{x}_{s,a}}$$

Uncertainty relation, for any state

$$\sum_t p(t) p(x^t_{s,a} | t)_{\rho_{\vec{x}_{s,a}}} \leq \zeta_{\vec{x}_{s,a}}$$

Steering to the maximally certain states

1. Fix Bob’s measurements

2. For these measurements, for the maximally certain states $\rho_{\vec{x}_{s,a}}$

$$\sum_t p(t)p(x^t_{s,a}|t) = \zeta_{\vec{x}_{s,a}}$$

3. If Alice can steer to the maximally certain states

$$p_{\text{win}} = \sum_{s,t} p(s,t) \sum_{a,b} p(a, b = x^t_{s,a}|s,t)$$

$$= \sum_{s,a} p(s)p(a|s)\zeta_{\vec{x}_{s,a}}$$

For XOR games, quantum Alice can steer to the maximally certain states of Bob’s optimal measurements:

More non-locality would require a violation of the uncertainty principle.

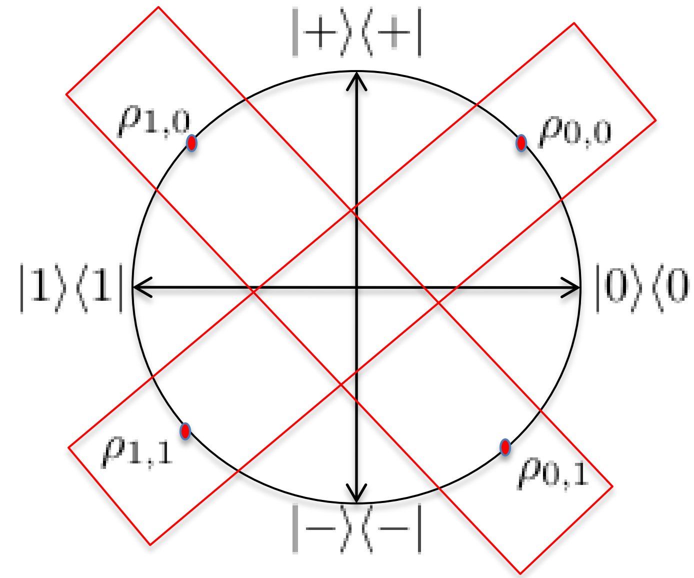
The amount of non-locality is exactly determined by the amount of ‘’uncertainty’’ and our ability to steer:

In any theory, going beyond the optimum winning probability would require a violation of the uncertainty principle with respect to the set of steerable states.

Example CHSH

Rules
$$x_{0,0} = (0,0)$$
$$x_{0,1} = (1,1)$$
$$x_{1,0} = (0,1)$$
$$x_{1,1} = (1,0)$$

distribution over questions $p(s) = p(t) = \frac{1}{2}$



Optimal measurements for Bob: measure Z and X.

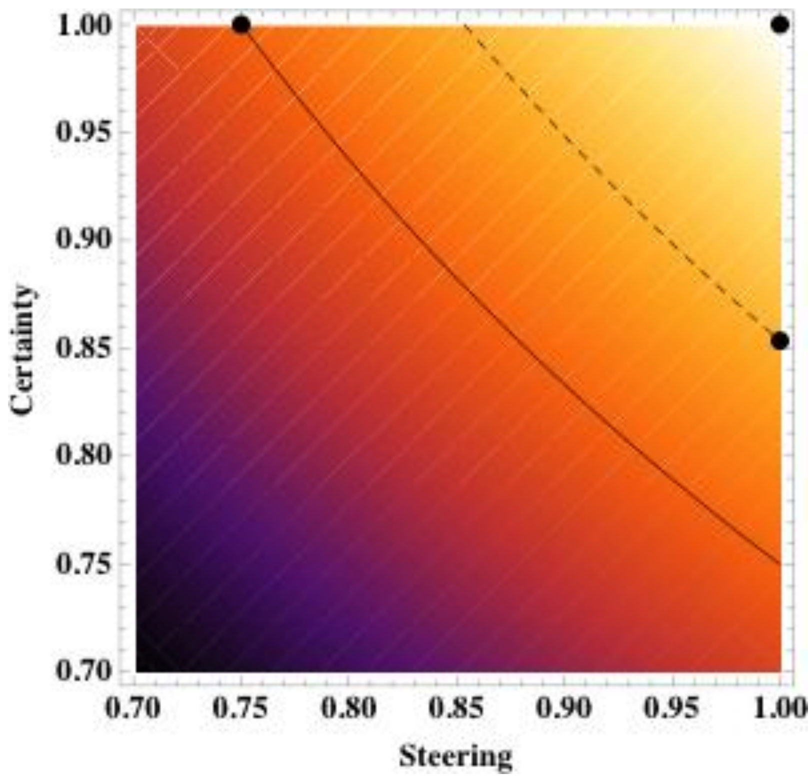
$$\frac{1}{2}\mathrm{tr}\left[\rho\left(|0\rangle\langle 0| + |+\rangle\langle+|\right)\right] \leq \frac{1}{2} + \frac{1}{2\sqrt{2}}$$
$$\frac{1}{2}\mathrm{tr}\left[\rho\left(|0\rangle\langle 0| + |-\rangle\langle-|\right)\right] \leq \frac{1}{2} + \frac{1}{2\sqrt{2}}$$
$$\vdots$$

For the maximally certain states

$$\frac{1}{2}\left(\rho_{0,0} + \rho_{1,1}\right) = \frac{\mathrm{id}}{2} \qquad \frac{1}{2}\left(\rho_{1,0} + \rho_{0,1}\right) = \frac{\mathrm{id}}{2}$$

Non-locality for different theories

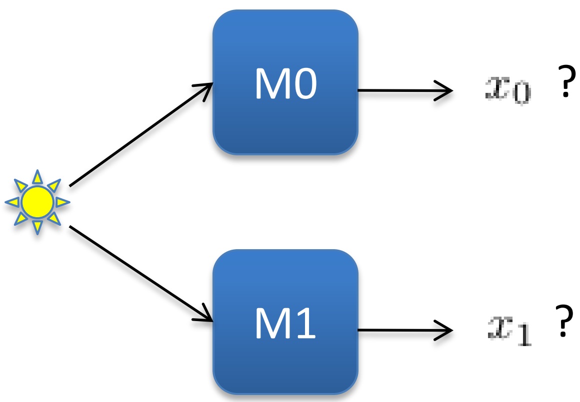
- Classical deterministic: No uncertainty, but also no steering.
- Quantum: Uncertainty, but perfect steering.
- No-signaling: No uncertainty, and perfect steering.



Uncertainty and non-locality are linked in **any** possible physical theory.

Uncertainty vs. complementarity

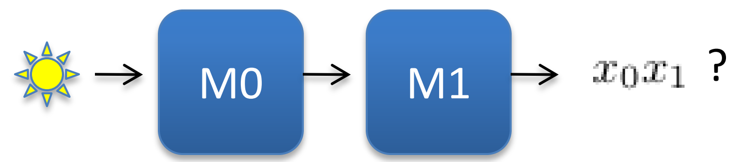
Uncertainty



CHSH example:

$$\frac{1}{2}\left(p(x_0|t=0)+p(x_1|t=1)\right)$$
$$\leq \frac{1}{2}+\frac{1}{2\sqrt{2}}$$

Complementarity



CHSH example: Cannot learn

$$x_0 \oplus x_1 = s$$

Could be equally uncertain (and non-local) but less complementary!

Summary and open questions

- Linked two fundamental concepts
- Degree of non-locality is already determined by two concepts: uncertainty relations and steering
- Holds for *any* physical theory.

Summary and open questions

- What properties fully characterize quantum theory?
- How strong can uncertainty relations for multiple measurements be in quantum theory?
- Uncertainty vs. complementarity?

Thank you!

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Postdoc Positions!