Exact entanglement renormalization for string-net models

Robert König, Ben Reichardt and Guifre Vidal

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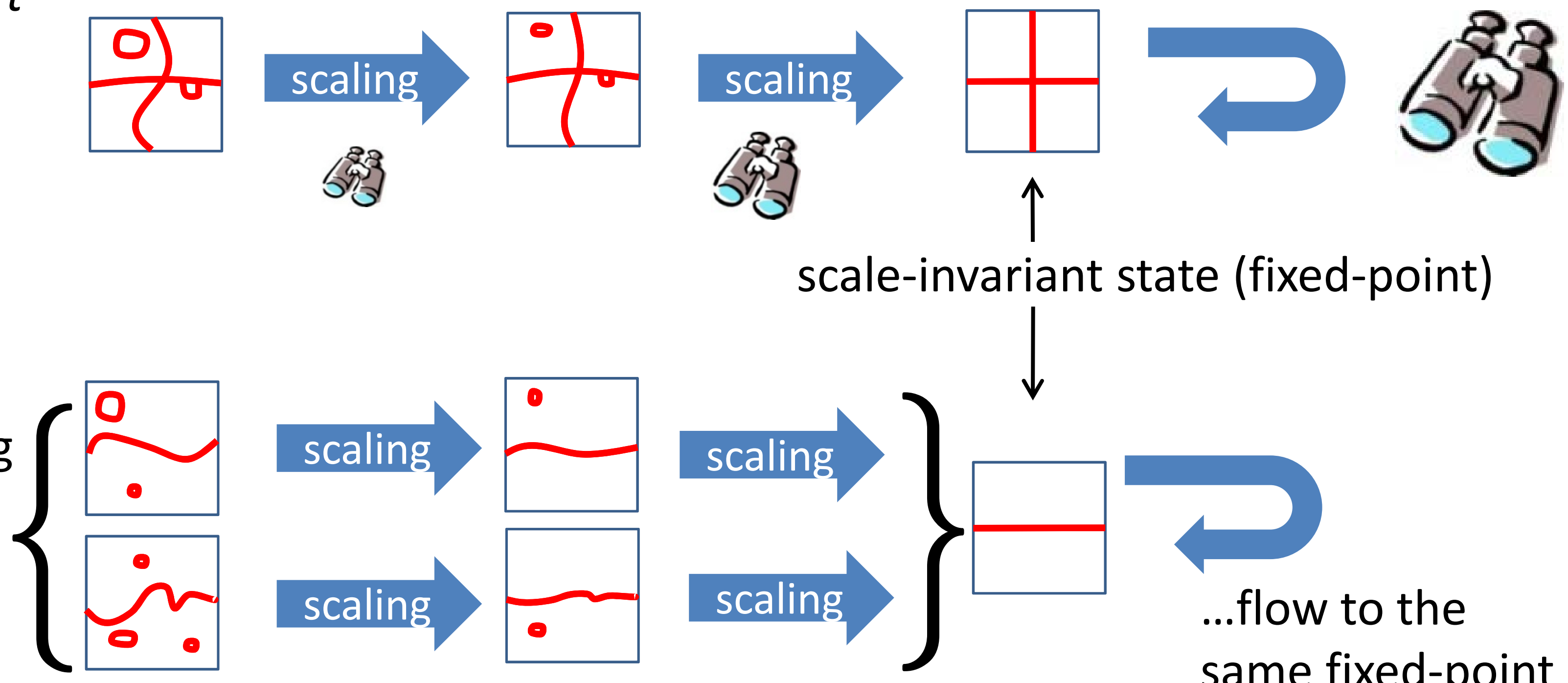
Santa Fe

Long-distance structure and scale-invariance

Eliminate short-distance details. Distill long-distance structures .



States differing only in short-distance properties

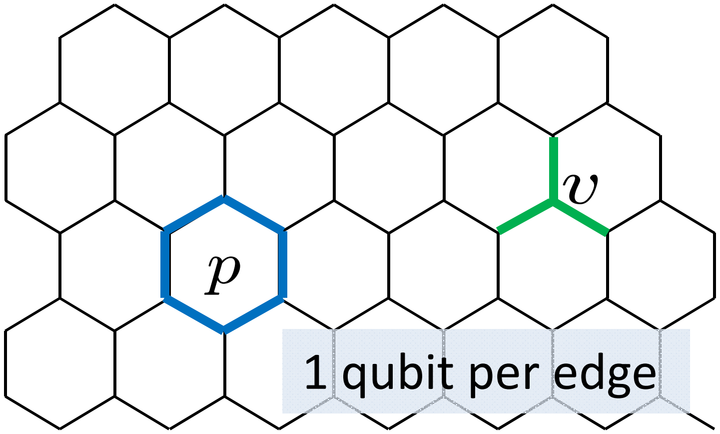
...flow to the same fixed-point under repeated coarse-graining

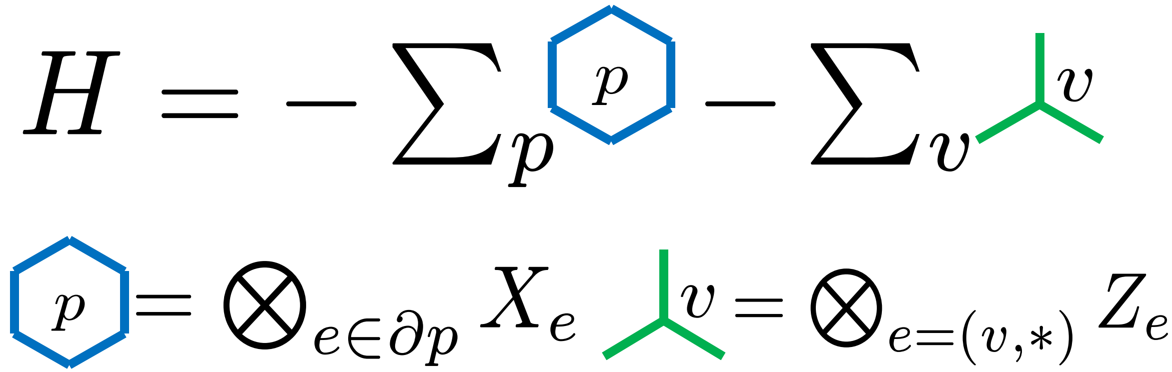
This talk:

- precise definition of scaling
- scale-invariance of ground states

for Levin and Wen's string-net models (2004)

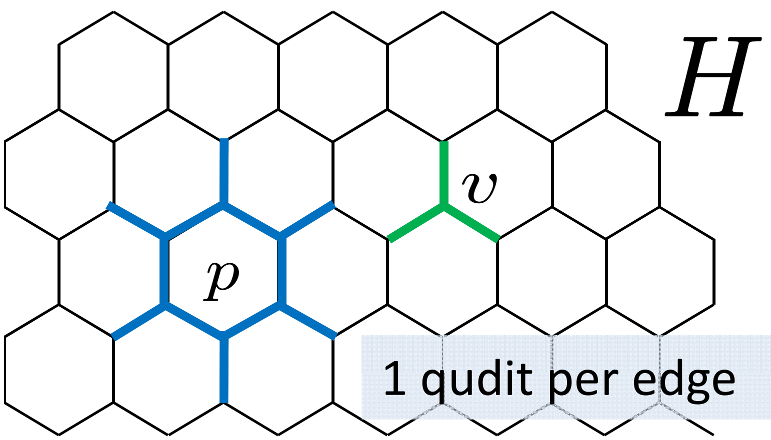
Example: Kitaev's toric code

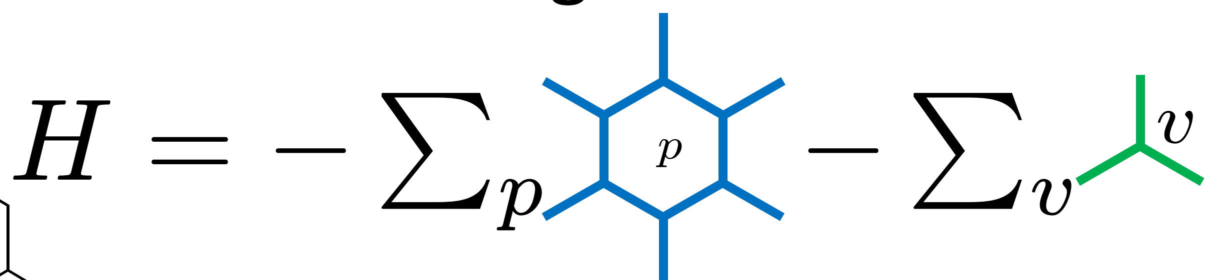




only has abelian anyons, not universal

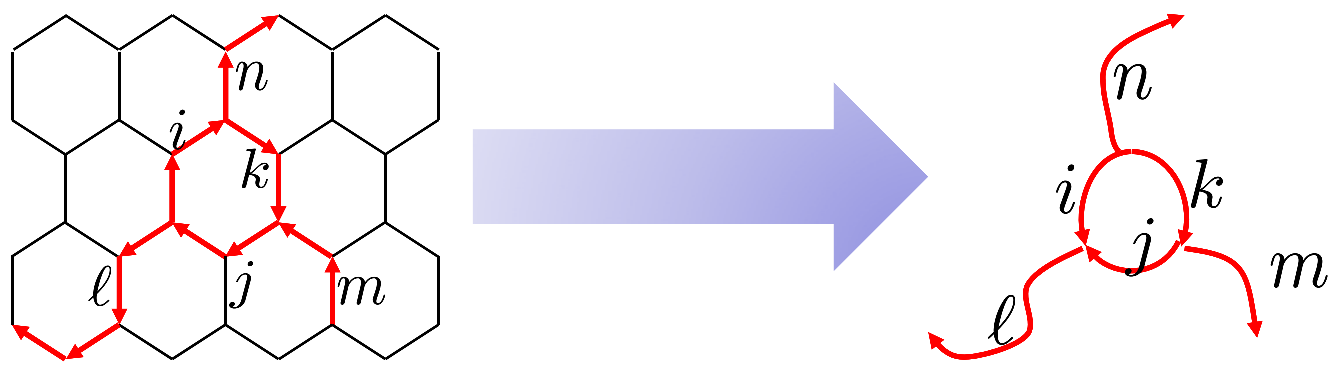
Generalization: Levin-Wen string-net model





The plaquette- and star-operators are defined in terms of a modular tensor category.

has **anyonic excitations described by the doubled theory**

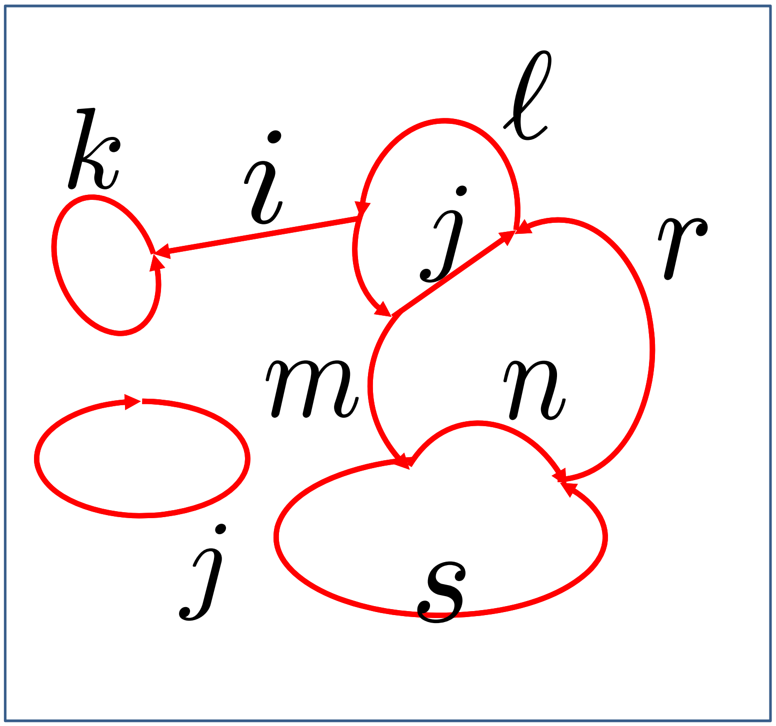


n
i
k
ℓ
j
m
i
k
n
j
m
ℓ

Smooth string-nets (we'll discretize later)

Levin & Wen 2004

Example:

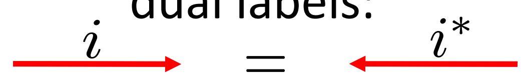


Features: directed trivalent "graph" with allowed triples at each vertex

label set:

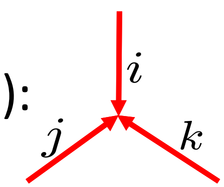
$$\{0, i, i^*, j, j^*, k, k^*, \ldots, \}$$

dual labels:

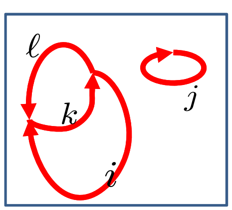


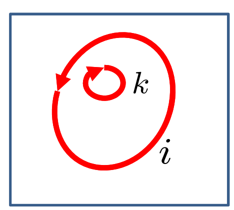
0-label (absence of string):

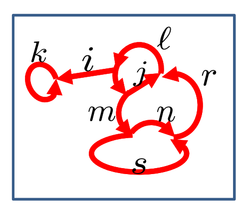
branching rules (set of allowed triples):



State: superposition of string-net configurations

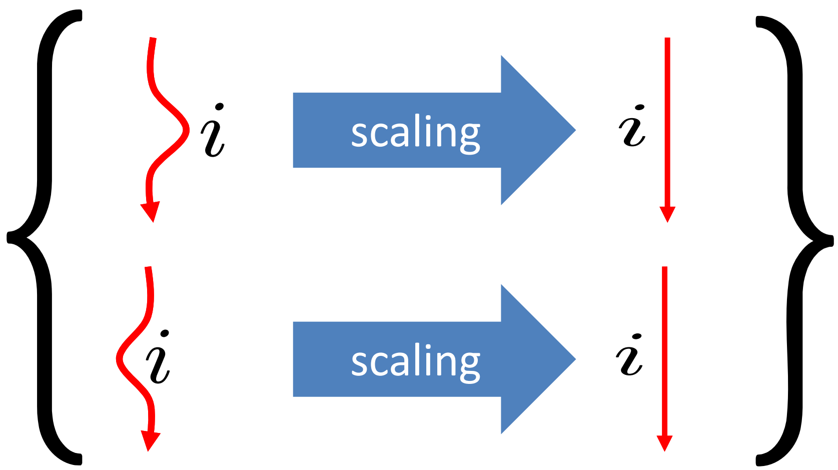
α 

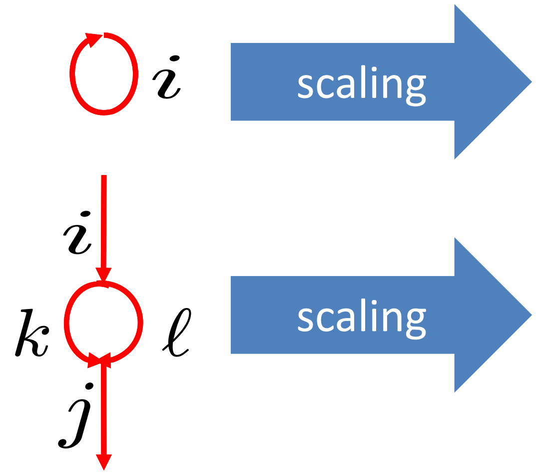
$+\beta$ 

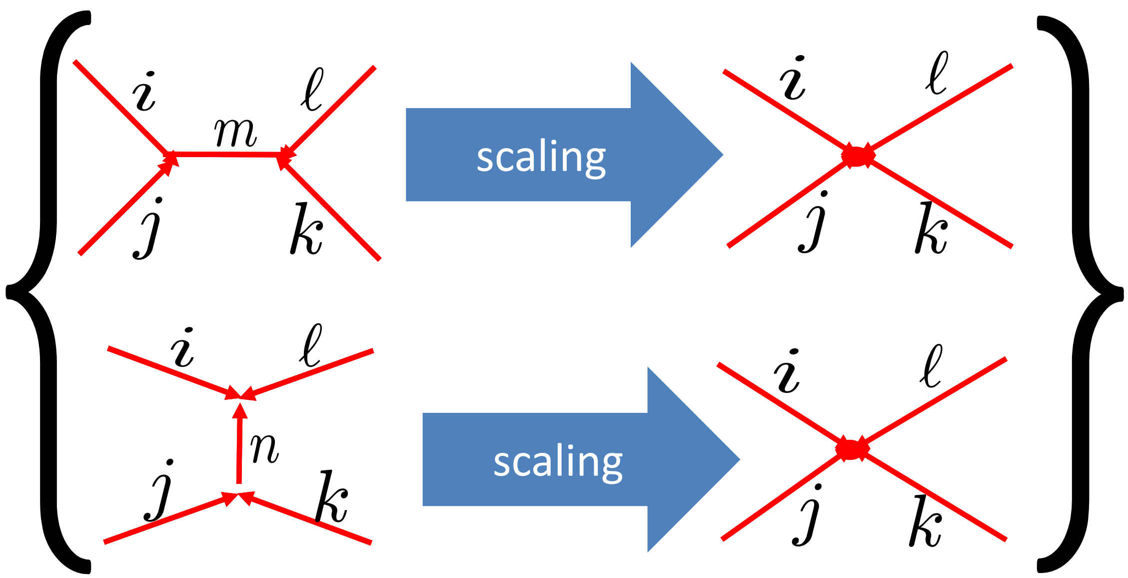
$+\gamma$ 

$+ \ldots$

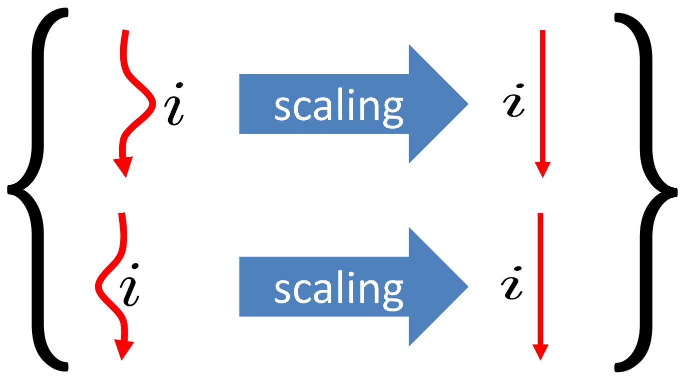
String-net scaling behavior



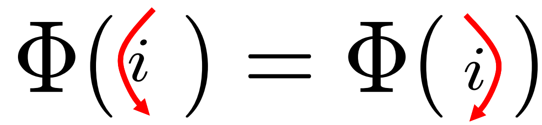


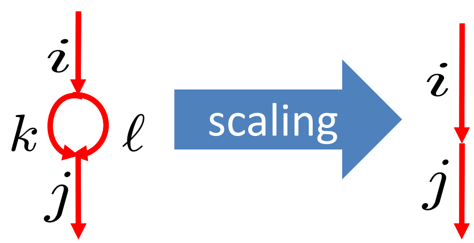


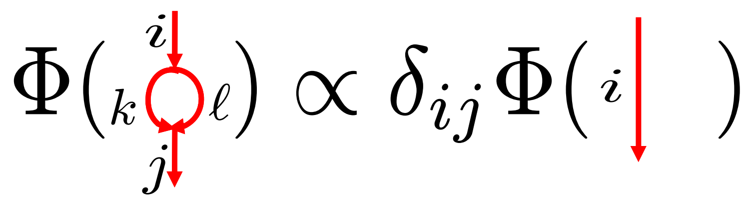
Consider a scale-invariant state $|\Phi\rangle = \sum_C \Phi(C)|C\rangle$

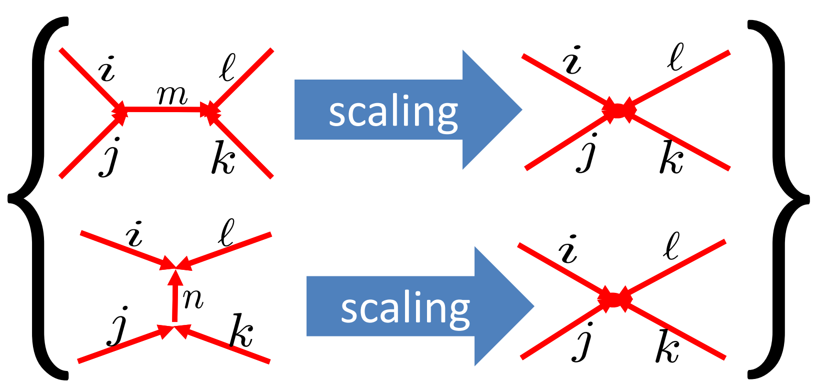


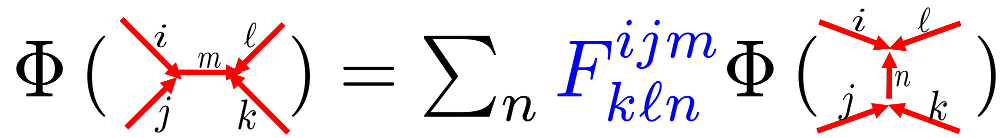
Implications in terms of **local rules:**





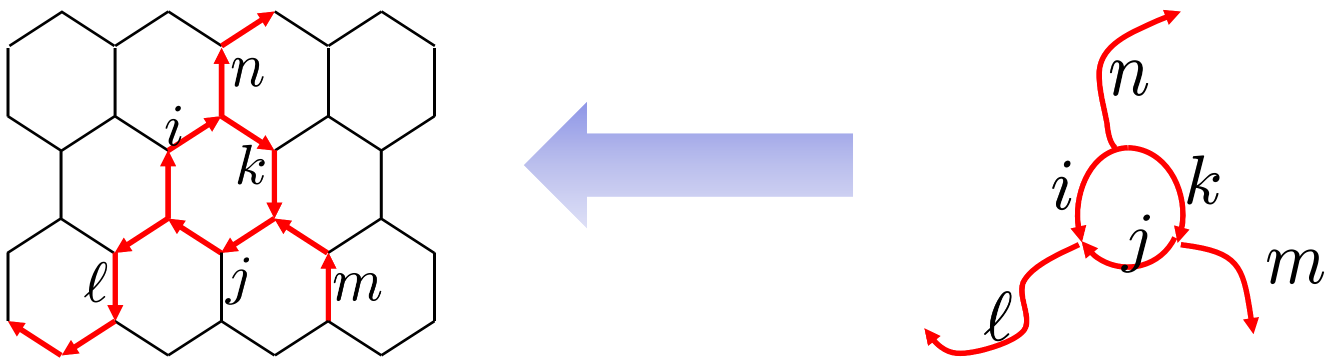


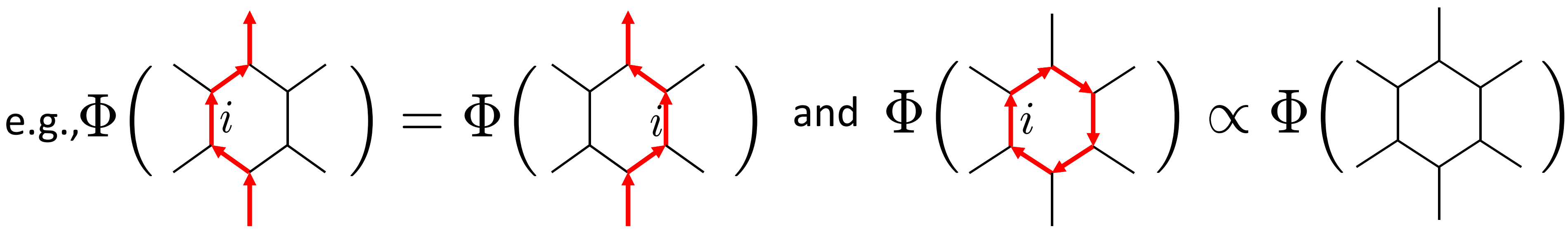




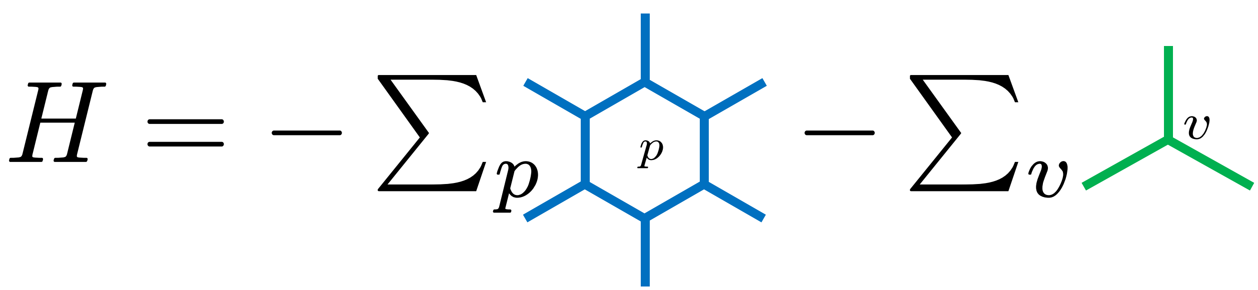
Note: $F^{ij\cdot}_{k\ell\cdot}$ is unitary for all i, j, k, ℓ

Fact: Levin-Wen Hamiltonian terms enforce (lattice version) of local rules





The model is “derived” from the **assumption of scale-invariance.**

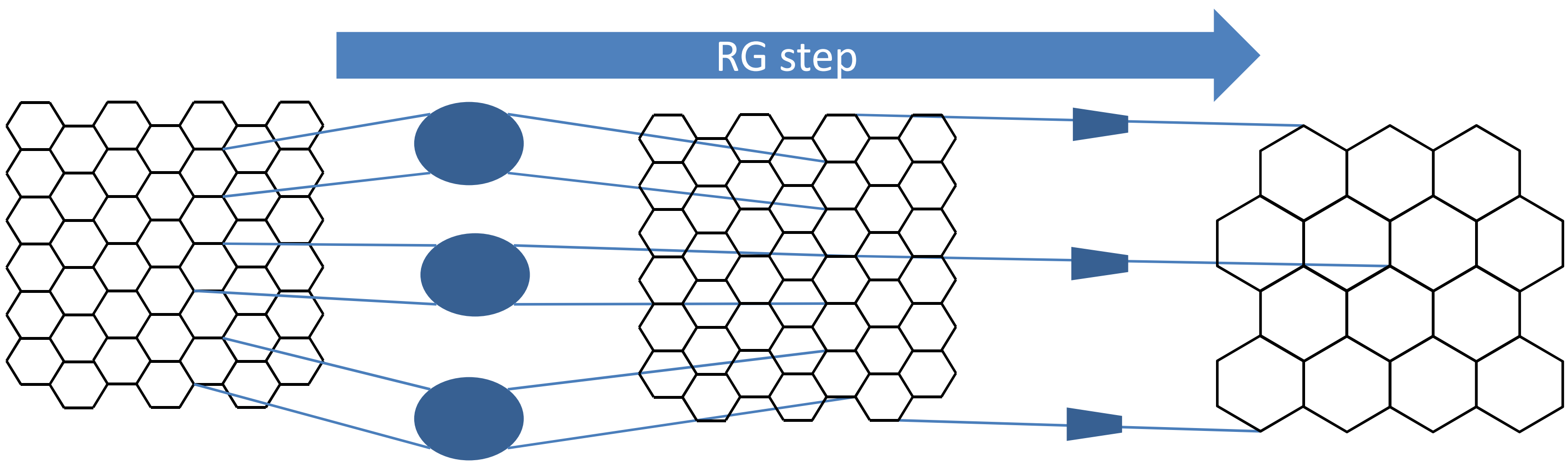


Levin-Wen models and RG flow

This talk: Can **scale-invariance** been shown formally?

i.e.,

What **renormalization group (RG) transformation fixes ground states** of the Levin-Wen model?

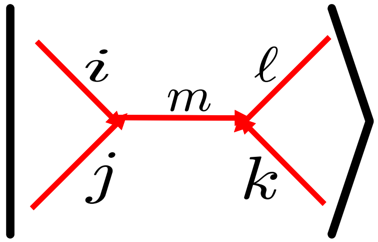


local isometries (unitaries)

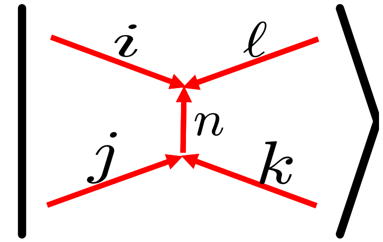
..before coarse-graining (local projections)

intuition: remove entanglement between neighboring regions before coarse-graining (Vidal'05)

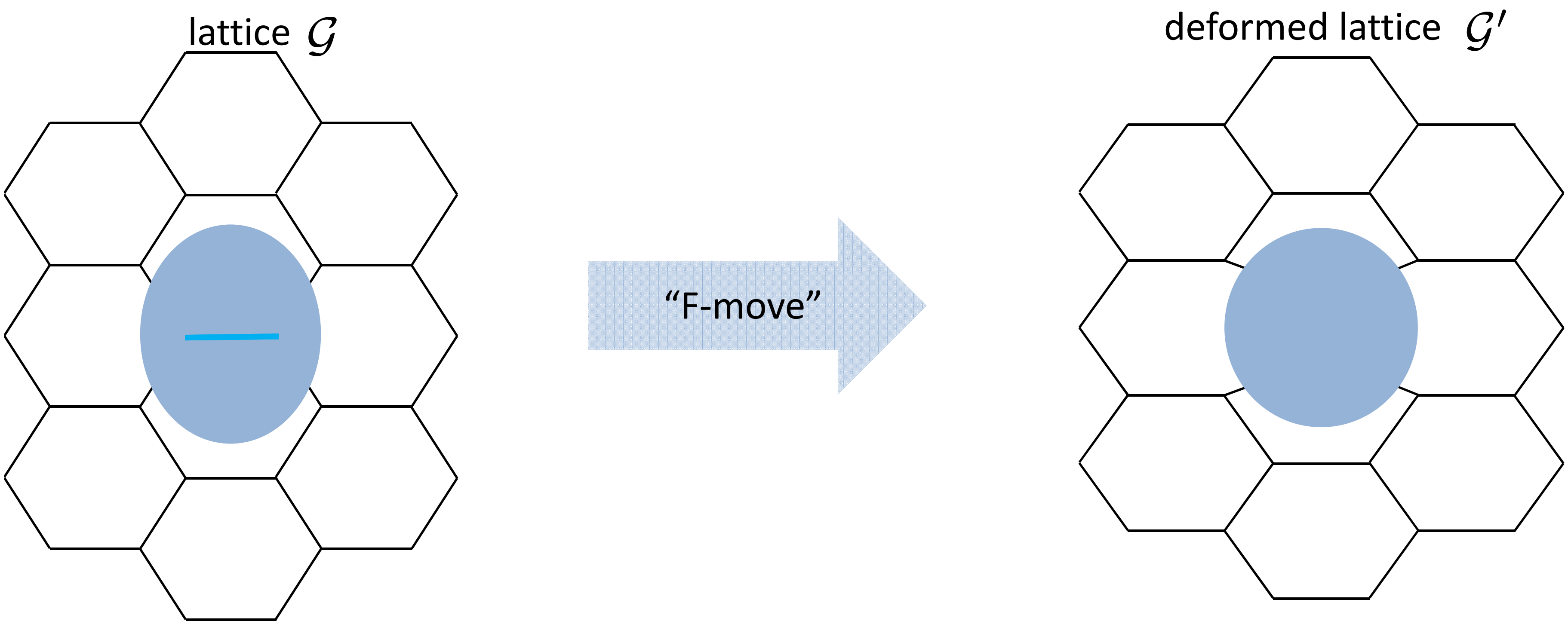
F-move:



$\longmapsto \sum_n F^{ijm}_{k\ell n}$

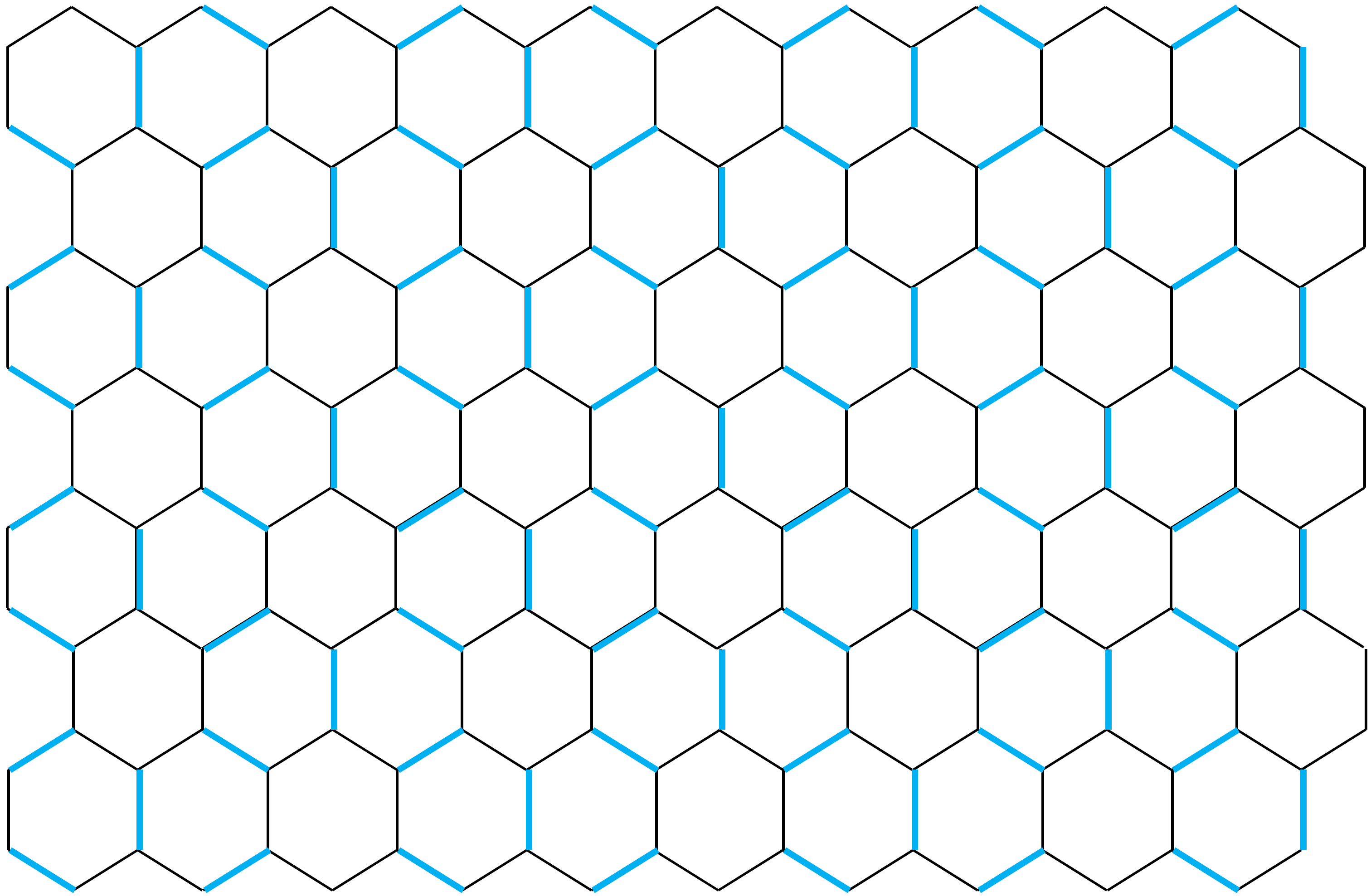


For unitary tensor categories, this is a **unitary.** The F-move provides a natural isomorphisms between the Hilbert spaces of the two lattices.



$\mathcal{H}_{\mathcal{G}}$ Hilbert space of spins on original lattice $\xrightarrow{\cong}$ $\mathcal{H}_{\mathcal{G}'}$ Hilbert space of spins on deformed lattice

renormalization step 1/4

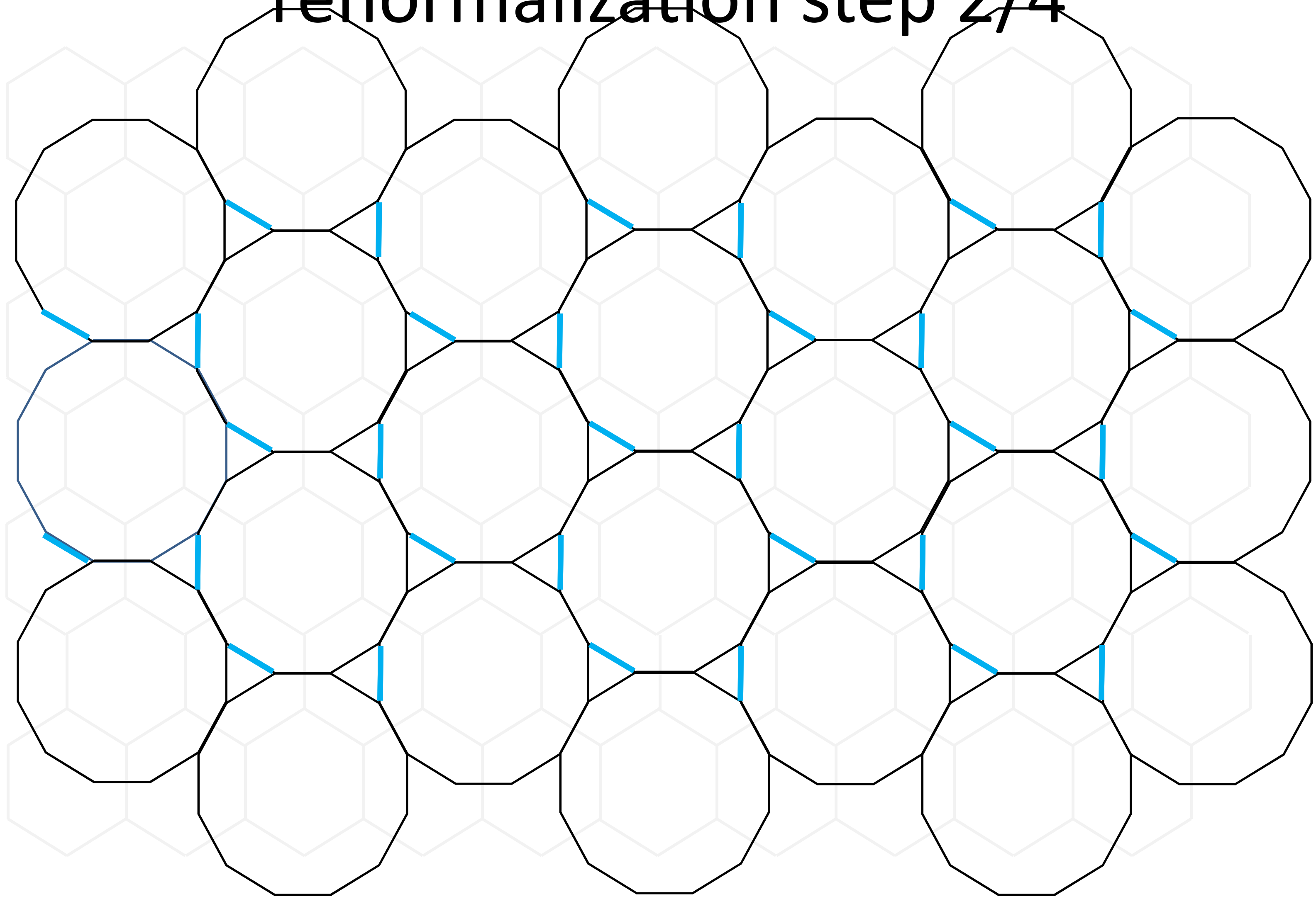


Basic idea: Simplify lattice using F-moves

First apply F-moves to blue edges

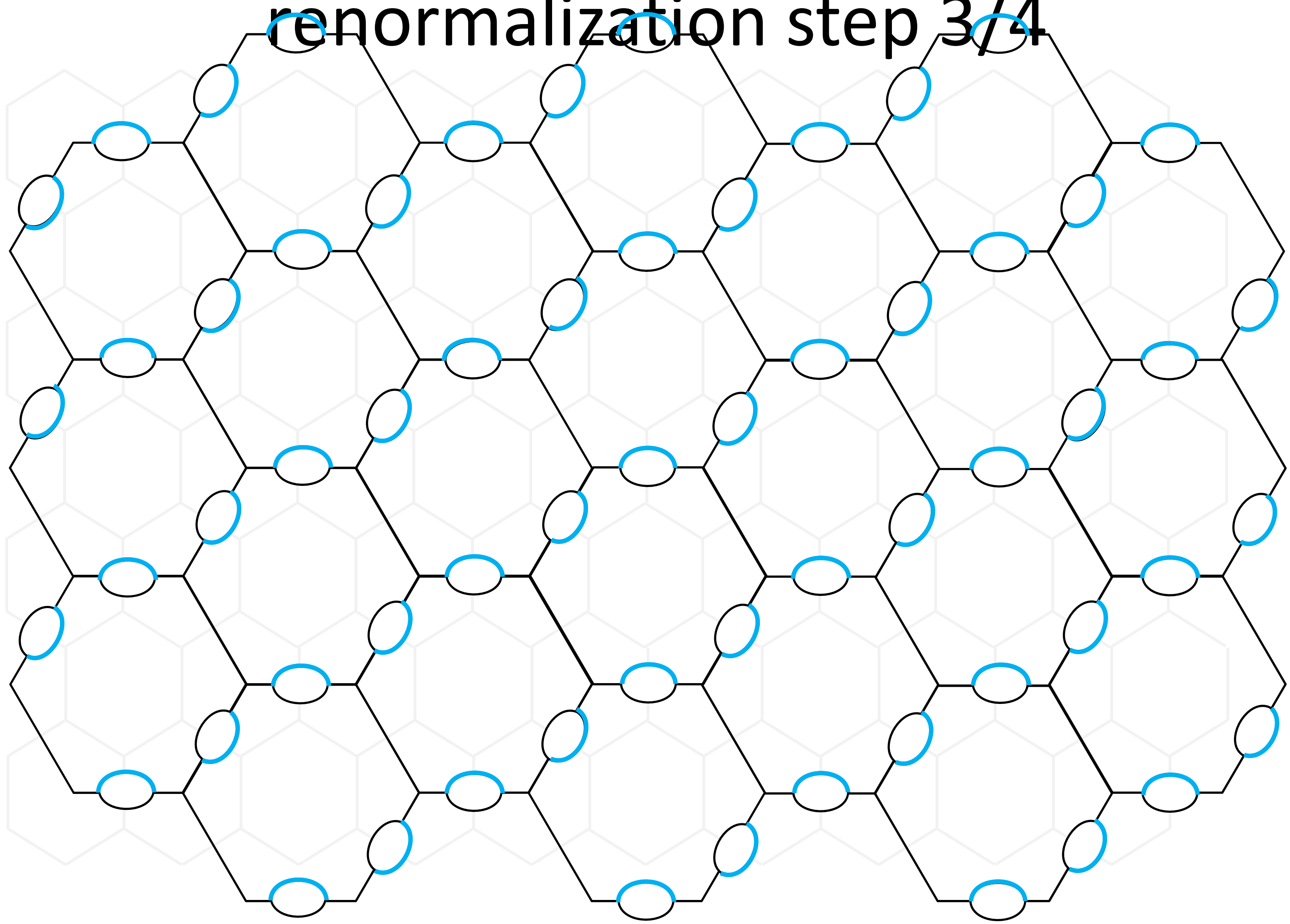
These moves commute.

renormalization step 2/4



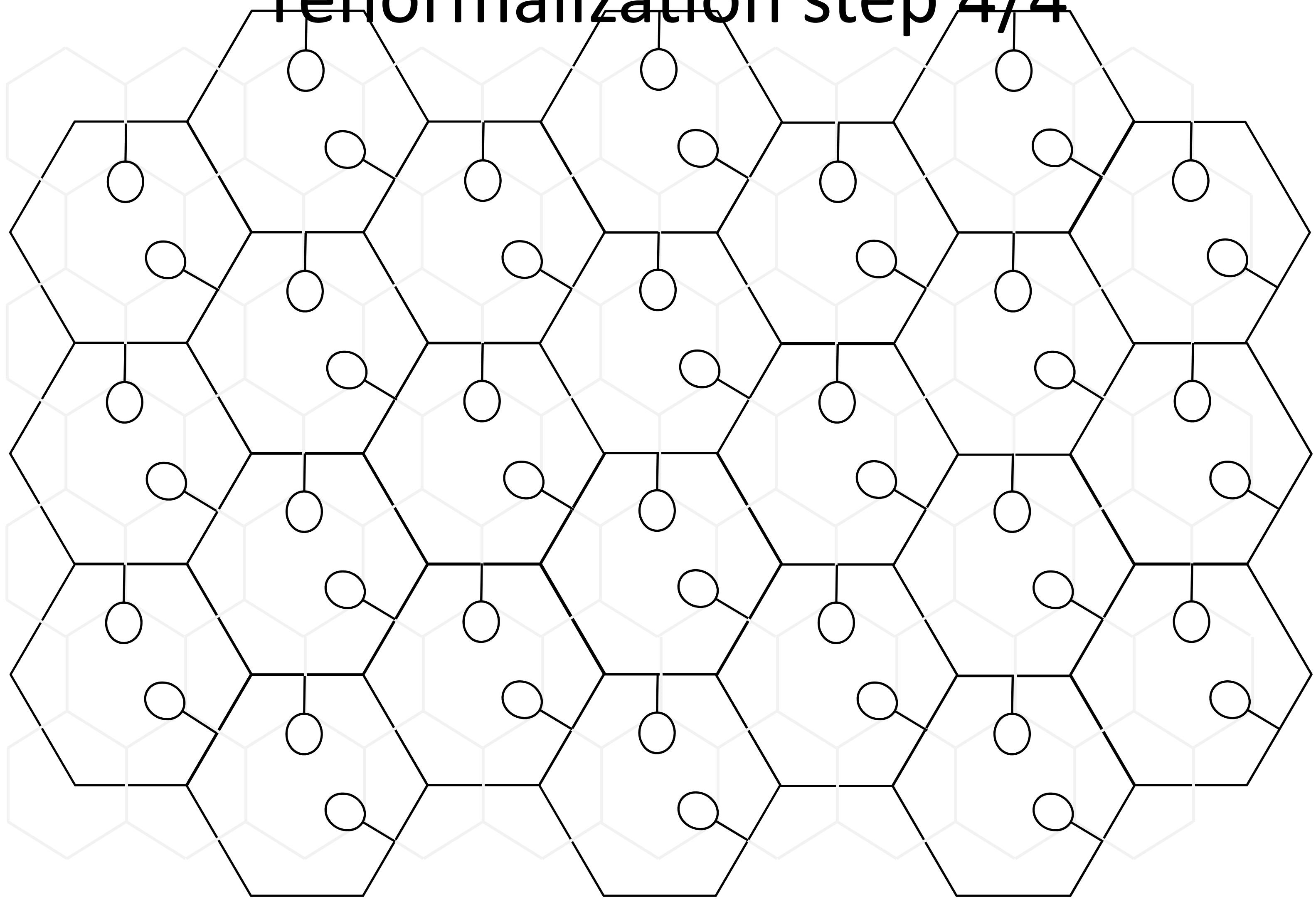
apply F-moves to blue edges

renormalization step 3/4

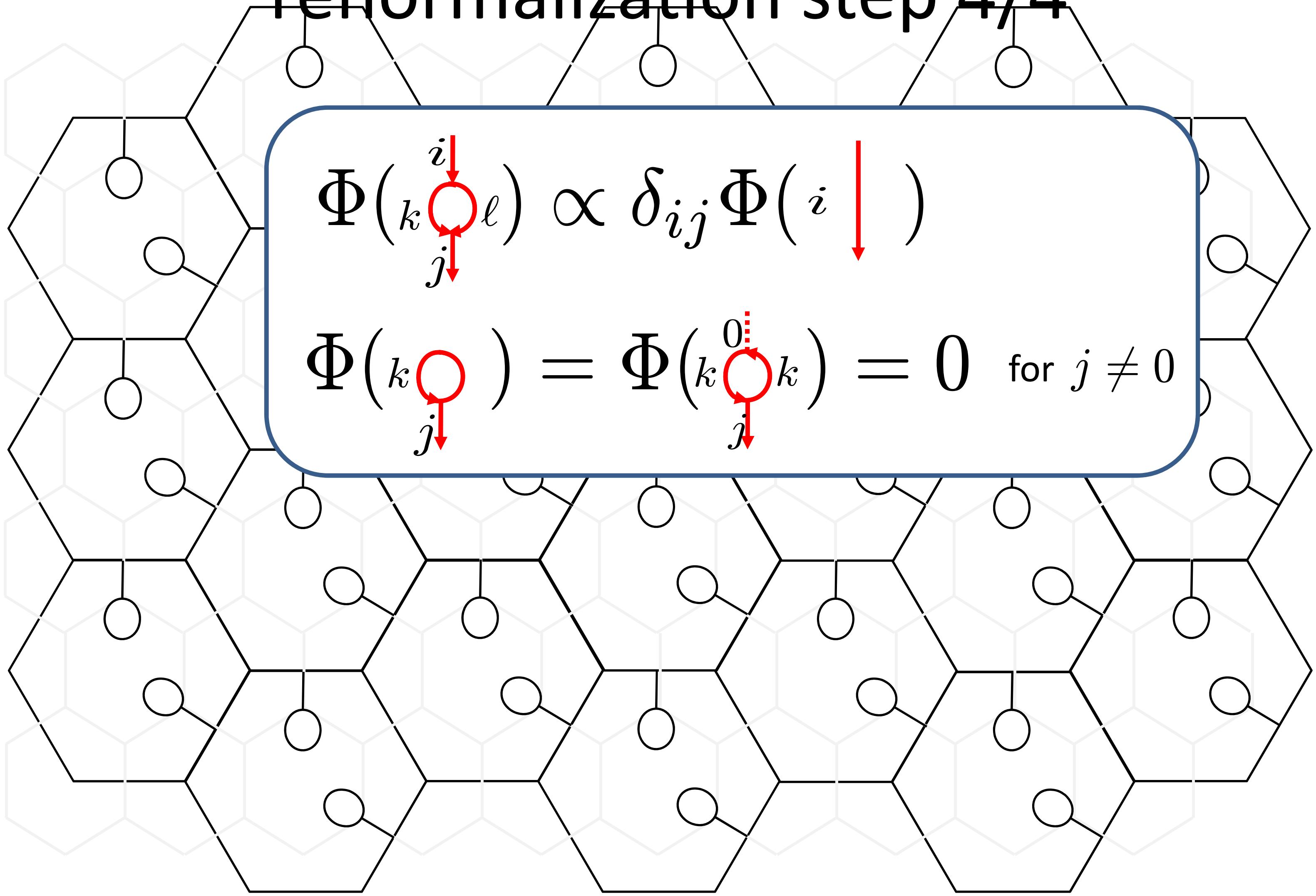


apply F-moves to edges 

renormalization step 4/4

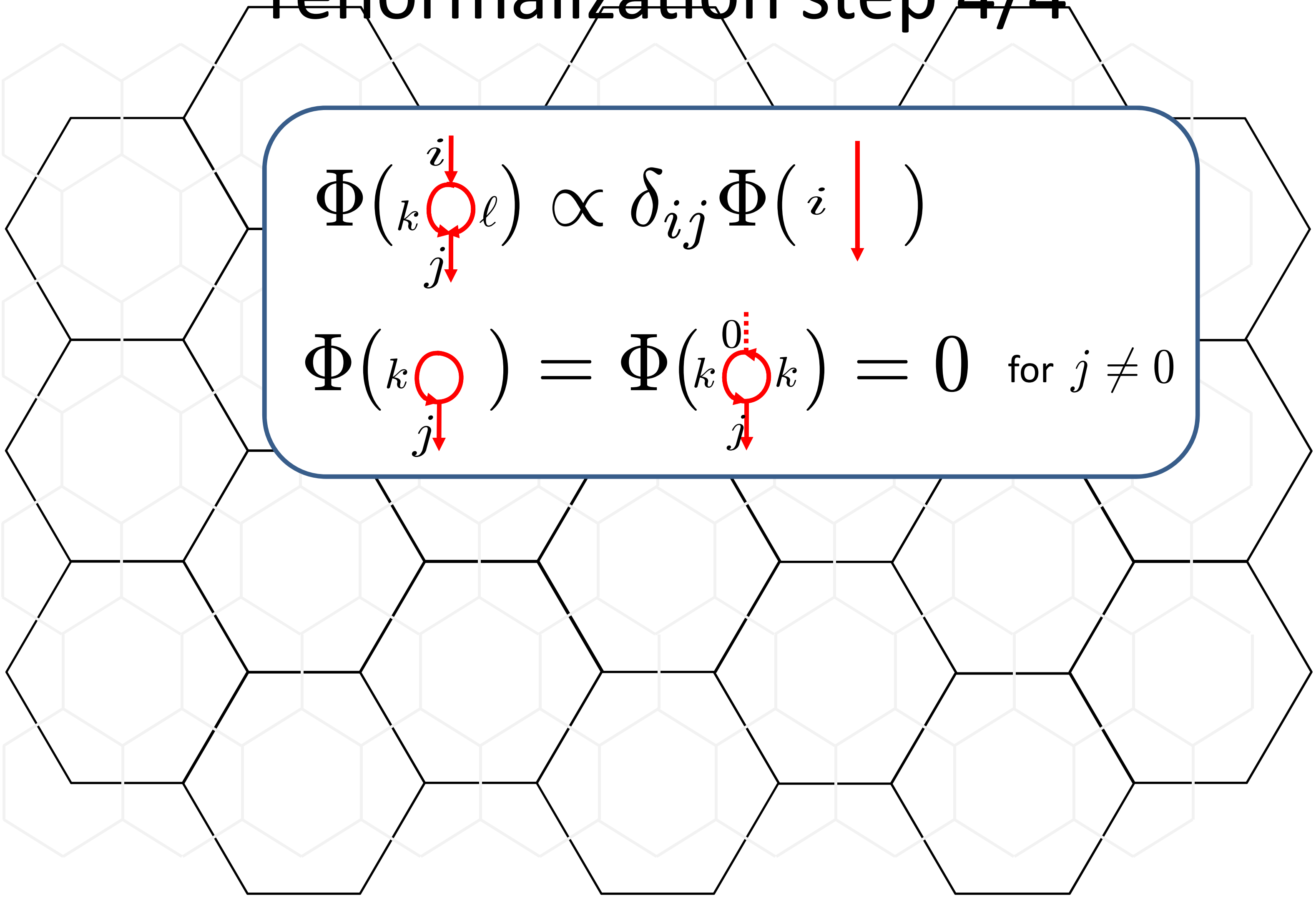


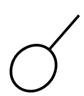
renormalization step 4/4



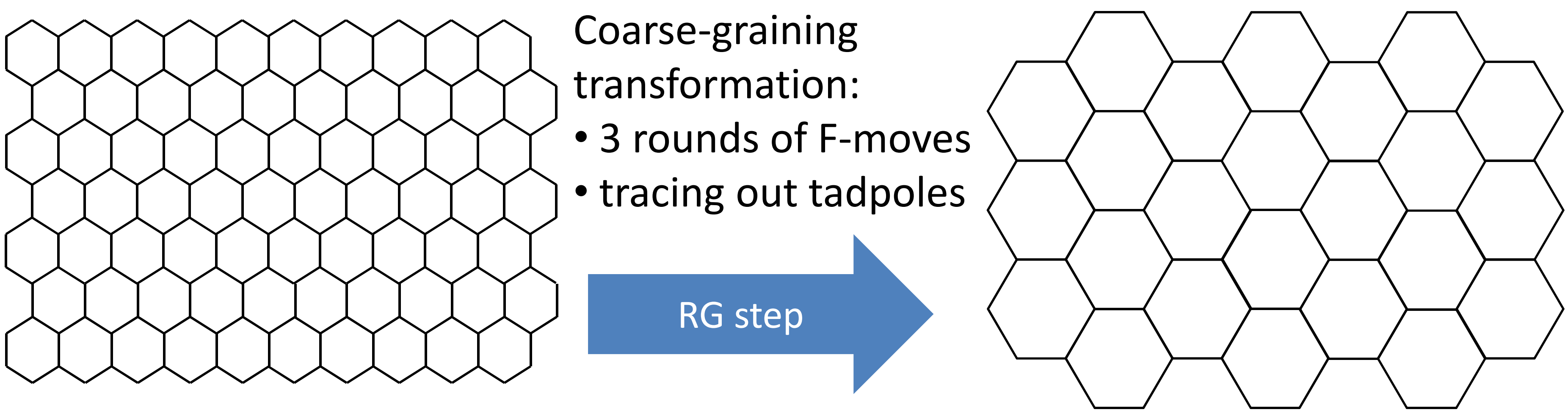
Project qudits on tadpoles onto a fixed state to eliminate degrees of freedom
(2 edges=2 qudits per tadpole)

renormalization step 4/4



Project qudits on tadpoles  onto a fixed state to eliminate degrees of freedom (2 edges=2 qudits per tadpole)

Summary of RG procedure



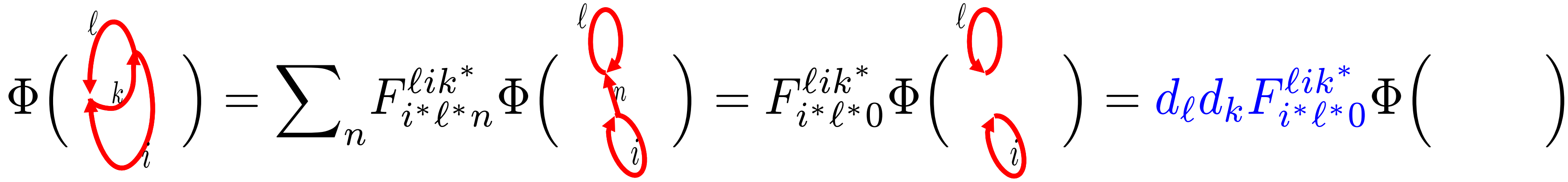
This RG procedure has the **ground state of the Levin-Wen model as fixed point**!

Proof: see arXiv: 0806.4583

Remark: RG procedure independent of topology

Fact: In the **plane**, there is a **unique** state satisfying the local rules (i.e., Levin-Wen ground state is unique)

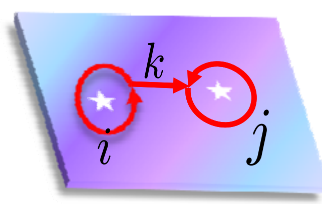
Proof: The coefficient of a string-net in the ground state is **proportional** to that of the empty configuration as **determined by the local rules**.



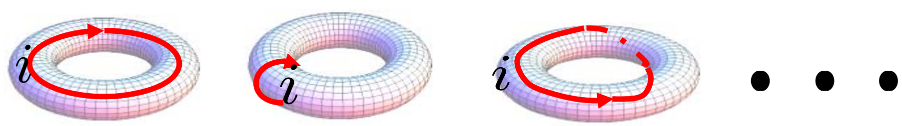
$$\Phi\left(\quad\right) = \sum_n F^{\ell i k^*}_{i^* \ell^* n} \Phi\left(\quad\right) = F^{\ell i k^*}_{i^* \ell^* 0} \Phi\left(\quad\right) = d_\ell d_k F^{\ell i k^*}_{i^* \ell^* 0} \Phi\left(\quad\right)$$

Topological ground space degeneracy: local rules define higher dimensional subspace for nontrivial surfaces.

punctured sphere



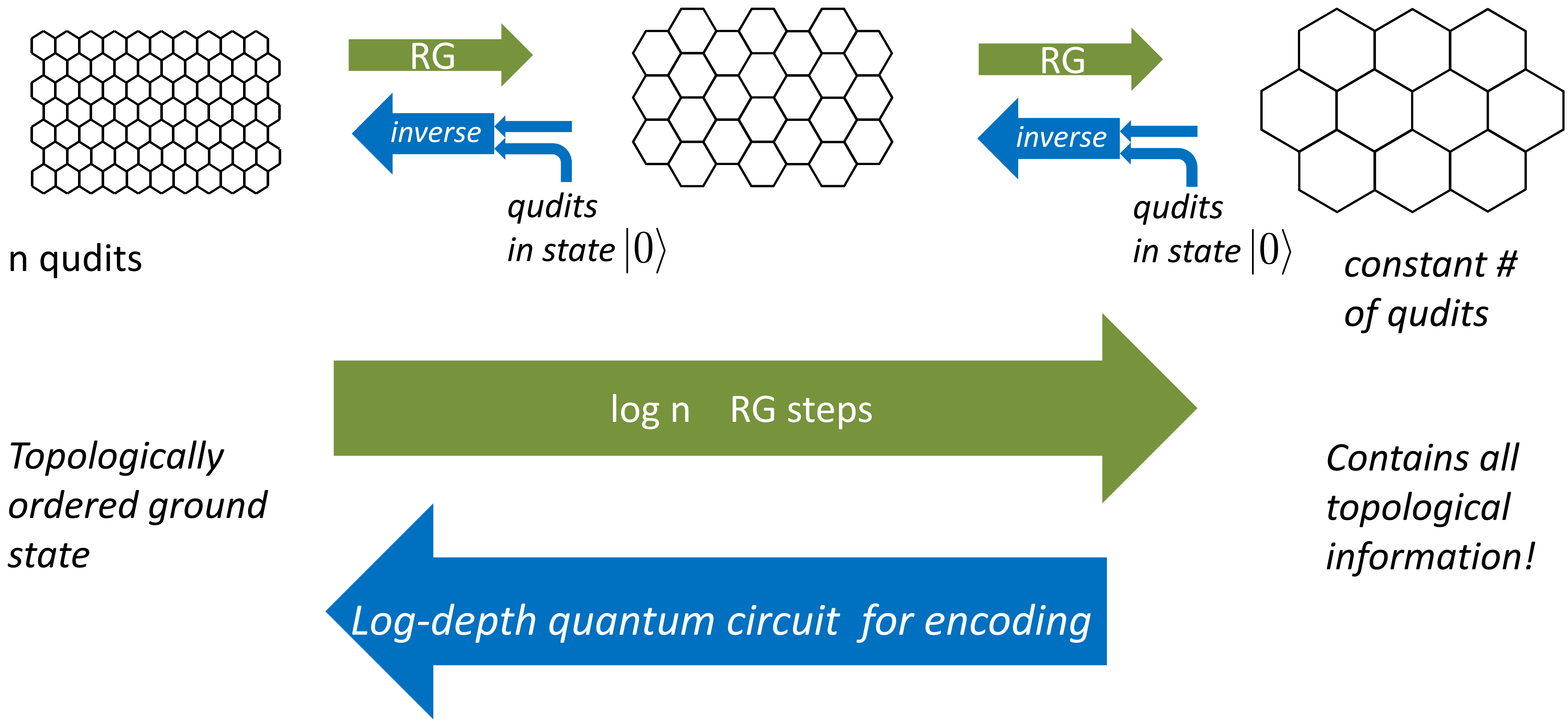
2-punctured plane



torus

Ground space defines a code, for example the toric code from the toric model.
Universal anyonic computation is possible inside the code space.

Further properties/uses of the RG procedure



- RG procedure "distills" topological information into a small system
- provides efficient MERA-description of ground states (for, e.g., computation of expectation values)
- may be used to initialize numerical variational methods when studying perturbations (stability of topological order?)