Predictive Quantum Learning

Dmitry Gavinsky

NEC Labs, Princeton

Learning under Standard Setting

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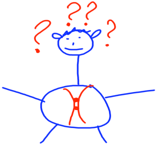

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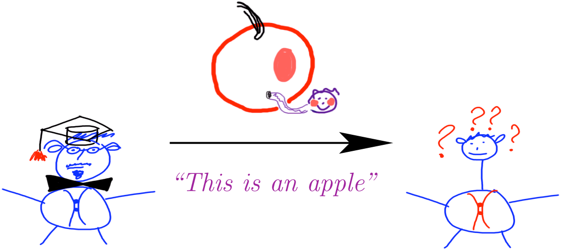

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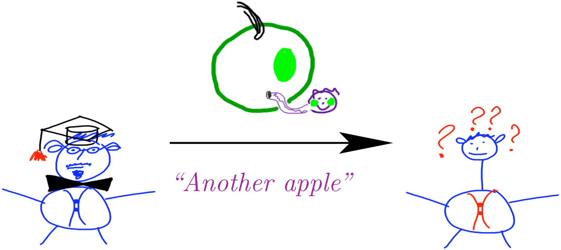

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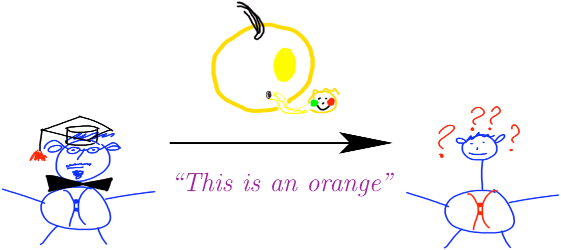

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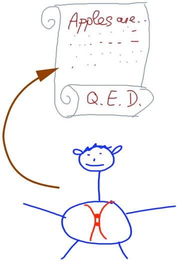

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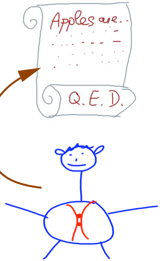

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► This model is called *Probably Approximately Correct (PAC)*, it has been introduced by Valiant [V84].

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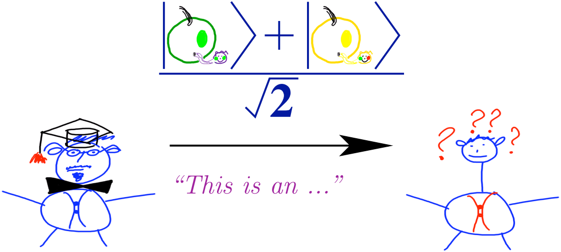

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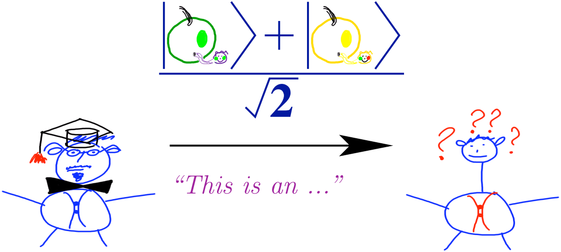

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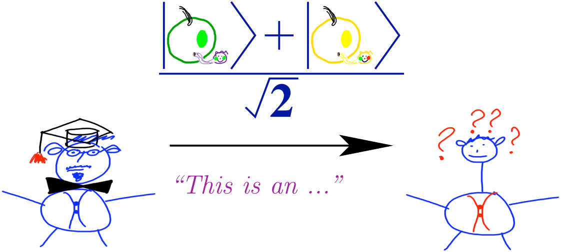

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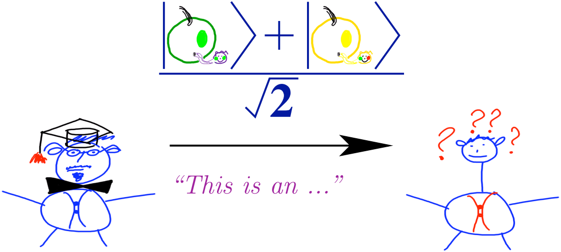

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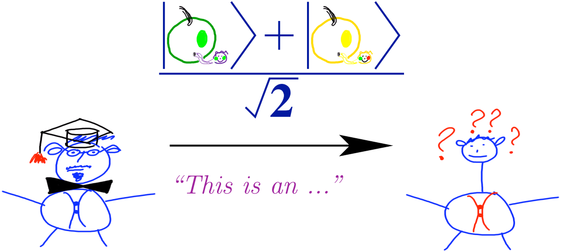

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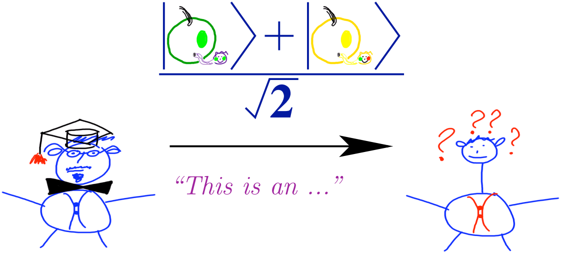
- Are quantum models stronger than classical?

Earlier Work



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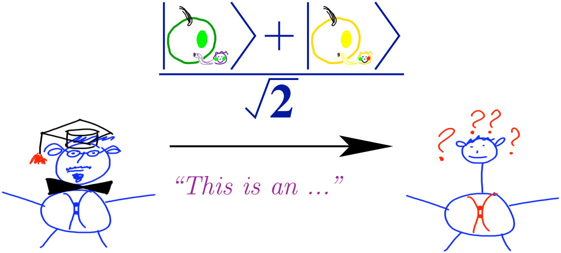

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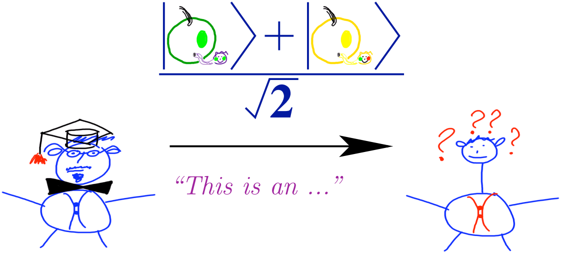

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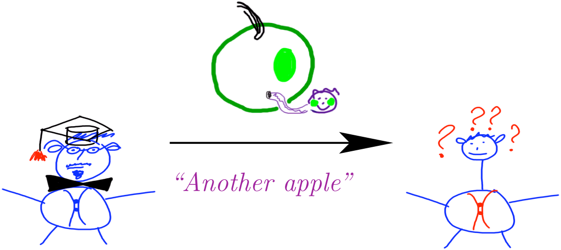

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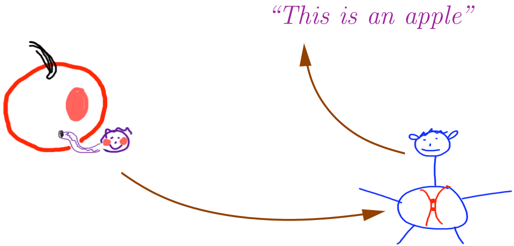

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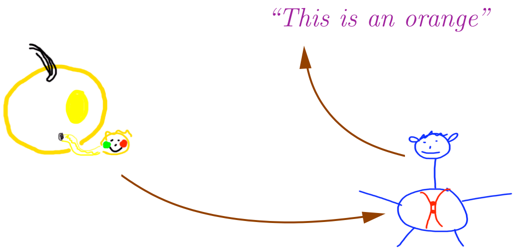

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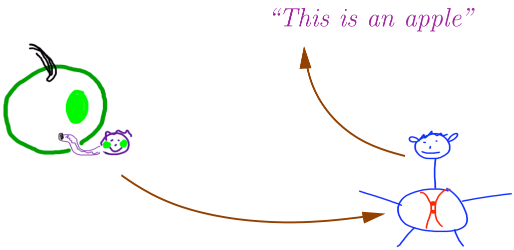

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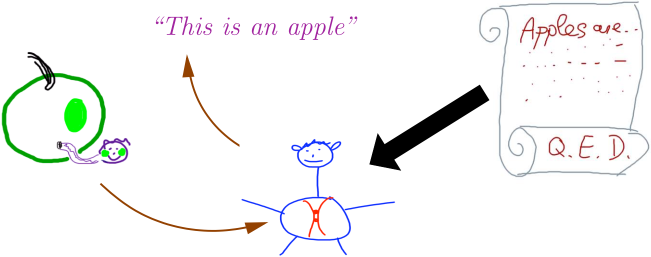

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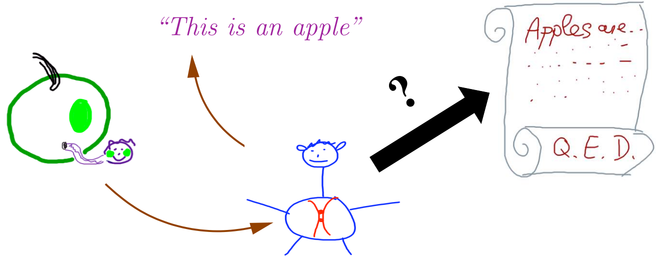
▶ Clearly, *standard learnability implies predictive learnability* (a hypothesis can be used as a distinguishing algorithm).

Does Predictive Learnability Equal Standard Learnability?



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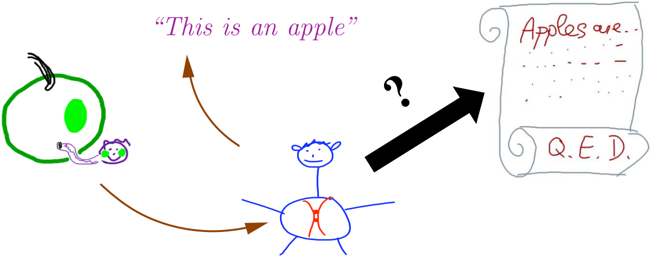

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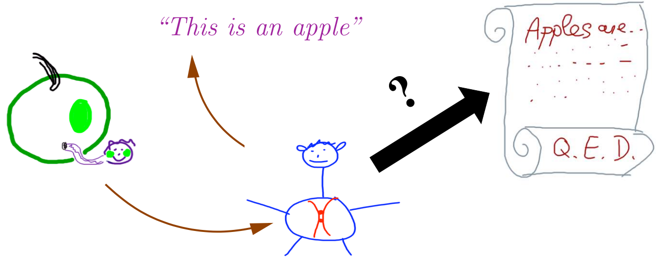

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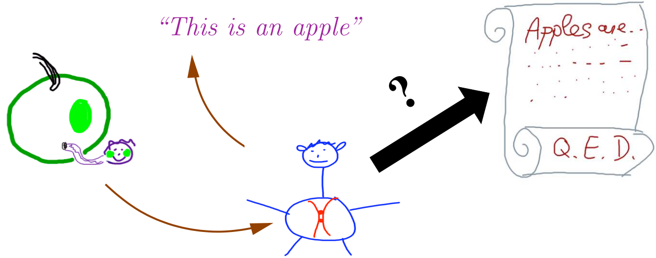

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- The argument *does not translate to the case of quantum models*.
- We will see that, indeed, *quantum predictive learning is qualitatively stronger than quantum standard learning*.
- ▶ Observe that *unconditional separation between quantum and classical learning* immediately follows (we will make a more formal statement later).

Relational Concept Classes

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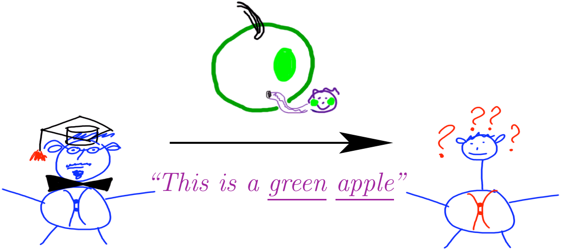

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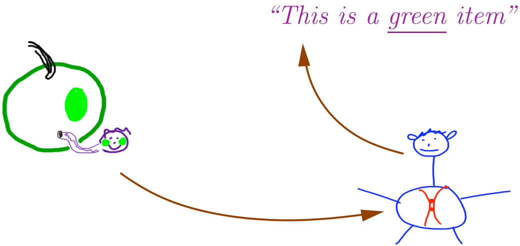

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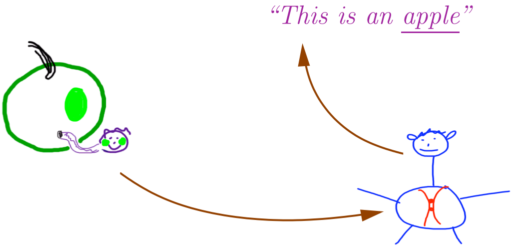

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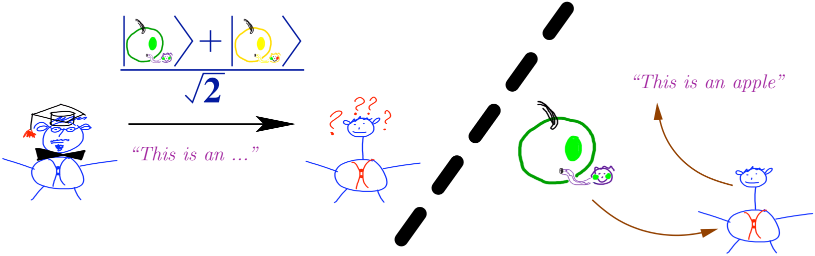
► Therefore, *willing to learn unspeakable concepts*, we can only hope to do so for a *relational* class, in a *quantum predictive* model.

Our Results



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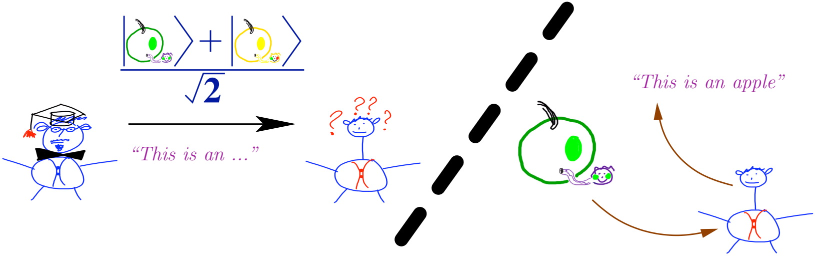

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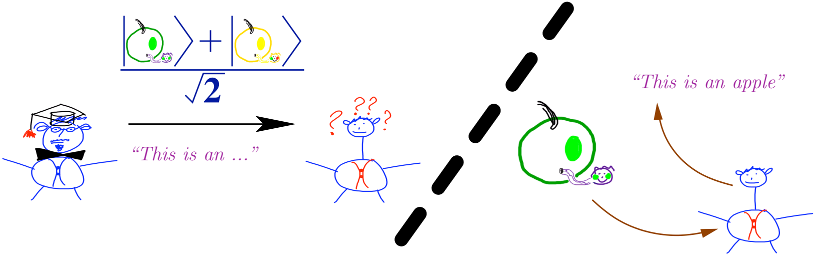

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► In particular, *$\mathcal{C}$ is not learnable classically*.

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- This construction has been inspired by a communication problem defined in [BJK04].

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▶ The student responds with $(a, c_a\oplus c_{a+q})$, as required.

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► We show that *each $h \in \mathcal{H}$ approximates at most $2^{O(\sqrt{N})} = 2^{O(2^{n/2})}$ concepts from $\mathcal{C}$* – this follows from the properties of $\mathcal{C}$, first observed in [BJK04].

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► Therefore, *$\mathcal{C}$ is unspeakable*.

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▶ *Assuming $BQP \not\subseteq P/poly$, the answer is trivial* (let $\mathcal{C} \stackrel{\text{def}}{=} \{f_L\}$, for any $L \in BQP \setminus P/poly$) – the goal is to *weaken the assumptions*.


Thank you!