Oracularization and 2-Prover 1-Round Interactive Proofs against Nonlocal Strategies

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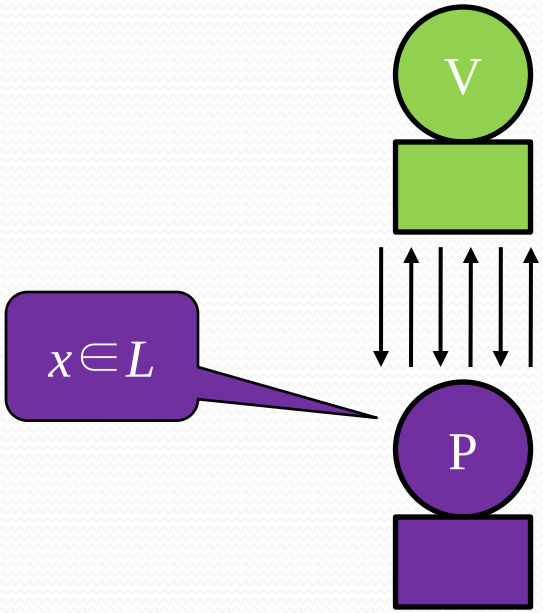
Interactive proof

[Babai 1985] [Goldwasser, Micali, Rackoff 1989]



Verifier: randomized poly-time

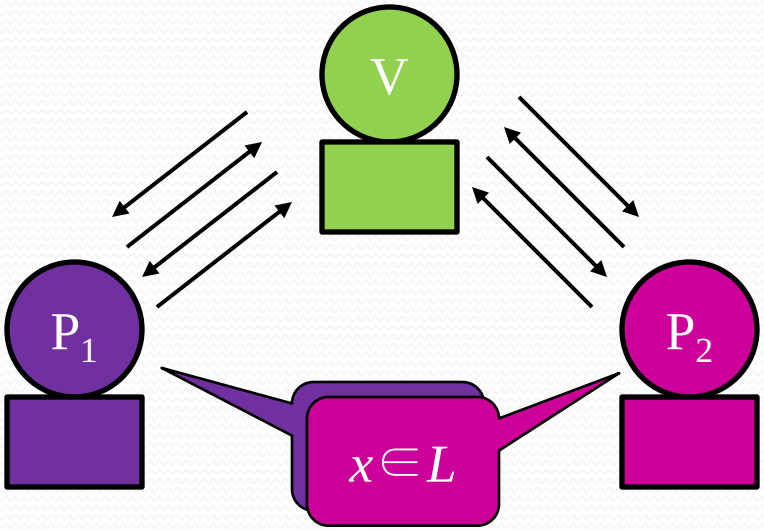
- $x \in L \Rightarrow$ prob. of acceptance must be $\geq c$
- $x \notin L \Rightarrow$ prob. of acceptance must be $\leq s$

Prover: infinitely powerful computationally

IP=PSPACE [Shamir 1992]

Multi-prover interactive proof (MIP)

[Ben-Or, Goldwasser, Kilian, Wigderson 1988]



2 or more provers are kept separated

Provers together try to convince V

V can "cross-check" the provers' answers

MIP=NEXP [Babai, Fortnow, Lund 1991]

Note: Shared randomness between provers does *not* change the computational power

Computational power of MIP

[Feige, Lovász 1992]

NEXP = MIP with
- Poly provers
- Poly rounds
- Bounded 2-sided error

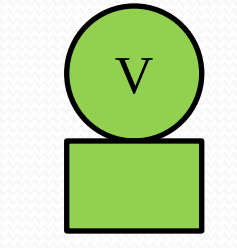
= MIP with
- 2 provers
- 1 rounds
- Exp-small 1-sided error

Oracularization technique:

Poly-prover poly-round (with some restriction) → 2-prover 1-round

Quantum nonlocality

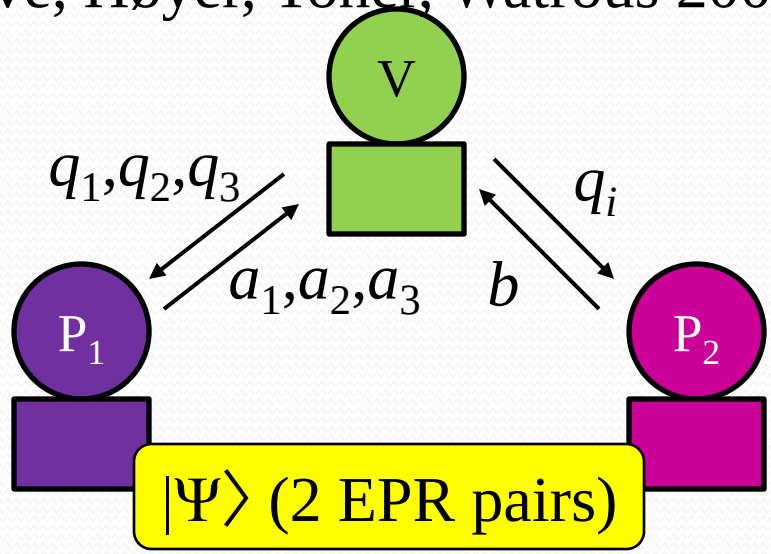
Magic Square game [Avavind 2002]
[Cleve, Høyer, Toner, Watrous 2004]



$q_1,q_2,q_3 \in \{1,\ldots,9\}$

	a_1	
	a_2	
	a_3	

Oracularize



- Each column has odd parity
- Each row has even parity

Max. winning probability =
17/18 in the classical world
1 using prior-entanglement

Effect of quantum nonlocality on MIP

Entanglement gives provers more power

- Honest provers use nonlocality
  → The power of MIP might increase
- Dishonest provers also use nonlocality
  → Existing MIP protocols become unsound

????

$$\mathrm{MIP}^* \overset{\subseteq}{\underset{\supseteq}{=}} \mathrm{MIP} = \mathrm{NEXP}$$

Related results about MIP in quantum world (1)

$\oplus$MIP(2,1), $\oplus$MIP*(2,1):
2 provers, 1 round, 1-bit answer,
verifier only look at the XOR of the answers

- With some constant 2-sided error,
  $\oplus$MIP*(2,1)$\subseteq$EXP$\subsetneq$NEXP=$\oplus$MIP(2,1)
  (unless EXP=NEXP)
  [Cleve, Høyer, Toner, Watrous 2004]

  Entanglement makes the class smaller!

- NP$\subseteq\oplus$MIP*(2,1) with constant 2-sided error
  [Cleve, Gavinsky, Jain 2007]

Related results about MIP in quantum world (2)

- Trivially, MIP* $\supseteq$ IP $=$ PSPACE
- [Kempe, Kobayashi, Matsumoto, Toner, Vidick 2008]:
  - PSPACE $\subseteq$ MIP* with
    2 provers, 1 round, $1-1/\mathrm{poly}$ soundness error
  - NEXP $\subseteq$ MIP* with
    3 provers, 1 round, $1-1/\exp$ soundness error
  - NEXP $\subseteq$ QMIP (quantum messages) with
    2 provers, 1 round, $1-1/\exp$ soundness error
- NEXP $\subseteq$ MIP* with
  3 provers, 1 round, $1-1/\exp$ soundness error, 1-bit answer
  [Ito, Kobayashi, Preda, Sun, Yao 2008]

Related results about MIP in quantum world (3)

- [Ben-Or, Hassidim, Pilpel 2008]:

  NEXP has 2-prover 2-round protocol
  with constant soundness
  in new model with
  - Quantum interaction
  - Classical communication between provers
  - Without prior-entanglement

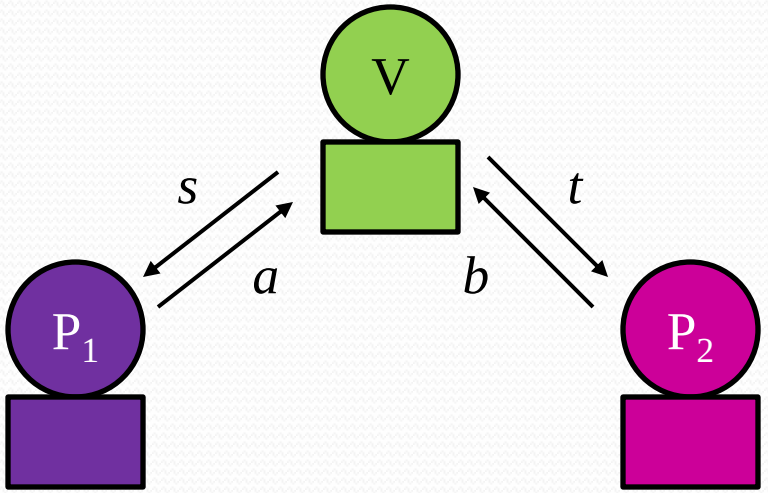
Our results

- PSPACE$\subseteq$MIP*
  with 2 provers, 1 round, **exp-small** 1-sided error
  - 2 provers are more useful than 1, even with entanglement
  - Soundness holds for more powerful **no-signaling** provers
- NEXP$\subseteq$MIP*
  with **2** provers, 1 round, $1-1/\exp$ 1-sided error
- Limitation of independent sampling:
  Known 2-prover protocols for NEXP
  really has error probability $1-1/\exp$ in some cases

No-signaling provers

$p(a,b|s,t)$ is called **no-signaling** when

- $p(a,b|s,t)\geq 0$
- $\sum_{a,b} p(a,b|s,t)=1$
- $\sum_{a} p(a,b|s,t)$ does not depend on s
- $\sum_{b} p(a,b|s,t)$ does not depend on t

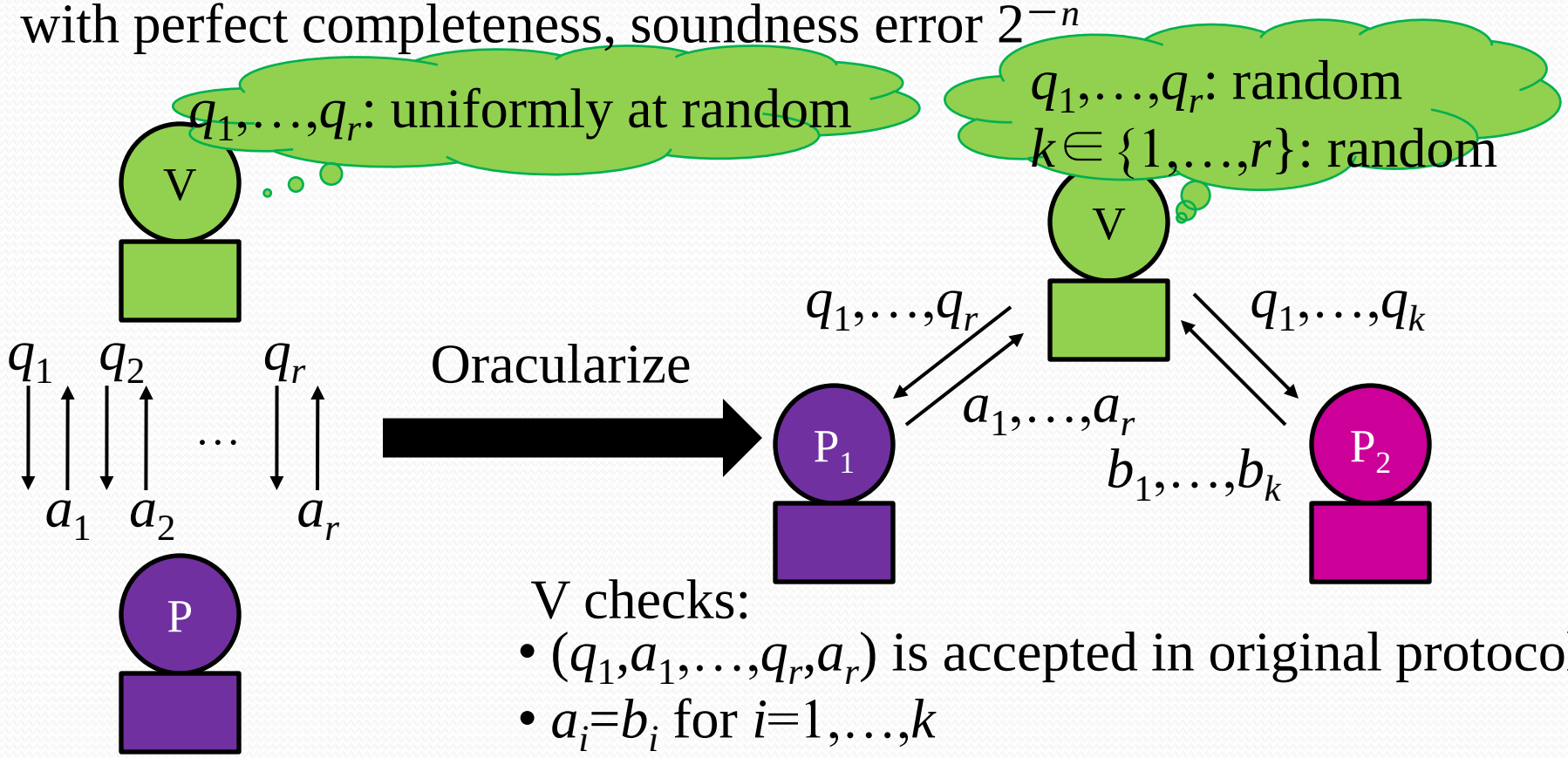


Unentangled provers $\subseteq$ Entangled provers $\subseteq$ No-signaling provers

MIP with no-signaling provers $\subseteq$ EXP [Preda]

Protocol for PSPACE by [KKMTV08]

Public-coin 1-prover r-round protocol for PSPACE (r = poly(n)) with perfect completeness, soundness error 2^{-n}



V checks:

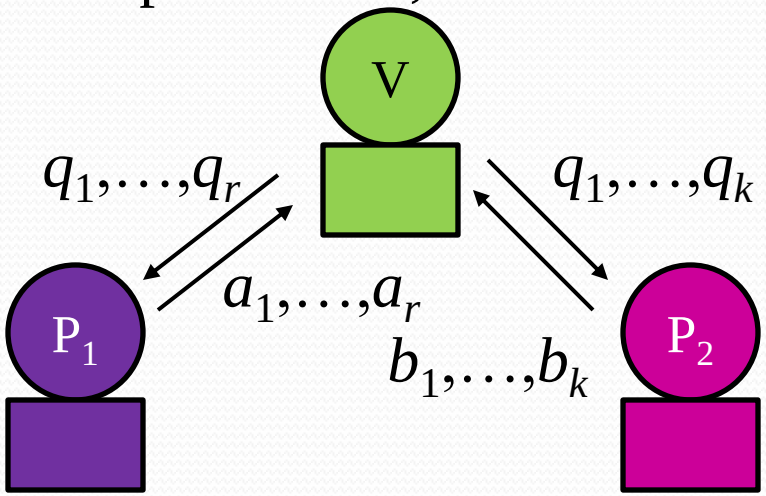
- $(q_1,a_1,\ldots,q_r,a_r)$ is accepted in original protocol
- $a_i=b_i$ for $i=1,\ldots,k$

[KKMTV] proved $1-1/O(r^2)$ soundness error against entangled provers

We prove $1-1/O(r)$ soundness error against **no-signaling** provers

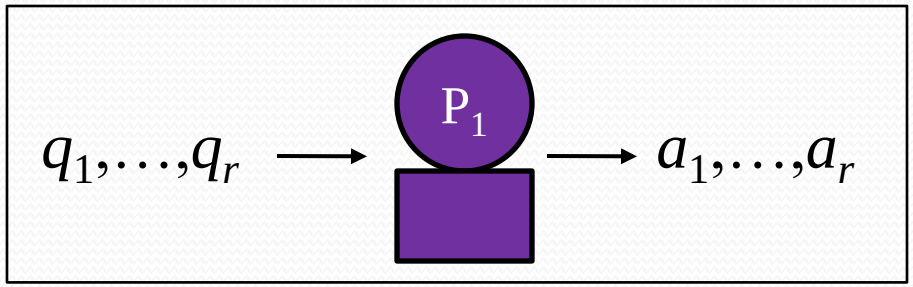
Analysis of soundness (1)

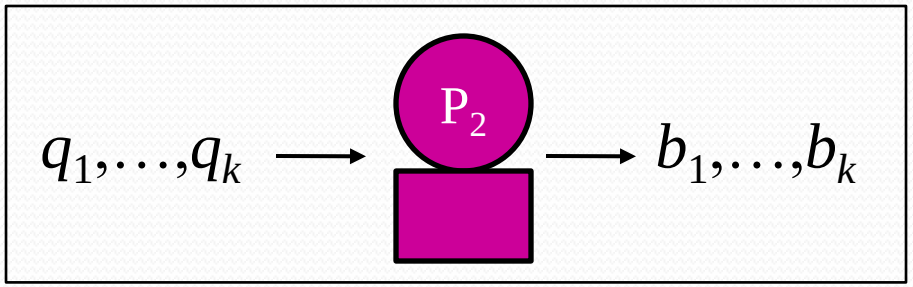
Suppose: P_1 and P_2 have a no-signaling strategy to convince V with prob. $1-\varepsilon$, with small ε



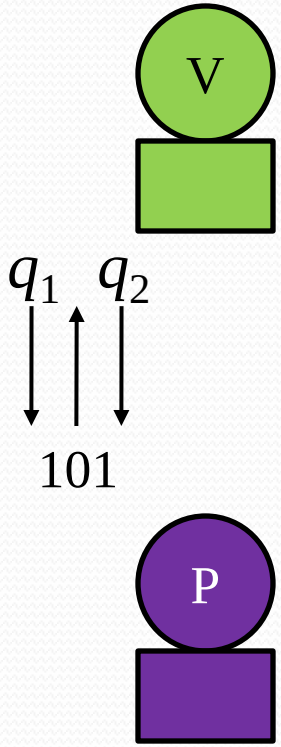
$p(a_1,\ldots,a_r; b_1,\ldots,b_k \mid q_1,\ldots,q_r; q_1,\ldots,q_k)$ no-signaling

$\Rightarrow p_1(a_1,\ldots,a_r \mid q_1,\ldots,q_r)$ and $p_2(b_1,\ldots,b_k \mid q_1,\ldots,q_k)$ are well-defined

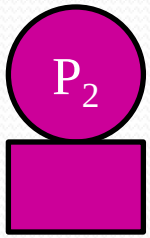




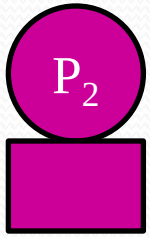
Analysis of soundness (2)



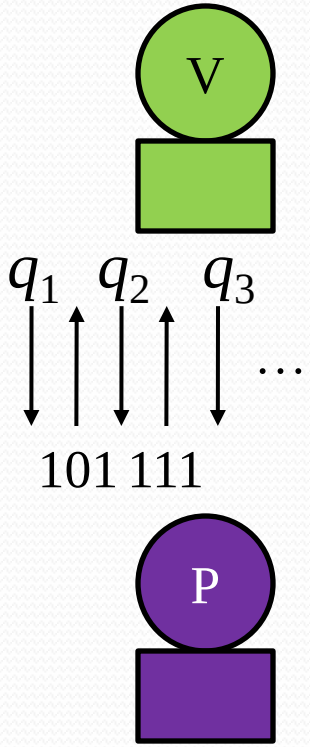
Construct P's strategy using distribution p_2

1st round: $q_1 \rightarrow$  $\rightarrow$ 101

$\neq$

2nd round: $q_1, q_2 \rightarrow$  $\rightarrow$ 001,111

Analysis of soundness (2)



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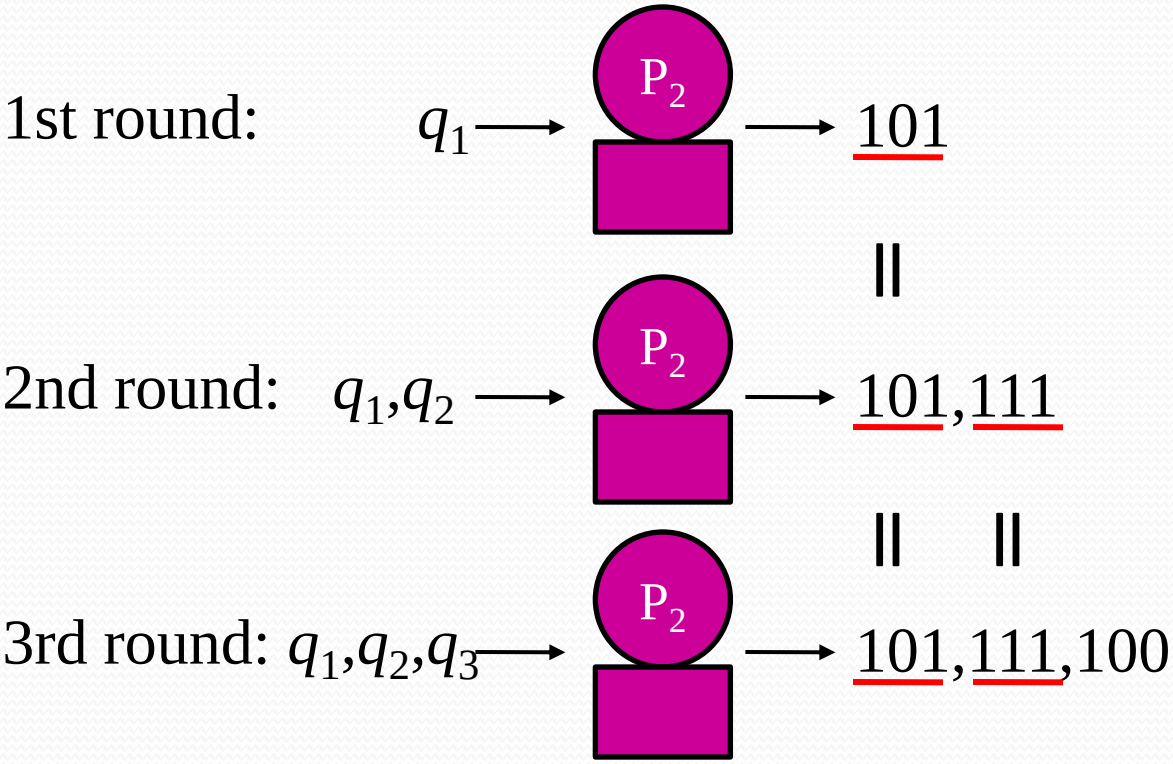


This P behaves similarly to P_1

$\Rightarrow$ If $x \notin L$, P_1 and P_2 cannot be accepted w.p. much higher than 2^{-n} (Contradiction!) $\Rightarrow x \in L$

Final step: Parallel repetition

Running the protocol poly times in parallel
→ Soundness error becomes exp-small [Holenstein 2007]

Resulting protocol exactly the same as [Cai, Condon, Lipton 1994]

Implication

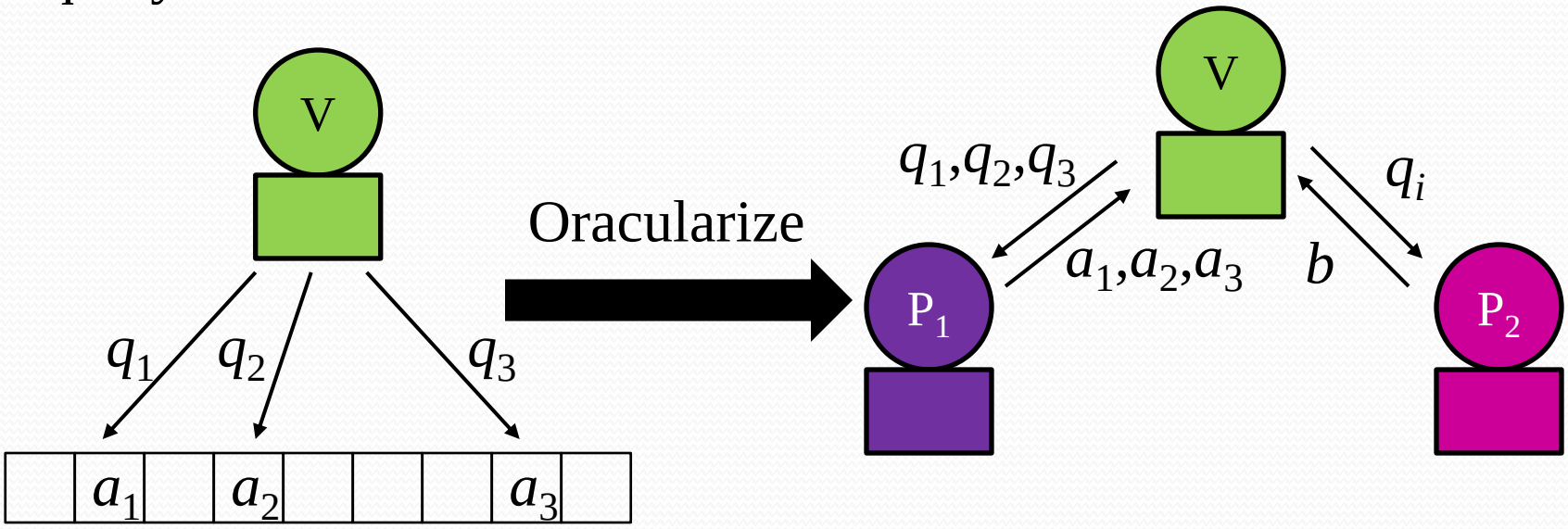
Oracularization of 1-prover IP protocols works
even if 2 provers are just no-signaling

Cf. 1-prover constant-round IP is weak: IP(k)=AM$\subseteq \Pi_2$P
[Goldwasser, Sipser 1986 & Babai, Moran 1988]

If we want constant-round interactive proof with exp-small error,
asking 2 provers is more powerful than asking 1 prover
even if 2 provers are entangled
(unless the polynomial hierarchy collapses)

2-prover 1-round protocol for NEXP

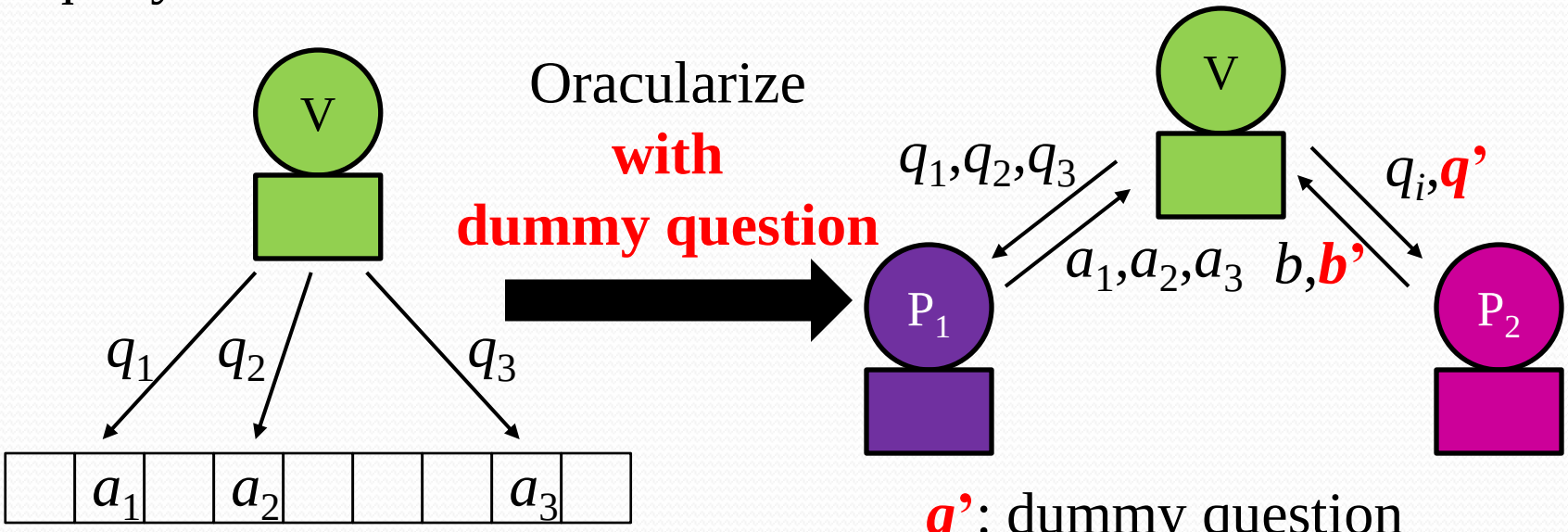
3-query PCP for $L \in$ NEXP



Provers can cheat with entanglement
(Kochen-Specker game, Magic Square game)
[Cleve, Høyer, Toner, Watrous 2004]

Dummy question prevents perfect cheating

3-query PCP for $L \in$ NEXP



q': dummy question chosen **independently**

High acceptance prob.

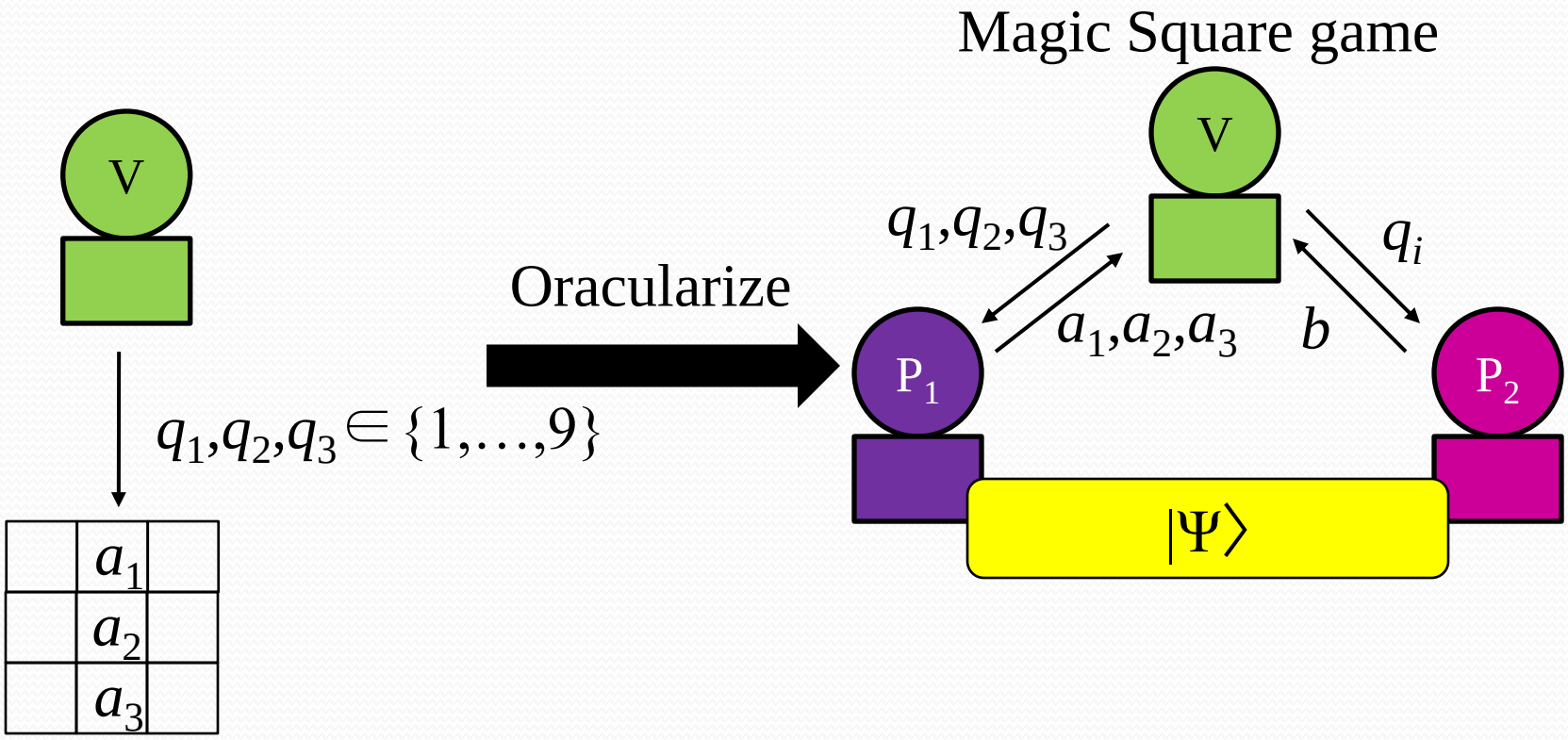
$\Rightarrow$ All the measurements by provers are almost commuting

$\Rightarrow$ Soundness error at most $1-1/O(|Q|^2)$ = **$1-1/\exp$**

↑ against entangled provers

Similar to [Kempe, Kobayashi, Matsumoto, Toner, Vidick 2008]

Limit of independent sampling



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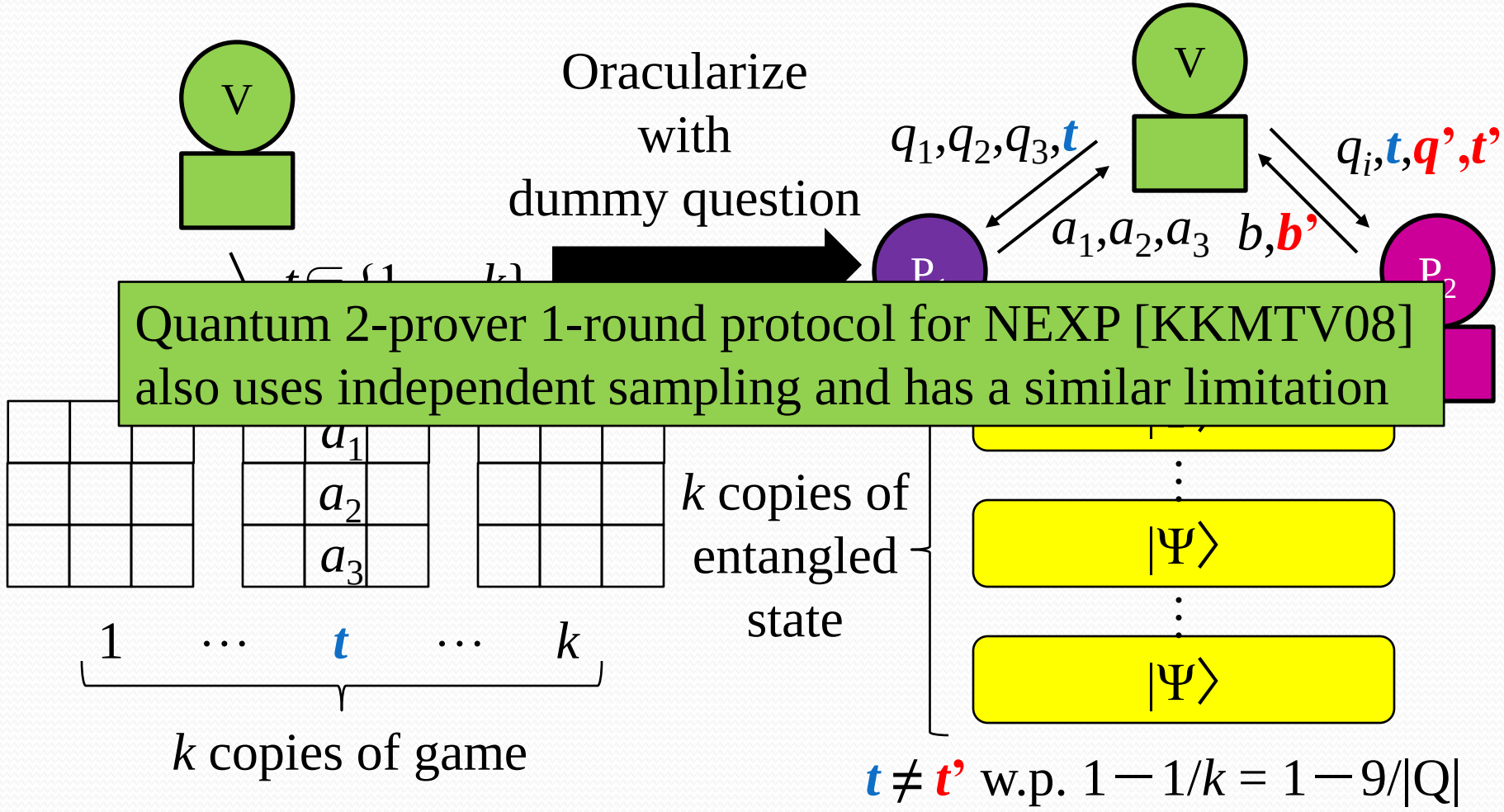
Limit of independent sampling



Summary

2-prover 1-round protocol for PSPACE
with **exp-small** soundness error against **no-signaling** provers
based on oracularization technique

2-prover 1-round protocol for NEXP
with 1－1/exp soundness error against entangled provers
using oracularization with dummy question

Independent sampling seems to impose limitation on soundness
- The above protocol for NEXP
- Quantum 2-prover 1-round protocol for NEXP by [KKMTV08]

Open problems

- Better soundness for EXP & NEXP
- Upper bound for MIP*
  - [Doherty, Liang, Toner, Wehner] [Navascués, Pironio, Acín] imply 2-prover 1-round MIP* $\subseteq$ Recursive assuming finite-dim entanglement suffices
- Characterization of MIP^{ns}, MIP with no-signaling provers
  - PSPACE $\subseteq$ MIP^{ns} $\subseteq$ EXP (upper bound based on LP [Preda])
- Parallel repetition for MIP*
- Alternative to oracularization
  - Parallelization
    - Possible using quantum answers from provers [Kempe, Kobayashi, Matsumoto, Vidick 2007]
  - Reducing the number of provers