# Quantum boolean functions

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# Introduction

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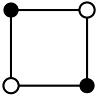
Many interpretations:

- Truth table
- Subset of $[2^n] = \{1, \ldots, 2^n\}$
- Family of subsets of $[n]$
- Colouring of the $n$-cube
- Voting system
- Decision tree
- ...

| $x_1$ | $x_2$ | $f(x_1, x_2)$ |
|---|---|---|
| 0 | 0 | 0 |
| 0 | 1 | 1 |
| 1 | 0 | 1 |
| 1 | 1 | 0 |

$$\mathcal{F} = \{\{1\}, \{2\}\}$$



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Any $f : \{0,1\}^n \to \mathbb{R}$ has the expansion

$$f = \sum_{S \subseteq [n]} \hat{f}_S \chi_S$$

for some real $\{\hat{f}_S\}$ – the Fourier coefficients of $f$.

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This talk: quantum generalisations of these results.

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- The *bit oracle* $|x\rangle|y\rangle \mapsto |x\rangle|y + f(x)\rangle$,
- The *phase oracle* $|x\rangle \mapsto (-1)^{f(x)}|x\rangle$

...and both of these give QBFs.

# Other examples of QBFs

A projector $P$ onto any subspace gives rise to a QBF: take $f = \mathbb{I} - 2P$. Thus:

- Any quantum algorithm solving a decision problem gives rise to a QBF (consider it as a projector onto the "yes" inputs)
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There are uncountably many QBFs, even on one qubit: for any real $\theta$, consider

$$f = \begin{pmatrix} \cos\theta & \sin\theta \\ \sin\theta & -\cos\theta \end{pmatrix}$$

# “Fourier analysis” for QBFs

It turns out that a natural analogue of the characters of $\mathbb{Z}_2$ are the Pauli matrices:

$$\sigma^0 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \ \sigma^1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \ \sigma^2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \text{ and } \sigma^3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}.$$

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We write a tensor product of Paulis (a stabiliser operator) as $\chi_{\mathbf{s}} \equiv \sigma^{s_1} \otimes \sigma^{s_2} \otimes \cdots \otimes \sigma^{s_n}$, where $s_j \in \{0, 1, 2, 3\}$.

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It's well known that any $n$ qubit Hermitian operator $f$ has an expansion

$$f = \sum_{\mathbf{s} \in \{0,1,2,3\}^n} \hat{f}_{\mathbf{s}} \chi_{\mathbf{s}},$$

with all the $\hat{f}_{\mathbf{s}}$ real. This is our analogue of the Fourier expansion of a function $f : \{0, 1\}^n \to \mathbb{R}$.

# Norms and closeness

Some definitions we'll need later:

- The (normalised) Schatten $p$-norm: for any $d$-dimensional operator $f$,

  $$\|f\|_p \equiv \left(\frac{1}{d}\sum_{j=1}^{d}\sigma_j^p\right)^{\frac{1}{p}},$$

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- Closeness: Let $f$ and $g$ be two QBFs. Then we say that $f$ and $g$ are $\epsilon$-close if $\|f - g\|_2^2 \leqslant 4\epsilon$.

# Quantum property testing

Consider the following representative example:

**Stabiliser testing**

Given oracle access to an unknown QBF $f$ on $n$ qubits, determine whether $f$ is a stabiliser operator $\chi_{\mathbf{s}}$ for some $\mathbf{s}$.

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We give a test (the quantum stabiliser test) that has the following property.

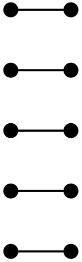
**Proposition**

Suppose that a QBF $f$ passes the quantum stabiliser test with probability $1 - \epsilon$. Then $f$ is $\epsilon$-close to a stabiliser operator $\chi_{\mathbf{s}}$.

The test uses 2 queries (best known classical test uses 3).

# Quantum stabiliser testing algorithm (sketch)

1. Apply $f$ to the halves of $n$ maximally entangled states $|\Phi\rangle^{\otimes n}$ resulting in a quantum state $|f\rangle = f \otimes \mathbb{I}|\Phi\rangle^{\otimes n}$.



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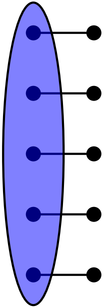

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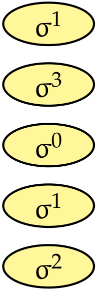

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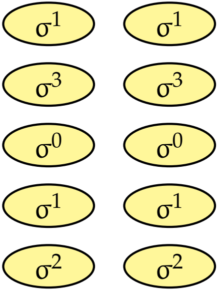

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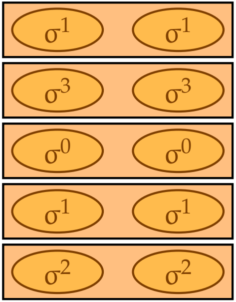

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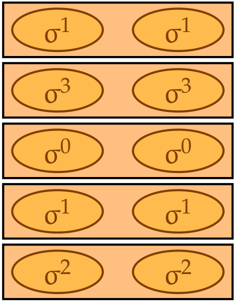
5. Accept if all measurements say "yes".




It turns out that for the stabiliser test $\Pr[\text{test accepts}] = \sum_{\mathbf{s}} \hat{f}_{\mathbf{s}}^4$, which implies the proposition.

# Hypercontractivity and noise

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- We state and prove a different generalisation, to Hermitian operators on $n$ qubits, which has an interpretation in terms of the qubit depolarising channel.

# Quantum hypercontractivity

- Let $\mathcal{D}_\epsilon$ be the qubit depolarising channel with noise rate $1-\epsilon$, i.e. $\mathcal{D}_\epsilon(f) = \frac{(1-\epsilon)}{2}\operatorname{tr}(f)\mathbb{I} + \epsilon f$.

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Notes on the proof:

- Not a simple generalisation of the classical proof, but would be if the maximum output $p \to q$ norm were multiplicative!
- Proof is by induction on $n$ and relies on a non-commutative generalisation of Hanner's inequality by King.

# Application: A quantum FKN theorem

- The classical FKN (Friedgut-Kalai-Naor) theorem: Let $f$ be a boolean function. Then, if $\sum_{|S|>1} \hat{f}_S^2 < \epsilon$, $f$ is $O(\epsilon)$-close to depending on 1 variable (being a dictator).

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- This result is the first stab at understanding the structure of the Fourier expansion of QBFs.
- Applications? “Quantum voting”?

# Computational learning of QBFs

What does it mean to approximately learn a quantum boolean function $f$?

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We give a quantum algorithm that outputs the large Fourier coefficients of $f$. If $f$ is almost completely determined by these, this is sufficient to approximately learn $f$.

# Computational learning of QBFs

**Quantum Goldreich-Levin algorithm**

Given oracle access to a quantum boolean function $f$, and given $\gamma, \delta > 0$, there is a poly $\left(n, \frac{1}{\gamma}\right) \log\left(\frac{1}{\delta}\right)$-time algorithm which outputs a list $L = \{\mathbf{s}_1, \mathbf{s}_2, \ldots, \mathbf{s}_m\}$ such that with prob. $1 - \delta$: (1) if $|\hat{f}_{\mathbf{s}}| \geqslant \gamma$, then $\mathbf{s} \in L$; and (2) if $\mathbf{s} \in L$, $|\hat{f}_{\mathbf{s}}| \geqslant \gamma/2$.

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Example: learning dynamics of a 1D spin chain. Informally:

## Theorem

Let $H$ be a Hamiltonian corresponding to an $n$-site spin chain, and let $t = O(\log n)$. Then we can approximately learn the QBFs $\sigma_j^s(t) \equiv e^{-itH} \sigma_j^s e^{itH}$ with $\mathrm{poly}(n)$ uses of $e^{itH}$.

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So we can predict the outcome of measuring $\sigma^s$ on site $j$ after a short time, on average over all input states.

# Conclusions

Summary:

- We've defined a quantum generalisation of the concept of a boolean function.
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We still have many open conjectures... such as:

- Conjecture: There exists an efficient quantum property tester for dictators.
- Conjecture: Every traceless quantum boolean function has an influential qubit: there is a $j$ such that $\| \operatorname{tr}_j f \otimes \mathbb{I}/2 - f \|_2^2 = \Omega((\log n)/n)$.
- ...

# The end

Further reading:

- Our paper: arXiv:0810.2435.
- Survey paper by Ronald de Wolf: http://theoryofcomputing.org/articles/gs001/gs001.pdf
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Thanks for your time!